Classification of the Energy Levels of Odd-Mass Nuclei in the Heavy-Element Region*

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Most of the data available on energy levels in odd-mass nuclei of the heaviest elements have been summarized and evaluated. The observed levels have been classified according to the level diagrams calculated by Nilsson for nuclei with prolate spheroidal deformations. Qualitatively, the agreement between the data and the Nilsson diagrams is very good.

INTRODUCTION

I N the system of nuclei, regions of pronounced spheroidal deformation have been clearly recognized. In these regions, the Bohr-Mottelson unified nuclear model¹ enjoyed an early great success in describing the energy levels and associated spectra, particularly in delineating the effects of collective modes of internal nuclear motion. One major area of applicability is in the heavy-element region delimited approximately by A > 225.

More recently Nilsson² and others^{3,4} have characterized the energy levels of a particle moving in an axially-symmetric but nonspherical potential. In the limit of high nuclear deformation Nilsson has found it possible to define independent-particle states in terms of a set of quantum numbers not applicable to spherical nuclei. This picture of these states and the expected order of filling have already proved highly useful in correlating experimental information pertaining to the the appropriate odd-mass nuclei.

It is the purpose of this communication to summarize, in terms of this system of classification, energy-level assignments in the heavy element region. A number of the assignments to be listed have already been discussed in other publications, and for these, detailed justification will be omitted.

The Nilsson representations for neutron and proton states in the region A > 225 are shown in the diagrams of Figs. 1 and 2. The energies at which particular states lie appear on the ordinate axis, but this scale will only be used here as a means of visualizing the order of filling of levels. The abscissa scale indicates the deformation of a prolate sphereoid in terms of a parameter, δ , which also will not be used further. At zero deformation we see the typical shell-model level structure for a spherically symmetric potential. The (2j+1)-fold degeneracy is seen removed as permanent deformation sets in, and at high deformation the levels (now twofold degenerate) are described in terms of a new set of quantum numbers indicated by the indices in the margins.

The origin of these numbers will not be described here (see reference 2) other than to define formally the nomenclature. At high deformation the projection of the particle angular momentum, j, along the nuclear symmetry axis becomes a good quantum number, and this number, Ω , is the observed spin in the absence of rotational motion. The parity is designated (+) or (-) and depends upon the *even* or *odd* character of the principle quantum number, N, the total number of nodes in the wave function. The numerical value of Nis the first integer in the brackets and corresponds to the oscillator shell of the shell model. The second number in the brackets is n_z , the number of nodal planes perpendicular to the symmetry axis; and the third number is Λ , the component of the orbital angular momentum along the symmetry axis. We shall also refer from time to time to the quantum number, K, which represents the projection of the nuclear spin, I, on the symmetry axis. For low-lying states, K and Ω should be equal, and in the following discussions we use the two more or less interchangeably.

It should be pointed out that for a particular state, these *asymptotic* quantum numbers are fully descriptive only in the limit of high deformation. The absence of "purity" of these quantum numbers is not of concern here because (a) we shall not be dealing with the consequences of their impurity as they apply to transition selection rules, and (b) we are confining our attention to cases of large deformation where these numbers do serve as the best means of labeling different states.

An example will serve to show how this system of nomenclature operates and its application to specific assignments. Consider the proton states designated 5/2+ [642] and 5/2- [523] which lie close to each other at a nuclear deformation described by $\delta \cong 0.25$. (Author references for the assignments of these orbitals to the particular nuclear states will appear later in this paper.) By counting from proton number 82, it is seen that these levels come in at approximately proton numbers 93 and 95 which characterize the elements neptunium and americium. It is found that the state 5/2+ [642] is undoubtedly the ground state of Np²³⁷,

^{*} This work was performed under the auspices of the U. S. Atomic Energy Commission.

¹ A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 27, No. 16 (1953); Phys. Rev. 89, 316 (1953).

 ²S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 29, No. 16 (1955).
 ⁸K. Gottfried, thesis, Massachusetts Institute of Technology,

June, 1955 (unpublished). ⁴ Marion Rich, University of California Radiation Laboratory

Report UCRL-3587, November 16, 1956 (unpublished).



FIG. 1. Nilsson diagram for neutrons in the region $126 \le N \le 160$. The abscissa is the nuclear deformation (prolate), and the ordinate indicates the energy of the various levels. The dashed line indicates very roughly the deformations thought to pertain for most of the nuclei discussed.

and 5/2-[523] is the ground state of Am^{241} . The term [642] defines the state as follows: It comes from the sixth oscillator shell as can be seen by its connection to the $i_{13/2}$ level in the spherically symmetric potential. The other 5/2+ states for N=6 all lie much higher and none is shown in Fig. 2. The highest has $n_z=0$ descending to the maximum value $n_z=4$ which applies to the state under discussion. The value of Λ can differ from Ω only by 1/2 and takes an even or odd value as $N-n_z$ is even or odd. As a means simply of designating states, the Λ quantum number is redundant as used here because there can only be one 5/2+ state which has N=6 and $n_z=4$.

The ground state of Am²⁴¹, 5/2-[523] is seen to connect with the $h_{9/2}$ state of spherical nuclei which is in the fifth oscillator shell. The highest-lying 5/2state possible for N=5 has $n_z=0$ and connects with the $f_{5/2}$ state, the next lowest has $n_z=1$ ([512]), and the one under discussion here has $n_z=2$. Since we have $N-n_z=3$ (odd), Λ must be 3, i.e., $\Omega+1/2$.

Recapitulating, the asymptotic quantum numbers appearing in brackets in Fig. 2 serve to identify the doubly-degenerate states shown in the diagrams. Because each state is only twofold degenerate, it would be expected to appear as an unpaired orbital in a ground state only for a single nucleon number. However, the same state would appear as excited levels for neighboring nuclei. One might also expect from time to time that the same configuration would repeat for ground



FIG. 2. Nilsson diagram for protons in the region $82 \leq Z \leq 126$. The abscissa is the nuclear deformation (prolate), and the ordinate indicates the energy of the various levels. The dashed line indicates very roughly the deformations thought to pertain for most of the nuclei discussed.

states as the result of an appreciable change in deformation.

RESULTS

Tables I and II summarize assignments of Nilsson levels for odd-neutron and odd-proton configurations, respectively. In these tables the Nilsson levels are listed across the top in order of increasing energy (appropriate to the deformations shown by the broken lines in Figs. 1 and 2) and the numbers appearing in the tables are the energies of the states (in kev) relative to their ground states. The pattern is quite evident and shows that a particular assignment for the ground state of one nucleus appears in excited states of neighboring nuclei.

The method of coding the reliability of the assignments is by means of the parentheses enclosing the energies of the states. Absence of parentheses indicates rather conclusive evidence; a single set implies a tentative assignment based on substantial evidence; and double parentheses are used to signify that some evidence is available but that much information is yet lacking. For those entries which are italicized, the corresponding spin has been determined directly, but it should be pointed out that this in itself does not constitute proof that the assignment is correct.

TABLE I. Odd-neutron level assignments. The table contains level assignments for nuclei with neutron numbers 137-151. At the head of the table (reading from left to right) is the expected order of orbital filling (see Fig. 1). The orbitals are identified by Ω_i^a parity, and the bracketed indices which refer, respectively, to N, n_{z_1} and Λ as defined in the text. The numeral 0 signifies that the orbital identified at the head of the column appears as the ground state configuration for the particular nucleus. Other numbers are the energies (in kev) above the ground state at which the indicated states lie. (For example, the ground state of Th²³¹ is 5/2 + [633], and the configurations 5/2 - [752] and 7/2 - [743] appear at energies of 185 and 390 kev, respectively.) Italicized zeros indicate that the corresponding ground state spins have been measured by one or more direct methods. Parentheses are used to indicate uncertainties of the assignments: single parentheses indicate rather good evidence and double parentheses, less conclusive evidence. Where no parentheses appear, the assignment is considered certain.

| Isotope | 3/2+ [631] | 5/2 — [752] | 5/2+ [633] | 7/2 — [743] | 1/2 + [631] | 5/2+ [622] | 7/2 + [624] | 9/2 — [734] | Neutron number |
|--|---------------|-----------------|---------------|----------------|-------------------|------------------------|----------------|----------------|-------------------|
| $\substack{\text{Ra}^{225}\\\text{Th}^{227}\\\text{U}^{229}}$ | | | | | | | | | 137 |
| Ra ²²⁷ Th ²²⁹ U ²³¹ Pu ²³³ | | ((0)) | 0 | | | | | | 139 |
| ${\begin{array}{c}{}{}{}{}{}{}{}{}{}{}{}{}{}{}{}{}{}{$ | (313) | 185 ((~300)) | 0 0 | 390 | ((400)) | | | | 141 |
| Th ²³³ U ²³⁵ Pu ²³⁷ Cm ²³⁹ | | | | 0 0 | ~ 0.1 145 | | | | 143 |
| U ²³⁷ Pu ²³⁹ Cm ²⁴¹ | | | | 392 | 0 0 0 | (145) 286 | (512) | | 145 |
| U ²³⁹ Pu ²⁴¹ Cm ²⁴³ Cf ²⁴⁵ | | | | | | $((0)) \\ 0 \\ 0 \\ 0$ | 172 | | 147 |
| Pu ²⁴³ Cm ²⁴⁵ Cf ²⁴⁷ Fm ²⁴⁹ | | | | | ((631)) | 255 | (0) 0 | 394 | 149 |
| Pu ²⁴⁵ Cm ²⁴⁷ Cf ²⁴⁹ Fm ²⁵¹ | | | | | | | | 0 | 151 |

^a The spin of the lowest-lying member of a rotational band is generally the same as the value of Ω for that band. For some $\Omega = 1/2$ bands, however, the higher-spin rotational states have lower energy than the spin-1/2 state. Examples of this are the spin-3/2 ground states of Pa²³¹ and Pa²³³ which have been assigned $\Omega = 1/2$.

