Inelastic Proton-Proton Collisions at 6.2 Bev*

R. M. KALBACH,† J. J. LORD, AND C. H. TSAO
Department of Physics, University of Washington, Seattle, Washington
(Received September 8, 1958)

Inelastic, 6.2-Bev proton-proton collisions in nuclear emulsions are examined using the internal beam of the Berkeley Bevatron. Multiple scattering, grain density, range, and angle measurements yield the momentum spectra and angular distributions of secondary pions and protons together with the cross sections for accessible final states. The results indicate a cross section of 7.3±4.6 mb for two prong events, 12.1±2.4 mb for four prong events, 2.7±0.6 mb for six prong events, 0.3±0.3 mb for eight prong events, and 0.2±0.2 mb for ten prong events, giving a total inelastic cross section of 22.6±5.3 mb. The average charged pion multiplicity was found to be 1.9±0.3 and the value of K, the average degree of inelasticity, 0.49±0.05. Comparison of observed partial inelastic cross sections with the predictions of the Fermi statistical theory indicates that this theory underestimates the relative probability for states with high meson multiplicity. Considerable forward and backward peaking was observed in the center-of-mass system angular distributions for secondary protons and, to a lesser extent, for secondary charged pions. Center-of-mass system momentum distributions for secondary charged pions peak at lower momenta than predicted by the statistical theory, while those for protons peak somewhat higher than predicted. Effects are discussed which could account for these discrepancies.

I. INTRODUCTION

THIS paper contains the results of an investigation of inelastic proton-proton collisions at a bombarding energy of 6.2 Bev using nuclear emulsions. The ultimate aim of such an experiment is to furnish information on the nature of nucleon-nucleon collisions in an energy region only recently provided by particle accelerators. Since, in this experiment, the total kinetic energy of the two colliding protons in the center-of-mass system is approximately fourteen times the pion rest energy, multiple meson production will contribute heavily to the observed interactions.

The simplest approach to the problem of multiple meson production is the Fermi statistical theory of nucleon-nucleon collisions.[1] The basic assumption of the statistical theory is that at the instant of collision, the two colliding nucleons coalesce and deposit their total energy in the space occupied by their meson fields. It is further assumed that since the interactions of pions and nucleons are strong, the system attains statistical equilibrium during the time of the collision so that accessible final states are formed with probabilities governed by the laws of statistics. In an attempt to formulate a more realistic theory of the collision, Fermi and others[2–6] subsequently included more precise formulations of the various conservation laws, modified the original concept of the interaction volume, and examined the effects of interactions among final-state particles.

The results of the present experiment are then intended to be a test of the original statistical theory with its later modifications as well as to provide a value for the inelastic proton-proton cross section in an energy region intermediate to previous accelerator results and cosmic-ray measurements.

II. EXPERIMENTAL PROCEDURE

A. General

A small stack of 600-micron, Ilford G-5 emulsions was exposed to one reduced-intensity pulse of the 6.2-Bev internal proton beam of the Berkeley Bevatron. Details of the exposure, preliminary scanning, and angle measurements of secondary tracks are given in the preceding paper devoted to an analysis of the elastic interactions obtained with the same exposure.

In order to select inelastic events for analysis the following criteria were established:

(a) There must be an even number of secondary tracks, or prongs.

(b) An event must not have a recoil blob or low-energy electron, indicating collision with a heavy nucleus. (One event having a low-energy electron was discarded).

(c) Secondary protons must have a kinetic energy less than that of an elastically scattered proton emerging at the same scattering angle.

It is evident that these criteria are not adequate to eliminate all edge-on collisions of beam protons with nuclei other than protons. However, it was concluded that at a bombarding energy at 6.2 Bev, the Fermi momentum of such bound protons would not materially effect the momentum spectrum or angular distribution of secondary pions and protons. On the other hand, it is necessary for a determination of cross section to know the fraction of inelastic events satisfying the

* Assisted by a joint program of the National Science Foundation, the Office of Naval Research, and the U. S. Atomic Energy Commission.
† Now at the University of Arizona, Tucson, Arizona.
1 E. Fermi, Progr. Theoret. Phys. Japan **5**, 570 (1950).
2 E. Fermi, Phys. Rev. **92**, 452 (1953); **93**, 1435 (1954).
3 J. V. Lepore and M. Neuman, Phys. Rev. **98**, 1484 (1955).
4 J. S. Kovacs, Phys. Rev. **101**, 397 (1956).
5 G. E. A. Fialho, Phys. Rev. **105**, 328 (1957).
6 J. V. Lepore and R. N. Stuart, Phys. Rev. **94**, 1724 (1954).

criteria listed above which are actually collisions with heavy nuclei. In order to do this, a plot was made of the number of events having an odd number of secondaries which satisfy criteria (b) and (c) as a function of the number of secondaries. With the assumption that collisions with protons and neutrons are equally probable in large nuclei, this plot allows one to estimate the number collisions involving bound protons which are included among the events which satisfy all the selection criteria.

