Time Reversal in Nuclear Interactions*

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The limitations imposed by time-reversal invariance of nuclear forces have been examined for nuclear reactions, elastic double scattering (polarization) experiments, and angular correlations of gamma rays emitted from unoriented nuclei. For each of these, we have found that certain experiments, which may superficially appear to be sensitive tests of time-reversal invariance, are actually completely or partially insensitive to this symmetry. For example, in certain cases, the unitarity of the S matrix is sufficient to assure detailed balance. Such insensitivity to time-reversal invariance operates in some of the experimentally best-investigated problems. Those experiments which may be expected to be sensitive tests yield an upper limit of about 10% for that fraction of the Hamiltonian which is odd with respect to a time inversion. We have suggested experiments which may lower this limit.

I. INTRODUCTION

THE classic work of Lee and Yang[1] in 1956 has focused attention on various symmetries of physical laws, namely conservation of parity (P), invariance under charge conjugation (C), invariance under time reversal (T), and various combinations thereof. The experiments of Wu, Ambler, *et al.*[2] and of Garwin, Lederman, and Weinrich[3] were the first of many to demonstrate conclusively that invariance under P and C do not hold in weak interactions. However, it has not yet been shown conclusively that such interactions are invariant under time reversal.[4]

Another consequence of Lee and Yang's work has been a re-examination of the foundations of our beliefs that strong interactions are invariant with respect to the symmetries enumerated above. If the breakdown of invariance under P, C, or T in strong interactions were due solely to the weak forces which are present, then the admixture, F, of forces odd under these symmetries should not exceed 10^{-12}. It is legitimate, however, to ask whether there are forces of nuclear or electromagnetic strength which do not satisfy invariance under P, C, or T. Lee and Yang[1] summarized some of the then existing evidence for P conservation. Since that time, the forbiddenness of certain reactions[5,6] and the nonappearance of circular polarization[6] in nuclear gamma-ray transitions have been used by Tanner and Wilkinson to set an upper limit of about 3×10^{-4} for the admixture of nuclear forces that are odd under parity transformations, a result which enables one to blame the absence of the neutron's electric dipole moment on nonconservation of parity alone, although it is by this time well known[7,8] that time-reversal invariance also forbids the appearance of electric dipole moments, magnetic quadrupole moments, etc.

Charge conjugation invariance, except for the implications of the PCT theorem, plays no direct role in the low- and medium-energy nuclear physics phenomena discussed here, and will not be considered in this paper.

Herein, we examine in some detail the basis of our knowledge of time-reversal invariance in nuclear interactions. Our conclusion is that it has not been established to a very high accuracy, in fact, to no better than about one part in ten.[9]† On the other hand, the very precise experimental checks of the predictions of quantum electrodynamics certainly show that our present ideas on the nature of the "bare electromagnetic interaction" (which include of course the assumption of time-reversal invariance) are valid. It is possible that the structure of the system, whether nucleon or nucleus, interacting with the electromagnetic field may introduce some lack of time-reversal invariance in the effective interaction. Such effects of the nuclear wave functions on the emission of gamma rays and their possible detection by means of angular correlations are the subject of a separate paper.[10] However, it should be clear that all atomic and molecular phenomena which involve

* Supported in part by the U. S. Atomic Energy Commission.

[1] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).

[2] Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. **105**, 1413 (1957).

[3] Garwin, Lederman, and Weinrich, Phys. Rev. **105**, 1415 (1957).

[4] Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. **110**, 787 (1958).

[5] N. Tanner, Phys. Rev. **107**, 1203 (1957); D. H. Wilkinson, Phys. Rev. **109**, 1603 (1958).

[6] D. H. Wilkinson, Phys. Rev. **109**, 1610 (1958).

[7] L. Landau, Zhur. Eksptl. i Teoret. Fiz. **32**, 405 (1957) [translation: Soviet Phys. JETP **5**, 336 (1957)], and Nuclear Phys. **3**, 127 (1957).

[8] T. D. Lee and C. N. Yang, Brookhaven National Laboratory Report BNL-443 (T-91), 1957 (unpublished).

[9] E. M. Henley and B. A. Jacobsohn, Phys. Rev. **108**, 502 (1957).

† *Note added in proof.*—Since this article was submitted, reports of several experiments which reduce this limit to a few percent have been published or submitted for publication. Polarization-asymmetry comparisons in $p-p$ scattering at energies near 200 Mev are described by Hillman, Johansson, and Tibell [Phys. Rev. **110**, 1218 (1958)], and A. Abashian and E. M. Hafner [Phys. Rev. Letters **1**, 255 (1958)]. Detailed balance in nuclear reactions at lower energies is examined by Bodansky, Eccles, Farwell, Rickey, and Robison (submitted to Phys. Rev. Letters); and by L. Rosen (private communication) and J. N. Bradbury and L. Stewart [Bull. Am. Phys. Soc. Ser. II, **3**, 417 (1958)].

[10] B. A. Jacobsohn and E. M. Henley, Phys. Rev. **113**, 234 (1959), following paper.

only the exchange of virtual or real quanta with wavelengths very long compared to the size of the nucleus must reflect time-reversal invariance.

In a sense similar to that discussed above for the electromagnetic case, a demonstration of the reality of the weak-interaction coupling constants, involving as it does the structure of the nucleon in addition to the "bare" interaction, would shed some light on time-reversal invariance for those strong interactions involved in this structure.[11]

We believe that the independent tests of time-reversal invariance in nuclear forces discussed in this pair of papers are of interest for several reasons: (a) some of the experiments discussed here may yield more precision than the beta-decay measurements; (b) if time-reversal invariance held for the bare weak interactions but broke down for the strong, it is possible that the effective weak-coupling constants might be less sensitive to this breakdown than are the nuclear forces; and (c) if it should turn out that there were some violation of time-reversal invariance in the effective beta interaction, it would then become imperative to find how much of this could be blamed on the strong interactions.