In the following paragraphs the assignments made in Tables I and II are discussed briefly. A number of known nuclei in the region have not been sufficiently investigated to warrant inclusion; in particular, a large proportion of known excited states have not been tabulated for want of sufficient information of the type needed to make meaningful assignments. In a sense, the general reliability of the model can be gauged both by the orderly sequence of assignments and by the fact that in no case were accurate data available that indicated an assignment significantly at variance with the expectations of the theory.

In the discussions which follow, the odd-neutron cases are considered first and are grouped according to element. Following these are the odd-proton nuclei, again grouped by element. The tables summarize the data in terms of increasing neutron or proton numbers.

ODD-NEUTRON NUCLEI

Th²²⁹ (neutron number 139).—The ground state of Th²²⁹ is assigned 5/2 + [633] which comes in at about the expected place for neutron number 139. This assignment will be seen to be the same as that for the ground state of U²³³ and was arrived at on the basis that the "favored alpha decay" of U²³³ leads to the ground state of Th²²⁹.^{5–7} Bohr, Fröman and Mottelson⁶ were the first to give theoretical grounds for assigning identical structures to states that are connected by an alpha emission process whose rate obevs simple one-body alpha-decay theory. The ground-state transitions of

⁵ I. Perlman and F. Asaro, Annual Review of Nuclear Science Annual Reviews, Inc., Starford, 1954), Vol. 4, p. 157.
 Bohr, Fröman, and Mottelson, Kgl. Danske Videnskab.
 Selskab, Mat.-fys. Medd. 29, No. 10 (1955).
 Gol'din, Novikova, and Tretyakov, Phys. Rev. 103, 1004 (1955).

^{(1956).}

TABLE II. Odd-proton level assignments. The table contains level assignments for nuclei with proton numbers 89–99. At the head of the table (reading from left to right) is the expected order of orbital filling (see Fig. 2). The orbitals are identified by Ω_i^a parity, and the bracketed indices which refer, respectively, to N, n_z , and A as defined in the text. The numeral 0 signifies that the orbital identified at the head of the column appears as the ground state configuration for the particular nucleus. Other numbers are the energies (in kev) above the ground state at which the indicated states lie. (For example, the ground state of Pa²³¹ is 1/2 - [530] and the configurations 3/2 + [651] and 5/2 + [642] appear at energies of 166 and 84 kev, respectively.) Italicized zeros indicate that the corresponding ground state spins have been measured by one or more direct methods. Parentheses are used to indicate uncertainties of the assignments: single parentheses indicate rather good evidence and double parentheses, less conclusive evidence. Where no parentheses appear, the assignment is considered certain.

| Isotope | 3/2 — [532] | 3/2+ [651] | 1/2 — [530] | 5/2+ [642] | 5/2 — [523] | 3/2 — [521] | 7/2+ [633] | 7/2 — [514] | Proton number |
|---|----------------|-----------------|-------------------------------|----------------------|--|----------------------------------|---------------|----------------|------------------|
| Ac ²²⁵ Ac ²²⁷ Ac ²²⁹ | | (0) | (330) | | | | | | 89 |
| Pa ²²⁷ Pa ²²⁹ Pa ²³¹ Pa ²³³ Pa ²³⁵ | | (166) (~200) | <i>0</i> 0 | (84) (86) | | | | | 91 |
| Np ²³¹ Np ²³³ Np ²³⁵ Np ²³⁷ Np ²³⁹ | | | ((267)) | 0 0 0 | 48 60 75 | | | | 93 |
| Am ²³⁷ Am ²³⁹ Am ²⁴¹ Am ²⁴³ Am ²⁴⁵ | | | ((540)) ((480)) ((265)) | (187) (206) 84 | 0 0 0 | ((630)) | 465 | | 95 |
| Bk ²⁴³ Bk ²⁴⁵ Bk ²⁴⁷ Bk ²⁴⁹ | | | | (393) | | $(0) \\ 0 \\ (0) \\ ((\sim 10))$ | 0 | | 97 |
| ${f E}^{249}_{{f E}^{251}}_{{f E}^{253}}_{{f E}^{255}}$ | | | | | | | 0 | | 99 |
| Mv | | | | | ······································ | | | | 101 |

^a The spin of the lowest-lying member of a rotational band is generally the same as the value of Ω for that band. For some $\Omega = 1/2$ bands, however, the higher-spin rotational states have lower energy than the spin-1/2 state. Examples of this are the spin-3/2 ground states of Pa²²¹ and Pa²²³ which have been assigned $\Omega = 1/2$.

even-even alpha emitters (both states 0+) provide the empirical basis for defining "favored" or "unhindered" alpha emission.

The reader is referred to the section below on U²³³ for the assignment of 5/2+ [633] for its ground state. The fact that Th²²⁹ and U²³³ both have the same ground-state configurations implies that the odd-particle state filled for Th²²⁹ is vacated when the particle becomes paired. Further information on low-lying states around this neutron number should help explain this repetition in configuration.

Th²²⁹ has a number of low-lying excited states defined by the alpha-particle spectrum of U²³³.^{5,7,8} These levels, up to about 350 kev, include members of the groundstate rotational band, but in addition there are a few which undoubtedly consist of other intrinsic configurations. By analogy with U²³³ the configurations 5/2- [752], 3/2+ [631] and 1/2+ [631] might be represented, but there is not enough information available to permit making assignments.

The states associated with neutron number 139 are discussed further under the section on U^{231} .

Th²³¹ (*neutron number 141*).—The assignments for the levels of Th²³¹ have already been reported in brief form by Pilger *et al.*⁹ and are based on studies of the alpha decay of U²³⁵ and the beta decay of Th²³¹. Briefly, the arguments are as follows:

The ground state of U²³⁵ is almost surely 7/2-[743](see below), and its favored alpha decay goes to a level in Th²³¹ at 390 kev which is, accordingly, given this assignment. At about 190 kev there is a group of levels (at least 3) which probably represents a highly compressed rotational band. The 7/2-[743] state at 390 kev drops to two or more members of this band by M1transitions; therefore this band has *odd*-parity and spin 5/2 or 7/2 for the base state. The only Nilsson

⁸ Carl Ruiz (private communication, 1958).

⁹ Pilger, Stephens, Asaro, and Perlman, Bull. Am. Phys. Soc. Ser. II, 2, 394 (1957).

level near the 7/2 - [743] state satisfying these conditions is $5/2 - \lceil 752 \rceil$. It is interesting to note that alpha decay to this band is only slightly hindered and this is probably related to the fact that $7/2 - \lceil 743 \rceil$ (ground state of U^{235}) and $5/2 - \lceil 752 \rceil$ are structurally similar. The compressed nature of the band at about 190 kev is thought to be due to the close proximity and strong interaction of these two odd-parity bands. The effect referred to here was first worked out for another nucleus by Kerman.¹⁰

The states at about 190 key drop to the ground state band by E1 transitions, therefore the ground state has even parity and the most logical choice among the Nilsson levels is 5/2 + [633]. This is the same as the ground state of U²³³ which is not unexpected because both have the same neutron number. Supporting evidence for this ground-state assignment comes from the spacing of members of the ground-state band⁹ and from the beta-decay properties of $Th^{231} \rightarrow Pa^{231}$.¹¹

Recapitulating, the ground state of Th²³¹ is the expected state, $5/2 + \lceil 633 \rceil$; the state at about 190 kev is $5/2 - \lceil 752 \rceil$ produced by creating a hole in a filled level (see Fig. 1); and the state at 390 kev is $7/2 - \lceil 743 \rceil$ which is expected to be near because it will appear as the ground state for neutron number 143.

