view recently discussed.[14,57] These interactions are chirality-invariant,[2] can be cast more readily into a two-component form,[3] and are invariant under strong and (or) weak mass reversal.[4] Although these speculations are somewhat formal at present, they might not necessarily be void of physical content.

Perhaps the most disappointing feature of our whole investigations is that we have been forced to use the language of local field theory, and in particular to rely heavily on the Lagrangian formalism. Whether we regard $CP$ invariance or $G$ invariance as a fundamental invariance principle in order to obtain the parity restrictions, we must assume that the interaction Lagrangian contains either nonderivative-type couplings only or derivative-type couplings only. We feel that such assumptions are extremely unsatisfactory.

However, the possibility exists that the use of field theory is unjustified and yet symmetries or relations among symmetries implied by the theory are still valid. For example, the requirement imposed by the $CPT$ theorem may turn out to be of greater generality than our *present* field theory by means of which the theorem has been proved. Another example of this kind is the empirical fact that parity conservation holds at least to an accuracy of one part in $10^8$ in intensity[58] whereas the inadequacy of local field theory is already reflected in that, in order to account for various self-energy effects, some sort of Feynman cutoff becomes necessary at energies not too high in comparison with the nucleon rest energy.[59] We believe that in elementary-particle physics today only those arguments that are based on symmetry principles are on a firm and permanent footing. We may hope that relations between internal symmetry laws and space-time symmetry laws similar to the ones discussed in this paper are still valid in a more satisfactory theory of elementary particles.

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[57] R. P. Feynman, Bull. Am. Phys. Soc. Ser. II, 3, 55 (1958).

[58] D. H. Wilkinson, Phys. Rev. **109**, 1603, 1614, 1610 (1958).

[59] R. P. Feynman and G. Speisman, Phys. Rev. **94**, 500 (1954).

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# Indication for $K$, $\Lambda$, $\Sigma$ Relative Parities from Effective-Range Analysis of $K^+p$ Scattering

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From dispersion relations an effective range formula is derived for $K^+p$ scattering. In the expression for the effective range, the integrals over the cross sections are certainly convergent and weighted against the contributions from the unphysical region. This expression is then analyzed under the experimentally suggested hypothesis of rather constant $K^+p$ cross sections up to $\sim$110 Mev; it is observed that the effective range is rather energy independent and the integrals contributing to it are estimated to be all of the same sign. The expression for the effective range is then quantitatively evaluated, and it is shown that the comparison with the low-energy dependence of $\sigma^+$ indicates equal $\Lambda$ and $\Sigma$ parities with opposite $K$ parity ($K$ pseudoscalar). The possibility of evaluating the coupling constants from the low-energy behavior of the $K^+p$ cross section is then briefly discussed.

## (1) INTRODUCTION

SEVERAL attempts have been performed in the last year[1–3] to obtain the relative parities of the strange particles from the analysis of the meson-nucleon dispersion relations. The results were, however, rather ambiguous; no definite conclusions were possible at least in the absence of information about the sign of the $K^-$-$p$ potential (sign of the corresponding scattering length). The main difficulty for the low-energy dispersion relation analysis lies in the fact, generally believed, that only subtracted dispersion relations can be used in order to have convergent integrals. These relations involve the scattering lengths as undetermined parameters and are tautological at zero kinetic energy ($T_K=0$).† Matthews and Salam[1] noted, however, that if $\sigma^- - \sigma^+$ vanishes for infinite energies

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[1] P. T. Matthews and A. Salam, Phys. Rev. **110**, 569 (1958). See also C. Goebel, Phys. Rev. **110**, 572 (1958).

[2] K. Igi, Progr. Theoret. Phys. (Kyoto) **19**, 238 (1958).

[3] S. Barshay, Phys. Rev. Letters **1**, 177 (1958).

† *Note added in proof.*—In a recent paper by K. Igi [Progr. Theoret. Phys. (Kyoto) **20**, 403 (1958)] an analysis of $K^+$-$p$ scattering cross section with such dispersion relations is performed. However, the indetermination of the sign of the $K^-$-$p$ scattering length do not allow for definite conclusions.

sufficiently quickly, then a relation between the $K^-$-$p$ and $K^+$-$p$ scattering lengths can be obtained by subtracting the corresponding dispersion relation for $T_K=0$. This relation is not sufficient, at present, to specify the parities of the strange particles.