In the light of our present ideas of the origin of nuclear forces, it is probably more sensible to look for breakdowns of space-time symmetry laws in the still mysterious central core region of the nucleon—in other words, at very high energies—rather than in the relatively well-explored fringe region made up of π mesons. Despite this obvious fact, we have preferred to restrict the orientation of the experimental parts of these papers to phenomena which do not explore this core mainly because these are the only ones where relevant experiments seem to have been done. (However, part of the multiple-scattering discussion of Sec. IV is applicable to high energy.) For these processes, the nuclear force can be described by a possibly velocity-dependent point interaction. Examples of terms in a phenomenological Hamiltonian which violate time-reversal invariance are easy to invent[12]; one must however use caution since some of the apparently more obvious ones can be transformed away by a gauge transformation.[13]

In the following sections we first briefly review the time-reversal operation. This is followed in Sec. III by a discussion of the consequences of time-reversal invariance for nuclear reactions. This section is subdivided into several parts: In part A we examine the limitations imposed on detailed balance tests of time-reversal invariance by various restrictions such as the unitarity of the S matrix. In part B we summarize our present experimental knowledge and in part C we propose several specific experiments to improve this knowledge.

A similar discussion is given in Sec. IV for polarization experiments and in Sec. V for electromagnetic and β-decay tests of time-reversal invariance.

II. BRIEF REVIEW OF THE TIME-REVERSAL OPERATION

The consequences of time-reversal invariance for various physical phenomena were first pointed out by Wigner[14] who also developed the formalism for time reversal in quantum mechanics. Coester[15] has shown that time-reversal invariance implies the symmetry of the S matrix. We give here a brief review of the time-reversal operation since we shall want to go into some detail concerning its implications and since it is necessary to make clear our notation and phase conventions. The first few statements below are contained in Coester's article; many of the rest can be obtained from it with a minor amount of manipulation.

In a given representation, there exists a unitary matrix U which relates a matrix Q to its time-reversed matrix Q^τ by the relation

$$U^\dagger Q U = (Q^\tau)_{\text{trans}}, \tag{1}$$

where Q_{trans} is the transpose of Q. A consequence is that $(AB)^\tau = B^\tau A^\tau$. From the definition of ψ^τ,

$$\langle \phi^\tau(t)|Q|\psi^\tau(t)\rangle = \langle \psi(-t)|Q^\tau|\phi(-t)\rangle, \tag{2}$$

one finds immediately that

$$\psi(t) = \textstyle\sum_r |r\rangle a_r(t) \tag{3}$$

gives

$$\psi^\tau(t) = \textstyle\sum_r |r\rangle_\tau a_r^*(-t), \tag{4}$$

where

$$|r\rangle_\tau \equiv |r^\tau\rangle = \textstyle\sum_s |s\rangle\langle s|U|r\rangle. \tag{5}$$

With Coester's choice of phase and in a representation in which J and M are diagonal, Eq. (5) reads

$$|\alpha, J, M\rangle_\tau = (-1)^{J+M}|\alpha^\tau, J, -M\rangle, \tag{6}$$

where α includes all other labels. Equations (5) and (1) together yield

$$\langle m|Q|n\rangle = \langle n^\tau|Q^\tau|m^\tau\rangle. \tag{7}$$

It is straightforward to show that, if A is any operator, then

$$[A\psi(t)]^\tau = (UU^*)(A^\dagger)^\tau(UU^*)_{\text{trans}}\psi^\tau(-t) = \epsilon(A^\dagger)^\tau\psi^\tau(-t), \tag{8}$$

where $\epsilon=1$ except if A connects states of integral with states of half-integral angular momentum, in which case it is -1. Application of Eq. (8) to a scattering

[11] In addition, there are those time-reversal invariance tests in beta decay which involve the products of the weak coupling constants with the Gamow-Teller and Fermi nuclear matrix elements. These involve the structure of the nucleus rather than the nucleon. See Sec. V.

[12] A specific example is $V(r)=f(r)\{\boldsymbol{\sigma}_a\cdot\mathbf{r}\ \boldsymbol{\sigma}_b\cdot\mathbf{p}+\boldsymbol{\sigma}_b\cdot\mathbf{r}\ \boldsymbol{\sigma}_a\cdot\mathbf{p}\}$ +H.c.

[13] G. Morpurgo and B. F. Touschek, Nuovo cimento 12, 677 (1954).

[14] E. P. Wigner, Nachr. Akad. Wiss. Göttingen, Math.-physik. Kl. IIa 31, 546 (1932).

[15] F. Coester, Phys. Rev. 89, 619 (1953).

state gives the well-known result that time reversal changes an outgoing into an incoming wave:

$$\psi_V{}^{(+)\tau}(E,\alpha,\mathbf{k},s_1,m_1,\cdots)$$
$$\equiv\left\{\left(1+\frac{1}{E+i\delta-K-V}V\right)\phi(E,\alpha,\mathbf{k},s_1,m_1,\cdots)\right\}^\tau$$
$$=(-1)^{s_1+m_1+\cdots}\psi_{V^\tau}{}^{(-)}(E,\alpha^\tau,-\mathbf{k},s_1,-m_1,\cdots)$$
$$=(-1)^{s_1+m_1+\cdots}\left\{1+\frac{1}{E-i\delta-K-V^\tau}V^\tau\right\}$$
$$\times\phi(E,\alpha^\tau,-\mathbf{k},s_1,-m_1,\cdots). \quad (9)$$

The label α refers to the internal wave function of bound states of the scattering particles, K is the kinetic energy operator, and V is the scattering interaction. For any bound system it follows immediately from time-reversal invariance that the Hamiltonian satisfies $H=H^\tau$, and that one can always choose the physically irrelevant phase so that (recalling that the external coordinates such as spin direction have been explicitly separated off)

$$|\alpha\rangle=|\alpha^\tau\rangle. \quad (10)$$

An equivalent statement of this result is that the coefficients of the bound-state wave function are real in any representation for which $|r\rangle=|r\rangle_\tau$.

