Hyperon-Antihyperon Production in Nucleon-Antinucleon Collisions and the Relative Σ-Λ Parity*

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(Received September 8, 1958)

A discussion is given of hyperon-antihyperon production in nucleon-antinucleon collisions near threshold. The rate for the reaction and the polarization of the outgoing baryons are calculated in terms of three amplitudes characterizing the transition operators in the two cases of even and odd relative Σ-Λ parity. In terms of these amplitudes, a correlation function which gives the angular correlations between the pion momenta from the subsequent decay of the hyperon and antihyperon, and the vectors involved in the production process, is given for each relative parity. The possibility of this class of experiments being useful in determining the relative Σ^0-Λ parity, as well as the relative charged Σ-Λ parity, is discussed.

I. INTRODUCTION

EXPERIMENTS with high-energy antinucleons may well become possible in the not too distant future, in particular, at the Bevatron or at the alternating gradient synchrotron under construction at Brookhaven. Experiments with antinucleons of energies of ~1 Bev will give us information on whether the very large annihilation cross section at the low and moderate energies persists[1]; and will investigate the pion production processes initiated by antinucleons.[2] Another very interesting class of reactions are those in which hyperon-antihyperon states are produced. Examples of such reactions are

$$\bar{p}+n\rightarrow\Sigma^-+\bar{\Lambda} \quad\text{or}\quad \bar{\Sigma}^+ +\Lambda, \tag{1a}$$

$$\bar{n}+p\rightarrow\Sigma^+ +\bar{\Lambda} \quad\text{or}\quad \bar{\Sigma}^-+\Lambda, \tag{1b}$$

$$\bar{p}+p\rightarrow\Lambda+\bar{\Lambda}, \tag{1c}$$

$$\bar{p}+p\rightarrow\Sigma^0+\bar{\Lambda} \quad\text{or}\quad \bar{\Sigma}^0+\Lambda. \tag{1d}$$

The threshold for reaction (1d), for example, is at about 950 Mev. The observation of the decay characteristics of such antihyperons, or of their annihilation into K mesons and pions will be most interesting. In this note we would like to remark that the Σ-$\bar{\Lambda}$ (or $\bar{\Sigma}$-Λ) states produced near threshold may be useful in the determination of the relative Σ-Λ parity. The great advantage of these states lies, of course, in the fact that they may contain, for example, only a Σ and a $\bar{\Lambda}$ and these particles, via their subsequent decays serve as natural analyzers of polarization patterns. At present, the experiment discussed by Pais and Treiman,[3] in which the capture reactions for Σ^- on protons leading to Λ-neutron states is studied, seems to be the closest experimental contact we have with the relative Σ-Λ parity. It is certainly to be hoped that this experiment will soon give some indication of possible success, which will mean that obstacles to such success (the most important being the need for polarized incident Σ^- and for some information on the relative importance of initial S and P waves in the reaction) can be overcome. The reactions to be discussed here also have their obstacles to usefulness, but in the light of the great importance of this relative parity to several recent attempts[4–6] at a qualitative theoretical understanding of the experiments involving the strong interactions of K mesons and hyperons, it may be well to discuss their potential usefulness, as well as these obstacles.

II. FORMULAS

Consider, for example, reaction (1a) say within ~25 Mev of threshold. If we may reasonably assume that the final particles are predominantly in an S state, the complex of incident states from which the reaction can proceed is considerably reduced.§ For both even and odd

TABLE I. Initial and final state configurations for hyperon-antihyperon production in nucleon-antinucleon collisions near threshold. (The amplitude c contains a statistical factor of $\sqrt{5}$.) The amplitudes are assumed to be essentially constant over a small range of beam energies away from the threshold. The operator, T, is to be taken between final spin state on the right and initial spin state on the left.

Initial state	Final state	Amplitude
	Even relative parity	
$^1S_0$	$^1S_0$	a
$^3S_1$	$^3S_1$	b'
$^3D_1$	$^3S_1$	c
	Odd relative parity	
$^3P_1$	$^3S_1$	A
$^1P_1$	$^3S_1$	B
$^3P_0$	$^1S_0$	C

* Work performed under the auspices of the U. S. Atomic Energy Commission.

† A National Science Foundation Postdoctoral Fellow.

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[2] Saul Barshay, Phys. Rev. **109**, 554 (1958).

[3] A. Pais and S. Treiman, Phys. Rev. **109**, 1759 (1958).

[4] Saul Barshay, Phys. Rev. Letters **1**, 97 (1958). In the second paragraph of this note, the symbols a_0 and a_1 should be interchanged.

[5] Feza Gürsey, Phys. Rev. Letters **1**, 98 (1958).

[6] A. Pais, Phys. Rev. **112**, 624 (1958).