Grain density and multiple scattering or range measurements were made on all inelastic secondaries. With the aid of the known dependence of the rate of energy loss on momentum times velocity,[7] such data allow one to determine the mass and momentum of the secondaries. The analysis was carried out by classifying events according to the number of secondary tracks rather than according to a scheme involving knowledge of neutral secondary particles. Experimental data then yields the momentum spectrum and angular distribution of secondary protons and charged pions together with the average charged-pion multiplicity for events with various even numbers of secondary tracks. Since many predictions of the statistical theory are given as a function of the total number of pions produced, a reformulation of these predictions is then necessary in order to permit a comparison with the data of the present experiment.

B. Measurement of Mass and Momentum

Determination of the mass and momentum of secondary tracks was accomplished by a combination of grain density and range or multiple-scattering measurements. In instances where a secondary comes to rest in the emulsion, measurement of range and examination of the track ending is sufficient to establish its identity and energy.

Secondaries which did not stop in the emulsion were first subjected to a grain density measurement. To make such measurements independent of the properties of the emulsion, a relative grain density, g, was assigned to each secondary track. This quantity is defined as the ratio of the average number of grains per 100 microns for the track in question to the average number of grains per 100 microns for a 6.2-Bev proton track passing through the emulsion at the same average depth.

In addition to a measurement of g, multiple-scattering measurements were also made on fast secondary tracks. This was done according to the method of Fowler,[8] the relation between the mean absolute second difference due to multiple scattering, $\bar{D}_m$, the momentum, p, and the velocity, v, being

$$pv=(K_z Z L^{\frac{3}{2}})/(573\bar{D}_m). \qquad (1)$$

In Eq. (1), K_z is the scattering factor, Z is the atomic number of the moving particle, and L is the cell length in microns. When $\bar{D}_m$ is in microns, pv is in Mev. For this experiment, the value of K_z is taken to be 25, a value consistant with available data for G-5 emulsions.[9]

After discarding values of the experimentally determined second differences, D_e, whose absolute values were greater than four times the mean, contributions to $\bar{D}_e$ due to the following effects were considered: (a) curvature of the track due to differential shrinkage of the emulsions during processing, (b) spurious scattering, and (c) noise in the scattering stage and associated measuring apparatus.

The statistical nature of the scattering process will cause random fluctuations of an emulsion track about an imaginary straight line which the track would define in the absence of multiple scattering. This is equivalent to the statement that the algebraic sum of N measured values of D_e should equal zero. However, it is found that differential shrinkage of the emulsion during development often introduces nearly a constant curvature in a track which initially varies about a straight line. This being true, the algebraic sum of the N values of D_e will be equal to some number, α. The contribution of such distortion to the value $\bar{D}_e$ may then be reduced by subtracting α/N from each measured D_e and recalculating the mean absolute second difference to obtain the corrected value $\bar{D}$.

Spurious scattering is a term applied to the random fluctuations of an emulsion track similar to those caused by multiple Coulomb scattering. However, the mean absolute second difference due to spurious scattering, $\bar{D}_{ss}$, has a different dependence on cell length than that associated with the later effect. Spurious scattering is presently believed to be caused by local distortions within the emulsions unrelated to those caused by differential shrinkage during development. The contribution to $\bar{D}$ due to spurious scattering in the plates for this investigation was carried out by Fischer.[10] The results of Fischer and others[11,12] indicates that $\bar{D}_{ss}$ is given rather closely by the relation

$$\bar{D}_{ss}=5.24\times10^{-4}L^{0.95}, \qquad (2)$$

where L and $\bar{D}_{ss}$ are expressed in microns.

The quantity D may then be decomposed as follows:

$$\bar{D}^2=\bar{D}_m{}^2+\bar{D}_{ss}{}^2+\bar{D}_n{}^2, \qquad (3)$$

where $\bar{D}_n$ is the contribution due to noise in the measuring apparatus. This was ascertained by performing multiple-scattering measurements on beam proton tracks using various cell lengths. Since pv is known in this case, $\bar{D}_n$, may be calculated by means of Eqs. (1),

[7] R. M. Sternheimer, Phys. Rev. **88**, 851 (1952); **91**, 256 (1953).
[8] P. H. Fowler, Phil. Mag. **41**, 169 (1950).
[9] Backus, Lord, and Schein, Phys. Rev. **88**, 1431 (1952).
[10] F. W. Fischer, Masters thesis, University of Washington, 1954 (unpublished).
[11] Biswas, Peters, and Rama, Proc. Indian. Acad. Sci. **A41**, 156 (1955).
[12] Brisbout, Dahanayake, Engler, Fowler, and Jones, Nuovo cimento **3**, 1400 (1956).