The implications of the above for the scattering matrix[16,17] $R(E)$ on the energy shell are

$$\langle b|R(E)|a\rangle\equiv\left\langle b\left|V+W\frac{1}{E+i\delta-H}V\right|a\right\rangle$$
$$=\langle a^\tau|R^\tau(E)|b^\tau\rangle. \quad (11)$$

Here V is the scattering interaction for the state a, W is the same for the state b; they differ for exchange reactions. If H satisfied time-reversal invariance (H contains the bound-state interactions as well as the scattering interactions), and if a, b, refer to plane wave states, then

$$\langle\beta,\mathbf{k}',s_1',m_1',s_2',m_2'|R(E)|\alpha,\mathbf{k},s_1,m_1,s_2,m_2\rangle$$
$$=(-1)^{s_1+s_2+s_1'+s_2'+m_1+m_2+m_1'+m_2'}$$
$$\times\langle\alpha,-\mathbf{k},s_1,-m_1,s_2,-m_2|R(E)|\beta,$$
$$-\mathbf{k}',s_1',-m_1',s_2',-m_2'\rangle. \quad (12)$$

Possible confusion about the role of incoming and outgoing waves (i.e., "final state interaction") in the scattering arises only when the scattering interaction is split into two parts $W=W_1+W_2$, or $V=V_1+V_2$, and the initial or final wave functions are taken as eigenfunctions of $K+V_1$ or $K+W_1$.

[16] B. A. Lippmann and J. Schwinger, Phys. Rev. **79**, 469 (1950).
[17] M. Gell-Mann and M. L. Goldberger, Phys. Rev. **91**, 398 (1953).

III. NUCLEAR REACTIONS AND DETAILED BALANCE

A. Restrictions; "Two-State Theorem"

Although time-reversal invariance implies detailed balance, the inverse statement is not always true, and therefore the implications of experiments which test detailed balance should be deduced with some caution. A well-known case in which the latter holds irrespective of time-reversal invariance is that of the Born approximation[18]; for most real processes involving strong interactions, this fact is of academic interest. However, as we shall show in this section, there are certain types of reaction for which unitarity alone predicts the validity of detailed balance.

In its simplest form, the theorem is trivial. It states that if the S matrix (which is defined only on the energy shell) breaks up into 2×2 matrices, then $|\langle a|S|b\rangle|^2=|\langle b|S|a\rangle|^2$ for all states a and b. This follows immediately when one writes down the most general 2×2 unitary matrix with determinant equal to one,

$$\begin{pmatrix}\cos\theta e^{i\phi} & i\sin\theta e^{i\eta}\\ i\sin\theta e^{-i\eta} & \cos\theta e^{-i\phi}\end{pmatrix}. \quad (13)$$

If the states a, b depend only on internal coordinates (i.e., in a representation in which J is diagonal), then time-reversal invariance demands that $\langle a|S|b\rangle=\langle b|S|a\rangle$. Thus in the cases discussed here the only possible effect of time-reversal invariance violations is the appearance of complex phases. It is a separate question to investigate whether or not a particular experiment is sensitive to these phases. If the two-state theorem is applicable and if only forward and backward total cross sections are measured, it is clear that time-reversal invariance is not being tested.

The usefulness of this theorem is considerably enhanced by the following generalization. Suppose that the S matrix breaks into separate blocks, and consider the states making up one block. Suppose further that these can be classified into two types of states $a_1,a_2,\cdots$, $b_1,b_2,\cdots$ and that the matrix elements connecting the a group with the b group are of the form

$$\langle a_j|S|b_k\rangle=\Lambda_{jk}\langle a|M|b\rangle,$$
$$\langle b_k|S|a_j\rangle=\lambda_{kj}\langle b|M|a\rangle, \quad (14)$$

with $|\Lambda_{jk}|=|\lambda_{kj}|$. We make no further restriction on the S-matrix other than unitarity, from which it follows that

$$\textstyle\sum_{j'}|\langle a_j|S|a_{j'}\rangle|^2+\sum_k|\Lambda_{jk}|^2|\langle a|M|b\rangle|^2=1,$$
$$\textstyle\sum_{j'}|\langle a_{j'}|S|a_j\rangle|^2+\sum_k|\lambda_{kj}|^2|\langle b|M|a\rangle|^2=1. \quad (15)$$

Summing these equations over j and subtracting, we

[18] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), Chap. X.

find immediately that

$$|\langle a|M|b\rangle|^2=|\langle b|M|a\rangle|^2,$$

or

$$|\langle a_j|S|b_k\rangle|=|\langle b_k|S|a_j|. \tag{16}$$

We shall encounter in part B3 of this section a case in which the matrix elements take the form of Eq. (14).

In addition to the above theorems, there are models which, if applicable to a nuclear reaction, would imply lack of sensitivity of detailed balance to the presence or absence of time-reversal invariance. An example is the direct surface-interaction model[19] used to describe the rapid angular variation of some medium-energy (p,n) and stripping reactions. The rapid angular dependence is then given by the square of the Hermitian matrix

$$\langle\phi_f(\mathbf{r})|\ \exp(i\mathbf{Q}\cdot\mathbf{r})|\phi_i(\mathbf{r})\rangle_{|\mathbf{r}|=R},$$

where $\mathbf{Q}$ is the momentum transfer. In this model, the time-reversal-odd forces (those which are odd under time reversal) would modify a slow modulation factor which does not affect the forward angular distribution. The parameter R depends on the effective radii for the two channels i and f, and should not differ for the forward and backward reactions. Thus, we do not believe that forward angular distributions are sensitive tests of time-reversal invariance.

In general, then, the most useful tests of time-reversal invariance in nuclear interactions can be expected to be those which are difficult to explain by simple models such as the above, and in which there are many open competing channels.

B. Experimental Information

Of necessity, almost all experimental tests of detailed balance have been performed on light nuclei. The reason is that generally only for these targets are the nuclear levels widely enough separated that both backward and forward reactions can be performed to the ground states of the nuclei involved.