In this paper we shall make use of the information on the low-energy behavior of $K^+$-$p$ scattering: we shall show that this will allow for rather clear-cut indications on the parity values.[4]

In the next paragraph we shall derive an effective range expression for $K^+$-$p$ (or $K^-$-$p$) scattering by direct subtraction of the corresponding dispersion relation. It will be shown that for low energies the effective range $r^+$ is very weakly dependent on the energy. Besides, in the expression for $r^+$, the integrals over the cross sections are convergent and are weighted against contributions from the unphysical region. It is shown that the contributions of the integrals over $\sigma^+$ and $\sigma^-$ are both of the same sign; these contributions are then evaluated by approximating in a simple way the experimental cross sections. Then it is seen that the isotropy and the very weak energy dependence of $K^+$-$p$ scattering for low energies gives a clear indication for equal $\Lambda$ and $\Sigma$ parities and opposite $K$ parity. It is also possible to guess, with the support of good arguments, that the high energy contributions (where there are no experimental data) would not spoil the conclusions obtained, but rather add weight to their correctness.

In the following, the relative $K$-$\Lambda$ and $K$-$\Sigma$ parities will be called $P_\Lambda$ and $P_\Sigma$, respectively. If $P_\Lambda=P_\Sigma$ ($P_\Lambda=-P_\Sigma$) the relative $\Lambda$-$\Sigma$ parity will be $+1$ $(-1)$.

The conclusions reached here are completely independent of the ventilated possibility of opposite $K^+$-$K^0$ parity. In such a case, the parity we are dealing with in this paper is that of the charged heavy meson.

We shall call $\pi$, $K$, $N$, $\Lambda$, and $\Sigma$ the masses of the corresponding particles and use units in which $\hbar=c=1$. If not specified otherwise, the energies and momenta are those of the heavy meson in the laboratory system; we shall use frequently $K$ as an energy unit.

## (2) EFFECTIVE RANGE RELATION

The unsubtracted relativistic dispersion relations for forward $K^+$-$p$ and $K^-$-$p$ scattering can be written in the following way.[5]

$$D^\pm(\omega)=B_\Lambda^\pm(\omega)+B_\Sigma^\pm(\omega)+\frac{1}{\pi}\mathcal{P}\int\frac{k'\sigma^\pm(\omega')}{\omega'-\omega}d\omega'+\frac{1}{\pi}\int\frac{k'\sigma^\mp(\omega')}{\omega'+\omega}d\omega',\quad(1)$$

where $D^+(\omega)$ and $D^-(\omega)$ are the real parts of the forward $K_p^+$ and $K_p^-$ scattering amplitudes at energy $\omega$; $\sigma^+$ and $\sigma^-$ are the total cross sections, and $\mathcal{P}$ denotes the principal value of the integral. The lower limit of the integrations is $K$ for the integrals containing $\sigma^+$, while it is

$$\omega_0=\frac{(\Lambda+\pi)^2-N^2-K^2}{2N}\sim\tfrac{1}{2}K\quad(2)$$

for those containing $\sigma^-$, owing to the possibility of $K^-$ absorption processes. The bound-state contributions $B_Y$ ($Y$ stands for $\Lambda$ or $\Sigma$) are given by

$$B_Y^\pm(\omega)=\pm\frac{g_Y^2}{2Y}\left(\frac{Y+N+\omega_Y}{\omega\pm\omega_Y}\right)\quad\text{if}\quad P_Y=+1,\quad(3)$$

$$B_Y^\pm(\omega)=\mp\frac{g_Y^2}{2Y}\left(\frac{Y-N-\omega_Y}{\omega\pm\omega_Y}\right)\quad\text{if}\quad P_Y=-1,\quad(4)$$

where

$$\omega_Y=(Y^2-N^2-K^2)/2N.\quad(5)$$

By performing a subtraction in the dispersion relations (1), we obtain[6]

$$D^\pm(\omega)=D^\pm(K)+(\omega-K)r^\pm(\omega),\quad(6)$$

where

$$r^\pm(\omega)=R_\Lambda^\pm(\omega)+R_\Sigma^\pm(\omega)+\frac{1}{\pi}\mathcal{P}\int\frac{k'\sigma^\pm(\omega')d\omega'}{(\omega'-\omega)(\omega'-K)}-\frac{1}{\pi}\int\frac{k'\sigma^\mp(\omega')d\omega'}{(\omega'+\omega)(\omega'+K)},\quad(7)$$

the bound-state contributions $R_Y(\omega)$ being given by

$$R_Y^\pm(\omega)=\mp\frac{g_Y^2}{2Y}\left(\frac{Y+N+\omega_Y}{(\omega\pm\omega_Y)(K\pm\omega_Y)}\right)\quad\text{for}\quad P_Y=+1,\quad(8)$$

$$R_Y^\pm(\omega)=\pm\frac{g_Y^2}{2Y}\left(\frac{Y-N-\omega_Y}{(\omega\pm\omega_Y)(K\pm\omega_Y)}\right)\quad\text{for}\quad P_Y=-1.\quad(9)$$