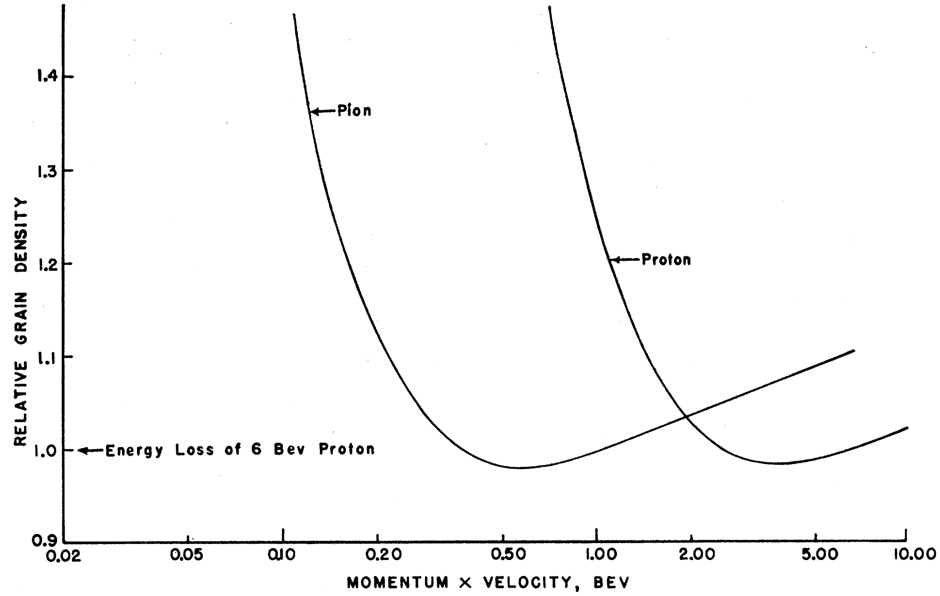

FIG. 1. Relative grain density *versus* momentum times velocity. The curves are normalized so that the grain density of a 6.2-Bev proton is unity.

(2), and (3). This quantity was found to vary only slightly with cell length and was thus considered to be constant throughout the measurements.

The Sternheimer equation[7] for g *vs* pv was used to determine the mass of secondaries. This equation is plotted in Fig. 1 for pions and protons. K mesons are not shown because of the relatively low production cross section and the experimental difficulty in distinguishing between protons and K mesons. Due to statistical error in the measured values of grain density and multiple scattering, the experimental point (g,pv) will not, in general, fall on the pion or proton curve. One must then determine the most probable mass of the particle and the corresponding value of pv.

For given values of the standard deviation in the measured value of g, σ_g, and pv, σ_{pv}, the probability that the values for g and pv are displaced by amounts ϵ_g and ϵ_{pv}, respectively, is proportional to the quantity

$$P=\exp[-\tfrac{1}{2}(\epsilon_g^2/\sigma_g^2)-\tfrac{1}{2}(\epsilon_{pv}^2/\sigma_{pv}^2)]. \quad (4)$$

To find the most probable location of the experimental point on both the pion and proton curves, we must maximize P, first with the assumption that the particle is a pion, and second, that it is a proton. When the points have been determined for which these probabilities are maximized, the ratio of the P values will be a measure of the certainty with which the particle in question is a pion (or proton). The associated values of pv for both pion and proton assumptions may then be read from the curves of Fig. 1. Standard deviations associated with the pv values may be determined by locating those points on the pion and proton curves for which P drops to $1/e$ of its maximum value. These calculations were carried out with an IBM-650 machine.

With a knowledge of the scattering angle, identity, and momentum of a particular secondary, the scattering angle and momentum in the center-of-mass system, subsequently referred to as the c.m. system, may be determined with the aid of standard transformation equations. In cases where the identification is not obvious, the momentum and scattering angle in the c.m. system is examined under both pion and proton assumptions. The correct choice is often more obvious when such values are considered in the light of scatter-

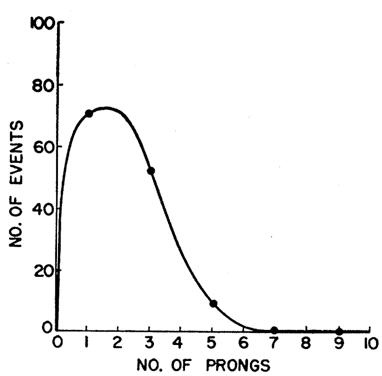

FIG. 2. Distribution of the number of charged secondaries in collisions of beam protons with bound neutrons.

TABLE I. Correction for scanning efficiency.

Number of secondaries	Number of events located	Number of events corrected for scanning efficiency
1	38	71.1
2 (elastic)	20	37.0
2 (inelastic)	112	86.2
3	91	52.3
4	153	65.3
5	23	8.5
6	34	11.2
7	1	0.29
8	6	1.5
9	4	0.88
10	3	0.60

ing angles and momentum values for the remaining secondaries of a particular event, while in some cases this procedure leads to the rejection of an event because of an obvious momentum unbalance for either assumption.