§ The relevant parameter here is the momentum in the final state times the dimension of the volume within which the strange particles are produced; this dimension may be of the order of the K meson Compton wavelength. Of course, the angular distribution in the final state may provide a check of the assumption of S state production.

relative Σ-Λ parity we deal with a transition operator characterized by three amplitudes, these being given in Table I for the two cases. The transition operators may then be written as follows:

If relative Σ-Λ parity is even,

$$\begin{aligned} T &= aX_s+(b'-\tfrac{1}{3}c)X_t+c\boldsymbol{\sigma}_1\cdot\mathbf{n}\,\boldsymbol{\sigma}_2\cdot\mathbf{n}X_t \\ &= aX_s+bX_t+c\boldsymbol{\sigma}_1\cdot\mathbf{n}\,\boldsymbol{\sigma}_2\cdot\mathbf{n}X_t, \end{aligned} \tag{2a}$$

If relative Σ-Λ parity is odd,

$$T=(3/2)^{\frac{1}{2}}A\mathbf{S}\cdot\mathbf{n}+\sqrt{3}B\mathbf{S}'\cdot\mathbf{n}X_t+C\mathbf{S}'\cdot\mathbf{n}X_s, \tag{2b}$$

where

$X_s=\tfrac{1}{4}(1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)$,

$X_t=\tfrac{1}{4}(3+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)$,

$\mathbf{S}=\tfrac{1}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)$,

$\mathbf{S}'=\tfrac{1}{2}(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)$,

$\mathbf{n}$= a unit vector defined by the beam direction in the incident state,

and where we denote the spin operator for the $\bar{\Lambda}$ by $\boldsymbol{\sigma}_1$, and that for the Σ by $\boldsymbol{\sigma}_2$.

We may now compute for case (2a) and case (2b) the square of the matrix element, respectively, summed and averaged over final and initial spins, which we call R, as well as the polarization of the outgoing particles, in terms of the three amplitudes which define the transition operators. Since we have considered only outgoing S waves, the final particles will, of course, be polarized only if the incident antinucleon is polarized. We find

$$R=\tfrac{1}{4}\{|a|^2+3|b|^2+3|c|^2+2\operatorname{Re} b^*c\}, \tag{3a}$$

$$R\mathbf{P}_{\bar{\Lambda}}=\tfrac{1}{2}\mathbf{P}[|b|^2+\operatorname{Re} a^*(b+c)]+2\mathbf{nP}\cdot\mathbf{n}\operatorname{Re} c^*(b-a),$$

$R\mathbf{P}_\Sigma$= the same as $R\mathbf{P}_{\bar{\Lambda}}$ with $a\rightarrow -a$;

$$R'=\tfrac{1}{4}\{3|A|^2+3|B|^2+|C|^2\}, \tag{3b}$$

$$\begin{aligned} R'\mathbf{P}'_{\bar{\Lambda}} &= -\mathbf{P}(3/8)^{\frac{1}{2}}\operatorname{Re} A^*(\sqrt{3}B+C) \\ &\quad +\mathbf{nP}\cdot\mathbf{n}\tfrac{1}{4}\{3|A|^2+3\operatorname{Re}[\sqrt{2}AB^* \\ &\quad +(2/3)^{\frac{1}{2}}AC^*+(4/3)^{\frac{1}{2}}BC^*]\}, \end{aligned}$$

$R'\mathbf{P}'_\Sigma$= the same as $R\mathbf{P}'_{\bar{\Lambda}}$ with $B\rightarrow -B$,

where $\mathbf{P}$ is the polarization of the incident antinucleon, and the primed quantities refer to odd relative Σ-Λ parity.

A further very interesting function is that which describes the correlation between the directions involved in the production process and the pion momenta from the subsequent decay of the $\bar{\Lambda}$ and Σ. The $\bar{\Lambda}$ and Σ decays proceed through the transition operators $(x+y\boldsymbol{\sigma}_1\cdot\mathbf{p}_1)$ and $(x'+y'\boldsymbol{\sigma}_2\cdot\mathbf{p}_2)$, respectively, where $\mathbf{p}_1$ and $\mathbf{p}_2$ are the decay pion momenta measured in the rest system of the decaying particle. The asymmetry parameters for the $\bar{\Lambda}$ and Σ decays are then

$$\alpha_1=2\operatorname{Re} x^*y/(|x|^2+|y|^2)$$

and

$$\alpha_2=2\operatorname{Re} x'^*y'/(|x'|^2+|y'|^2),$$

respectively. The correlation functions F, in terms of the amplitudes describing the production process, are then given by

