Gamma-Ray Angular Correlation Tests for Time-Reversal Invariance in Nuclear Forces*

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A systematic method for constructing tests of time-reversal invariance in nuclear forces by means of gamma-ray angular correlations is described. The transition supplying the test consists of a mixed multipole; it must be preceded by a polarizer to produce a nuclear orientation and (in all but one case) followed by a second photon which serves as an analyzer of the final nuclear orientation. The exception is a test in which one detects the linear polarization and direction of a quantum from a nucleus with initial orientation of third degree.

A general formula for the direction of a single photon with arbitrary polarization and involving an arbitrary orientation of both the initial and final nuclear states is presented. Expressions of this type have the advantage that individual ones can be combined together in a simple way to form an arbitrary correlation formula. This is carried through numerically in one case for beta-gamma-gamma direction correlation, where the first photon is mixed $E2$ and $M1$. It is shown that such measurements must be carried out to 1 or 2% in order to better the present limit of our knowledge concerning time-reversal invariance.

I. INTRODUCTION

IN the preceding paper[1] we have shown that our present knowledge of the validity of time-reversal invariance in nuclear forces limits the fraction of time-reversal-odd interaction to $\sim 10\%$, although the corresponding number for the parity-odd force is $\sim 3\times 10^{-4}$. The desirability of lowering the former limit justifies the detailed discussion given here of one method likely to accomplish this end for phenomena in the 1–10 Mev range—e.g., angular correlations involving gamma rays.

One consequence of time-reversal invariance that was first pointed out by Lloyd[2] is the reality of certain matrix elements which occur in the theory of angular correlations of successive gamma rays. Should time-reversal invariance not hold, then there could be a nonvanishing sine of a phase η for each matrix element. It is probably unnecessary to remind the reader that these phases need not be connected with a breakdown of time-reversal invariance in the basic electromagnetic interaction, but could arise from a small imaginary part (in a certain standard representation) of the nuclear wave function. The difference between two such phases, $\Delta\eta$, in principle can be detected from the interference terms arising in a mixed transition. Certain experiments measure $\cos\Delta\eta$ and therefore detect only terms of second order in $\Delta\eta$; these have been discussed in the preceding paper. This paper concerns itself with those angular correlations which would detect the first-order $\sin\Delta\eta$ terms,[3] which are directly proportional to the fraction $\mathfrak{F}$ of time-reversal-odd nuclear interaction.

Since we only discuss tests of time-reversal invariance which make use of electromagnetic transitions between bound states of nuclei, we avoid the troublesome question of final-state interactions.[4] For gamma rays of ~ 1 Mev, the phase shift for nonresonant elastic scattering is expected to be $\sim 5\times 10^{-4}$ radians or less; this then marks the lower limit of $\mathfrak{F}$ which can be reached by the methods given in the succeeding sections.

We endeavor to present a systematic approach to the problem of constructing experiments to measure $\sin\Delta\eta$. Rather than attempting to list a series of such experiments, we break the angular correlation problem into steps; the first and last steps described in Sec. II play the role of polarizer and analyzer of the nuclear orientation which is needed to measure $\sin\Delta\eta$. The middle step is the time-reversal invariance test itself and is treated separately in Sec. III.

In Sec. IV, we present formulas for calculating the coefficients of the various terms in the angular correlation. These formulas are sufficiently general to handle all polarizations of the radiation and all nuclear orientations. They represent some improvement over angular correlation treatments which we have found in the published literature[5] inasmuch as with their aid each radiation can be treated separately, and as a final step successive ones can be put together. The results given in Sec. IV may also prove helpful in the interpretation of angular correlation experiments which have nothing to do with testing time-reversal invariance.

In Sec. V we discuss several concrete experiments and present a few numbers, and in the appendix we sketch the proof of the formulas of Sec. IV.

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[1] E. M. Henley and B. A. Jacobsohn, Phys. Rev. 113, 225 (1959), preceding paper.

[2] S. P. Lloyd, Phys. Rev. 81, 161 (1951).

[3] The first suggestion of testing time-reversal invariance in strong interactions by means of one such measurement appears in T. D. Lee and C. N. Yang, Brookhaven National Laboratory Report BNL-443, 1957 (unpublished).

[4] A misleading statement concerning alpha-gamma correlations appears in B. A. Jacobsohn and E. M. Henley, Bull. Am. Phys. Soc. Ser. II, 2, 393 (1957). This question is treated in a forthcoming paper by B. A. Jacobsohn and L. W. Miller.