III. RESULTS

A. Cross-Section Measurement

Upon preliminary inspection of the identity and momentum of secondary tracks from a total of 315 accepted inelastic events, two of the four-prong and five of the six-prong events were determined to be interactions with heavy nuclei and discarded. Table I shows the number of events located which satisfy acceptance criteria (b) and (c) of Sec. II-A having both even and odd numbers of secondary tracks, while Fig. 2 shows the number of proton-neutron events from this table plotted as a function of the number of secondary tracks. The number of collisions with heavy nuclei included in the events satisfying all selection criteria is then to be estimated from this graph. The resulting correction of accepted events for edge-on collisions is shown in Table II.

TABLE II. Correction of cross section for collisions with bound protons.

Number of secondary prongs	2	4	6	8	10
Total number of events from scanning	123.2	65.3	11.2	1.5	0.6
Number of peripheral collisions from Fig. 2	68.0	24.0	2.0	0.5	0.0
Number of free proton-proton collisions	55.2	41.3	9.2	1.0	0.6
Cross section, mb	16.1	12.1	2.7	0.3	0.2

TABLE III. Final cross-section results.

Number of secondary prongs	Cross section, mb
2 (elastic)	8.8 ± 2.0
2 (inelastic)	7.3 ± 4.6
4	12.1 ± 2.4
6	2.7 ± 0.6
8	0.3 ± 0.3
10	0.2 ± 0.2

Cross sections for observed final states, compared to an elastic cross section of 8.8 ± 2.0 mb,[13] are listed in Table III. The observed ratio of the elastic to the inelastic cross section is 0.4 ± 0.1.

Those events for which all the secondaries could be analyzed are listed in Table IV. When the results of this table are corrected for scanning efficiency and edge-on collisions, an average charged-pion multiplicity of 1.9 ± 0.3 results. If the ratio of charged to neutral pions is assumed to be 2:1, the average pion multiplicity is 2.8 ± 0.4.

[13] See Kalbach, Lord, and Tsao, Phys. Rev. **113**, 325 (1959), preceding paper.

TABLE IV. A comparison of the relative abundance of various final states with the predictions of the Fermi theory.

Final state charged particles	Number of events located	Number of events expected according to Fermi theory
p, p	5	5.3
p, π	22	22.5
π, π	8	7.2
p, p, π, π	5	9.4
p, π, π, π	9	5.3
π, π, π, π	1	0.3
p, p, π, π, π, π	1	3.0
$p, \pi, \pi, \pi, \pi, \pi$	2	0
$\pi, \pi, \pi, \pi, \pi, \pi$	0	0

B. Momentum Spectra and Angular Distributions

The c.m. system angular distributions and momentum spectra for pions and protons emitted in accepted two-, four-, and six-prong events are given in Figs. 3–6. Figure 3 shows the angular distribution of secondary protons from two-, four-, and six-prong events. Here, the most outstanding feature is the sharp peaking in

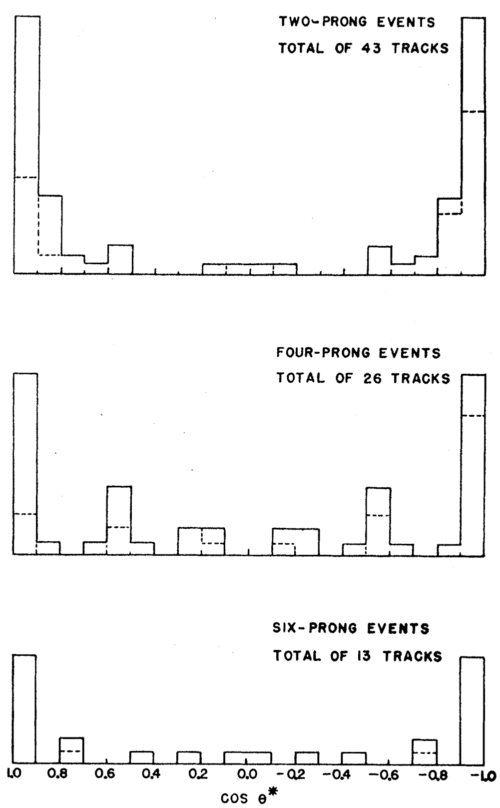

FIG. 3. Angular distribution of secondary protons from accepted proton-proton collisions plotted in the c.m. system, where θ^* is the scattering angle in the c.m. system. The dotted histogram corresponds to the experimental data, and the solid histogram is the result of averaging this data about the line $\cos\theta^*=0$.