The vanishing of $\sigma^+(\omega')-\sigma^-(\omega')$ for $\omega'\to\infty$, which is a rather general statement,[7] ensures the convergence of the integral expression in (7) and, therefore, the meaning of (6).

From now on we shall fix our attention on $K^+$-$p$ scattering and restrict the discussion to not too high values of the energy $\omega$ (let us say $T_K\lesssim 110$ Mev).

We first note that for small values of the energy, $r^+(\omega)$ is very weakly dependent on $\omega$. This fact is clear in the expressions for $R_Y(\omega)$ and the second integral in (7). As for the first integral, we note that since $\sigma^+$

[4] In a forthcoming paper [Amati, Galzenati, and Vitale, Nuovo cimento (to be published)] dispersion relation analysis of low energy $K^-$-$p$ scattering and absorption will be performed. The conclusions reached are in agreement with the present ones.

[5] For the conventional derivation see D. Amati and B. Vitale, Nuovo cimento **7**, 190 (1958). A rigorous proof for heavy-meson-nucleon dispersion relation has not yet been obtained; see, however, Bremermann, Oehme, and Taylor, Phys. Rev. **109**, 2178 (1958).

[6] We note that the subtracted forward dispersion relation, and therefore their analysis, are completely independent of the existence or nonexistence of a direct $K\pi$ interaction.

[7] I. Ia. Pomeranchuk, Zhur. Eksptl. i Teoret. Fiz. U.R.S.S. **34**, 725 (1958) [translation: Soviet Phys. JETP **34**(7), 499 (1958)].

is practically constant at small energies,[8,9] the principal value integral is also rather independent of $\omega$. Then the expression (6) is really a conventional effective range relation; the difference from the normal potential scattering lies in the fact that $r^+$ can well be negative. We shall see later that this seems to be the case for $K^+$-$p$ scattering.

As for the contribution of the unphysical region to $r^+$, we see that the denominators of the last integral of (7) are slowly varying and rather large in that region. Then even if $A^-(\omega')=k'\sigma^-(\omega')$ has cusps or kinks[2] there, the integral over the unphysical region will be rather, small and, in some sense, proportional to the mean value of $A^-$.

We shall discuss now, in a qualitative manner, the signs of the integrals contributing to $r^+(\omega)$. The contribution of the last integral in (7), for the physical region, is clearly negative. As for the first integral, let us begin by noting that the very-high-energy contribution can be grouped together with the very-high-energy contribution of the integral over $\sigma^-$. But since $\sigma^+-\sigma^-$ is quite probably negative for all energies (owing to the greater number of channels at disposal for a $K^-$-$p$ reaction), then the high-energy contribution to $r^+(\omega)$ is also expected to be negative, even if probably small. Let us now discuss the low- and intermediate-energy contribution of $\sigma^+(\omega')$ to the first integral in (7). We noted before that

$$\mathcal{P}\int\frac{k'\sigma^+(\omega')}{\omega'-\omega}d\omega',$$

extended to the low- and intermediate-energy range, will decrease when $\omega$ increases provided that $\omega$ is always contained in the range of energies for which $\sigma^+$ is very slowly varying. Then, by considering the subtraction made to obtain (6), it is clear that the principal-value integral in (7) is expected to give a negative contribution. This conclusion can also be reached by the direct analysis of that term on the basis of the low-energy dependence of $\sigma^+$. Then, just by qualitative analysis, we are able to estimate that the integral contributions to $r^+$ are all negative. Since the value of $R_Y^+(\omega)$ is negative for $P_Y=+1$ and positive for $P_Y=-1$, the possibility of relating the sign and magnitude of $r^+(\omega)$ to the relative parities of the strange particles is evident. But to carry out this comparison, it is better to give some more quantitative details of the magnitudes of the different terms of (7).