[5] However, there is some overlap between this work and that

TABLE I. Various methods of detecting the degree of orientation (Ω_b) of a nucleus of spin j_b with the aid of a direction, **k**, and polarization measurement of a photon emitted by b; this is referred to in the text as the analyzer.

Polarization measurement	Quantity measured	Degree of orientation detected
None	$(\mathbf{k}\cdot\mathbf{j}_b)^{2n}$	$0, 2, 4, \cdots$
Circular	$(\mathbf{k}\cdot\mathbf{s})(\mathbf{k}\cdot\mathbf{j}_b)^{2n+1}=(\mathbf{j}_b\cdot\mathbf{s})(\mathbf{k}\cdot\mathbf{j}_b)^{2n}$	$1, 3, 5, \cdots$
Linear	$(\boldsymbol{\varepsilon}\cdot\mathbf{j}_b)^2(\mathbf{k}\cdot\mathbf{j}_b)^{2n}$	$2, 4, 6, \cdots$

TABLE II. Methods of producing an initial nuclear orientation.

Method	Degree of orientation (Ω_a)
Strong magnetic field	$1, 2, 3, \cdots$
Crystal field	2
Allowed β decay (Gamow-Teller)	1
First-forbidden β decay	1, 2, 3
Magnetic field followed by long-lived unoriented radiation	$1, 2, 3, \cdots$
γ ray, oriented	$2, 4, \cdots$
γ ray, oriented and circularly polarized	$1, 3, \cdots$

II. POLARIZERS AND ANALYZERS

In order to test time-reversal invariance, use shall be made of a transition in which a nuclear state of spin j_a, oriented to a degree Ω_a, emits a (mixed) radiation with momentum **k** and arbitrary polarization, leaving the nucleus in a state with spin j_b, oriented to a degree Ω_b. The "degree of orientation" Ω labels the representation of the rotation group according to which the statistical tensor transforms; in other words, Ω is the power of **j** which must be measured. Thus $\Omega=0$ refers to a totally unoriented state, $\Omega=1$ to polarization, $\Omega=2$ to alignment, etc.

We defer to the next section the consideration of experiments that can be used to test time-reversal invariance, and the role of nuclear orientations in these tests. Here we discuss the detection (analyzer) and production (polarizer) of the orientations, Ω_b and Ω_a, in the final and initial states, respectively. A gamma ray emitted by the state b is used as the analyzer, since this function must be performed in a time that is short relative to the relaxation time of the state. Table I presents the various ways in which a photon can be used as an analyzer, depending on the type of polarization one wishes to measure. In the second column under "Quantity measured" are listed of course only quantities invariant under time reversal, since any breakdown of time-reversal invariance is small and therefore should not be used in the analyzer or polarizer; the appearance of only even powers of **k** ensures parity conservation. The third column simply counts the power of $\mathbf{j}_b$ appearing in the second. It is clear that one can detect even orientations without a polarization measurement, that odd orientations require a circular polarization measurement, and there is nothing to be gained in detecting the direction of linear polarization in the analyzer.

Table II lists some ways of producing the initial orientation Ω_a, together with the degree of orientation achievable by each method. While most entries in this table are self-evident, a few points may be singled out for special comment. First, although from angular momentum considerations alone one might expect an allowed β decay to be able to produce $\Omega_a=2$, this is missing because the transition probability is at most linear in the electron momentum, $\mathbf{p}_e$, and one finds no power of $\mathbf{j}_a$ higher than that contained in $\mathbf{p}_e\cdot\mathbf{j}_a$; an analogous reason rules out $\Omega=4$ for first-forbidden transitions.[6] Second, although a magnetic field can produce arbitrary orientations, the lifetime of the state a will in most instances be too short to allow the nuclei to come to thermal equilibrium in the field; if this is the case, then the test radiation can be preceded by a long-lived radiation (such as an alpha particle, a beta particle, or a strongly forbidden gamma quantum) the direction of which is not measured and which plays the role only of a delaying agent. Then whatever degrees of orientation are present in the long-lived state can also be present in the state a, although in general with different coefficients. In fact, if $R(\Omega)$ is the coefficient of the term in the density matrix of the initial state (j) which corresponds to the orientation Ω, an unoriented 2^L-pole radiation will reduce this coefficient to the following for the state a:

$$R_a(\Omega)=R(\Omega)\times\begin{Bmatrix}\Omega & j & j\\ L & j_a & j_a\end{Bmatrix}\times(-1)^{j+j_a+L+\Omega}[(2j+1)(2j_a+1)]^{\frac{1}{2}},\quad (1)$$

where the symbol in brackets is a Wigner 6-j coefficient,[7] equal except for sign to a Racah coefficient.