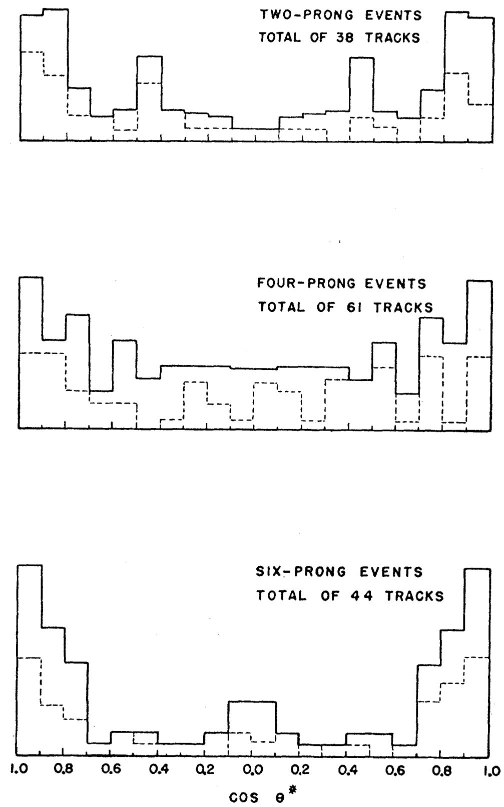

FIG. 4. Angular distribution of secondary pions from accepted proton-proton collisions plotted in the c.m. system. The dotted histogram corresponds to the experimental data while the solid histogram is the result of averaging these data about the line $\cos\theta^*=0$.

the forward and backward directions. The two-prong distribution differs only slightly from that of elastically scattered protons given in the previous paper. Although forward and backward peaking is appreciable in four-prong events, this effect is reduced compared with that observed for protons from two-prong events. Protons from six-prong events are emitted with slight preference in the forward and backward directions, although this distribution does not differ greatly from isotropy if one takes statistical errors into consideration.

The angular distribution of pions emitted in two-, four-, and six-prong events, shown in Fig. 4, are in general less peaked than those for protons from such events. The pion angular distribution for six-prong events displays greater peaking than the distribution for pions from events with less secondaries. Statistics are best for four-prong events, where it is apparent that the angular distribution is more nearly isotropic.

Figure 5 shows the momentum distributions of protons from two-, four-, and six-prong events. As the average multiplicity increases, the maximum of the momentum distribution shifts slightly to lower momenta.

Pion momentum distributions for two-, four-, and six-prong events are shown in Fig. 6. The most striking feature is that the maxima of the observed distributions do not shift appreciably as the average multiplicity increases, and are considerably lower than predicted by the statistical theory. Although considerable asymmetry about 90° is apparent in some of the angular distributions, three of the six distributions are weighted more heavily in the backward direction and three are weighted in the opposite sense. It is therefore presumed that these asymmetries are not evidence of appreciable bias in selecting or identifying the tracks plotted. The distributions were thus symmetrized about 90° to give histograms outlined by the solid lines, while the unsymmetrized data is indicated by dashed lines. The average momentum in the center-of-mass system of the charged pions emitted in all inelastic events was calculated to be 0.32 Bev/c. From this value and the average pion multiplicity of 2.3, the average total energy carried off by pions in a single event is calculated to be 0.98 Bev in the center-of-mass system. The ratio of the average total energy carried off by pions to the total

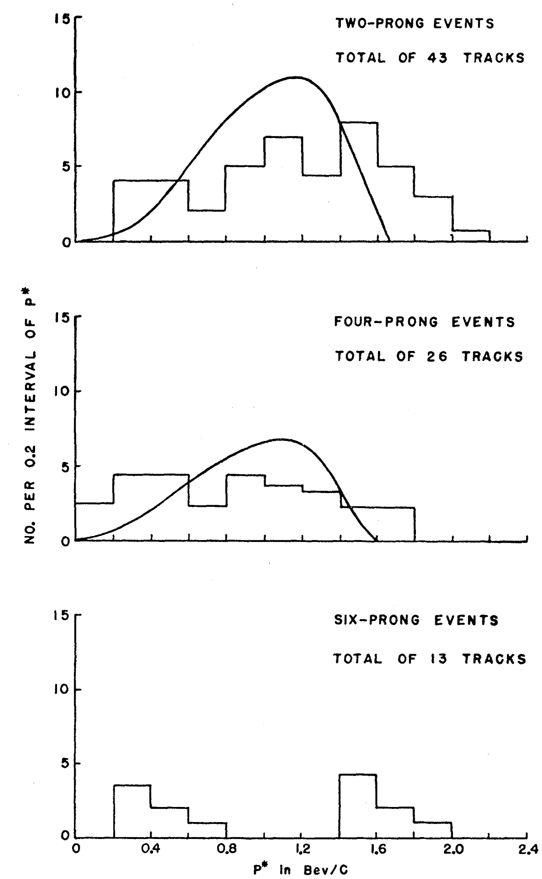

FIG. 5. Momentum distribution of secondary protons from accepted proton-proton collisions plotted in the c.m. system. The histogram represents the experimental data while the smooth curve is the result of the statistical theory.

available kinetic energy in the c.m. system was calculated to be 0.49±0.05. This quantity is subsequently referred to as K, the degree of inelasticity.

