High-Energy Maxima in the π-p Cross Sections

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It is pointed out that the consideration of causality requires that any description of the maximum in the π^--p total cross section at 800-Mev π^- energy, and the maximum in the π^+-p cross section at 1200-Mev, is necessarily equivalent to a description in terms of resonances and hence in terms of nucleon isobars.

I. INTRODUCTION

ATTEMPTS to provide an explanation of the behavior of the π,p cross sections at the high-energy maxima observed by Cool, Piccioni, and Clark,[1] can be separated into two groups. There have been attempts to describe the behavior of the cross sections with energy in terms of resonances affecting one or more partial waves, each representing a particular total angular momentum, parity, and isotopic spin. Cool *et al.*[1] have shown that if such a resonance in a single partial wave is responsible for the π^--p cross section maxima at 800-Mev π^- energy, the total angular momentum of the partial wave must be at least $\frac{5}{2}\hbar$. Recently Wilson[2] has suggested that the photonucleon production[3–5] of π mesons at energies near 400 Mev in the center-of-mass system is consistent with a simple description in terms of the existence of a nucleon isobar at this energy, which has a total angular momentum of $\frac{3}{2}$ and an isotopic spin of $\frac{1}{2}$. The broad maximum in the π^--p total cross section might then result from the resonance associated with this isobar and one or more resonances at a somewhat higher energy. Feld[6] also considers that several resonances may contribute to this maximum. In particular he suggests that states of even parity and spins $\frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$, dominate the cross section.[7]

These maxima have also been discussed in terms which do not specifically involve nucleon isobars. Piccioni[8] pointed out that the interaction of the incident π meson with the virtual π mesons in the nucleon field involves small center-of-mass energies and large wavelengths of the pion-pion system. Therefore a resonance of this system could result in a large maximum in the π-p cross section though only small values of the angular momentum of the pion-pion system be involved. Consequences of this idea were studied by Cool *et al.*,[1] Takeda,[9] and Dyson.[10] Lindenbaum and Sternheimer[11] have suggested that these maxima might result from the formation of the $(\frac{3}{2},\frac{3}{2})$ isobar in the final state.

It is the purpose of this note to point out that at any sufficiently sharp fluctuation in the total π-p cross sections, the behavior of the scattering matrix required by the imposition of causality and the existence of dispersion relations is similar to the behavior of the scattering matrix at the energy of the formation of an isobar.

II. BEHAVIOR OF THE SCATTERING MATRIX NEAR THE ENERGY CORRESPONDING TO THE FORMATION OF AN ISOBAR

Since there appears to exist no universally accepted precise description of scattering behavior near the energy corresponding to isobar formation when the particles involved are at relativistic energies, it is necessary to adopt a particular complete description for the sake of definiteness. The definitions implied in Eqs. (1) and (2) follow from the work of Wigner and Eisenbud.[12] Though this work is concerned with the nonrelativistic problems of nuclear level structure, the qualitative features used here have a wider degree of validity.

In particular we examine the behavior of the scattering matrix near a pole of the Wigner R matrix. Equation (2) then represents the elements of the scattering matrix near a resonance under the condition that the nonresonant scattering and absorption may, be large. Primarily in order to simplify the discussion we consider a situation where only two channels labeled a and b are open. Ignoring statistical factors, spins, and angular dependences, the scattering amplitudes for a specified total angular momentum and

[1] Cool, Piccioni, and Clark, Phys. Rev. **103**, 1082 (1956).
[2] R. R. Wilson, Phys. Rev. **110**, 1212 (1958).
[3] DeWire, Jackson, and Littauer, Phys. Rev. **110**, 1208 (1958).
[4] P. C. Stein and K. C. Rogers, Phys. Rev. **110**, 1209 (1958).
[5] Heinberg, McClelland, Turkot, Woodward, Wilson, and Zipoy, Phys. Rev. **110**, 1211 (1958).
[6] B. T. Feld, Ann. Phys. N. Y. 4, 189 (1958).
[7] This view is, however, difficult to reconcile with the large forward-backward asymmetry in the π^--p elastic scattering found by Erwin and Kopp [A. Erwin and J. Kopp, Phys. Rev. **109**, 1364 (1958)].
[8] O. Piccioni (private communication, 1954).
[9] G. Takeda, Phys. Rev. **100**, 440 (1955).
[10] F. J. Dyson, Phys. Rev. **99**, 1037 (1955).
[11] S. J. Lindenbaum and R. Sternheimer, Phys. Rev. **106**, 1107 (1957).
[12] E. P. Wigner and L. Eisenbud, Phys. Rev. **72**, 29 (1947).