III. TEST OF TIME-REVERSAL INVARIANCE

We turn our attention now to the transition involving the mixed radiation which is to serve as the test of time-reversal invariance. The possibilities are summarized in Table III, the second column of which lists the lowest-order quantities (even with respect to parity and odd with respect to time reversal) which one might attempt to measure. The failure of any one of these quantities to vanish would unambiguously demonstrate a violation of time-reversal invariance. To understand the entries in this column, one must remember that the operation of time reversal consists

of F. Coester, Argonne National Laboratory Report ANL-5316, Chicago, 1954 (unpublished).

[6] We are grateful to Dr. R. R. Lewis for pointing this out to us.

[7] Many of the properties of the 3-j (essentially Clebsch-Gordan), 6-j (Racah), and 9-j coefficients are summarized in A. de-Shalit, Phys. Rev. **91**, 1479 (1953); in M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley and Sons, Inc., New York, 1957); and in A. R. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton University Press, Princeton, 1957). We refer to these in the appendix as (dS), (R) and (E), respectively. The D matrices we use in Eq. (6) are those of (R), and are the inverse of those given by (E).

TABLE III. Tests of time-reversal invariance using a mixed γ-ray transition.

γ-ray polarization	Quantity measured	Degree of orientation Ω_a	Ω_b
None	$(\mathbf{k}\cdot\mathbf{j}_b)(\mathbf{k}\cdot\mathbf{j}_b\times\mathbf{j}_a)$	1	2
	$(\mathbf{k}\cdot\mathbf{j}_a)(\mathbf{k}\cdot\mathbf{j}_a\times\mathbf{j}_b)$	2	1
Circular	$(\mathbf{k}\cdot\boldsymbol{\sigma})(\mathbf{k}\cdot\mathbf{j}_a\times\mathbf{j}_b)$	1	1
	$(\mathbf{k}\cdot\boldsymbol{\sigma})(\mathbf{k}\cdot\mathbf{j}_a\times\mathbf{j}_b)(\mathbf{j}_a\cdot\mathbf{j}_b)$	2	2
Linear	$(\mathbf{k}\cdot\mathbf{j}\times\boldsymbol{\varepsilon})(\mathbf{k}\cdot\mathbf{j})(\boldsymbol{\varepsilon}\cdot\mathbf{j})$,	3	0
	with a and b in various	2	1
	arrangements	1	2
		0	3

of changing the sign of all momenta and angular momenta. Since the direction of linear polarization possesses no positive or negative sense, the unit vector $\boldsymbol{\varepsilon}$ representing the direction of linear polarization must occur an even number of times in any expression having physical meaning, and is unaffected by time reversal. In addition, time reversal interchanges incoming and outgoing waves but, as explained in the Introduction, this makes a negligible difference in the case of gamma emission.

The last two columns give the degree of orientation needed for the initial (a) and final (b) states. We see from the table that one and only one kind of measurement can be used if the mixed radiation is not to be followed by one of the analyzing gamma-rays listed in Table I. This is the case in which one measures the direction of emission as well as the linear polarization of the gamma; the initial spin j_a must be oriented to the third degree.[8] (This requires $j_a \geqslant \frac{3}{2}$.) Table II shows that this initial orientation can be supplied by a first-forbidden β decay or a strong magnetic field at sufficiently low temperatures; it is conceivable that the advantage of eliminating one radiation direction measurement after the test of time-reversal invariance might compensate for the difficulty of producing such a high-order initial orientation and of simultaneously detecting the linear polarization.