Th²³³ (neutron number 143).—Th²³³ is a short-lived beta emitter about which very little is known. Nothing is known about its excited states, and the only information available concerning its ground state is that derived from its beta-decay properties.

By analogy with U²³⁵ and Pu²³⁷ one might guess that the ground state is either 7/2 - [743] or 1/2 + [631]. The data of Freedman and co-workers¹² indicate that an appreciable proportion of the decay of Th²³³ goes to the ground-state rotational band of Pa²³³ which is believed to have K=1/2 as will be discussed in a later section. This information would seem to favor the 1/2+ [631] assignment for Th²³³ if the choice rests between the two states mentioned. Possible difficulty exists, however, because there also seems to be direct population of the 5/2+ band of Pa²³³, although this is by no means certain. In view of the paucity of information, no assignment is entered in Table I. Still, it is interesting to note the relative positions of the abovementioned two states in other nuclei with 143 neutrons. In Pu²³⁷, the state 1/2+ [631] is 145 kev above the 7/2- [743] ground state; in U²³⁵ the two states lie within 100 ev of each other; and if the above-mentioned evidence is significant, the 1/2+ state becomes the ground state in Th²³³.

U²³¹ (neutron number 139).—At present very little is known about the energy levels of U²³¹ because information comes only from its electron-capture decay

properties. Something possibly may be learned about its excited states when its alpha-emitting parent, Pu²³⁵, can be studied.

The ground state would be expected to be 5/2-[752] (185-kev state in Th²³¹), 5/2+ [633] (ground state of Th²³¹ and U²³³), or $3/2+ \lceil 631 \rceil$ (an excited state in U^{233}). The 5/2+ assignment can probably be ruled out because Th²³¹ and U²³¹ both decay to Pa²³¹, and there are selective differences in the levels occupied.¹¹ Of the two remaining assignments, $5/2 - \lceil 752 \rceil$ is slightly preferred on the basis that no population to the ground state, K=1/2 band of Pa²³¹, could be detected in the U²³¹ electron-capture decay. Table III and its discussion (see below) go into the beta-decay selection processes as related to the Nilsson levels, and it will be seen that the $\log ft$ values for decay of U²³¹ to the K=1/2 band in Pa²³¹ probably would be such as to have permitted the observation of this transition if U^{231} were $3/2 + \lceil 631 \rceil$.

U²³³ (neutron number 141).—The energy levels of U²³³ have recently been reviewed by Newton.¹³ The assignments suggested by him have been adopted (Table I) although different conclusions were reached regarding the related subject of the Pa²³³ assignment which is discussed below. As seen in Table I, the ground state of U^{233} has been assigned by Newton to 5/2+ $\lceil 633 \rceil$ (see also Th²³¹); a state at 313 kev is assigned to 3/2+ [631] (which implies excitation of a particle from a previously filled level); and a state at 400 kev is labeled 1/2 + [631]. This does not mean that other expected levels (Table I) are not present at even lower energies because it could easily be that such levels are not populated by the beta decay of Pa²³³.

Some light is shed on this subject by the recently investigated alpha branching of Pu237.14,15 The alpha group of highest energy so far observed (5.65 Mev) probably leads to the ground state of U²³³ or at least to a level near the ground state because this energy corresponds well with the total available energy calculated from closed decay cycles.¹⁴ The transition is rather highly hindered in agreement with the assignment of $7/2 - \lceil 743 \rceil$ for Pu²³⁷ (see below) and of 5/2 + $\lceil 633 \rceil$ for the ground state of U²³³ as made by Newton. The principal alpha group (or groups) leads to a level (or levels) some 300 kev higher and has a hindrance factor of 7. If the state at ~ 300 kev were the 3/2+[631] state observed from Pa²³³ decay, the hindrance factor seems much too low because, among other factors, the alpha wave would be limited to $l \ge 3$. This suggests that there is another level at ~ 300 kev. If it were the 7/2- state (same as the parent, Pu²³⁷), the hindrance factor is too high, hence this state might be 5/2- [752] which is expected in this region, and seen at 185 kev in the analogous nucleus, Th²³¹ (see above).

¹⁰ A. K. Kerman, Kgl. Danske Videnskab, Selskab, Mat.-fys. Medd. 30, No. 15 (1956).
¹¹ Hollander, Stephens, Asaro, and Perlman (unpublished).
¹² Freedman, Engelkemeir, Porter, Wagner, and Day (private communication to J. M. Hollander, 1957).

 ¹³ J. O. Newton, Nuclear Phys. 5, 218 (1958).
 ¹⁴ D. C. Hoffman, J. Inorg. Nuclear Chem. 4, 383 (1957).
 ¹⁵ Thomas, Vandenbosch, Glass, and Seaborg, Phys. Rev. 106, 1007 (1967). 1228 (1957).

TABLE III. Beta decay log*ft* values and transition classifications. The beta-emitting species are shown in the table by isotopic symbols followed immediately by the logit value. For each entry, the proton and neutron states of beta-emitter and daughter are designated in the first column and head row by Ω_i^a parity, and the bracketed asymptotic quantum numbers which are defined in the text. The symbols above each entry classify the beta-transitions according to the Nordheim selection rules as follows: *a* (allowed), 1*f* (first forbidden), $1f\Delta I = 2$ (first forbidden unique), 2*f* (second forbidden). The symbols following the commas further classify the transitions in terms of the asymptotic quantum number selection rules of Alagab: *u* (unhindered), *k*(hindered), *kAN* = 2 (hindered with change of 2 in the principal quantum number). Parentheses are used to code the reliability of the orbital assignments for the parent or daughter states and apply to whichever of the pair is least certain. A single set of parentheses indicates that the level assignments are tentative but based on substantial data: double parentheses indicate tentative assignments based on more meager data. No parentheses appear where assignments are considered certain.

| · · · | Proton state | | | | | | | | |
|------------------|----------------------------------|---|--|--|--|---------------------------------------|--|--|--|
| Neutron state | 3/2+[651] | 1/2 - [530] | 5/2+[642] | 5/2 - [523] | 3/2 [521] | 7/2+[633] | | | |
| 3/2+ [631] | | 1 <i>f</i> , <i>u</i> (Pa ²³³ 7.1) | - | | | | | | |
| 5/2-[752] | | 2f ((U ²³¹ >7.3)) | $\frac{1f, u}{((U^{231} 5.9))}$ | | | | | | |
| 5/2+ [633] | a, h (Th ²³¹ ~5.8) | $\begin{array}{c} 1 f \Delta I = 2, \ h \\ \mathrm{Th}^{231} > 7 \\ \mathrm{Pa}^{233} \ 9.3 \end{array}$ | a, h (Th ²³¹ 5.7) | | | | | | |
| 7/2- [743] | | | $\begin{array}{c} 1f, u \\ \mathrm{Np^{235}} 6.6 \\ \mathrm{Np^{239}} 6.5 \\ \mathrm{Pu^{237}} \sim 6.8 \end{array}$ | $a, h \Delta N = 2$ Am ²³⁹ >8 | | | | | |
| | <u></u> | $ \begin{array}{c} 1f, u \\ ((\operatorname{Pa^{233} \sim 6.5})) \\ ((\operatorname{H337} 6.2)) \end{array} $ | $2f U^{237} > 10 Np^{239} \ge 9.1$ | $1f\Delta I = 2, h$ U ²³⁷ >8.9 Am ²³⁹ >7 | $\begin{array}{c} 1f,u\\((\mathrm{Cm}^{241}7.3))\\(\mathrm{Bk}^{245}7.0)\end{array}$ | | | | |
| 1/2+ [631] | | $((Cm^{241} \ 7.1))$ | (Cm ²⁴¹ >9) | Cm ²⁴¹ >9 | | | | | |
| 5/2+ [622] | | | a, h Np ²³⁹ 7.0 | $\begin{array}{c} 1 f, u \\ ((\mathrm{U}^{239} \sim 5.8)) \\ \mathrm{Pu}^{241} 5.7 \\ \mathrm{Am}^{239} 5.9 \end{array}$ | 1 f, и Вк ²⁴⁵ 7.0 | | | | |
| 7/2+ [624] | | | a, h (Np ²³⁹ 6.8) (Pu ²⁴³ 5.9) | $\begin{array}{c} 1f, \ u \\ (\mathrm{Pu}^{243} \ 6.1) \\ (\mathrm{Am}^{239} \ 6.1) \end{array}$ | $1f\Delta I = 2, h$ Bk ²⁴⁵ >8.3 | a, h (Pu ²⁴³ 5.5) | | | |
| 9/2- [734] | | | | · | | $\frac{1f, u}{\mathrm{Bk}^{249} 7.0}$ | | | |

^a The spin of the lowest-lying member of a rotational band is generally the same as the value of Ω for that band. For some $\Omega = 1/2$ bands, however, the higher-spin rotational states have lower energy than the spin-1/2 state. Examples of this are the spin-3/2 grounds tates of Pa²²¹ and Pa²²³ which have been assigned $\Omega = 1/2$. ^b G. Alaga, Phys. Rev. 100, 432 (1955).