If the ratio of the forward and backward reactions is

$$p_i^2g_i\sigma_{i\to f}/p_f^2g_f\sigma_{f\to i}\equiv 1+\Delta, \tag{17}$$

where g_i and g_f are statistical weight factors and p_i and p_f are the relative momenta in the initial and final state, then Δ is of first order in H_{odd}, the time-reversal-odd interaction. In fact,

$$\Delta\approx 2[\langle f|R_e|i\rangle\langle f|R_o|i\rangle^*+\text{c.c.}]/|\langle f|R_e|i\rangle|^2, \tag{18}$$

where R_e is the time-reversal-even part of the scattering matrix. If the exact Hamiltonian is $H=J+W+J_o+W_o=K+V+K_o+V_o$, with $(W+W_o)$, the scattering interaction for the state f and $(V+V_o)$ the same for the state i (and all subscripts "o" refer to time-reversal-odd terms), then

$$\langle f|R_o|i\rangle\approx\left\langle f\left|W_o+W\frac{1}{E+i\delta-H}V_0\right|\psi_i^{(+)}\right\rangle$$

$$+\left\langle f\left|W\frac{1}{E+i\delta-H}K_o\frac{1}{E+i\delta-H}V\right|i\right\rangle \tag{18$'$}$$

to first order in H_o. We have assumed that the kinetic energy operators $J+J_o$ and $K+K_o$ include bound-state interactions. The connection supplied by Eq. (18) between a measured value of Δ and the fraction of time-reversal-odd force $\mathfrak{F}=H_{\text{odd}}/H_{\text{even}}$ is not immediate. Arguments given in the following sections will show that in certain cases $\Delta/\mathfrak{F}$ is small. Equation (18) interpreted in the most naive manner would predict $\Delta/\mathfrak{F}\approx 4$. For the sake of definiteness, in all cases where we are unable to make arguments to cut down the $\Delta/\mathfrak{F}$ ratio, we shall arbitrarily set it equal to 2 to give us, for purposes of comparison between experiments, an upper limit to $\mathfrak{F}$.

1. Photodisintegration of the deuteron

The capture reaction[20]

$$n+p\to d+\gamma \tag{19}$$

has been performed only at very low energies, where the magnetic dipole $({}^1S_0\to{}^3S_1)$ and the electric dipole $({}^3P_{2,1,0}\to{}^3S_1)$ are the sole transitions of importance. The matrix elements for electric and magnetic processes do not interfere in the angular distribution as long as we can consider the total spin to be a conserved quantity for the two-nucleon interaction. In the ${}^3P$ states, nuclear forces play little role since the angular momentum barrier keeps the nucleons outside their interaction range. The $E1$ matrix elements from the three ${}^3P$ states are thus related by simple geometrical factors and any phase they carry is common to all three; it will therefore have no effect on either forward or backward angular distributions. For the magnetic dipole transition the two-state theorem is applicable because the only states connected on the energy shell are the $(\gamma+d)$ and the ${}^1S_0$. (Invoking simply the Hermiticity of the weak electromagnetic interaction is not sufficient to establish the equality of the absolute values of the forward and backward matrix elements, since one involves an outgoing and the other an incoming ${}^1S_0$ wave; these are not simply related if time-reversal invariance is violated.)

These conclusions are hardly changed by the inclusion of a very small matrix element connecting the ${}^3D_1$ component of the deuteron to a ${}^1D_2$. Hence, the unitarity of the S matrix and the validity of the Born approximation for electric dipole capture tell us that

[19] Austern, Butler, and McManus, Phys. Rev. 92, 350 (1953).

[20] For references see J. M. Blatt and V. F. Weisskopf, reference 18, Chap. XII.

the process (19) at low energies together with its inverse cannot be used to test time-reversal invariance, whether total cross sections or angular distributions are measured.

2. Photodisintegration of He^4

The reaction

$$\gamma+He^4 \rightarrow p+H^3, \quad (20)$$

and its inverse can be sensitive to time-reversal invariance because of the many competing reactions which occur on the energy shell:

$$\begin{aligned} p+H^3 &\rightarrow p+H^3, \\ &\rightarrow n+He^3, \\ &\rightarrow d+d, \\ &\rightarrow \gamma+He^4, \\ &\text{etc.} \end{aligned}$$

Unfortunately, the experimental errors in the photodisintegration experiments[21] performed to date are large (22%). The forward[21] and backward[22] cross sections satisfy detailed balance to within this accuracy, so that an upper limit on $\mathfrak{F}$ of $\sim$11% is obtained. Angular distributions have been measured in both directions, but the errors are of the order of 30%; they are therefore not presently useful.

3. Pion Production and Capture by Deuterium

The reaction

$$p+p \rightarrow \pi^+ + d \quad (21)$$

has been extensively studied[23] in the region slightly above threshold; its inverse has also received experimental attention.[24] We shall show that here, as in case (1), there are influences which tend to make the connection between detailed balance and time-reversal invariance a weak one.

Since the argument depends on the detailed analysis of reaction (21) and its inverse, we must briefly state the results of the phenomenological theory.[25] Close to the production threshold only S- and P-wave pions and S-wave low-energy nucleons are involved and, if we neglect the relatively unimportant gamma-ray processes, we find that the S matrix splits into blocks of the interconnected states listed in Table I, where we have listed only those blocks which contain the state $(d+\pi^+)$. No further progress in the time-reversal analysis would be possible without the assumption explained by Gell-Mann and Watson[25] concerning the continuum matrix elements which involve a meson of momentum $\mathbf{p}$ and angular momentum l with accompanying nucleons of relative momentum $\mathbf{k}$:

TABLE I. Table of angular momentum states connected by the reaction $p+p \rightarrow \pi^+ + d$.

pp	d (or np)
$^3P_1$	$^3S_1+S$-wave π
$^1D_2$	$^3S_1+P$-wave π
$^1S_0$	$^3S_1+P$-wave π

$$\langle \pi^+ np|S|pp\rangle \approx \frac{\psi_{\mathbf{k}}^*(R)}{\psi_d^*(R)} \langle \pi^+ d|S|pp\rangle \approx \frac{\psi_{\mathbf{k}}^*(R)}{\psi_d^*(R)} \kappa p^l, \quad (22a)$$

$$\langle pp|S|\pi^+ np\rangle \approx \frac{\psi_{\mathbf{k}}(R')}{\psi_d(R')} \kappa' p^l, \quad (22b)$$

where ψ_d and $\psi_{\mathbf{k}}$ refer to the two-nucleon system. If time-reversal invariance is not assumed to hold, there is no immediate connection between κ and κ', and between the effective internucleon interaction distances, R and R'. However, as soon as we make the additional assumption that both R and R' are $\lesssim \hbar/\mu c$ (μ is the mass of the π meson), and that the relative energy of the two nucleons is sufficiently small so that it can be neglected inside the range of nuclear forces, then we can apply the extended two-state theorem, Eqs. (14) and (16). The result is that

$$\langle pp|S|\pi^+ d\rangle_J = \langle \pi^+ d|S|pp\rangle_J e^{i\eta_J}, \quad (22c)$$

for each of the groups of Table I.