parity take the form:

$$A_{ab}=\frac{i}{2k_a}(\delta_{ab}-U_{ab}), \tag{1}$$

where

$$U_{aa}=\frac{e^{-2i\omega_a}[1+i(\Gamma_a+g_{aa}\Delta)+\frac{1}{2}(\Gamma_a g_{bb}+\Gamma_b g_{aa}-2\Gamma_a^{\frac{1}{2}}\Gamma_b^{\frac{1}{2}}g_{ab})+\frac{1}{2}(g_{aa}g_{bb}-g_{ab}^2)\Delta]}{\Delta-\frac{1}{2}i(\Gamma_a+\Gamma_b+g_{aa}\Delta+g_{bb}\Delta)-\frac{1}{4}(\Gamma_a g_{bb}+\Gamma_b g_{aa}-2\Gamma_a^{\frac{1}{2}}\Gamma_b^{\frac{1}{2}}g_{ab})-\frac{1}{4}(g_{aa}g_{bb}-g_{ab}^2)\Delta},$$

$$U_{ab}=U_{ba}=\frac{e^{-i\omega_a}i(\Gamma_a^{\frac{1}{2}}\Gamma_b^{\frac{1}{2}}+g_{ab}\Delta)e^{-i\omega_b}}{\Delta-\frac{1}{2}i(\Gamma_a+\Gamma_b+g_{aa}\Delta+g_{bb}\Delta)-\frac{1}{4}(\Gamma_a g_{bb}+\Gamma_b g_{aa}-2\Gamma_a^{\frac{1}{2}}\Gamma_b^{\frac{1}{2}}g_{ab})-\frac{1}{4}(g_{aa}g_{bb}-g_{ab}^2)\Delta}; \tag{2}$$

Γ_a and Γ_b are the resonance widths for channels a and b, respectively, and the dimensionless quantities g are parameters which represent the nonresonant backgrounds. The symbol Δ is equal to $E_\lambda - E$ where the terms E_λ and E represent the resonance energy and the total energy of the system, and the ω_a and ω_b are phase angles. The quantities Γ, g, and ω, will in general vary slowly with energy. The elastic scattering amplitude element U_{bb} takes the same form as U_{aa} with appropriate interchanges of subscripts.

Using Eqs. (1) and (2), it can be shown that the dimensionless scattering amplitude $2k_aA_{aa}$ can be put into the form

$$2k_aA_{aa}=\rho_0e^{i\alpha}+\rho e^{i(\beta+\delta)},$$

where

$$\beta=2\tan^{-1}[(\Gamma_a+\Gamma_b)/(E_\lambda-E)]$$

and ρ_0, ρ, α, and δ are slowly varying functions of energy. Figure 1 shows the variation of $2k_aA_{aa}$ and $2k_bA_{bb}$ on the complex plane as a function of energy. If we write $2kA$ in the familiar form $2kA=1-\eta e^{2i\delta}$ and recall that the scattering cross section is proportional to $|A|^2$, the inelastic cross section to $1-\eta^2$, and the total cross section is proportional to ImA; we see that on Fig. 1, b represents the point of minimum absorption, c the maximum scattering cross section, d the maximum total cross section, f the maximum absorption cross section, g the minimum scattering cross section, and h the minimum total cross section. As the energy increases the locus of the values of $2k_aA_{aa}$ follows the curve in a counter-clockwise direction; a represents an energy far below resonance; c represents the value of $2k_aA_{aa}$ at $E=E_\lambda$, while i is the point reached at an energy far above the resonance. The primed letters represent the same quantities for $2k_bA_{bb}$. Note that the maxima and minima fall at different energies. The absorption maxima and minima will fall at the same energy for both incident channels, however, as they are connected by detailed balance.