U²³⁵ (neutron number 143).—The puzzle surrounding the low-lying energy levels of U²³⁵ has recently been solved.^{16,17} The principle (unhindered) alpha group of Pu²³⁹, which has spin 1/2, apparently led to the ground state of U^{235} , which has spin 7/2 in contradiction to our major selection rules, but it was found that this favored alpha group instead went to the expected spin-1/2state, which is an isomer lying less than 0.1 key above the ground state. The Nilsson level assignments as shown in Table I have been discussed by Asaro and Perlman.¹⁶ Higher-lying levels populated by Pu²³⁹ alpha decay have also been seen,^{18,19} but there are not yet sufficient data to make assignments.

U²³⁷ (neutron number 145).—The ground-state spin of U²³⁷ has not been measured, but it is almost certainly 1/2 or 3/2 because in its beta decay to Np²³⁷ no state with spin higher than 3/2 is populated.²⁰ Rasmussen et al. have suggested possible assignments $1/2 - \lceil 501 \rceil$ and $1/2 + \lceil 631 \rceil$ ²⁰ Because there is no evidence for the $1/2 - \lceil 501 \rceil$ state in other nuclei in this vicinity, and since the $\log ft$ values can be reasonably well explained on the basis of the $1/2 + \lceil 631 \rceil$ assignment, we definitely prefer this latter one. It will be seen that two other nuclei with the same neutron number, Pu²³⁹ and Cm²⁴¹, also have this configuration in their ground states.

The alpha decay of Pu²⁴¹ has been investigated^{21,22} and the favored group goes to a level in U^{237} at 145 kev. The ground state of Pu²⁴¹ has been assigned $5/2 + \lceil 622 \rceil$ as will be discussed below, hence the U²³⁷ level at 145

¹⁶ F. Asaro and I. Perlman, Phys. Rev. 107, 318 (1957)

 ¹⁷ Huizenga, Rao, and Engelkemeir, Phys. Rev. 107, 318 (1957).
 ¹⁷ Huizenga, Rao, and Engelkemeir, Phys. Rev. 107, 319 (1957).
 ¹⁸ F. Asaro and I. Perlman (unpublished data, 1956).
 ¹⁹ Novikova, Kondrat'ev, Sobalev, and Gol'din, J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 1018 (1957) [translation: Soviet Phys. JETP 5, 832 (1957)].

²⁰ Rasmussen, Canavan, and Hollander, Phys. Rev. 107, 141 (1957). ²¹ Asaro, Stephens, and Perlman (unpublished data, 1957).

²² Freedman, Wagner, and Engelkemeir, Phys. Rev. 88, 1155 (1952).

kev is ascribed to the same orbital. The gamma-ray de-excitation properties of this level support the idea that it has the same parity as the ground state. The transitions to the ground-state rotational band appear to be largely M1 (presumably K forbidden). It will be seen that the same situation is found in Pu²³⁹ in which the 5/2+ state appears at a somewhat higher energy. As has been mentioned, this state $5/2 + \lceil 622 \rceil$ shows up as the ground state for neutron number 147.

U²³⁹ (neutron number 147).—There is little experimental information on this short-lived beta emitter, but it might be expected in analogy to Pu²⁴¹ that the ground state is 5/2 + [622]. This assignment is consistent with the beta-decay properties of U^{239} , $2^{23,24}$ in which a 5/2state in Np²³⁹ of 74 kev receives most of the beta population.

Pu²³⁷ (neutron number 143).—Although the spin of Pu²³⁷ has not been measured directly, the assignments of the ground state to 7/2- [743] and a state at 145 kev to 1/2+ [631] are considered to be on rather firm ground. These states occur very close together in U²³⁵ and result in an isomeric transition with a half-life of 26.5 min. On the basis that the structure would repeat in Pu²³⁷, Stephens et al.²⁵ searched for and found an E3 isomeric transition, with a 0.18-sec half-life and energy as indicated, resulting from the alpha decay of Cm²⁴¹. The ground-state assignment 7/2 - [743] has also been made by Hoffman and Dropesky²⁶ from a study of the electron-capture decay of Pu²³⁷ to Np²³⁷.

Pu²³⁹ (neutron number 145).—The levels of Pu²³⁹ are populated following the decay of Np²³⁹,²⁷ Am²³⁹,²⁸ and Cm²⁴³, ^{29,30} and the Coulomb excitation of Pu^{239 31} itself. The level scheme is thus quite well worked out, and orbital assignments have been given by Hollander^{28,32} and by Asaro et al.30

There is a well-defined rotational band based upon the K=1/2 ground state $(1/2+ \lceil 631 \rceil)$ which orbital appears as the first excited state in Pu²³⁷ and U²³⁵ as already discussed. The $7/2 - \lceil 743 \rceil$ configuration which is the ground state for U²³⁵ and Pu²³⁷ occurs at 392 kev and the 5/2+ [622] configuration which will appear as the ground state for neutron number 147, occurs here at 286 kev. These assignments have been discussed in detail,^{27,25} but additional comment seems to be in order concerning a level at 512 key assigned by

Hollander et al.²⁷ as either 5/2+ or 7/2+. As pointed out,^{28,32} the only two assignments that seem reasonable are 5/2 + [633] and 7/2 + [624]. The state 5/2 + [633]would result from opening a filled level that appeared as the ground state for neutron number 141, and the 7/2+ [624] would be a new level to appear for higher neutron numbers as the ground state.

Of these, the choice of the 7/2+ assignment seems preferable to us. As will be seen, $5/2 + \lceil 622 \rceil$ (which is the 286-kev level in Pu^{239}) and 7/2+ [624] are found in Pu²⁴¹ and Cm²⁴⁵ (see Table I), and the spacings between these levels are similar, as might be expected because the energies of these orbitals have a rather flat dependence on nuclear deformation (see Fig. 1). A similar spacing also occurs in Pu²³⁹ between the state $5/2+ \lceil 622 \rceil$ and the one of 512 kev, hence this may be taken as some evidence that this latter state is 7/2+[624]. Also the absence of radiation from the 512-kev state to the ground state is nicely explained by the $7/2 + \lceil 624 \rceil$ assignment, although even for $5/2 + \lceil 633 \rceil$ the E2 transition to the ground state might not compete favorably with M1 transitions to the $5/2 + \lceil 622 \rceil$ band.

Pu²⁴¹ (neutron number 147).—The ground-state spin of Pu²⁴¹ has been measured as 5/2.³³ The 5/2+ [633] orbital was the ground state for neutron number 141, so the ground state of Pu²⁴¹ is almost surely $5/2 + \lceil 622 \rceil$ (see Fig. 1). The only information on excited states is that obtained from the alpha decay of Cm²⁴⁵.^{34,35} The favored alpha group leads to a state 172 key above ground. Another state of 58 kev higher energy has been observed and interpreted as the first member of the rotational band based upon the 172-kev state.³⁵ This spacing suggests that the band has K=7/2 or higher. The parity is fixed as even from the observation that the 172-kev state decays to the ground-state band by M1 transitions. In particular, the transition to the 5/2+ ground state is definitely M1. This fact not only fixes the parity of the 172-kev state, but also is consistent with the spin assignment of 7/2. The only Nilsson level in this region with these properties is 7/2+ [624], and the assignment is considered to be reasonably certain.

Pu²⁴³ (neutron number 149).—Information is available only on the ground state of Pu²⁴³ and is derived from the decay of this isotope to Am²⁴³.³⁶⁻³⁸ States in Am²⁴³ having spins 5/2, 7/2, and probably 9/2 seem to receive direct beta population from Pu²⁴³ (see discussion of Am²⁴³) so that a spin of 7/2 for Pu²⁴³ seems most reasonable. Because this coincides with the expected

²³ Freedman, Wagner, Engelkemeir, Huizenga, and Magnussen, quoted by Hollander, Perlman, and Seaborg, Revs. Modern Phys. **25**, 469 (1953).

²⁴ R. Lessler and G. T. Seaborg (unpublished).

²⁵ Stephens, Asaro, Amiel, and Perlman, Phys. Rev. 107, 1456 (1957)

²⁶ D. C. Hoffman and B. J. Dropesky, Phys. Rev. 109, 1282 (1958). ²⁷ Hollander, Smith, and Mihelich, Phys. Rev. 102, 740 (1956).