Other experiments can be devised by combining the information of Table III with various methods of analyzing and polarizing listed in Tables I and II. For example, as suggested by Lee and Yang,[4] a triple angular correlation experiment can be performed by using a beta decay followed by two gamma rays, the *first* of which is mixed. We give as an example the explicit method of constructing this case from our tables. The third line of Table II corresponds to producing the initial orientation by measuring $(\mathbf{p}\cdot\mathbf{j}_a)$; the first line of Table III ($\mathbf{k}_1$ gives the direction of this gamma-ray) is the time-reversal invariance test used; and finally, the detection of Ω_b is effected via the measurement of $(\mathbf{k}_2\cdot\mathbf{j}_b)^2$ (first line of Table I). Combining the three terms leads to the expression for the over-all quantity to be measured: $(\mathbf{k}_1\cdot\mathbf{k}_2)(\mathbf{k}_1\cdot\mathbf{k}_2\times\mathbf{p})$. One could, of course, have written down this expression without breaking the discussion into the separate steps; it would then have been less clear that it is necessary for $\mathbf{k}_1$ rather than $\mathbf{k}_2$ to be the mixed radiation.

By means of the three tables one is able to construct an exhaustive list of all possible gamma-ray angular correlation tests to first order in η. The only way to extend this list is to add to the methods of polarizing and analyzing. For example, one can in principle analyze odd orientations with a beta decay; we have not listed such possibilities because of relaxation-time limitations.

IV. PHOTON DISTRIBUTION FUNCTION

In order to be able to interpret the results of an experiment, one needs quantitative expressions to replace the qualitative ones of Tables I, II, and III. Although formulas for angular correlations from oriented nuclei exist in the literature,[9] to the best of our knowledge they do not appear in a form which possesses sufficient flexibility to be immediately useful for our purposes. What is needed is a formula for the angular distribution and polarization of a single transition which can be combined with arbitrary preceding and succeeding transitions so as to allow one to construct various correlation formulas at will. The final formula is thus assembled in a manner completely analogous to the construction in the discussion at the end of the preceding section.

This is achieved in the following way for an electromagnetic transition. If $(j_b m_b \mathbf{k} P|R|j_a m_a)$ denotes the amplitude for going from a nuclear state a with $j_z = m_a$ to a state in which the nucleus has quantum numbers j_b, m_b and in which there is one quantum with momentum $\mathbf{k}$ and circular polarization $P(=\pm 1)$, we define a "probability" for emission from the nuclear state initially oriented in the manner Ω_a, ω_a to that state oriented by Ω_b, ω_b by multiplication with Clebsch-Gordan coefficients as shown:

$$\begin{aligned} W(j_b\Omega_b\omega_b;\, P'\mathbf{k}P;\, j_a\Omega_a\omega_a) &\equiv \sum_{m_b'}\sum_{m_b}\sum_{m_a'}\sum_{m_a} (-1)^{j_b+m_b}(j_b j_b - m_b' m_b|\Omega_b\omega_b) \\ &\times\langle j_b m_b'\mathbf{k}P'|R|j_a m_a'\rangle^*\langle j_b m_b \mathbf{k}P|R|j_a m_a\rangle \\ &\times(-1)^{j_a+m_a}(j_a j_a - m_a' m_a|\Omega_a\omega_a). \quad (2) \end{aligned}$$

The closure property for the Clebsch-Gordan coefficients allows one to obtain correlations between successive transitions, if the intermediate states are undisturbed by external influences, simply by multiplying the relevant W's together and summing over all intermediate orientations. For example, one obtains

$$\begin{aligned} W_{\mathrm{corr}}(j_c\Omega_c\omega_c;\, P_2\mathbf{k}_2 j_b \mathbf{k}_1;\, j_a\Omega_a\omega_a) &= \sum_{\Omega_b}\sum_{\omega_b}\sum_{P_1=\pm1} W(j_c\Omega_c\omega_c;\, P_2\mathbf{k}_2P_2;\, j_b\Omega_b\omega_b) \\ &\times W(j_b\Omega_b\omega_b;\, P_1\mathbf{k}_1P_1;\, j_a\Omega_a\omega_a), \quad (3) \end{aligned}$$

[8] See also P. Stichel, Z. Physik **150**, 264 (1958).

[9] H. A. Tolhoek and J. A. M. Cox, Physica **19**, 101 (1953); F. Mandl, Proc. Phys. Soc. (London) **71**, 177 (1958).

for the angular correlations between an unpolarized gamma ray emitted in the direction $\mathbf{k}_1$ and a polarized gamma ray with momentum $\mathbf{k}_2$, the initial and final nuclear orientations being as indicated. If c is the final state and is therefore unoriented, then $\Omega_c=0$; otherwise c can be combined with the next transition. The initial transition of a series is to be multiplied by the density matrix for the initial nuclear state in the following manner.

