Pion Spectrum in Radiative K_π^+ Decay*

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The π^+ energy spectrum is calculated for the process $K^+\rightarrow\pi^++\pi^0+\gamma$, taking into account the processes of internal bremsstrahlung and of direct dipole emission, both $E1$ and $M1$ transitions being permitted with parity nonconservation for weak decays. For the π^+ energy range 55–75 Mev, which has been examined in some recent investigations, the internal bremsstrahlung process alone leads to an expectation of 1.0 anomalous K_π^+ decay in the 8653 K^+ decays examined, quite compatible with the observation of two anomalous K_π^+ decays in these data. For reasonable values of the dimensionless parameters $A(\mu/M)^4$ and $B(\mu/M)^4$ which specify the dipole transition strengths, (that is, with $A=B=1$ and M equal to the K-particle mass), it appears that their main contribution will arise from the interference of the $E1$ direct transition with the internal bremsstrahlung term, increasing or decreasing the above estimate by about 20%.

1. INTRODUCTION

The well-established K_π^+ decay modes are the θ^+ mode $(\pi^++\pi^0)$ and the τ' mode $(\pi^++\pi^0+\pi^0)$ which account for $25.6(\pm1.7)\%$ and $1.70(\pm0.32)\%$, respectively, of K^+ decay events.[1] The τ' mode gives a spectrum of π^+ energies up to 53.3 Mev, while the π^+ mesons from the θ^+ decay have the unique energy 107.7 Mev.[1] Recently two anomalous K^+ decay events giving secondaries of energy intermediate between these limits have been reported. In an investigation of the secondaries of 5000 K^+ decays, Harris *et al.*[2] have found one K_π^+ decay giving a π^+ secondary of energy $60.0(\pm1.0)$ Mev. Also Prowse and Evans,[3] in a scan of 353 K^+ decays, found one event with a π^+ secondary of energy $61.7(\pm1.5)$ Mev. In addition a systematic investigation of 3300 K^+ decays has been made by O'Ceallaigh,[4] who has not observed any anomalous decays. These investigations did not include π^+ secondaries of energy greater than about 75 Mev, since these pions could not be distinguished near the decay origin from the usual 107.7-Mev secondary of the θ^+ decay nor from the high-energy muons in the $K_{\mu2}^+$ and $K_{\mu3}^+$ decays. To distinguish these, considerable track length would have to be followed on more than 75% of all K^+ decays.

Harris *et al.*[2] have suggested that these events may represent examples of a new decay mode,[5] a direct radiative K^+ decay

$$K^+\rightarrow\pi^++\pi^0+\gamma, \tag{1.1}$$

since this direct mode will give π^+ energies predominantly in the range 30 to 70 Mev. The fact that the two observed π^+ energies agree within experimental error has been noted by Prowse and Evans who remark that a two-body K^+ decay, $K^+\rightarrow\pi^++X_0$, would give π^+ mesons of this energy for an X_0 particle of mass 500 m_e.[6] Events of the type (1.1) could also arise from the process of internal bremsstrahlung associated with θ^+ decay. Our purpose here is to show that at present the observed events can be satisfactorily interpreted in terms of the internal bremsstrahlung process, although an additional direct radiative decay is by no means excluded.

2. DISCUSSION OF THE $K^+\rightarrow\pi^++\pi^0+\gamma$ PROCESS

The radiative K^+ decay (1.1) can occur through two physically distinct mechanisms.[7] The first of these is the internal bremsstrahlung process in which the photon is emitted from the outgoing charged pion. The matrix element for this process is

$$M_b=eGM\left(\frac{P}{P\cdot K}-\frac{p}{p\cdot k}\right)\cdot\epsilon. \tag{2.1}$$

In this expression P,p,k denote the four-vector momenta of the initial K^+ meson, the π^+ meson, and the photon, respectively, and ϵ denotes the polarization vector of the photon. G is the dimensionless coupling parameter (assumed momentum-independent) responsible for $K^+\rightarrow\pi^++\pi^0$ decay [defined precisely by expression (3.3) later], while M is some mass characteristic of the volume of interaction (with linear dimensions $R=\hbar/Mc$) of the $K^+\rightarrow\pi^++\pi^0$ decay.