IV. COMPARISON WITH THEORY

Partial inelastic cross sections for final states with up to four mesons, computed according to the Fermi theory,[1,2] the method of Lepore and Stuart,[6] and the theory of Lepore and Neuman,[3] are presented in Table V.

Table VI shows a reformulation of the results of the first two theories according to the number of secondary prongs, together with the corresponding experimental values.

The total cross section was measured to be 31.4±5.1 mb, compared to the assumption of approximately 60 mb from the statistical theory. The Fermi predictions for the occurrence of various final states are given in Table VI along with the experimental results. The values shown in this table together with the observed charged-meson multiplicity of 1.9±0.3, the Fermi value of 1.61, and the Lepore-Stuart value of 1.21 lead to the conclusion that the statistical theory apparently underestimates the contributions from states of high multiplicity.

The statistical theory also predicts the momentum distributions of pions and nucleons. A method for evaluating the necessary phase-space integrals using the saddle point approximation was given by Fialho.[5] In these calculations, all particles are treated relativistically and momentum is conserved for both nucleons and pions.

Comparison of the results of Fialho with those of the present experiment is most direct when the momentum distributions of pions and protons are formulated so that they correspond to events with two, four, and six charged secondaries rather than to one-, two-, three-, and four-meson final states. If we consider only final states with two charged secondaries in which up to four mesons are produced including at least one charged meson, reactions (c), (f), (g), (k), (l), (r), and (s) of Table V will contribute. Mesons produced by reaction (c) will be emitted with a distribution of momenta characteristic of single-meson production events. Likewise, charged pions emitted in the reactions (f) and (g), (k) and (l), and (r) and (s) will have distributions of momenta corresponding to double,

TABLE V. Probabilities for different final states in 6.2-Bev proton-proton collisions.

	Final state	Number of mesons	Fermi[a]	Lepore-Stuart[a]	Lepore-Neuman[b]
(a)	$p+p$	0	0.0014	0.0055	0.1869
(b)	$p+p+\pi^0$	1	0.0311	0.0856	0.0822
(c)	$p+n+\pi^+$	1	0.0933	0.2567	0.4934
(d)	$p+p+\pi^++\pi^-$	2	0.1378	0.1640	0.0538
(e)	$p+p+\pi^0+\pi^0$	2	0.0459	0.0547	0.0090
(f)	$p+n+\pi^++\pi^0$	2	0.2067	0.2459	0.1615
(g)	$n+n+\pi^++\pi^+$	2	0.0689	0.0820	0.0135
(h)	$p+p+\pi^0+\pi^0+\pi^0$	3	0.0964	0.0293	
(i)	$p+p+\pi^++\pi^-+\pi^0$	3	0.0113	0.0034	
(j)	$p+n+\pi^++\pi^++\pi^-$	3	0.1095	0.0332	
(k)	$p+n+\pi^++\pi^0+\pi^0$	3	0.0757	0.0226	
(l)	$n+n+\pi^++\pi^++\pi^0$	3	0.0451	0.0136	
(m)	$p+p+\pi^++\pi^++\pi^-+\pi^-$	4	0.0089	0.0004	
(n)	$p+p+\pi^++\pi^0+\pi^0+\pi^-$	4	0.0136	0.0006	
(o)	$p+n+\pi^++\pi^++\pi^-+\pi^0$	4	0.0318	0.0013	
(p)	$n+n+\pi^++\pi^++\pi^++\pi^-$	4	0.0064	0.0003	
(q)	$p+p+\pi^0+\pi^0+\pi^0+\pi^0$	4	0.0010	0.0001	
(r)	$p+n+\pi^++\pi^0+\pi^0+\pi^0$	4	0.0083	0.0004	
(s)	$n+n+\pi^++\pi^++\pi^0+\pi^0$	4	0.0069	0.0003	

[a] Computed for meson multiplicities up to 4.
[b] Computed for meson multiplicities up to 2.

TABLE VI. Probabilities for final states with different numbers of charged particles.