In summary, we see that the characteristics of a resonance so defined, require that the locus of the scattering amplitude for each of the two channels moves, as a function of energy, counter-clockwise in an approximately circular path in the complex plane. Total absorption cross sections, total scattering cross sections, and total cross sections pass through maxima and minima in the energy region near resonance. These maxima and these minima do not necessarily occur at the same energy.

III. CAUSALITY REQUIREMENTS ON THE BEHAVIOR OF THE SCATTERING MATRIX NEAR A CROSS-SECTION MAXIMUM

Goldberger and others[13–15] have used conditions imposed by microcausality to develop dispersion relations relating the real and imaginary parts of the scattering amplitude in the forward direction. For our purpose we can write

$$R(E)=a(E)+\frac{k^2}{\pi}\int_\pi^\infty \frac{I(E)dE'}{k'^2(E'-E)}, \tag{3}$$

where $R(E)$ and $I(E)$ are the real and imaginary parts of the forward scattering amplitude at the total energy E,$a(E)$ is a slowly varying function of E,k' is the wave number at energy E and the lower bound to the integral is the π-meson rest mass. The form of the equation is the same for π^+ and π^- mesons. From the optical theorem, the imaginary part of the forward scattering amplitude, $I(E)=\sigma(E)/4\pi k$, where $\sigma(E)$ is the total cross section. We are concerned with the behavior of the real part of the scattering amplitude near the energy corresponding to a sharp maxima or minima in the total cross section. We can simulate such behavior by writing $I=I_0-b\cos\{2\tan^{-1}[\Gamma(E_\lambda-E)^{-1}]+\alpha\}$, where I_0 and b are real positive functions of energy varying slowly over the width Γ,E_λ is a "resonance" energy, and α is a phase angle, again varying slowly with energy. From Eq. (3) it can be shown then that $R=R_0+b'\sin\{2\tan^{-1}[\Gamma(E_\lambda-E)^{-1}]+\alpha\}$, where b' and α' are approximately equal to b and α, and R_0 is again a slowly varying function of energy. As an increasing function of energy the locus of the forward scattering amplitude traverses a counter-clockwise circle of radius b in the complex plane. Although the high-energy π-p maxima are neither sharp, nor isolated, the detailed calculations of Sternheimer[16] concerning values of the forward scattering

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[14] R. Karplus and M. A. Ruderman, Phys. Rev. **98**, 771 (1955).

[15] Goldberger, Miyazawa, and Oehme, Phys. Rev. **99**, 986 (1955).

[16] R. M. Sternheimer, Phys. Rev. **101**, 384 (1956), and calculations in reference 1.