$$RF=(RF)_0+(RF)_\mathrm{P}, \tag{4a}$$

where

$$\begin{aligned} (RF)_0 &= R+\tfrac{1}{4}\alpha_1\alpha_2\{\mathbf{p}_1\cdot\mathbf{p}_2(-|a|^2+|b|^2 \\ &\quad -2\operatorname{Re} b^*c+|c|^2)+\mathbf{p}_1\cdot\mathbf{np}_2\cdot\mathbf{n}8\operatorname{Re} b^*c\}, \end{aligned}$$

$$\begin{aligned} (RF)_\mathrm{P} &= \alpha_1\mathbf{P}_{\bar{\Lambda}}\cdot\mathbf{p}_1+\alpha_2\mathbf{P}_\Sigma\cdot\mathbf{p}_2-\tfrac{1}{2}\alpha_1\alpha_2 \\ &\quad \times[\mathbf{P}\cdot\mathbf{p}_1\times\mathbf{p}_2\operatorname{Im} ab^*+\mathbf{p}_2\cdot\mathbf{np}_1\cdot\mathbf{n}\times\mathbf{P} \\ &\quad \times\operatorname{Im} c^*(a+2b)+\mathbf{p}_1\cdot\mathbf{np}_2\cdot\mathbf{n}\times\mathbf{P}\operatorname{Im} c^*(2b-a) \\ &\quad +\mathbf{P}\cdot\mathbf{nn}\cdot\mathbf{p}_2\times\mathbf{p}_1\operatorname{Im} c^*a]. \end{aligned}$$

$$R'F'=(R'F')_0+(R'F')_\mathrm{P}, \tag{4b}$$

where

$$\begin{aligned} (R'F')_0 &= R'+\tfrac{1}{4}\alpha_1\alpha_2\{\mathbf{p}_1\cdot\mathbf{p}_2(-3|B|^2+|C|^2) \\ &\quad +\mathbf{p}_1\cdot\mathbf{np}_2\cdot\mathbf{n}(3|A|^2-2|C|^2)\}, \end{aligned}$$

$$\begin{aligned} (R'F')_\mathrm{P} &= \alpha_1\mathbf{P}'_{\bar{\Lambda}}\cdot\mathbf{p}_1+\alpha_2\mathbf{P}'_\Sigma\cdot\mathbf{p}_2-\tfrac{1}{2}\alpha_1\alpha_2 \\ &\quad \times[(3/\sqrt{2})(-\mathbf{p}_1\cdot\mathbf{np}_2\cdot\mathbf{n}\times\mathbf{P}+\mathbf{p}_2\cdot\mathbf{np}_1\cdot\mathbf{n}\times\mathbf{P}) \\ &\quad \times\operatorname{Im} AB^*+(3/2)^{\frac{1}{2}}(\mathbf{p}_1\cdot\mathbf{np}_2\cdot\mathbf{n}\times\mathbf{P} \\ &\quad +\mathbf{p}_2\cdot\mathbf{np}_1\cdot\mathbf{n}\times\mathbf{P})\operatorname{Im} AC^* \\ &\quad +\sqrt{3}\mathbf{P}\cdot\mathbf{nn}\cdot\mathbf{p}_1\times\mathbf{p}_2\operatorname{Im} B^*C]. \end{aligned}$$

III. DISCUSSION

We see that the correlation functions, even for unpolarized incident antinucleons, show quite interesting effects. In particular, there can be a correlation of each decay pion with the beam direction. This correlation would, of course, go over to one of the form $\mathbf{p}_1\cdot\mathbf{p}_2$ if one averages over $\mathbf{n}$ by considering all hyperon-antihyperon states with fixed angles between decay and production planes, irrespective of the center of mass production angle.‖ If the incident antinucleon is polarized (normal to its motion), further correlation effects are described by the functions $(RF)_\mathrm{P}$ and $(R'F')_\mathrm{P}$. Present antiproton beams produced in high-energy nucleon-nucleus collisions at small angles to the incident projectile may have little polarization. It is clear from Eqs. (4a) and (4b) that the three-amplitude transition operators (2a) and (2b) lead to three- and two-parameter correlation functions $(FR)_0$, $(F'R')_0$, for even and odd relative Σ-Λ parity, respectively. In each case, the coefficients of $\mathbf{p}_1\cdot\mathbf{p}_2$, and $\mathbf{p}_1\cdot\mathbf{np}_2\cdot\mathbf{n}$ in the correlation function comprise two ex-

‖ *Note added in proof.*—The correlation function is of the form $1+a\alpha_1\alpha_2\mathbf{p}_1\cdot\mathbf{p}_2+b\alpha_1\alpha_2\mathbf{p}_1\cdot\mathbf{np}_2\cdot\mathbf{n}$. If the triplet final state predominates, then for odd relative parity, $a<0$, $b>0$; for even relative parity, if $b<0$ then $a>0$ and if $b>0$ then a can be either greater or less than zero. If the singlet final state predominates, then for odd relative parity, $a>0$, $b<0$; for even relative parity, $a<0$, $b=0$. For any observation there are at least two choices, therefore the analysis is not definitive in that it hinges on the admixture of triplet and singlet final states. If the final states were predominantly singlet, we would have a distinction; if predominantly triplet, we would have a distinction only if $b<0$. The analysis can be extended by doing the experiment with polarized antiprotons.