²⁸ Smith, Gibson, and Hollander, Phys. Rev. 105, 1514 (1957).

 ²⁹ Asaro, Thompson, and Perlman, Phys. Rev. 92, 694 (1953).
 ³⁰ Asaro, Thompson, Stephens, and Perlman, Bull. Am. Phys.

 ³¹ J. O. Newton, Nuclear Phys. 3, 345 (1957).
 ³² Jack M. Hollander, Phys. Rev. 105, 1518 (1957).

³³ Bleaney, Llewellyn, Pryce, and Hall, Phil. Mag. 45, 773, 991

^{(1954).} ³⁴ Hulet, Thompson, and Ghiorso, Phys. Rev. **95**, 1703 (1954). ²⁶ Perlman (unpublished data, 1954) ³⁵ Hulet, Holmpson, and Griotso, Filys. Rev. 20, 1105 (1994).
 ³⁵ Asaro, Thompson, and Perlman (unpublished data, 1954) reported by I. Perlman and J. O. Rasmussen, in *Handbuch der Physik* (Springer-Verlag, Berlin, 1957), Vol. 42.
 ³⁶ Engelkemeir, Fields, and Huizenga, Phys. Rev. 90, 6 (1953).

³⁷ Freedman, Porter, Wagner, Day, and Engelkemeir (private communication to Jack M. Hollander).

³⁸ Asaro, Stephens, and Perlman (unpublished data, 1956).

Nilsson level, $7/2 + \lceil 624 \rceil$, this assignment is given to Pu²⁴³.

Cm²⁴¹ (neutron number 145).—The ground state of Cm²⁴¹ is almost surely the same as that of Pu²³⁹ (also with 145 neutrons), 1/2+ [631]. This assignment was made by Stephens et al.25 and the arguments will be reviewed briefly here. Cm²⁴¹ decays by electron capture to Am^{241} ^{39,40} and although there are low-lying 5/2+and 5/2- states, neither of these is directly populated. Instead decay takes place to states that seem to have spins of 3/2 or 1/2.

As already mentioned when Pu²³⁷ was discussed, the favored alpha decay of Cm²⁴¹ populates a 0.18-sec isomeric state of Pu²³⁷ which drops to the ground state by an E3 transition.²⁵ The evidence is excellent that the isomeric state has the assignment $1/2 + \lceil 631 \rceil$. hence the ground state of Cm²⁴¹ is almost surely the same.

There is no information at present on the excited states of Cm²⁴¹, although applicable data might be obtained from studies of the alpha decay of Cf²⁴⁵.

Cm²⁴³ (neutron number 147).—The ground state of Cm^{243} might be expected to be 5/2+ [622], the same as Pu²⁴¹. There is some experimental evidence for this assignment from the study of the alpha spectrum of Cm²⁴³.^{29,30} The favored alpha group populates a level in Pu²³⁹ at 286 kev and the assignment of 5/2+ [622]has been made for this state.^{29,30}

Excited states of Cm²⁴³ are known from the electroncapture decay of Bk²⁴³, but no information is available that is suitable for making assignments.

 Cm^{245} (neutron number 149).—As seen in Fig. 1, the expectations for neutron number 149 are either 7/2+[624] and 9/2- [734]. In a brief earlier report resulting from the study of the alpha decay of Cf²⁴⁹, Stephens et al.⁴¹ made the 7/2+ [624] assignment to the ground state of Cm^{245} , while the 9/2- [734] orbital appeared at a level 394 key above ground. At 255 kev above ground, the orbital $5/2+ \lceil 622 \rceil$ reappeared through the opening of a filled level. (See Cm²⁴³ and Pu²⁴¹, where this orbital represents the ground state.) Because no discussion was presented in that report,⁴¹ a brief account of these assignments and one other will be given here.

The states of Cm²⁴⁵ have been studied through the electron-capture decay^{42,43} of Bk²⁴⁵ and the beta decay⁴⁴ of Am²⁴⁵ as well as from the alpha decay of Cf²⁴⁹.^{41,45} In addition, the alpha decay of Cm²⁴⁵ has been studied.^{34,35}

and as has been already discussed under Pu²⁴¹, the 7/2+ [624] assignment is likely for the Cm²⁴⁵ ground state.

Am²⁴⁵ presumably has spin and parity 5/2 - or 5/2 +, and in its decay to Cm245 populates both the ground state and a level at 255 kev. Bk²⁴⁵, which we believe to have spin 3/2, does not populate the ground state of Cm²⁴⁵, but only the 255-kev level. This 255-kev level decays only to the ground state of Cm²⁴⁵ (not to higher members of the ground-state rotational band) by an M1 transition. Furthermore, four members of the rotational band based on the 255-kev level have been observed in the alpha decay of Cf²⁴⁹, and the spacing and alpha population of these states suggest a spin 5/2for the 255-kev level. All these data rather strongly suggest a spin and parity of 5/2+ for the 255-kev level. and its assignment as the $5/2 + \lceil 622 \rceil$ Nilsson level seems almost certain.

The favored alpha decay of Cf²⁴⁹ populates a rotational band whose base level is 394 key above the ground state of Cm²⁴⁵. This level decays by transitions that appear to be E1 to the 7/2 and 9/2 members of the ground-state rotational band, with no detectable branching to the 5/2+ [622] band at 255 kev. No branching to this (394-kev) level was observed in the decay of either Am²⁴⁵ or Bk²⁴⁵ (spins 5/2 and 3/2). From these data we conclude that the spin and parity of the 394-kev level is 7/2- or 9/2-, with 9/2somewhat more likely (see Fig. 1). An alpha-gamma angular-distribution measurement was made to distinguish between these two choices,⁴¹ and the results, while not absolutely definitive, also favored the 9/2spin and parity. We thus conclude that the 394-key level (and hence the ground state of Cf^{249}) is very likely the $9/2 - \lceil 734 \rceil$ Nilsson level. The only other level observed in Cm²⁴⁵ is one at about 630 kev populated in the decay of Bk²⁴⁵. This level decays by a single gamma ray to the 5/2+ [622] state and therefore presumably has low spin. We have very tentatively assigned it as the 1/2+ [631] state which comes in as the ground state for neutron number 145.

Cf²⁴⁵ (neutron number 147).—Chetham-Strode and co-workers⁴⁶ have obtained evidence that the alphadecay of Cf²⁴⁵ leads predominantly to the ground state of Cm²⁴¹, and we can say that the transition is unhindered or only slightly hindered. In the absence of other information, it might be inferred that the ground state of Cf²⁴⁵ is the same as that of Cm²⁴¹, 1/2+ [631]. This assignment is possible but not expected. Because there is no evidence bearing on this assignment other than the observation cited, we have not made an entry in Table I.

Cf²⁴⁹ (neutron number 151).—It has already been suggested under the Cm²⁴⁵ discussion that Cf²⁴⁹ ground state has the assignment $9/2 - \lceil 734 \rceil$. Consistent with

³⁹ Amiel, Albridge, and Asaro (unpublished data, 1956).

⁴⁰ Amiel, Stephens, and Asaro (unpublished data, 1957).

⁴¹ Stephens, Asaro, Thompson, and Perlman, Bull. Am. Phys. Soc. Ser. II, **2**, 394 (1957).

 ⁴² Magnussen, Freedman, Engelkemeir, Fields, and Wagner, Phys. Rev. 102, 1097 (1956).
 ⁴³ Chetham-Strode, Stephens, Asaro, and Perlman (unpublished)

data, 1956).

 ⁴⁴ Brown, Hoffman, Crane, Balagna, Higgins, Barnes, Hoff, Smith, Mize, and Bunker, J. Inorg. Nuclear Chem. 1, 254 (1955).
 ⁴⁵ L. B. Magnussen (private communication to Jack M. Hollander, 1957). See also reference 42.

⁴⁶ Chetham-Strode, Choppin, and Harvey, Phys. Rev. 102, 747 (1956).

this assignment is the value log ft=7.0 for the beta decay of Bk²⁴⁹,⁴⁷ since Bk²⁴⁹ is thought to have spin and parity 7/2+ (see Table III).

ODD-PROTON NUCLEI

It has been seen (Table I) that the same level repeats for ground states of nuclei having the same (odd) neutron number. This is a reflection of the adequacy of the model employed. Similar behavior for unpaired protons allows grouping of isotopes of each *odd* element for discussion. The applicable Nilsson diagram is shown in Fig. 2, and the summary of assignments in Table II.