Secondly it is possible, if the $K^+\rightarrow\pi^++\pi^0$ decay proceeds through virtual intermediate states, that the photon may be emitted from one of the intermediate charged particles. These processes, which we will refer to as direct emission processes, may be viewed as radiative transitions from an initial K^+ state to a final $(\pi^++\pi^0)$ state. Taking into account parity non-

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[1] M. Gell-Mann and A. H. Rosenfeld, Ann. Rev. Nuclear Sci. 7 (1957).

[2] Harris, Lee, Orear, and Taylor, Phys. Rev. 108, 1561 (1957).

[3] D. J. Prowse and D. Evans, Bull. Am. Phys. Soc. Ser. II, 3, 163 (1958).

[4] C. O'Ceallaigh, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics, 1957* (Interscience Publishers, Inc., New York, 1957), Sec. 8, p. 23.

[5] R. H. Dalitz, Phys. Rev. 99, 915 (1955).

[6] The phase space available for this process is 70% that for the θ^+ mode so the matrix element would have to be quite small to account for only two events in 8653 being observed.

[7] These mechanisms have been discussed for the case of radiative τ^+ decay by Dalitz (see reference 4).

conservation for these weak decay processes, the simplest radiative transitions are $E1$ and $M1$ transitions to a final two-pion state of p-wave relative motion. Of these the $E1$ transition is coherent with the internal bremsstrahlung process whereas, in the total decay rate, the $M1$ transition amplitude does not interfere with either of the other two amplitudes. In general the direct radiative processes would be expected to be small relative to the internal bremsstrahlung since the former depend on the finiteness of the extent of the spatial region over which the virtual processes leading to K^+ decay extend, whereas contributions to the internal bremsstrahlung process for a photon k come from regions with linear dimensions of order $1/k$. However, for the θ^+ decay mode, there is a special situation since the θ^+ decay mode is weak relative to the θ_0 decay modes, its partial lifetime being longer by a factor of about 500. This may be accounted for by the operation of a $\Delta T=\frac{1}{2}$ selection rule, as suggested by Gell-Mann[8] for the decay processes of K mesons and hyperons. If this selection rule were rigorously valid, the θ^+ decay could not occur. With zero spin for the K^+ meson, the two pions are emitted with S wave relative motion and can have only isotopic spin $T=2$ ($T=0$ being excluded since $T_3=1$ for the two pions of total charge $+e$, and $T=1$ is not allowed by the symmetry requirements for the wave function of two bosons with total angular momentum zero). In this case the internal bremsstrahlung process would also be forbidden, but the direct $E1$ and $M1$ radiative processes would still be permitted since the electromagnetic interaction allows the transifrom an initial $T=\frac{1}{2}$ K^+ meson to a final $(\pi^++\pi^0)$ p-wave state of isotopic spin $T=1$. In the actual situation where the θ^+ decay is strongly suppressed, it is possible that the direct emission process might well be relatively prominent.

The matrix elements which represent the direct emission processes must each be gauge invariant and can therefore be expressed in terms of the field strength $F_{\mu\nu}$ associated with the photon. The direct emission process which interferes with the internal bremsstrahlung (2.1) has an amplitude of the form

$$M_e=AgeM^{-3}p^\mu q^\nu F_{\mu\nu}, \tag{2.2}$$

where q denotes the four-vector momentum of the π^0 meson and $F_{\mu\nu}=k_\mu\epsilon_\nu-k_\nu\epsilon_\mu$. Expression (2.2) is the only nonzero, gauge-invariant, scalar expression which is linear in $F_{\mu\nu}$. g pertains to the θ_0 decay [see precise explanation following (3.4)]. In general A could be a function of the quantities $k\cdot p$ and $k\cdot q$; here A will be assumed to depend only slowly on these quantities and to remain finite for $k\cdot p=k\cdot q=0$. This corresponds to the assumptions that the source of the electromagnetic radiation has a dimension R small relative to the outgoing wavelengths. The factor M^{-3} has been introduced in the amplitude (2.2) for dimensional reasons, to make the quantity A a pure number. For later calculation, it is convenient to express these matrix elements in the $(\pi^++\pi^0)$ c.m. system; in this system (2.2) reduces to

$$M_e=AgeM^{-3}(p_0+q_0)(\mathbf{p}\cdot\mathbf{E}), \tag{2.3}$$

where $\mathbf{E}=k\boldsymbol{\varepsilon}$ is the electric vector so that the transition is clearly an electric dipole transition to a p state of the $\pi^++\pi^0$ system.