Since the total effect of a breakdown of time-reversal invariance appears in the phases η_J, it is immediately clear that comparison of the total cross sections for the reaction (21) and its inverse is irrelevant, and it is satisfactory but quite unhelpful to note that[24] the Δ of Eq. (17) is 0.0 ± 0.1.

The phase combination $(\eta_0-\eta_2)$ of Eq. (22c) can in principle be determined by comparing the forward and backward angular distributions but, as we now show, the low-energy experiments give little information about it. In the notation of Rosenfeld,[25] $-(5)^{\frac{1}{2}} r_0 = S_{J=0}/S_{J=2}$ for the reaction $p+p \rightarrow \pi^+ + d$, and we call r_0' the corresponding quantity for the reverse reaction. If τ_0 and τ_0' are the arguments of these complex numbers, then as we have seen, we expect $|r_0| = |r_0'|$ in any case, but $\tau_0 = \tau_0'$ only if time-reversal invariance holds.

The situation is summarized in Fig. 1. The angular distribution experiments can only determine a circle in the complex plane on which r_0 must lie. We have drawn the limiting circles deduced from the measurements of Crawford and Stevenson.[23] Tripp[26] has observed the polarization of the resulting deuterons; his work

[21] E. G. Fuller, Phys. Rev. **96**, 1306 (1954).
[22] J. E. Perry and S. J. Bame, Jr., Phys. Rev. **99**, 1368 (1955).
[23] F. S. Crawford, Jr., and M. L. Stevenson, Phys. Rev. **97**, 1305 (1955), and others; the most recent collection of references will be found in Fields, Fox, Kane, Stallwood, and Sutton, Phys. Rev. **109**, 1704 (1958).
[24] Durbin, Loar, and Steinberger, Phys. Rev. **84**, 581 (1951); Sachs, Winick, and Wooten, Phys. Rev. **109**, 1733 (1958).
[25] M. Gell-Mann and K. M. Watson, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Stanford, 1954), Vol. 4, p. 219.
[26] R. D. Tripp, Phys. Rev. **102**, 862 (1956).

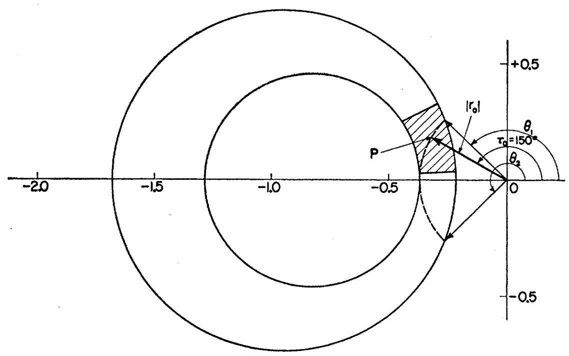

FIG. 1. Complex r_0 plane. The full circles bound the region located by the low-energy $p+p\rightarrow\pi^{+}+d$ experiments of Crawford and Stevenson. The shaded area shows the limits set by Tripp with measurements of deuteron polarization; the center P of this region corresponds to a phase $\tau_0\sim150°$. The experiments of Sachs, Winick, and Wooten, when corrected to the same energy region, yield almost the same two circles to bound r_0'. The dotted curve marks the locus of r_0' consistent with the latter's results and the unitarity requirement, explained in the text, that $|r_0|=|r_0'|$. The values of τ_0' range from $\theta_1=135°$ to $\theta_2=225°$.

narrows the allowed region of r_0 to the shaded area in the figure, with the best value of $\tau_0\approx150°$.

The recent angular distribution measurements[24] by Sachs, Winick, and Wooten on the reverse process are done at a slightly higher energy than the above. If one applies a small empirical correction to reduce these data to the threshold region, an allowed region for r_0' is found which is almost identical with the r_0 circles. We have preferred not to clutter the figure by drawing them. Lacking an experiment with pions on polarized deuterons, one has only the requirement that $|r_0|=|r_0'|$, and that r_0' lie within its allowed region. This only restricts τ_0' to be within the rather wide limits $135°<\tau_0'<225°$; the upper limit on $|\eta_0-\eta_2|$ is thus ~1 radian, which does not provide a close limit on the maximum allowable time-reversal-odd interaction.

We see that everything has conspired in these experiments to reduce the sensitivity of detailed balance to time-reversal invariance. Had the shaded region of Fig. 1 turned out to lie at the top of the circle instead of where it did, one would have found that τ_0 and τ_0' differed by $\sim10°$. It is clear that a small value of r_0 implies lack of sensitivity to time-reversal invariance; indeed $r_0=0$ means pure $J=2$ interaction and no interference at all of the matrix elements connecting the states listed in Table I.

In order for the reaction (21) and its inverse to provide sensitive time-reversal invariance tests, it appears that pions should be allowed to interact with polarized deuterons, or that the reactions should be examined far enough above threshold so that many channels are open and the restrictions imposed by the two-state theorem disappear.

4. *Stripping and Pickup Reactions*

Whereas many stripping reactions have been performed, few pickup reactions appear in the literature because of the large energy released in the latter processes. Detailed balance is applicable to two pairs of published experiments[27,28]:

$$p+\mathrm{Li}^7\rightleftarrows d+\mathrm{Li}^6, \qquad (23)$$

$$p+\mathrm{He}^4\rightleftarrows d+\mathrm{He}^3. \qquad (24)$$

In these reactions there are generally many competing open channels on the energy shell, so that the unitarity of the S matrix is not an effective limiting condition for testing time-reversal invariance. However, for near-forward angles, the angular distributions can generally (the above examples are no exception) be understood by a simple one-parameter surface interaction, and as explained in Sec. IIIA may not be sensitive to time-reversal invariance; reactions (23) have been measured only over the first peak in the diffraction pattern. For (24), angular distributions have been obtained between 20° and 100° in both directions, and at the larger angles do not agree with the simple model discussed in Sec. IIIA. Comparing arbitrarily normalized differential cross sections, one finds agreement to approximately 15% so that time-reversal-odd forces are limited to a relative strength of 8% of the total nuclear force.

Comparison of forward and backward absolute cross sections gives agreement to within the accuracy of the measurements, which is 25% for (23) and 40% for (24).