Actinium (proton number 89).—It should perhaps be pointed out at the start that the lack of apparent rotational structure in the actinium (and, for that matter, the protactinium) isotopes has for some time been noted, and initially we felt that these isotopes were outside the region of stable spheroidal deformation and could not be described by the strong-coupling approximation of the unified nuclear model. We have recently concluded that this absence of simple rotational bands is probably due rather to the many anomalous K=1/2rotational bands in this region, and the influence of these bands through rotational-particle coupling on the K=3/2 and even in some cases, the K=5/2 bands in this vicinity. The assignments made for the actinium (and protactinium) isotopes are based on this conclusion, and therefore are perhaps somewhat less certain than those made in regions where the unified nuclear model is certainly applicable.

The energy levels of Ac²²⁷ have been studied following the beta decay of Ra²²⁷,^{48,49} and also rather thoroughly following the alpha decay of Pa231.50-56 The levels of Ac²²⁵ have thus far been studied only as populated by the beta decay of Ra²²⁵, ^{57,58} although some information from the alpha decay of Pa²²⁹ is being obtained.⁵⁹ For this reason considerably more is known about the energy levels in Ac²²⁷, and this isotope will be discussed first.

1957)

level in Ac²²⁷ at 330 kev. Because the ground state of Pa²³¹ is believed to be the 3/2 member of the K=1/2rotational band based on the state, $1/2 - \lceil 530 \rceil$, this assignment is also given to the 330-kev level in Ac²²⁷. It should be emphasized that the state at 330 kev has spin 3/2 according to this assignment but is properly designated by the K quantum number of the orbital which is 1/2. An unambiguous designation would be 3/2, 1/2 - [530], showing that it is the I = 3/2 member of the K=1/2 band. The I=1/2 and I=7/2 members of this band are also presumably seen in Ac²²⁷ at energies of 356 and 386 kev, respectively. The alpha populations from Pa²³¹ are in good agreement with those expected for favored decay to this band.

The ground state of Ac²²⁷ has a measured spin of 3/2 and is probably connected with the 330-kev level by what appears to be an M^2 transition. The ground state must therefore have even parity, and because the two states both have spin 3/2 there is implied a strong retardation of the permitted E1 transition. The most likely 3/2 + assignment is 3/2 + $\lceil 651 \rceil$ (see Fig. 2).

Between about 25 and 125 kev above the ground state of Ac²²⁷ there are at least six levels observed, most of which can be shown to have odd parity if the two previous assignments are correct. There is no apparent rotational structure among these levels, but, as will be pointed out, this does not necessarily mean that these states all have different intrinsic configurations. Under conditions which could apply here, there can be severe distortions in level spacings in a rotational band.

Returning to the assignments of these levels, it will be noted from Fig. 2 that aside from $1/2 - \lceil 530 \rceil$ the only odd-parity states in the vicinity of 3/2+ [651] are those connected with the $h_{9/2}$ orbital: 1/2 - [541], 3/2- [532], and 5/2- [523]. The positions of these levels for proton number 89 are not known, but it will be seen that the state $5/2 - \lceil 523 \rceil$ comes in as shown in Fig. 2 as the ground state for proton number 95. In order for it to lie below 1/2 - [530] (at 330 kev in Ac²²⁷), the nuclear deformation would have to be considerably less for this state in Ac²²⁷ than that indicated by the dashed line in Fig. 2. In this regard, the spin 5/2for Pa²²⁹ suggested by Hill⁵⁹ (see section on protactinium) would make this assumption not unreasonable. From the foregoing arguments we might expect the 3/2- [532] and 5/2- [523] levels to lie closest to the 3/2+ [651] ground state of Ac²²⁷ and suggest tentatively that a group of levels around 27 kev be assigned the orbital $3/2 - \lceil 532 \rceil$ and those around 110 kev, 5/2- [532]. The apparent absence of rotational structure is attributed to the displacement of levels by the mechanism discussed by Kerman.¹⁰ However, these assignments do not seem definite enough for inclusion in Table II.

Very little is known about Ac²²⁵ but there is one piece of information which suggests similarity to Ac²²⁷. In Ac^{227} there is a 27-kev E1 transition between an

The favored alpha decay of Pa²³¹ seems to populate a

 ⁴⁷ Magnussen, Studier, Fields, Stephens, Meek, Friedman, Diamond, and Huizenga, Phys. Rev. 96, 1576 (1954).
 ⁴⁸ J. P. Butler and J. S. Adam, Phys. Rev. 91, 1219 (1953).

⁴⁹ Asaro, Stephens, and Perlman (unpublished data, 1956

⁵⁰ Rosenblum, Cotton, and Bouissières, Compt. rend. 229, 825 (1949).

⁵¹ Gol'din, Tret'yakov, and Novikova, Froceedings of the Con-ference of the Academy of Sciences of the U.S.S.R. on the Peaceful Uses of Atomic Energy, Moscow, July, 1955 (Akademiia Nauk, S.S.S.R., Moscow, 1955), p. 226 [English translation by Con-sultants Bureau, New York: U. S. Atomic Energy Commission Report TR-2435, 1956]. ⁵² D. Fully Main and Perlman (unpublished data, 1955). ⁵³ D. Fully Main and M. Piou, J. phys. radium 14, 65 (1953). ⁵¹ Gol'din, Tret'yakov, and Novikova, Proceedings of the Con-

 ⁵³ P. Falk-Vairant and M. Riou, J. phys. radium 14, 65 (1953).
 ⁵⁴ A. Mauhasser and R. Riou, Compt. rend. 238, 2520 (1954).
 ⁵⁵ Teillac, Riou, and Desneiges, Compt. rend. 237, 41 (1953).

⁵⁶ Stephens, Asaro, and Perlman (unpublished data, 1956-

⁸⁷ Magnussen, Wagner, Engelkemeir, and Freedman, Argonne National Laboratory Report ANL-5386, January, 1955 (unpublished).

⁵⁸ Perlman, Stephens, and Asaro, Phys. Rev. 98, 262(A) (1955). ⁵⁹ Max Hill (private communication, 1958).

excited state of 27 kev and ground. In Ac²²⁵ a 40-kev E1 transition is found and is the only prominent gamma transition following Ra²²⁵ decay. This implies that there is a level in Ac²²⁵ at 40 kev bearing the same relation to the ground state as the 27-kev level in Ac²²⁷.

Protactinium (proton number 91).---A considerable number of protactinium isotopes are known but substantial data are available for only Pa²³¹ and Pa²³³. The energy levels of Pa²³¹ have been studied from the beta decay of Th²³¹,^{11,60-62} U²³¹ electron capture,^{11,63} alpha decay of Np²³⁵,⁶⁴ and Coulomb excitation of Pa²³¹ itself.65 The states of Pa233 have been studied in conjunction with Th²³³ beta decay¹² and Np²³⁷ alpha decay.66,67

The ground-state spin of Pa²³¹ has been measured as 3/2 and that for Pa²³³ is deduced to be the same from the decay properties. Each is probably the I=3/2member of the K=1/2 band, 1/2- [530], which has already been mentioned in the discussion of Ac²²⁷ where this state appears at 330 kev above ground. In Pa²³³, the I=1/2, 3/2, 5/2, and 7/2 members have been observed, and these lie at $\sim 6, 0, 69$, and 56 kev, respectively. The structure and spacings in this anomalous rotational band are probably much the same in Pa²³¹.

In both Pa²³¹ and Pa²³³ there is another intrinsic state at about 85 kev. Each drops to the 3/2 and 7/2members of the ground-state band by E1 transitions and has been assigned $5/2 + \lceil 642 \rceil$ partly on the basis of these gamma transitions and also because this level in Pa²³³ receives the favored alpha transition from Np²³⁷ decay and 5/2+ [642] is almost surely the ground state of Np²³⁷.

A somewhat uncertain assignment of $3/2 + \lceil 651 \rceil$ has been made for levels in Pa²³¹ and Pa²³³ at 166 kev and ~ 200 kev, respectively. There is no obvious rotational-band structure based on either this state or the 5/2+ [642] state. However, this would be expected, since such similar states lying so close to each other might be expected to interact in such a way as to distort the normal rotational-level spacings. Calculations are underway at present to see if the experimental level pattern can be reproduced, and the preliminary results seem favorable.

There is also some information available for assigning the ground states of Pa²³⁵ and Pa²²⁹. Pa²³⁵ might be expected to have the $1/2 - \lceil 530 \rceil$ band as its ground

state in analogy to Pa²³¹ and Pa²³³. This would be consistent with the observed decay of Pa235 without gamma-ray emission to U^{235} , ⁶⁸ since a 1/2+ state lies within 0.1 key of the ground state of U²³⁵. Also the log ft value for this decay is very similar to that observed for decay between these two states in other nuclei (see Table III). Nevertheless, because the ground state of U^{235} has a spin of 7/2-, almost any spin up to 9/2 would be possible for Pa²³⁵. An enlightening experiment would be a determination of whether the 26-min isomeric state (spin 1/2) of U²³⁵ is populated in the decay of Pa²³⁵, but this has not yet been done.