As shown by Fano,[10] the density matrix for a state with angular momentum j can always be decomposed into a sum of statistical tensors

$$(jm'|\rho|jm)=\sum_{\Omega}\sum_{\omega}(-1)^{j+m'}(2\Omega+1)^{\frac{1}{2}} \times\begin{pmatrix} j & j & \Omega \\ -m' & m & -\omega\end{pmatrix}\rho_j(\Omega,\omega). \quad (4)$$

Then a direct substitution shows that the first transition of the series must be written

$$\sum_{\Omega}\sum_{\omega}W(\cdots;j\Omega\omega)\rho_j(\Omega,\omega). \quad (5)$$

The derivation of the formula for W is given in the Appendix. The result is

$$\begin{aligned}
&W(j_b\Omega_b\omega_b;P'\mathbf{k}P;j_a\Omega_a\omega_a)\\
&=N_{ba}^2\sum_{\lambda}\sum_{\nu}\sum_{\mu}(2\lambda+1)(\nu|D^{\lambda}(\phi,\theta,\alpha)|\mu)\\
&\times[(2\Omega_b+1)(2\Omega_a+1)]^{\frac{1}{2}}\begin{pmatrix}\Omega_b & \Omega_a & \lambda\\ -\omega_b & \omega_a & \nu\end{pmatrix}\\
&\times\sum_{L}\sum_{L'}[(2L'+1)(2L+1)]^{\frac{1}{2}}(a\|L'\|b)(a\|L\|b)^*\\
&\times(-P')^{L'+p'}(-P)^{L+p}\begin{pmatrix}L' & L & \lambda\\ P' & -P & -\mu\end{pmatrix}\\
&\times(-1)^{L'+\lambda+\Omega_a+\omega_a+1}\begin{Bmatrix} j_b & j_a & L'\\ j_b & j_a & L\\ \Omega_b & \Omega_a & \lambda\end{Bmatrix}. \quad (6)
\end{aligned}$$

In Eq. (6), N_{ba}^2 is a positive normalization constant depending only on the internal states a and b and not on their orientations. The rotation group matrices D^{λ}, and the Wigner 3-j and 9-j coefficients are defined as in reference 7. P is $+1$ or -1 according to whether $\boldsymbol{\sigma}\cdot\mathbf{k}$ for the quantum is positive (now called right circular polarization) or negative, respectively. The sign of the reduced matrix element for a transition from a nuclear state a to b, $[(a\|L\|b)]$, has been chosen to agree with the final formulas in Biedenharn and Rose.[11] The number p is zero if the radiation emitted has even parity, one if odd. We have written Eq. (6) so that it can be used to test parity conservation as well as time-reversal invariance; with parity conserved, as we assume it to be throughout this paper, $p'=p$.

[10] U. Fano, Phys. Rev. **90**, 577 (1953).

[11] L. C. Biedenharn and M. E. Rose, Revs. Modern Phys. **25**, 729 (1953).

The angles ϕ, θ, α appearing as arguments in D^{λ} are the Euler angles of a coordinate system the z axis of which points along the direction of the photon. It is then completely general to give the probability for light linearly polarized in the x direction:

$$W(\epsilon_x)\sim\sum_{P'=\pm1}\sum_{P=\pm1}W(\cdots P'P\cdots). \quad (7)$$

The selection rules for each term can be read off directly from the 9-j coefficient appearing in (6), for each row and each column must satisfy the triangle inequality. Furthermore, the symmetry properties of the Wigner coefficients can be used to separate the terms in $\cos[\eta(L)-\eta(L')]$, and $\sin[\eta(L)-\eta(L')]$, where $(a\|L\|b)\equiv|(a\|L\|b)|\exp[i\eta(L)]$. For example, in the cases of unpolarized or linearly polarized radiation, the interchange of L and L' in any single term of Eq. (6) multiplies the coefficients of the matrix elements by $(-1)^{\Omega_a+\Omega_b}$. Thus, if $\Omega_a+\Omega_b$ is odd, the only terms surviving in the sum are those involving $\sin[\eta(L)-\eta(L')]$; if even, those involving $\cos[\eta(L)-\eta(L')]$ remain. It is readily seen that these results agree with those found earlier in a simpler way and listed in Tables I and III.

Equation (1) is found by integrating (6) over all directions and summing over polarizations. The normalization constant is determined by observing that a (hypothetical) $L=0$ photon would not change the nuclear orientation.