5. *Charge-Exchange Reactions*

The available experiments which fall into this category and to which detailed balance can be applied are[29,30]

$$p+\mathrm{H}^3\rightleftarrows n+\mathrm{He}^3. \qquad (25)$$

These experiments have been performed over a range of energies for which the two-state theorem is not expected to be applicable. Angular distributions have not been obtained for the (n,p) reaction, and the accuracy of the magnitude of the total cross sections is only approximately 30%. Hence these researches do not, in their present state, serve as a useful test of time-reversal invariance, although if the accuracy were improved, the reactions could be used as such.

C. Proposed Experiments

It is clear that when many competing channels are available on the energy shell, and when a Born approximation analysis is not valid, any nuclear reaction can, in principle, be used to test time-reversal invariance. If these conditions are not met, care must be taken that an experiment is sensitive to the existence of complex

[27] J. B. Reynolds and K. G. Standing, Phys. Rev. **101**, 158 (1956).

[28] J. Benveniste and B. Cork, Phys. Rev. **89**, 422 (1953); J. C. Allred, Phys. Rev. 84, 695 (1951).

[29] Willard, Bair, and Kingston, Phys. Rev. **90**, 865 (1953); Vlasov, Kalinin, Ogloblin, Samoilov, Sidorov, and Chuev, Zhur. Eksptl. i Teoret, Fiz. **28**, 639 (1955) [translation: Soviet Phys. JETP **1**, 500 (1955)].

[30] J. H. Coon, Phys. Rev. **80**, 488 (1950).

phases, which are then the only remaining possible manifestations of the lack of time-reversal invariance. If the conditions are met, then measurements of the angular distribution or cross section of a reaction performed both forward and backward with light nuclei as targets (for purposes of energy resolution) to an accuracy of better than $\sim 20\%$ would improve our present knowledge of the presence of time-reversal-odd forces in the nuclear Hamiltonian. As specific examples, we list

$$d+N^{14} \rightleftarrows \alpha+C^{12}, \tag{26}$$

$$d+C^{12} \rightleftarrows \alpha+B^{10}, \tag{27}$$

both of which satisfy the conditions stated above, and are especially useful because both forward and backward processes can be analyzed at the same laboratory. For the reactions (26), deuterons of 21.7 Mev can be used forwards and alpha particles of twice this energy backwards. For the reactions (27) it is possible to employ 14.9-Mev deuterons and 29.8-Mev alpha particles in the forward and backward processes, respectively. All of these energies are readily obtainable with a standard 60-in. cyclotron. The reactions (26) are being actively studied at the University of Washington by Bodansky, Eccles, Farwell, Rickey, and Robison.[31] Preliminary data between 30° and 150° in the center-of-mass system show agreement to within approximately 15%. Work is in progress to improve the accuracy and extend the angular range.

IV. POLARIZATION MEASUREMENTS IN ELASTIC COLLISIONS

In an elastic collision between particles of spins s and s', the relevant parts of the scattering matrix can be written as a matrix in the product spin space of the two particles, $R(\mathbf{k}_2,\mathbf{k}_1)$, where $\mathbf{k}_1$ and $\mathbf{k}_2$ are the relative momenta before and after the collision. Since in general many other reactions besides the elastic scattering may be connected to the initial state on the energy shell, only weak restrictions will be imposed on the form of R by the unitarity condition which will therefore not be further considered in this section. The existence of time-reversal invariance implies that the only terms which can appear in R must satisfy $U^\dagger R(\mathbf{k}_2,\mathbf{k}_1)U = [R(-\mathbf{k}_1, -\mathbf{k}_2)]_{\text{trans}}$, or in other words, $R(\mathbf{k}_2,\mathbf{k}_1) = R^\tau(-\mathbf{k}_1, -\mathbf{k}_2)$. The assumptions of rotational and parity invariance show that the terms of lowest order in spin which can violate the above condition are of the forms[32]

$$\begin{aligned}[\mathbf{s}\cdot(\mathbf{k}_1+\mathbf{k}_2)][\mathbf{s}\cdot(\mathbf{k}_1-\mathbf{k}_2)]+[\mathbf{s}\cdot(\mathbf{k}_1-\mathbf{k}_2)][\mathbf{s}\cdot(\mathbf{k}_1+\mathbf{k}_2)],\\ [\mathbf{s}\cdot(\mathbf{k}_1+\mathbf{k}_2)][\mathbf{s}'\cdot(\mathbf{k}_1-\mathbf{k}_2)], \quad \text{etc.}\end{aligned} \tag{28}$$

(If, as in the case of $p-p$ scattering, the total spin S must be a constant of motion, only one combination of the above terms survives.) This immediately shows the well-known result that time-reversal invariance is irrelevant for a system of total spin $\frac{1}{2}$, such as protons on carbon.

In order to see how a violation of the time-reversal invariance condition could be detected, let us abandon it for the moment and consider a double scattering, the first from a nucleus a and the second from nucleus b.[33] The total probability for the process is then proportional to

$$\begin{aligned}\mathcal{P}_{ba}(\mathbf{k}_3,\mathbf{k}_2,\mathbf{k}_1) &\equiv \text{Spur}\{R_b(\mathbf{k}_3,\mathbf{k}_2)R_a(\mathbf{k}_2,\mathbf{k}_1) \\ &\qquad \times R_a^\dagger(\mathbf{k}_2,\mathbf{k}_1)R_b^\dagger(\mathbf{k}_3,\mathbf{k}_2)\} \\ &= \text{Spur}\{[R_b^\dagger(\mathbf{k}_3,\mathbf{k}_2)R_b(\mathbf{k}_3,\mathbf{k}_2)] \\ &\qquad \times [R_a(\mathbf{k}_2,\mathbf{k}_1)R_a^\dagger(\mathbf{k}_2,\mathbf{k}_1)]\} \\ &= \text{Spur}\{\epsilon_b(\mathbf{k}_3,\mathbf{k}_2)\rho_a(\mathbf{k}_2,\mathbf{k}_1)\},\end{aligned} \tag{29}$$

where $\epsilon \equiv R^\dagger R$ and $\rho \equiv RR^\dagger$ can be considered to be (unnormalized) efficiency and density matrices, respectively. Defining $\epsilon' = R^{\tau\dagger}R^\tau$ which is equal to ϵ only in the case of time-reversal invariance, one finds immediately

$$\begin{aligned}\mathcal{P}_{ba}(\mathbf{k}_3,\mathbf{k}_2,\mathbf{k}_1) &= \text{Spur}\{\epsilon_b(\mathbf{k}_3,\mathbf{k}_2)\rho_a(\mathbf{k}_2,\mathbf{k}_1)\} \\ &= \text{Spur}\{\epsilon_a'(-\mathbf{k}_1, -\mathbf{k}_2)\rho_b'(-\mathbf{k}_2, -\mathbf{k}_3)\}.\end{aligned} \tag{30}$$