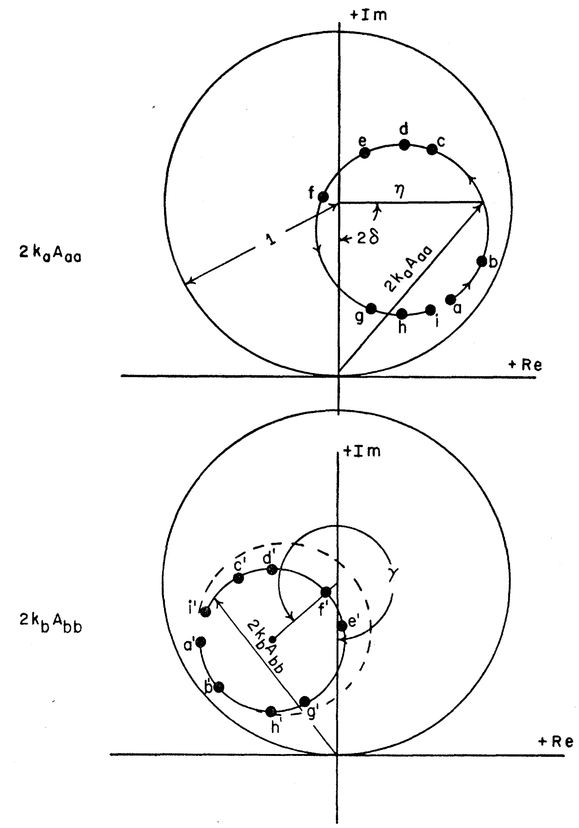

FIG. 1. Loci of the values of the scattering amplitudes A_{aa} and A_{bb} in the complex plane. As the energy increases through a resonance, the values of the scattering amplitudes move about a circle in a counter-clockwise direction. The larger solid circle on each diagram is the limit imposed by the conservation of reality.

amplitude at these resonances clearly illustrate this behavior.

While a resonance, defined according to the last section, would result in a variation with energy of the forward scattering amplitude equivalent to that described above, the inverse is not necessarily so, as the relative variation of the real and imaginary amplitudes are not correlated for individual partial waves corresponding to a particular angular momentum. From the forward scattering amplitudes alone we cannot exclude the possibility that the variation in the real amplitude is concerned with one partial wave, and the variation of the imaginary part with another.

There are essentially two possible descriptions of a maximum in a total cross section. Either one or more partial waves show maxima, or the maximum is the result of increases in the total cross sections due to some partial waves coupled with decreases in cross sections of other partial waves. The latter, rather pathological possibility, will not be considered in this report and we shall assume that the maxima in the π^--p total cross section at 800 Mev and the π^+-p cross section at 1200 Mev are the result in each case, of a total cross-section maximun in one or more partial waves. Since the optical theorem is valid for each partial wave separately, this is equivalent to maxima in the imaginary part of the scattering amplitude for these waves.

We must then consider the use of dispersion relations relating to one partial wave. Oehme[17] has shown that, in general, dispersion relations cannot be written for individual partial waves; however, he and others[18,19] have noted that if the interaction vanishes outside a radius a, the function $A_l(E)e^{2ik_a}$ where $A_l(E)$ is the scattering amplitude for a particular partial wave, is restricted by causality, and dispersion relations can be written for this function.

A value for a of $\hbar/M_\pi c$, where M_π is the mass of the π meson, would seem to be a reasonable interaction boundary for π-p scattering. If ka does not vary much over the width of a fluctuation in cross section, a relation similar to Eq. (3) can be written for A_l and the conclusions derived from Eq. (3) concerning the behavior of the forward scattering amplitude, will also hold for a scattering amplitude representing a particular angular momentum. At a maximum in the cross section the complex scattering amplitude, for at least one partial wave, will then take the same general form, $2k_aA_{aa}=\rho_0e^{i\alpha}+\rho e^{i(\beta+\epsilon)}$ as discussed in Sec. II and illustrated in Fig. 1. Restricting ourself again to two channels, we see that the values of A_{ab} are determined as a function of energy by the value of A_{aa}. Since $|A_{ab}|=|A_{ba}|$, restrictions are in turn placed upon the behavior of A_{bb}. Indeed the use of dispersion relations in channel b then determines that, within a phase factor γ, the behavior of A_{bb} will be equivalent to that described by either the solid or dotted circles in Fig. 1. This double valuedness can be expected since, in general, a specific absorption cross section at a resonance can result from either of two ratios of absorption and scattering widths.

We note that in the energy region of the cross-section maximum, the restrictions imposed by causality require that a maximum and minimum occur in the total cross section, the scattering cross section, the absorption cross section, and in the scattering cross section and total cross section associated with another channel. All of these cross sections then behave in essentially the same manner as those described in Sec. II, where the behavior was specifically described as resulting from the formation of an isobar.

IV. SUMMARY AND CONCLUSIONS

The result of Secs. II and III, that the properties of the scattering matrix at a cross section maximum imposed by considerations of causality are largely, if not wholly, equivalent to scattering matrix properties

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[18] N. G. van Kampen, Phys. Rev. **89**, 1072 (1953).
[19] J. M. Knight and J. S. Toll, Ann. Phys. N. Y. 3, 49 (1958).

near a pole in the R matrix, has been surmised by Schützer and Tiomno[20] and established by Wigner.[21] Since the object of the present work was to draw conclusions from the π-p cross sections, it seemed desirable to follow a somewhat different form in the discussion.