Hill⁵⁹ has suggested that the ground state of Pa²²⁹ is 5/2 on the grounds that states having spins of 7/2 and probably 5/2 are populated in the decay of this isotope to Th²²⁹, and also on preliminary evidence that a 5/2rotational band in Ac²²⁵ receives the favored alpha decay of Pa²²⁹. If this is the case, the only two reasonable assignments would be $5/2 + \lfloor 642 \rfloor$ or $5/2 - \lfloor 523 \rfloor$. It is not possible to make a clear choice between these assignments. We slightly prefer the latter, however, because it seems less likely (see Fig. 2) that the 1/2-[530] state (ground states of Pa²³¹ and Pa²³³) would cross the $5/2 + \lceil 642 \rceil$ state than the $5/2 - \lceil 523 \rceil$ state.

Neptunium (proton number 93).—At several places in the previous discussions, special use has been made of observed E1 transitions in identifying states (see, for example, protactinium) because such transitions limit considerably the possible choices. For low energies, (<100 kev), E1 transitions are particularly easy to identify because the conversion coefficients are small and unique. In fact, very often a rough intensity measurement of the photon is sufficient for identifying the transition unambiguously as E1. Three isotopes of neptunium, Np^{235} , Np^{237} , and Np^{239} , all have prominent E1 transitions following alpha decay of their respective parents,69 and the assignments of the states involved for two of these (Np²³⁷ and Np²³⁹) have been previously made.⁷⁰ These are $5/2 + \lfloor 642 \rfloor$ for the ground states and $5/2 - \lceil 523 \rceil$ for low-lying excited states. The arguments for these assignments are considered sound and will not be discussed further here. The energies may be seen in Table II. The supporting evidence in Np²³⁵ is not so extensive but the analogy in energy and intensity of the E1 transition is so close that these same assignments may be made with confidence. It may be mentioned that the electron-capture decay characteristics of Np²³⁵ are consistent with the ground-state assignment.71

Rotational states based upon these two intrinsic states have been identified in Np²³⁷ and Np²³⁹, but the only other well-studied intrinsic state in an odd-mass

⁶⁰ Jose O. Juliano, University of California Radiation Labora-tory Report UCRL-3733, April, 1957 (unpublished). ⁶¹ J. P. Mize and J. W. Starner, Bull. Am. Phys. Soc. Ser. II, 1, 171 (1956).

⁶² D. Strominger and J. O. Rasmussen, Phys. Rev. 100, 844 (1955).

 ⁽¹⁾ S5).
 ⁶³ W. W. T. Crane and I. Perlman (unpublished data, 1950).
 ⁶⁴ Hoff, Olsen, and Mann, Phys. Rev. 102, 805 (1956).
 ⁶⁵ J. O. Newton, Nuclear Phys. 3, 345 (1957); and private communication, 1957.

 ⁶⁶ Magnusson, Engelkemeir, Freedman, Porter, and Wagner, Phys. Rev. 100, 1237(A) (1955); and private communication from Jack M. Hollander, 1957.
 ⁶⁷ Stephens, Asaro, and Perlman (unpublished data, 1957).

 ⁶⁸ W. W. Meinke and G. T. Seaborg, Phys. Rev. 78, 475 (1950).
 ⁶⁹ Asaro, Stephens, Gibson, Glass, and Perlman, Phys. Rev. 100, 1541 (1955).

⁷⁰ Hollander, Smith, and Rasmussen, Phys. Rev. 102, 1372 (1956)

⁷¹ Hoff, Olsen, and Mann, Phys. Rev. 102, 805 (1956).

neptunium isotope is the 268-kev level of Np²³⁷, which very likely has spin and parity 3/2-, and was assigned as the $3/2 - \lceil 521 \rceil$ state by Rasmussen *et al.*²⁰ Although this assignment is certainly possible, we prefer that of 1/2- [530] with the 3/2- member of this band lying lowest, the same assignment made for the ground states of protactinium isotopes. This assignment is preferred for the following reasons: (1) An energy of 268 kev is already somewhat higher than might be expected for the 1/2- [530] band on the basis of its position relative to the 5/2+ [642] state in the protactinium isotopes, yet it is quite unlikely that the 1/2- [530] band could lie at lower energies in Np²³⁷ and not have been detected from the beta decay of U²³⁷. On the other hand, from the assignments made below for the americium and berkelium isotopes, it would be expected that the 3/2-[521] level would lie at energies somewhat higher than 268 kev in Np²³⁷. (2) De-excitation of the 267-kev level has been shown to have M2 decay in competition with E1, and E2 with $M1^{20}$ which can be best explained by the 1/2- [530] assignment, since this would involve K-forbidden restrictions for the dipole transitions. Neither of these arguments is very conclusive, however, so the preference for 1/2 - [530] over 3/2 - [521] is slight.

Americium (proton number 95).—The data on the americium isotopes come from the alpha decay of the berkelium isotopes, 42,43,72 the electron capture decay of Cm^{241} , 30,40 the beta decay of Pu^{243} , $^{36-38}$ and the alpha decay of the americium isotopes, themselves. $^{51,73-78}$ Americium-241 and Am^{243} have measured spins of 5/2, 79,80 and from studies of their alpha decay to Np^{237} and Np^{239} , it is reasonably certain that their ground state is the level 5/2— [523]. The ground state of Am^{239} is assigned as this same Nilsson state because its pattern of alpha decay seems to be quite similar to that of Am^{241} and Am^{243} , 69

We shall next turn to two excited states in Am²⁴³, at 84 and 465 kev, observed following the beta decay of Pu²⁴³. The 84-kev level decays by a prominent *E*1 transition to the ground state of Am²⁴³ and by a very weak *E*1 transition to an ~40-kev level—presumably the first member of the ground-state rotational band. This fixes the spin and parity of the 84-kev level at 5/2+ or 7/2+. Assignment is made to K=5/2, in particular 5/2+ [642], because the energy spacing with respect to the ground state, 5/2-[523], is similar to that seen in neptunium isotopes, except that the states are reversed. The 465-kev level decays by a predominantly *M*1 transition to the 84-kev level and to one, and possibly two, higher members of the rotational band based on the 84-kev level. Thus the parity of the 465-kev band is *even*, and the spin is probably 7/2 or 9/2, although 5/2 is also a possibility. The assignment 7/2+[633] is consistent with these data and with the proposed 7/2+ spin of Pu²⁴³. There is no other Nilsson level in the vicinity that seems to be satisfactory for this state.

The alpha decay properties of Bk²⁴³, Bk²⁴⁵, and Bk²⁴⁷ have all been studied, and provide an interesting comparison of the levels in Am²³⁹, Am²⁴¹, and Am²⁴³. In each case the ground-state transition is highly hindered, with hindrance factors of 660, 450, and \geq 500, respectively. The lowest intrinsic excited states observed in each of the americium isotopes also have rather similar alpha-hindrance factors. These states lie at energies of 187, 206, and 84 kev (in order of increasing mass number) and their alpha-hindrance factors are 54, 37, and 68, respectively. This suggests the same Nilsson level is involved for each isotope, and because the 84-kev level in Am^{243} has been assigned as the 5/2+ [642] state, we suggest this assignment for the other two levels as well. There is additional evidence in favor of these assignments. In both Am²³⁹ and Am²⁴¹ the levels (187 and 206 kev) drop to the I=5/2 and I=7/2members of the ground-state (5/2 - [523]) rotational band by transitions which are probably E1 (although E2 cannot be ruled out). These are just the two levels to which we would expect decay from the $5/2 + \lceil 642 \rceil$ state. This argument, of course, hinges on whether or not the radiations observed are indeed E1. Furthermore, Cm^{241} (spin 1/2+) has no observed population to the 206-kev level in Am²⁴¹, which is to be expected if the 5/2+ assignment of this level is correct. The differences in spacing of the $5/2 + \lceil 642 \rceil$ levels relative to the ground states in the three americium isotopes are rather large and will be discussed later.

A third state which seems to be populated systematically in the alpha decay of the berkelium isotopes lies at energies of 540, 480, and 265 kev in Am²³⁹, Am²⁴¹, and Am²⁴³, respectively. The hindrance factors to these states are 3.6, 2.3, and 4.2, respectively. In Am²³⁹ the 540-kev state decays to the ground state by a transition whose multipolarity is uncertain. In Am²⁴¹, however, the 480-kev level is also heavily populated in the electron-capture decay of Cm²⁴¹, and decays to the ground state of Am^{241} by a predominantly M1 transition (with apparently some E2 admixture). These data, together with the 1/2+ spin of Cm²⁴¹ and 5/2- groundstate spin of Am²⁴¹, fix the spin and parity of the 480-kev level at 3/2-. In Am²⁴³ the 265-kev level also decays to the ground state by a transition that has been shown to be predominantly M1. Thus the 540-, 480-, and

⁷² A. Chetham-Strode, University of California Radiation Laboratory Report UCRL-3322, June 26, 1956 (unpublished). ⁷³ Anone Reported and Replace Phys. Rev. 87, 277 (1952).