### (3) QUANTITATIVE EVALUATION OF $r^+(\omega)$

For the low- and intermediate-energy region we shall approximate $\sigma^+(\omega')$ with a constant $b$. Let us call $\Omega$ the upper limit of that energy region; then we put

$$\sigma^+(\omega')\approx b \quad \text{for} \quad K\leq\omega'\leq\Omega. \tag{10}$$

For $\sigma^-$, owing to the absorption processes, we shall use the approximation

$$\sigma^-(\omega')\approx c+d/k' \quad \text{for} \quad K\leq\omega'\leq\Omega, \tag{11}$$

with constants $c$ and $d$. For the unphysical region we shall simply continue $\sigma^-(\omega')\approx d/k'$ either to $\omega_0$ [Eq. (2)] or to some lower limit between $\omega_0$ and $K$.

The analysis of experimental data for $K^+$-$p$ and $K^-$-$p$ scattering gives for $b$, $c$, and $d$ the approximate values[10]:

$$b\sim 10/K^2, \quad c\sim 55/K^2, \quad d\sim 10/K. \tag{12}$$

The rather large experimental uncertainties make the values for $b$, $c$, and $d$ given in (12) rather dubious. We can, however, hope that the values of $\sigma^+$ and $\sigma^-$ given by (10), (11), and (12) are not too bad, especially at low energies. We estimate the error on the integrals over the cross sections to be probably not greater than 20 or 30%. It will be seen that the conclusions we shall reach are practically unchanged by it. We note that the contributions involving $b$, $c$, and $d$ all have the same sign, so that the error in each will not be enhanced by cancellations.

If we allow $\Omega$ to vary from $2K$ to $4K$, the computation shows that the total contribution of the integrals to $r^+(\omega)$ remains unchanged even though the separate contributions of the terms containing $\sigma^+$ and $\sigma^-$ are modified.

The contribution of the unphysical region, calculated as described before, turns out to be very small: less than 10% of the integral containing $\sigma^-$ over the physical region.

We shall write down the result of the computation both for very low values of the energy ($k^2/K^2\ll 1$, where $r^+$ is a constant), and for a higher value of the energy

[8] See, for instance, Proceedings of the Padua-Venice Conference on Fundamental Particles, 1958 (Suppl. Nuovo cimento, to be published) and *1958 Annual International Conference on High Energy Physics at CERN*, edited by B. Ferretti (CERN, Geneva, 1958).

[9] We want to stress that all the arguments of this paper rest on the evidence for small energy dependence of $\sigma^+$ at low energies. The experimental data, even though the errors are rather large, give a good basis for that evidence from $\sim$30 Mev to at least $\sim$120 Mev. If for energies lower than $\sim$30 Mev $\sigma^+$ would present some unexpected behavior (for instance some abrupt decrease), then the analysis of Eqs. (6) and (7) would be rather complicated and the evaluations given here would no longer be valid. Notwithstanding, a rather similar study would be possible for an equation similar to (6) but obtained by subtraction at an energy such that, from there on, $\sigma^+$ presents the characteristic slowly varying behavior. For such an expression, many of the qualitative arguments drawn here would be applicable so, probably, the conclusions on the parities would be similar to those obtained here. We note, however, the great interest that the experimental measurement of the $K^+$-$p$ total cross section at very low energies can have.

[10] This estimation is similar to that of Matthews and Salam[1] even if some new experimental results are introduced [see for instance the analysis of Ascoli, Hill, and Yoon, Nuovo cimento **5**, 813 (1958), and *Proceedings of the 1958 Annual Conference on High-Energy Physics at CERN*, edited by B. Ferretti (CERN, Geneva, 1958)].

which we choose to be $\omega=1.22K$ ($T_K=110$ Mev):

$$r^+(K)=\frac{1}{K^2}\begin{bmatrix}-11.6\ (g_\Lambda^2/4\pi)\\ 0.95(g_\Lambda^2/4\pi)\end{bmatrix}+\frac{1}{K^2}\begin{bmatrix}-10.3(g_\Sigma^2/4\pi)\\ 1.1(g_\Sigma^2/4\pi)\end{bmatrix}-\frac{10}{K^2},\quad(13)$$

$$r^+(1.22K)=\frac{1}{K^2}\begin{bmatrix}-9.5\ (g_\Lambda^2/4\pi)\\ 0.78(g_\Lambda^2/4\pi)\end{bmatrix}+\frac{1}{K^2}\begin{bmatrix}-8.5\ (g_\Sigma^2/4\pi)\\ 0.92(g_\Sigma^2/4\pi)\end{bmatrix}-\frac{11}{K^2}.\quad(14)$$

In the first (second) bracket of (13) and (14), the upper value must be considered if $P_\Lambda=+1$ ($P_\Sigma=+1$) and the lower one if $P_\Lambda=-1$ ($P_\Sigma=-1$).