The main conclusion which results from a comparison of Secs. II and III, is that the π-p total cross-section maxima must be associated with the formation of nucleon isobars. Such a statement is not, however, necessarily in conflict with other descriptions. The maxima could still be ascribed to resonances in the π-π interaction. This interaction would, however, manifest itself in resonances in the π-p system.

A subsidiary result of the calculations is the explicit exhibition of the effect of the nonresonant background in displacing the maxima of absorption cross sections with respect to the total cross-section maximum. This displacement can be as large as the width Γ. It is not certain then that the photonucleon maximum[2] near 400 Mev in the center-of-mass system might not be associated with the total π-p cross-section maximum which occurs near 600 Mev.

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[21] E. P. Wigner, Am. J. Phys. **23**, 371 (1955).

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Preliminary Results on the Momentum Dependence of the Asymmetry in Muon Decay*

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Positive muons from the Berkeley 184-in. synchrocyclotron were stopped in various materials and the asymmetry of positrons from their decay was determined by using a magnetic spectrometer method. The measurements of the polarization are in agreement with that predicted by the two-component neutrino theory.

The product of the muon beam polarization and of ξ, the parameter of the two-component neutrino theory, was measured as 0.89 ± 0.09.

THE two-component theory for muon decay[1,2] gives a predicted normalized spectrum for beta decay of completely polarized muons at rest

$$dN(x,\theta)=2x^2[3-2x+\xi\cos\theta(1-2x)]dxd\Omega(4\pi)^{-1}, \quad (1)$$

where x is essentially (for all energies considered in this measurement) the total β energy in units of the maximum total β energy, Ω is the solid angle of β momentum, and θ is the angle between the β momentum and the muon spin direction. The sign of ξ is positive for positive muons, and we have

$$|\xi|=\frac{g_Vg_A{}^*+g_V{}^*g_A}{|g_V|^2+|g_A|^2}=\frac{0.87\pm0.12}{R}, \quad (2)$$

where g_A and g_V are the polar vector and axial vector coupling constants, and R is a measure of the degree of depolarization in the stopping material and of the polarization of the beam.[3,4] Measurements of the energy variation by the use of an integral range measurement and a precessing magnetic field have been reported by Berley *et al.*[4] and Weinrich[4] and by Mukhin *et al.*[5] Telegdi and Wright[6] and Cassels *et al.*[7] have used total-absorption scintillation spectrometers. Low-energy bubble chamber data have also been obtained.[8]

To measure the asymmetric part of the spectrum, we have used the following technique. A polarized μ^+ beam produced by decay of pions made by the internal beam of the 184-in. cyclotron comes to rest in an absorber-counter sandwich. A magnetic field is produced by a Helmholtz coil giving either a depolarizing field of 0 ± 0.7 or 55 ± 10 gauss over the volume of the muon targets. The μ^+ are analyzed with a 180° $n=\frac{1}{2}$ magnetic spectrometer with a coincidence counting matrix to define several energy channels per field setting. The beam particles are put into anticoincidence with the events so that only particles produced in the targets and counters are detected. Figure 1 shows a side view of the equipment.

The spectrometer is set at an angle to the muon

* This work was done under the auspices of the U. S. Atomic Energy Commission.
[1] T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957).
[2] Four-component neutrino theory leads to

$$dN(x,\theta)\propto\{3(1-x)+2\rho(\tfrac{4}{3}x-1)\pm\xi\cos\theta[(1-x)+2\delta(\tfrac{4}{3}x-1)]\}x^2dxd\Omega,$$

in the usual notation; see, e.g., reference 9.
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[4] Berley, Coffin, Garwin, Lederman, and Weinrich, Phys. Rev. **106**, 835 (1957); M. Weinrich, Columbia University doctoral dissertation, February, 1958 (unpublished).
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