The $M1$ direct emission amplitude has opposite parity and takes the form

$$M_m=BgeM^{-3}p^\mu q^\nu \epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}. \tag{2.4}$$

In the c.m. system of the two pions, this reduces to

$$M_m=BgeM^{-3}(p_0+q_0)(\mathbf{p}\cdot\mathbf{H}), \tag{2.5}$$

where $\mathbf{H}=\mathbf{k}\times\boldsymbol{\varepsilon}$ is the magnetic vector of the photon. Again the number B is dimensionless and is assumed constant.

3. CALCULATION OF THE $K^+\rightarrow\pi^++\pi^0+\gamma$ BRANCHING RATIO

The probability per unit time for $K^+\rightarrow\pi^++\pi^0+\gamma$ decay is now given in terms of the amplitudes (2.1), (2.2), and (2.4) by the expression[9]

$$\frac{P}{T}=\int\left(\frac{1}{2m}\right)\frac{d^3p}{(2\pi)^3 2p_0}\frac{d^3q}{(2\pi)^3 2q_0}\frac{d^3k}{(2\pi)^3 2k_0} \times(2\pi)^4\delta^4(p+q+k-P)\{\sum|M_e+M_m+M_b|^2\}, \tag{3.1}$$

where $\sum$ denotes the sum over the photon polarization states. In this sum the interference terms between M_m and the other two matrix elements vanishes. The curly bracket then reduces to

$$B^2g^2e^2M^{-6}[(p\cdot k)q-(q\cdot k)p]^2 +\left\{[AgeM^{-3}][(p\cdot k)q-(q\cdot k)p] +MGe\left(\frac{P}{P\cdot k}-\frac{p}{p\cdot k}\right)\right\}^2. \tag{3.2}$$

This expression has a covariant form and since it is the differential decay probability (in the K^+ rest system) as a function of the π^+ energy (also in the K^+ rest system) that is desired, it proves convenient to evaluate the rest of the integral (3.1) in the $(\pi^0+\gamma)$ c.m. system.

The coupling parameter G for the $K^+\rightarrow\pi^++\pi^0$ process is given in terms of the experimental partial lifetime $\tau(\theta^+)$ by

[8] M. Gell-Mann, *Proceedings of the Sixth Annual Rochester Conference on High-Energy Physics* (Interscience Publishers, Inc., New York, 1956), Sec. 8, p. 23.

[9] Rationalized natural units are used such that $\hbar=c=1$ and $e^2/4\pi=1/137$.

$$\frac{1}{\tau(\theta^+)}=\int\left(\frac{1}{2m}\right)\frac{d^3p}{(2\pi)^3 2p_0}\frac{d^3q}{(2\pi)^3 2q_0}\times(2\pi)^4\delta^4(p+q-P)|MG|^2$$
$$=\frac{G^2}{16\pi}\{[1+(\mu/m)^2-(\mu_0/m)^2]^2-4(\mu/m)^2\}^{\frac{1}{2}}\left(\frac{M}{m}\right)M, \quad (3.3)$$

where μ,μ_0,m denote the π^+,π^0,K^+ masses, respectively. When the mass difference between the charged and neutral pions is neglected, this partial lifetime reduces to

$$\frac{G^2}{16\pi}(1-4\mu^2/m^2)^{\frac{1}{2}}\left(\frac{M}{m}\right)M. \quad (3.4)$$

This last expression holds also for the $\theta\to\pi^++\pi^-$ partial lifetime if G is replaced by g, the coupling parameter appropriate to θ^0 decay. However, the lifetime which is observed is that of the $\theta_1{}^0$ particle and this is obtained from (3.4) by replacing G by $g\sqrt{2}$; hence, finally,

$$\left(\frac{g}{G}\right)^2=\frac{(\theta^+\to\pi^++\pi^0)}{2(\theta_1{}^0\to\pi^++\pi^-)}=225, \quad (3.5)$$

where the values[1] $\tau(\theta^+\to\pi^++\pi^0)=4.78\times10^{-8}$ sec and $\tau(\theta_1\to\pi^++\pi^-)=1.1\times10^{-10}$ sec have been used.