If nuclei a and b are the same, then $(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$ and $(-\mathbf{k}_3, -\mathbf{k}_2, -\mathbf{k}_1)$ can differ only through the presence of a term which is noninvariant under time reversal, such as $\text{const}\times\mathbf{k}_2\cdot(\mathbf{k}_1-\mathbf{k}_3)$. All such terms are zero if the first and second polar angles are the same. However, one can think of performing one double-scattering experiment in which the polar angle of the first is θ and of the second is θ'; then of doing another double scattering which differs from this by moving the second scatterer, say, (but not the detector) in such a way that θ and θ' are interchanged. In other words, if $\mathbf{k}_1$, $\mathbf{k}_2$ and $\mathbf{k}_3$ are any three momenta (of equal magnitude), then instead of the three time-reversed momenta $\mathbf{k}_1' = -\mathbf{k}_3$, $\mathbf{k}_2' = -\mathbf{k}_2$, $\mathbf{k}_3' = -\mathbf{k}_1$ we can use $\mathbf{k}_1'' = \mathbf{k}_1$, $\mathbf{k}_2''$, $\mathbf{k}_3'' = \mathbf{k}_3$, where $\mathbf{k}_2''$ is determined by the two conditions $\mathbf{k}_2''\cdot\mathbf{k}_1 = \mathbf{k}_2\cdot\mathbf{k}_3$ and $\mathbf{k}_2''\cdot\mathbf{k}_3 = \mathbf{k}_2\cdot\mathbf{k}_1$. The $\mathbf{k}''$'s differ from the $\mathbf{k}'$'s by rotations only.

The interpretation of experiments of this type is always marred by the fact that the recoil of the target nuclei carries away some energy, for which a correction must be made in interpreting the inverse effect. Although this correction is only a few percent, so is the maximum possible violation of time-reversal invariance which is being sought. It may be advantageous instead to use Eq. (30) with protons on a nucleus a such as carbon, for which $R = R^\tau$, performing the time-reversal test on

[31] Bodansky, Eccles, Farwell, Rickey, and Robison, Cyclotron Research, University of Washington, Annual Progress Report, 1958 (unpublished); Bull. Am. Phys. Soc. Ser. II, 3, 327 (1958).

[32] See also L. Wolfenstein and J. Ashkin, Phys. Rev. 85, 947 (1952), and L. Wolfenstein, Phys. Rev. 96, 1654 (1954).

[33] Since an initial orientation in the target nucleus is related in the time-reversed situation to (minus) the final orientation, there seems little likelihood of obtaining experimental information about time-reversal invariance by looking for forbidden orientation terms; they are therefore omitted from further consideration here.

some other target b with spin >0. This has the additional helpful feature of involving only two unknown functions of energy E and $\cos\theta$ for carbon, for which the most general form of the R matrix is

$$R=f_1(E,\cos\theta)+if_2(E,\cos\theta)\boldsymbol{\sigma}\cdot\mathbf{k}_1\times\mathbf{k}_2. \quad (31)$$

The scattering from the nucleus b can be performed at the same energies and angles both forwards and backwards; the role of the carbon collision is that of a separately well-investigated polarizer or analyzer.[34]

Manipulation similar to that leading to Eq. (30) can be used for triple scattering as well. In the simplest case, in which a time-reversal test is done on a nucleus X, preceded and followed by a scattering on a carbon polarizer and analyzer, the analog of Eq. (30) is

$$\begin{aligned}\mathcal{P}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)&=\mathrm{Spur}\{\epsilon(\mathbf{k}_4,\mathbf{k}_3)R_X(\mathbf{k}_3,\mathbf{k}_2)\rho(\mathbf{k}_2,\mathbf{k}_1)R_X^\dagger(\mathbf{k}_3,\mathbf{k}_2)\}\\&=\mathrm{Spur}\{\epsilon(-\mathbf{k}_1,-\mathbf{k}_2)R_X^\tau(-\mathbf{k}_2,-\mathbf{k}_3)\\&\qquad\times\rho(-\mathbf{k}_3,-\mathbf{k}_4)R_X^{\tau\dagger}(-\mathbf{k}_2,-\mathbf{k}_3)\},\end{aligned} \quad (32)$$

which, if time-reversal invariance holds, is invariant with respect to the substitutions

$$\mathbf{k}_1\rightarrow-\mathbf{k}_4,\ \mathbf{k}_2\rightarrow-\mathbf{k}_3,\ \mathbf{k}_3\rightarrow-\mathbf{k}_2,\text{ and }\mathbf{k}_4\rightarrow-\mathbf{k}_1. \quad (33)$$

V. ELECTROMAGNETIC INTERACTIONS AND BETA DECAY

A. Electromagnetic Interactions

In addition to employing nuclear interactions to test the time reversibility of nuclear forces, it is possible to make use of both electromagnetic and weak forces to accomplish the same purpose. The matrix element between nuclear states of an interaction Hamiltonian which is invariant under time reversal has a phase factor, which is not equal to 0 or π [see Eq. (10)] unless the nuclear forces are also invariant under time reversal. Coincidence experiments of gamma-ray transitions between nuclear states can test time-reversal invariance by a measurement sensitive to either (1) $\cos\eta$ or (2) $\sin\eta$. The latter type of measurement is examined in full detail in the following paper. The former is included here because experiments already performed serve to set an upper limit on $\mathfrak{F}$. The correlation function for a double cascade, the first transition of which is mixed, between nuclear states of angular momentum j_1, j_2 and j_3, respectively, can be written as

$$W(\beta)\sim\omega_{\mathrm{I}}+|\delta|^2\omega_{\mathrm{II}}+2|\delta|\cos\eta\,\omega_{\mathrm{III}}, \quad (34)$$

where β is the angle defined by the momentum of the two gamma rays, and δ is defined by

$$\delta\equiv\frac{\langle j_1\|L'\|j_2\rangle}{\langle j_1\|L\|j_2\rangle}=|\delta|e^{i(\eta_{L'}-\eta_L)}\equiv|\delta|e^{i\eta} \quad (35)$$