Number of charged secondaries	2	4	6	8	10
Fermi probability	0.595	0.396	0.009	...	...
Lepore-Stuart probability	0.770	0.230	0.000	...	...
Experimental probability	0.325	0.538	0.118	0.113	0.006

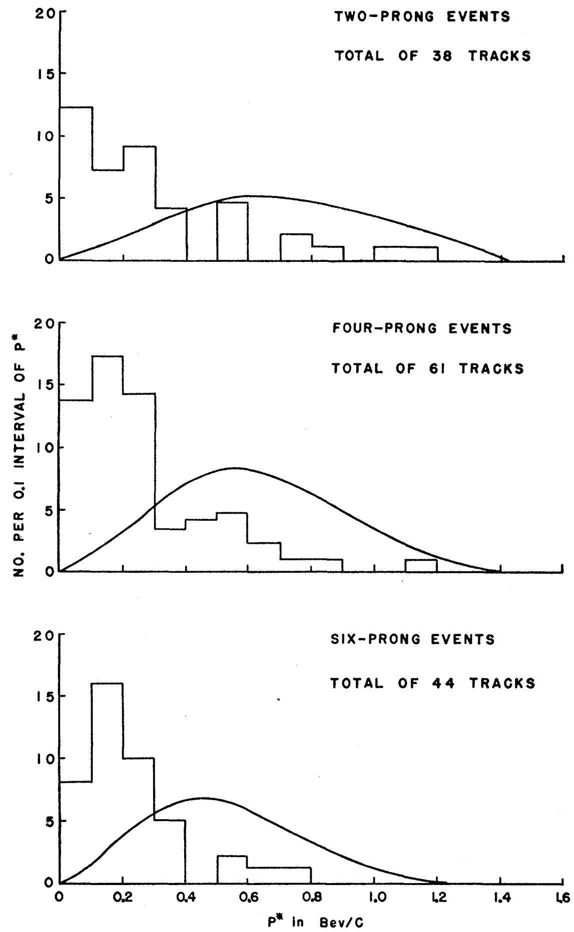

FIG. 6. Momentum distribution of secondary pions from accepted proton-proton collisions plotted in the c.m. system. The histogram represents the experimental data while the smooth curve is the prediction of the statistical theory.

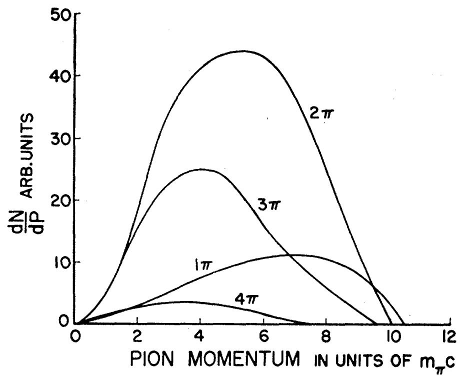

FIG. 7. Components of the momentum distribution of pions emitted in two-prong events computed according to the method of Fialho.

triple, and quadruple meson production events, respectively. The momentum distribution of charged mesons emitted in two-prong events will then be a weighted sum of the single, double, triple, and quadruple production, meson distributions of Fialho.

From Table V, the relative probability for the occurrence of the final state corresponding to reaction (c) is 0.0933. This is the relative weight for the single-meson production contribution. The weight for the double production contribution is 0.2067 for (f) in addition to 2×0.0689 for (g) giving a total 0.3445, the factor of two arising from the fact that reaction (g) will contribute two mesons to the experimentally observed distribution. Weights for the various contributions to the distribution for events with two charged secondaries are plotted in Fig. 7. After these curves are added, the resulting curve is normalized so that its area is equal to that of the experimental histogram and it is then plotted as a smooth curve along with this data in Fig. 6. A similar procedure is followed for the protons from two-prong events, the protons and pions from four-prong events, and the pions from six-prong events.

Although the relative probability for states with more than four mesons is rather small, such states contribute heavily to events with four and six charged secondaries. Unfortunately, using the method of Fialho, meson momentum distributions can be computed only for meson multiplicities up to 4 and proton distributions only for multiplicities up to 2. It is apparent from Fig. 7 that inclusion of states of higher multiplicity would shift the peak in the theoretical momentum distributions to lower values of momentum for both mesons and protons.

In general, the momentum spectra for the pions peak at lower momenta than predicted by the statistical theory of Fialho. This is most convincing for the pions emitted in events with two charged secondaries, since one can show that final states in which four mesons are produced contribute only 2.7% of the total probability for formation of a state with two charged secondaries from all reactions listed in Table V. This decreasing contribution from states of higher meson multiplicity is the result of the increasing ratio of neutral to charged pions necessary to give only two charged secondaries. It is therefore concluded that omission of states for which the meson multiplicity is greater than 4 does not seriously affect the theoretical momentum spectrum of pions from two-prong events. In general, the momentum spectra of the protons peak at higher momenta than predicted by the statistical theory. Here, the contribution to the predicted spectrum from events of higher multiplicity would increase the discrepancy between theory and experiment.

According to the statistical theory of Fermi, the angular distributions of all particles should be isotropic in the c.m. system. Fermi pointed out, however, that in case all particles are extreme relativistic in the c.m. system, a condition which does not apply in the present experiment, angular momentum conservation would account for a peaking in the forward and backward direction. Thus, in contrast to the prediction of the theory, experimental angular distributions are rather sharply peaked in the forward and backward directions for protons, while those for pions do not differ greatly from isotropy.