 ⁷⁸ Asaro, Reynolds, and Perlman, Phys. Rev. 87, 277 (1952).
 ⁷⁴ P. P. Day, Phys. Rev. 97, 689 (1955).

⁷⁵ Jaffe, Passell, Browne, and Perlman, Phys. Rev. 97, 142 (1955).

⁷⁶ F. Asaro and I. Perlman, Phys. Rev. 93, 1423 (1954).

⁷⁷ Stephens, Hummel, Asaro, and Perlman, Phys. Rev. 98, 261(A) (1955).

⁷⁸ Rosenblum, Valadares, and Milsted, J. phys. radium 18, 609 (1957).

⁷⁹ M. Fred and F. S. Tomkins, Phys. Rev. 89, 318 (1953).

⁸⁰ J. G. Conway and R. D. McLaughlin, Phys. Rev. 94, 498 (1954).

265-kev levels in the three americium isotopes seem to be quite similar, and probably all have spin and parity 3/2-. On this basis the Nilsson assignment for these levels is almost certainly either 1/2- [530] (3/2member lying lowest) or 3/2- [521]. We slightly prefer the former assignment because the level seems to vary in energy relative to the 5/2- [523] state in about the same manner as the 5/2+ [642] state. From the slopes of the levels on the Nilsson diagram this is more reasonable for the 1/2- [530] than the 3/2-[521] assignment.

The small hindrance factors for the alpha decay of berkelium isotopes to these states are worthy of note. It will be seen that the ground states of the berkelium isotopes are assigned 3/2- [521] and the states in americium I=3/2 member of 1/2- [530]. The small hindrance factors are probably associated with the structural similarity of these orbitals. Also the 3/2- [521] level in the americium isotopes is probably not too far away in energy from this level, so that mixing of the type described by Kerman¹⁰ would further lower the hindrance factor.

The large variation in spacing between the 5/2+ [642] and 1/2- [530] states in the three americium isotopes relative to the 5/2- [523] ground state is somewhat puzzling. This is possibly due to a larger nuclear deformation in Am²³⁹ and Am²⁴¹ than in Am²⁴³ (or probably Am²⁴⁵). The reason for larger nuclear deformations in Am²³⁹ and Am²⁴¹ might be found in the neutron levels filling in this vicinity, which are the steeply down-sloping (deforming) 7/2- [743] and 1/2+ [631] orbitals. On the other hand, around Am²⁴³ and Am²⁴⁵ the much flatter (less deforming) neutron levels, 5/2+ [622] and 7/2+ [624], are filling.

The only other level observed in an americium isotope is the 630-kev level populated in Am^{241} by Cm^{241} electron-capture decay. This state decays both to the ground and the 480-kev states, and we have very tentatively given it the assignment, 3/2- [521]. The assignment 3/2+ [651] is also possible but seems less likely because this state would be expected to decay to the 5/2+ [642] state, and no such gamma transition could be observed.

Berkelium (proton number 97).—The similarity of the alpha decay schemes of Bk²⁴³, Bk²⁴⁵, and Bk²⁴⁷ has already been described, and thus we believe these three isotopes all have the same Nilsson level for their ground states. Information as to the identity of this level may be derived from the electron-capture decay of Bk²⁴⁵ to Cm²⁴⁵.^{42,43} This decay goes predominantly to a level assigned spin and parity 5/2+ in Cm²⁴⁵, with no detectable branching to the 7/2+ (ground) or 9/2-(394 kev) states. This suggests a spin of 3/2 for Bk²⁴⁵, although 5/2 or 7/2 would also be possible. From the Nilsson diagrams, either the 3/2- [521] or the 7/2+ [633] level would be expected as the ground state of

the berkelium isotopes, and on the basis of the foregoing data, we definitely prefer the 3/2- state.

Considerably more information is available about the energy levels of Bk²⁴⁹, mostly from the alpha decay of E²⁵³.^{81,82} The favored alpha decay of E²⁵³ goes to the ground state of Bk²⁴⁹, and four members of the groundstate rotational band have been observed to receive alpha population. This means that the ground-state configurations of Bk249 and E253 are identical. The spacing of this band gives the best agreement with a 7/2spin, but the rotational constant $(\hbar^2/2\Im)$ is considerably smaller than usual for this region. However, for an even-parity odd-proton state in this region an abnormally small rotational constant is reasonable according to arguments given by Hollander, Smith, and Rasmussen in explaining the similarly small value for the $5/2 + \lceil 642 \rceil$ band in Np²³⁷.⁷⁰ We have accordingly assigned the ground state of Bk²⁴⁹ to the 7/2 + [633]orbital. The beta decay of Bk²⁴⁹ to Cf^{249 47} is consistent with this assignment.

Alpha population is also observed to three members of a rotational band whose base level is 393 kev above the ground state of Bk²⁴⁹. These levels decay by predominantly M1 transitions to the ground-state rotational band of Bk²⁴⁹. The pattern of gamma ravs de-exciting this band, and the pattern of alpha population to the band members suggests that this is the 5/2+ [642] band; however, the spacing of the levels is not in very good agreement with this conclusion, nor, in fact, with any other reasonable spin. Nevertheless we have tentatively given this assignment to the 393-kev level, because the unusual spacing of the band can possibly be explained in terms of rotational-particle coupling to the 3/2+ [651] band. Another group of levels has been observed in the alpha decay of E²⁵³, and these seem to comprise a rotational band whose lowest observed level is at 84 kev above the ground state of Bk²⁴⁹. The spacing of this band indicates that the 84-kev level has a spin of 7/2, and the rotational constant appears normal. Asaro et al.⁸¹ have been able to show that the 84-kev level is followed by two prompt (<2 μ sec) transitions, which are highly converted in the L shell. It is possible that the 84-kev level is the 7/2-[514] Nilsson state, which decays by an M2 transition to the 9/2+ member of the ground-state rotational band, and then on to the ground state. However, we feel that a somewhat more likely situation is that the rotational band is really based on the $3/2 - \lceil 521 \rceil$ Nilsson state, but that the alpha decay to the 3/2 and 5/2 members of this band is obscured (as it certainly would be) by the intense alpha groups to the first two members of the ground-state rotational band. The two prompt transitions following the 84-kev level would then be those de-exciting this level to the 3/2- state.

⁸¹ Asaro, Hummel, Stephens, and Perlman (unpublished data, 1955–1957).

⁸² Jones, Schuman, Butler, Cowper, Eastwood, and Jackson, Phys. Rev. **102**, 203 (1956).

which subsequently decays by an unobserved (~10 kev) M2 transition to the ground state of Bk²⁴⁹. A preliminary search has failed to reveal an isomeric state of Bk²⁴⁹ (the 10-kev level); however, we feel that the possibilities are not yet exhausted. Of course, one would like to find a low-lying 3/2 state in Bk²⁴⁹ because in the other berkelium isotopes the 3/2- [521] orbital represents the ground state.

Einsteinium (proton number 99).—The only einsteinium isotope for which sufficient data are available to make an assignment is E^{253} . This isotope very likely has a ground-state assignment of 7/2+ [633] because favored alpha decay was observed to a state in Bk²⁴⁹ that was given this assignment.

CLASSIFICATION OF BETA TRANSITIONS

The beta transitions upon which the Nilsson assignments in Tables I and II are partially based are summarized in Table III. Here the log ft value and the parent nucleus are listed at the intersection of the column and row corresponding to the two levels that the beta transition connects. The log ft values are only approximate, and in a few cases the value given very

likely represents that for decay to more than one member of the rotational band, rather than just the base level. These errors are probably not larger than a factor of 2 or 3 in the ft value, however. The classification of each beta transition is given at the top of each group. These classifications are allowed (a) or first or second forbidden (1f or 2f), according to the Nordheim selection rules, and hindered (h) or unhindered (u) in the asymptotic quantum numbers of deformed nuclei according to the rules given by Alaga.⁸³ The parentheses around the transitions indicate uncertainty in orbital assignment as in Tables I and II, and in this case the notation on the beta transition is that of the least certain of the levels which it connects. The range of log ft values for the various transition types is similar to that found by Alaga⁸³ for the spheroidal region 150<*A*<190.

ACKNOWLEDGMENT

The authors are indebted to Dr. S. G. Nilsson for many helpful discussions, and for permission to use the results of his calculations before publication.

83 G. Alaga, Phys. Rev. 100, 432 (1955).