[34] See also F. Mandl, Proc. Phys. Soc. (London) **71**, 686 (1958); R. J. N. Phillips, Nuovo cimento **8**, 265 (1958).

and is real if time-reversal invariance holds. Expressions for ω_{I}, ω_{II}, and ω_{III} are given by Biedenharn and Rose.[35] For a $2(E2,M1)2(E2)0$ transition, the correlation function is proportional to

$$W=1+A_2P_2(\cos\beta)+A_4P_4(\cos\beta), \quad (36)$$

with

$$|\delta|^2=A_4/(0.327-A_4), \quad (37a)$$

$$\cos\eta=[0.447(A_2+A_4)-0.112]/(0.327A_4-A_4^2)^{\frac{1}{2}}. \quad (37b)$$

In order to measure $\cos\eta$ successfully, it is clear that $|\delta|$ must be of order one. Transitions of the type indicated above are among the few which can be expected to have this property, since the shell model predicts that the transitions between states $j_1=2$ and $j_2=2$ proceeds predominantly by magnetic dipole radiation; the collective nuclear effects, however, enhance the electric quadrupole transition rate, so that fairly near closed shells, it is possible to find transitions with $|\delta|\sim1$. Experiments bear out this prediction. The purity of the second transition has the advantage of adding no further unknown parameters to the angular correlation expression.

The best examples in the literature of such transitions are listed in Table II together with the values of $|\delta|$ which are found. Of those listed, only the Hg^{198} measurements yield a precise enough value of $\cos\eta$ to be of use to us.[36] One finds $\cos\eta=-1.037\pm0.079$, corresponding to $\pi-\eta\approx0\pm0.3$ radian, or a fraction of time-reversal-odd nuclear force $\mathfrak{F}\lesssim30\%$.

It is clear that it is difficult to push downward the upper limit on $\mathfrak{F}$ through experiments such as these which measure only η^2.

TABLE II. Table of elements in which $2(E2+M1)2$ transitions take place with mixing ratios between $0.1\lesssim|\delta|\lesssim10$. The second column lists the absolute values of δ obtained with the assumption of time-reversal invariance.

Element	$\lvert\delta\rvert$	Reference
S^{34}	0.133 ± 0.024	a
Fe^{58}	2.2 ± 0.3	b
Se^{76}	>2.2	c, d
Pd^{106}	0.21 ± 0.07	e
Sb^{122}	3.2 ± 1.0	f
Os^{188}	15	g
Ir^{194}	7	h
Hg^{198}	0.82 ± 0.09	i, j

[a] H. E. Handler and J. R. Richardson, Phys. Rev. **102**, 833 (1956).
[b] Frauenfelder, Levine, Rossi, and Singer, Phys. Rev. **103**, 352 (1956).
[c] J. J. Kraushaar and M. Goldhaber, Phys. Rev. **89**, 1081 (1953).
[d] F. R. Metzger and W. B. Todd, J. Franklin Inst. **256**, 277 (1953).
[e] E. D. Klema and F. K. McGowan, Phys. Rev. **92**, 1469 (1953).
[f] M. J. Glaubman, Phys. Rev. **98**, 645 (1955).
[g] Potnis, Dubey, and Mandeville, Phys. Rev. **102**, 459 (1956).
[h] J. J. Kraushaar and M. Goldhaber, Phys. Rev. **89**, 1081 (1953).
[i] D. Schiff and F. R. Metzger, Phys. Rev. **90**, 849 (1953).
[j] C. D. Schrader, Phys. Rev. **92**, 928 (1953).

[35] L. C. Biedenharn and M. E. Rose, Revs. Modern Phys. **25**, 729 (1953). See in particular Sec. III A(2).

[36] The corrections of the spin assignments is unambiguous in this case and independent of time-reversal invariance. See C. D. Schrader, Phys. Rev. **92**, 928 (1953), and Elliott, Preston, and Wolfson, Can. J. Phys. **32**, 153 (1954).

B. Beta Decay

In beta decay the most useful measurements of time-reversal invariance in weak interactions depend on the presence of both Fermi and Gamow-Teller transitions.[37] Such experiments therefore only determine the relative phase of nuclear matrix elements and beta-decay coupling constants. The recent experiment of Ambler *et al.*[4] on beta-gamma correlations from polarized Mn^{52} attempt to detect terms of the type

$$\mathrm{Im}\,[(C_V C_A'^* + C_V' C_A^*) M_{\mathrm{F}} M_{\mathrm{GT}}^*], \qquad (38)$$

where C_V, C_A are vector and axial vector coupling constants, and M_{F}, M_{GT} are nuclear matrix elements for Fermi and Gamow-Teller transitions. The limits on η obtained from this investigation are $140° \lesssim \eta \lesssim 250°$. Only if nuclear forces are known to be invariant under time reversal do such experiments give direct information on time-reversal invariance of weak interactions. However, a small upper limit on the over-all phase in Eq. (38) would strongly indicate time-reversal invariance in both weak *and* strong forces.

[37] See, for example, Jackson, Treiman, and Wyld, Phys. Rev. **106**, 517 (1957); R. B. Curtis and R. R. Lewis, Phys. Rev. **107**, 1381 (1957); M. Morita and R. S. Morita, Phys. Rev. **107**, 1316 (1957).

VI. CONCLUSIONS

We have investigated the limits set on the presence of time-reversal-odd forces in the nuclear Hamiltonian by experiments already performed. On the basis of detailed-balance tests, polarization experiments, and angular correlation measurements in nuclear gamma-ray cascades, we find that the fraction of time-reversal-odd interaction is less than 10%. In addition we have shown that care must be exercised in choosing experiments to test time-reversal invariance sensitively, and have suggested several which could be used to lower the above limit. Among these are tests which employ angular correlations of gamma rays from oriented nuclei; these are described in detail in the following paper.

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