The π^+ energy spectra arising from the above transitions which lead to $K^+\to\pi^++\pi^0+\gamma$ decay may be expressed in the K^+ rest system as a branching ratio relative to the total K^+ decay rate (g and G have been assumed momentum-independent):

$$dB/dw=\alpha(\theta^+)\phi(w)[I_b(w)+(g/G)A(\mu/M)^4I_{\text{int}}(w)+(g/G)^2A^2(\mu/M)^8I_{E1}(w)+(g/G)^2B^2(\mu/M)^8I_{M1}(w)], \quad (3.6)$$

where the phase space factor $\phi(w)$, the internal bremsstrahlung term I_b, the electric dipole term I_{E1}, the magnetic dipole term I_{M1}, and the interference term I_{int} between the electric dipole and the internal bremsstrahlung are given by

$$\phi=4(e^2/4\pi)pk_0(2\pi\lambda m)^{-1}(1-4\mu^2/m^2)^{-\frac{1}{2}}(\mu^{-1}),$$
$$I_b=\frac{2}{k_0{}^2}\left[\frac{w}{p}\ln\left(\frac{\mu}{w-p}\right)-1\right](\mu^2),$$
$$I_{E1}=I_{M1}=\tfrac{2}{3}m^2p^2k_0{}^2\mu^{-6}, \quad (3.7)$$
$$I_{\text{int}}=2\lambda^2m^2\left\{\frac{1}{2pm}\left[m^2\ln\frac{m-w+p}{m-w-p}-\mu^2\ln\frac{m(w+p)-\mu^2}{m(w-p)-\mu^2}\right]-1\right\}(\mu^{-2}),$$

TABLE I. The expressions ϕI_b, ϕI_{E1}, and ϕI_{int} are given as functions of w, the π^+ energy. These expressions, apart from constant factors, are the internal bremsstrahlung, electric dipole, and bremsstrahlung-electric dipole interference contributions to the branching ratio (3.6) of radiative to total $K_\pi{}^+$ decay. Units are chosen such that μ(140 Mev) equals one.

(w-140) Mev	w/μ	$10^3\phi I_b$	$10^3\phi I_{E1}$	$10^3\phi I_{\text{int}}$
0	1.0	0	0	0
10	1.071	0.067	1.14	0.56
20	1.143	0.276	2.97	1.56
30	1.214	0.520	4.83	2.73
40	1.286	0.923	6.36	3.95
50	1.357	1.49	7.40	5.11
60	1.429	2.36	7.64	6.08
70	1.500	3.71	6.82	6.70
80	1.571	6.02	5.11	6.67
90	1.643	11.2	2.67	5.80
100	1.714	30.1	0.52	3.52
108	1.771	∞	0	0

with

$$k_0=\tfrac{1}{2}(m/\lambda)(1-2w/m)$$

and

$$\lambda^2=1+(\mu/m)^2-2(w/m),$$

where w and p are the π^+ energy and momentum in the K^+ rest system. $\alpha(\theta^+)$ is the proportion of K^+ decays which proceed via the θ^+ mode [the presently accepted value[1] is $\alpha(\theta^+)=0.256\pm0.017$]. Also note that k_0 is the photon energy in the $(\pi^++\gamma)$ c.m. system. The expressions (ϕI_b), (ϕI_{E1}), and $\phi I_{\text{int}})$ are tabulated as functions of w in Table I; in this tabulation the unit of energy has been taken to be the π^+ rest energy.

In the 8653 K^+ decays examined so far, the expected number of radiative decay events due to internal bremsstrahlung would be 1.0. With $A\sim B\sim1$; $(g/G)=15$, and $M=m$, this expectation would be corrected to 1.0 ± 0.2 event, since the interference may be either constructive or destructive depending on the sign of A. A value of M as small as μ is excluded, since the dipole terms would then dominate the internal bremsstrahlung over almost the entire spectrum and would lead to a very large expectation ($\sim$10%) for the proportion of K^+ decays proceeding through the radiative mode (1.1). With $M=2.17\mu$, the direct dipole processes with $A=B=1$ each contribute about the same as the internal bremsstrahlung in the energy range 55–75 Mev, the interference term being about 50% larger—with destructive interference, the dipole emission terms only add about 50% to the rate expected for internal bremsstrahlung alone, whereas constructive interference would mean an increase in the radiative decay by a factor about 4.5. The minimum number of radiative events possible in this energy range, according to the expression (3.7), is 0.5 for 8653 K^+ decay events; this corresponds to $M=2.40\mu$ and destructive interference.[10] If the direct emission processes are due mainly to virtual baryon-antibaryon pairs, M might be expected

[10] The minimum number is obtained with $B=0$. A is taken to have the value unity, only the combination $A(\mu/M)^4$ is significant.

to be of the order of a nucleon mass, in which case the direct emission processes are unlikely to contribute appreciably to the radiative $K_{\pi}{}^{+}$ process.