The theory of Lepore and Stuart[6] is similar to that of Fermi but differs in that momentum conservation for pions is included. In addition, the effects of pion indistinguishability are investigated. The results were extended to give predictions of the relative probabilities for various final charge states, the necessary weights being obtained from the isotopic spin considerations of Fermi. These results are given in Tables V and VI.

The theory of Lepore and Neuman[3] incorporates the conservation of center-of-energy into a statistical theory similar to that of Fermi. However, the interaction volume does not contain the Lorentz contraction term. Instead, energy-dependent cutoffs are introduced which give a uniform shrinkage of the configurational volume with increasing energy. The relative probabilities for final states with meson multiplicity up to 2 in proton-proton collisions are given in Table V. The insistance on the conservation of center-of-energy has the interesting effect of reducing the contributions from high momenta to phase-space integrals, an effect which would seem to bring closer agreement with experiment.

A theory developed by Kovacs[4] incorporates the interactions of final-state particles, the suppression of some one-meson states by virtue of the Pauli principle, conservation of angular momentum and parity, and the enhancement of two-meson states due to resonance effects into the statistical theory. Although neither the theory of Kovacs nor that of Lepore and Neuman has been extended to the high multiplicities of the present experiment, they would seem, qualitively, to give improved agreement with experiment.

It is also of interest to compare the results of the present experiment with the predictions of the Heisenberg theory.[14] According to this theory, Camerini *et al.*[15] plot the average meson multiplicity as a function of the incident proton energy in the c.m. system for various values of K, the degree of inelasticity. The experimental value of K is 0.49 ± 0.05, which corresponds to a predicted multiplicity of 2.2. Since the experimental value is 2.8 ± 0.4, it is apparent that the Heisenberg theory only slightly underestimates the average meson multiplicity. The total cross section predicted by this theory is of the order of 100 mb at the energy of the present experiment, contrasted with the experimental value of 31.4 ± 5.1.

V. DISCUSSION

Since the observed inelastic cross section of 22.6 ± 7.0 at 6.2 Bev, together with the data of Fowler *et al.*, indicates that this quantity does not change greatly from 0.8 to 6.2 Bev, there seems to be little indication of the large increase in total cross section predicted by the Heisenberg theory or implied by the cosmic-ray value of 120_{-20}^{+30} mb obtained by Williams[16] as an average of the proton-proton and neutron-proton cross sections in the neighborhood of 30 Bev.

The observed average pion multiplicity is greater than the Fermi statistical theory prediction as well as that of the Lepore and Stuart, a disagreement which is also apparent when the experimental results are compared with the corresponding theoretical values in Tables IV and VI.

Protons from inelastic collisions tend to be emitted in the forward and backward directions in the c.m. system with higher average momentum than predicted by the statistical theory. This would seem to indicate collisions of low momentum transfer in contrast to the large momentum transfer predicted by the Fermi theory. On the other hand, the pion momentum spectra are shifted toward lower momenta than predicted by the statistical theory, a result suggestive of strong interactions among final-state particles.

Although calculations have been made only for lower bombarding energies, certain qualitative statements can be made concerning the effects of including final state interactions upon the predictions of the statistical theory. The strong, low-energy pion-nucleon resonance is not considered in evaluating statistical weights in the Fermi theory and any theory should certainly include this in a modification. As the energy in the c.m. system is increased, it is possible to produce more mesons, those states being most heavily favored wherein the mesons produced have the requisite energy to rescatter off of the nucleons at resonance. According to this description, the average pion momentum will not vary greatly with the incident bombarding energy and will be lower than predicted by the statistical theory at the energy of this experiment.

Although a number of modifications of the statistical theory would account qualitatively for the discrepancy between theory and experiment, most have the disadvantage of destroying the basic simplicity of the Fermi theory. It would seem that one could retain this simplicity and at the same time obtain better agreement with experiment by properly weighting those final states known to involve strong interactions between the pions and nucleons.

ACKNOWLEDGMENTS

We wish to thank Professor E. Henley, Professor Y. B. Kim, Professor G. E. Masek, and Professor S. H. Neddermeyer for helpful discussions. Mrs. Richard Capps and Mr. Don Culbertson formulated the IBM-650 program for analysis of the events, and Mr. Gordon Williams provided helpful data on proton-nucleus collisions at 6.2 Bev. We are indebted to J. Siegwarth, J. Kirk, R. Gersten, and Miss J. Mouldenhour for scanning the plates and performing preliminary analysis. The exposure of the emulsions was made possible by Professor E. J. Lofgren and the staff of the Bevatron.

[14] W. Heisenberg, Z. Physik **133**, 65 (1952).
[15] Camerini, Davies, Fowler, Franzinetti, Muirhead, Lock, Perkins, and Yekutielli, Phil. Mag. **42**, 1261 (1951).
[16] R. W. Williams, Phys. Rev. 98, 1393 (1958).