perimental quantities. If the incident antinucleon is polarized, one has several more experimental coefficients in the correlation function and one also has the possibility of analyzing the hyperon polarizations from the up-down asymmetry in one of their decays (if the asymmetry parameter is known) and of relating this to the two real amplitudes and the two phases in the transition operators of Eqs. (2a) and (2b) (one over-all real amplitude is determined by R).

Evidently a major obstacle to the analysis of events of this kind is the limitation on the cross section imposed by being near threshold. Also the maximum cross section from the incident S wave is of the order of a millibarn. Of course, some of the incident P and D waves can contribute so that it may not be unreasonable to expect cross sections of the order of those that we have come to associate with strange particle production in the π^--p experiments, even if the annihilation probability is still high. A further limitation on the usefulness of reaction (1a) is imposed by the fact that the neutron will be in the deuteron, so that if the incident beam is at an energy such that one is within $\sim$25 Mev of threshold from a free nucleon, the deuteron momentum spread may be capable of pushing this up by a few tens of Mev. In principle, similar charged Σ-$\bar{\Lambda}$ final states can be obtained by striking protons with antineutrons cut off sharply at the high-energy end. It may, in fact, be best to utilize the $\bar{p}$-p reaction (1d). The rapid decay, $\Sigma^0 \rightarrow \Lambda+\gamma$, precludes the possibility of obtaining Σ^0 beams to perform the Pais-Treiman experiment.[3] Fulton and Feldman[7] have suggested a simultaneous measurement of the Λ and photon polarizations in the decay of polarized Σ^0 as a measure of the relative Σ-Λ parity. An analysis of the correlation functions for reaction (1d) would be of particular interest, especially since it is important to have a determination of the relative Σ^0-Λ parity *independent* of the relative charged Σ-Λ parity. For this case, the formulas of Sec. II must be modified owing to the action of the transition operator for Σ^0 decay to $\Lambda+\gamma$. For odd relative Σ-Λ parity this operator has the form $\boldsymbol{\sigma}_2\cdot\mathbf{u}$, with $\mathbf{u}=\mathbf{e}$, and for even relative Σ-Λ parity it has the form $\boldsymbol{\sigma}_2\cdot\mathbf{u}$, with $\mathbf{u}=\mathbf{k}\times\mathbf{e}$, where $\mathbf{e}$ is the photon polarization and $\mathbf{k}$ is a unit vector in the direction of the photon momentum, in the Σ^0 rest system. The prescription is to replace $\mathbf{p}_2$ in the correlation functions by $\mathbf{p}'_2=2\mathbf{p}_2\cdot\mathbf{uu}-\mathbf{p}_2$. After averaging over photon polarizations, one makes the following substitutions in Eqs. (4a) and (4b): $\mathbf{p}_1\cdot\mathbf{p}_2\rightarrow-\mathbf{p}_1\cdot\mathbf{k}\mathbf{p}_2\cdot\mathbf{k}$; $\mathbf{p}_2\cdot\mathbf{n}\rightarrow-\mathbf{p}_2\cdot\mathbf{k}\mathbf{n}\cdot\mathbf{k}$. Finally, if one averages over the photon direction, the substitutions in Eqs. (4a) and (4b), $\mathbf{p}_1\cdot\mathbf{p}_2\rightarrow-\frac{1}{3}\mathbf{p}_1\cdot\mathbf{p}_2$ and $\mathbf{p}_2\cdot\mathbf{n}\rightarrow-\frac{1}{3}\mathbf{p}_2\cdot\mathbf{n}$, represent an over-all reduction in the correlation coefficients.

In conclusion, we remark that these formulas could be applied to final Σ-Λ states produced near threshold when Ξ^- particles of $\sim$110 Mev are incident on protons (or deuterons) (if the Ξ^- spin is $\frac{1}{2}$). Of course, to obtain definite information on the relative Σ-Λ parity one must have the cascade particle-nucleon relative parity. This question has recently been studied by Treiman.[8]

ACKNOWLEDGMENTS

I would like to thank Professor S. Treiman, Professor M. Ruderman, and Dr. J. Sandweiss for helpful conversations. I would also like to thank Professor G. W. Ford for several most stimulating discussions.

[7] T. Fulton and G. Feldman (to be published).

[8] S. Treiman, Phys. Rev. (to be published).