V. SPECIFIC EXPERIMENTS

By means of Eqs. (3), (4), (5), (6) and existing tables,[12] in addition to a considerable amount of diligence and care, the reader can construct formulas for any gamma angular correlation he might wish.

As an example, and as an aid in estimating the precision required in a correlation test of time-reversal invariance, we have carried out calculations for the angular correlation of two successive gamma rays emitted in a $2(E2+M1)2(E2)0$ cascade from an initial polarized nucleus. The reasons for this particular choice are given in the preceding paper. We find

$$\begin{aligned}W=(1+|\delta|^2)+[0.250+0.732|\delta|\cos\eta-0.0765|\delta|^2]\\ \times P_2(\hat{k}_1\cdot\hat{k}_2)+0.327|\delta|^2P_4(\hat{k}_1\cdot\hat{k}_2)-0.368p|\delta|\sin\eta\\ \times[(\hat{k}_1\cdot\hat{k}_2)(\hat{k}_1\times\hat{k}_2)_z], \quad (8)\end{aligned}$$

in which $\hat{k}_1$ and $\hat{k}_2$ are unit vectors and the mixing parameters δ and η for the first photon are related to the reduced matrix elements of Eq. (6) via

$$(\|E2\|)/(\|M1\|)\equiv\delta\equiv|\delta|e^{i\eta}. \quad (8')$$

The quantity p has a maximum absolute value of one and appears in the density matrix of the initial state

[12] Albert Simon, Oak Ridge National Laboratory Report, ORNL-1718, 1954 (unpublished). Simon, Van der Sluis, and Biedenharn, Oak Ridge National Laboratory Report, ORNL-1679, 1954 (unpublished). Kenneth Smith and John W. Stevenson, Argonne National Laboratory Report, ANL-5776, 1957 (unpublished).

$(m'|\rho|m)=\delta_{m',m}(0.2)[1+(0.5)pm]$. If the polarization of this initial state with $j=2$ is produced by an allowed Gamow-Teller e^- decay in the z direction, the quantity p is $+v_{e^-}/c$ if the beta transition is $1\rightarrow 2$, whereas p is $-2v_{e^-}/3c$ if it is $3\rightarrow 2$.

If one assumes the most favorable case with $|\delta|=1$ and $\eta=\pi-\eta'$, one finds for both photons in the x-y plane

$$\frac{W(\phi_2-\phi_1=+45°)}{W(\phi_2-\phi_1=-45°)}\cong 1-0.213p\eta'. \qquad (9)$$

Since from the existing angular correlation data[1] one knows $\eta'=0\pm 0.3$, an experiment of the type discussed here in which $p=1$ and in which the numerator and denominator of (9) are each measured to 4% accuracy will equal the above precision. However, from the nuclear reaction data discussed in the preceding paper, one might estimate the error in η' to be $\frac{1}{3}$ to $\frac{1}{2}$ of the above.

VI. CONCLUSIONS

We have analyzed gamma-ray correlations in detail and have shown that experiments to detect time-reversal-odd terms in the nuclear Hamiltonian can be performed in several ways, if one starts with an arbitrarily aligned initial nuclear state. In order to demonstrate this we first derived a relation for the emission of an arbitrarily polarized gamma ray from an oriented nucleus, and then showed how it can be used together with connecting links, when several transitions are involved, as in tests of time-reversal invariance. The most useful of these are those which detect either (a) the linear polarization-angular correlation function in a mixed transition from a third-order oriented nucleus, and (b) the angular correlation of two successive gamma transitions (the first of which is mixed) from a polarized initial state. As an example of the latter, we have investigated the $2(E2+M1)2(E2)0$ cascade and have shown that an accurate determination of time-reversal invariance is feasible, but difficult.

APPENDIX. DERIVATION OF GENERALIZED INTENSITY FORMULA

The derivation of Eq. (6) from the definition (2) follows standard lines, but is considerably eased by first deriving a property of the 9-j symbol. From either chapter six of Edmonds[7] or Eqs. [dS: (12), (13), and (14)],[7] one can find the following equality:

$$\begin{pmatrix} j_{11} & j_{12} & j_{13} \\ \lambda_{11} & \lambda_{12} & \lambda_{13}\end{pmatrix}\begin{pmatrix} j_{21} & j_{22} & j_{23} \\ \lambda_{21} & \lambda_{22} & \lambda_{23}\end{pmatrix}\begin{pmatrix} j_{11} & j_{21} & j_{31} \\ \lambda_{11} & \lambda_{21} & \mu_{31}\end{pmatrix}$$
$$\times\begin{pmatrix} j_{12} & j_{22} & j_{32} \\ \lambda_{12} & \lambda_{22} & \mu_{32}\end{pmatrix}\begin{pmatrix} j_{13} & j_{23} & j_{33} \\ \lambda_{13} & \lambda_{23} & \mu_{33}\end{pmatrix}$$
$$=\begin{Bmatrix} j_{11} & j_{12} & j_{13} \\ j_{21} & j_{22} & j_{23} \\ j_{31} & j_{32} & j_{33}\end{Bmatrix}\begin{pmatrix} j_{31} & j_{32} & j_{33} \\ \mu_{31} & \mu_{32} & \mu_{33}\end{pmatrix}. \qquad \text{(A-1)}$$

All repeated Greek letters (i.e., the λ's) are summed over. (A-1) follows by substituting repeatedly into de-Shalit's Eqs. (12) and (13), and finally using his Eq. (14) once. The latter equation can also be used as a quick check on the above by multiplying both sides of (A-1) with the 3-j coefficient

$$\begin{pmatrix} j_{31} & j_{32} & j_{33} \\ \mu_{31} & \mu_{32} & \mu_{33}\end{pmatrix},$$

and summing over the μ's.

The amplitude for the emission of a circularly polarized quantum is

$$(j_bm_b\mathbf{k}P|R|j_am_a)=N\sum_L\sum_M\sum_p (M|D^L|P)^*$$
$$\times(2L+1)^{\frac{1}{2}}(-1)^{j_a+m_a}(-P)^{L+p}$$
$$\times\begin{pmatrix} j_a & L & j_b \\ m_a & -M & -m_b\end{pmatrix}(a\|L,p\|b)^*, \qquad \text{(A-2)}$$

in which $p=0$ or 1 refer to radiation which carries off even or odd parity, respectively. The nuclear matrix element of the particle operator Q, defined by

$$H_{\text{int}}\equiv\sum_L\sum_M\sum_p a(LMp)Q_L{}^M(p)+\text{H.c.}, \qquad \text{(A-3)}$$

with $a(LMp)$ the usual photon annihilation operator, is related to the symbol in (A-2) by

$$(j_am_a|Q_L{}^M(p)|j_bm_b)=(-1)^{j_a+m_a}(2j_b+1)^{\frac{1}{2}}$$
$$\times\begin{pmatrix} j_a & L & j_b \\ m_a & -M & m_b\end{pmatrix}(a\|L,p\|b). \qquad \text{(A-4)}$$

If we insert this expression for the amplitude into the definition of W [our Eq. (2)], then use (E:4.2.7) or (R:4.22) to rid ourselves of the complex conjugation of one D, and finally (E:4.3.2) or (R:4.25) to combine the two D's, we find

$$W=|N|^2(-1)^{j_b-j_a+\omega_b+1}[(2\Omega_a+1)(2\Omega_b+1)]^{\frac{1}{2}}$$
$$\times\sum_\lambda\sum_\nu\sum_\mu\sum_{L'}\sum_L\sum_{p'}\sum_p(2\lambda+1)$$
$$\times(-1)^\nu(a\|L'p'\|b)(a\|L,p\|b)^*(-P')^{L'+p'}(-P)^{L+p}$$
$$\times\begin{pmatrix} L' & L & \lambda \\ P' & -P & -\mu\end{pmatrix}\begin{pmatrix} j_a & L' & j_b \\ m_a' & -M' & -m_b'\end{pmatrix}$$
$$\times\begin{pmatrix} j_a & L & j_b \\ m_a & -M & -m_b\end{pmatrix}\begin{pmatrix} j_a & j_a & \Omega_a \\ -m_a' & m_a & -\omega_a\end{pmatrix}$$
$$\times\begin{pmatrix} L' & L & \lambda \\ M' & -M & -\nu\end{pmatrix}\begin{pmatrix} j_b & j_b & \Omega_b \\ -m_b' & m_b & -\omega_b\end{pmatrix}. \qquad \text{(A-5)}$$

After we make use of the symmetry properties of the 3-j coefficients with respect to the sign of the m's, we can compare the last five 3-j coefficients with (A-1) and immediately see that they give our Eq. (6), which is what was to be proven.