The contribution of the internal bremsstrahlung to the range 55–75 Mev is 1.2×10^{-4} per K^{+} decay, which leads to a prediction of 1.0 anomalous K^{+} event in the 8653 decays examined to date. With only two anomalous events observed in this energy range, it is clear that there is no necessity to invoke the direct emission processes to account for these events. The rate of anomalous $K_{\pi}{}^{+}$ events with pions in the range 75–100 Mev is 5.6×10^{-4} per K^{+} decay for the internal bremsstrahlung process alone, which predicts 4.8 such events should have been included in the 8653 examined so far. It would be of interest to check directly the internal-bremsstrahlung interpretation of these anomalous events by such observations

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Ξ^{-} Capture Reactions in Hydrogen*

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In the capture reaction $\Xi^{-}+p\rightarrow\Lambda+\Lambda$, the asymmetries in the Λ decays constitute an excellent analyzer for determining the polarization pattern of the Λ's and hence the nature of the final orbital states involved. Such information, together with evidence concerning alternate capture channels $\Xi^{-}+p\rightarrow\Xi^{0}+n$, $\Xi^{-}+p\rightarrow\Lambda+\Lambda+\gamma$, may permit a determination of the parity of Ξ relative to that of the nucleon.

A

THE time will no doubt come when machine-produced cascade particles are sufficiently numerous to permit quantitative experiments. Among these, capture of Ξ^{-} in hydrogen should prove especially fruitful. We consider in particular the reaction

$$\Xi^{-}+p\rightarrow\Lambda+\Lambda, \quad (1)$$

assuming the capture takes place from a low-lying atomic orbit. One has here an unusual opportunity to extract detailed information concerning an elementary process. The reasons are the following: (1) The Pauli principle introduces a simplification by reducing the variety of possible final states. (2) The polarization pattern of the final Λ particles can be detected experimentally by using Λ decay as a polarization analyzer. If circumstances are favorable it should be possible to determine unambiguously the parity of the cascade particle (defined relative to that of the nucleon).

We shall suppose that the capture reactions occur with appreciable probability only from atomic S and $2P$ states; or, in the case of capture in flight of slow Ξ^{-} particles, from initial S states. The possible transitions depend on the cascade parity and are further limited by the Pauli principle. They are indicated in spectroscopic notation in Table I.[1] It is important to notice, and we shall return to this later, that the reaction (1) is forbidden for certain initial states.[2] Other processes, however, may always occur—whatever the initial state; namely:

$$\Xi^{-}+p\rightarrow\Xi^{0}+n, \quad (2)$$

if energy conservation permits;

$$\Xi^{-}+p\rightarrow\Lambda+\Lambda+\gamma; \quad (3)$$

and "free" Ξ^{-} decay,

$$\Xi^{-}+(p)\rightarrow\Lambda+\pi^{-}+(p). \quad (4)$$

The last process would not be easily distinguishable from decay in flight of slow Ξ^{-} particles. The relative

TABLE I. Summary of $\Xi^{-}+p\rightarrow\Lambda+\Lambda$ transitions.

Even Ξ parity		Odd Ξ parity	
Initial state	Final state	Initial state	Final state
$^1S_0$	$^1S_0$	$^1S_0$	$^3P_0$
$^3S_1$	forbidden	$^3S_1$	$^3P_1$
$^3P_2$	$^3P_2$, $^3F_2$	$^3P_2$	$^1D_2$
$^3P_1$	$^3P_1$	$^3P_1$	forbidden
$^3P_0$	$^3P_0$	$^3P_0$	$^1S_0$
$^1P_1$	$^3P_1$	$^1P_1$	forbidden

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[1] We assume that Ξ has spin one-half.

[2] It is worth noting here, however, that two-body cascade-nucleon hyperfragments might be stable against decay via strong reactions, not only for isotopic spin unity, a well-known possibility, but also for $I=0$—in the latter case if the state is $^3S_1$. Decay via $\Lambda+\Lambda$ would then be forbidden. De-excitation would of course occur rapidly via $\Lambda+\Lambda+\gamma$; but, as in the case of the Σ^{0} particle, the fragment might live long enough to have a reasonably well-defined mass.