In writing (13) and (14) we have neglected the contributions of the high energies ($\omega'>\Omega$) that are obviously rather difficult to evaluate. But as discussed in the previous paragraph, this contribution is expected to be negative, even if probably small. Then the actual values of $r^+$ should be possibly somewhat lower than (13) and (14); we shall see that this can favor our analysis.

## (4) DISCUSSION AND CONCLUSIONS

The experimental evidence for a weak dependence of $\sigma^+$ on the energy implies a low value for $r^+$. To make a more quantitative statement we must find out which values of $r^+$ are compatible with the experimental results. It is not feasible from the present experimental data to obtain a value for $r^+$; it is possible, however, to find a maximum for its absolute value.

Let us first express $\sigma^+(\omega)$ as a function of $r^+(\omega)$, making use of the experimental fact that the angular distribution for $K^+$-$p$ scattering is essentially isotropic, in the c.m. system, for the energies we are interested in. Passing to the c.m. system and making use of the optical theorem, we get

$$D^{+2}(\omega)=\frac{k^2}{k_b^2}[4\pi\sigma^+(\omega)-k_b^2\sigma^{+2}(\omega)],\quad(15)$$

or

$$|D^+(K)|=\frac{N+K}{N}[4\pi\sigma^+(K)]^{\frac{1}{2}},\quad(15')$$

where $k_b$ is the momentum in the c.m. system. Inserting (6), we find

$$\sigma^+(\omega)-\frac{k_b^2\sigma^{+2}(\omega)}{4\pi}=\left(\frac{k_b(N+K)}{kN}\right)^2\sigma^+(K)+\frac{(\omega-K)k_b}{2\pi k}D^+(K)r^+(\omega)+\left(\frac{(\omega-K)k_b}{k}\right)^2\frac{r^{+2}(\omega)}{4\pi}.\quad(16)$$

As for the sign of $D^+(K)$, we know from experiment that it is negative due to the sign of the interference with the Coulomb potential. Then an increase in $\sigma^+(\omega)$ for increasing energy, as seems to be the case for higher energies, will essentially characterize a negative $r^+$. We are interested in finding some limitation for this negative value (i.e., a minimum value for $r^+$) in order to draw from (13) or (14) some condition on the signs of the bound state contributions.

The experimental data[8] indicate that up to $T_K\sim 110$ Mev, $\sigma^+$ is nearly 14 mb, with errors of about some millibarns. It is very difficult to judge which are the real limitations given by these numbers; however, the data allow us to believe, with sufficient confidence, that when going from very low energies ($T_K\sim 0$) up to 110 Mev, $\sigma^+$ has not increased by more than 5 mb from about its mean value of 14 mb. If this is indeed the case, we can obtain from (15′) and (16) the following limitation for $r^+$:

$$r^+(1.22K)\gtrsim -7.5/K^2.\quad(17)$$

We see then, comparing with (14), that the bound state contributions must be positive, and this is a clear indication of $P_\Lambda=P_\Sigma=-1$. The possibility of $P_\Lambda=P_\Sigma=+1$ is obviously inconsistent with (17); that of different $\Lambda$ and $\Sigma$ parities ($P_\Lambda=-P_\Sigma$) is possible only if $g_\Lambda^2$ and $g_\Sigma^2$ differ by nearly a couple of orders of magnitude, and this is really implausible.[11]

We see that the generally believed rather constant behavior of $\sigma^+$ at low energies, suggests clearly that the parities of $\Lambda$ and $\Sigma$ are most probably the same while the parity of the heavy meson is opposite. In order to confirm this result it would be very good to improve the experimental information on the detailed behavior of $K^+$-$p$ scattering at low energies (from $T_K\sim 0$ to $T_K\sim 110$ Mev for instance), especially by giving an upper bound for the variation of the total cross section and confirming the isotropy of the angular distribution. Indeed, if experimental data will show with better precision that the variation of $\sigma^+$ with energy is quite smaller than the maximum variation which we were confident to assume, all the arguments made previously will be strengthened.

We want also to note that the determination of $r^+$ can supply a good method for investigating the value of the coupling constants. As a poor indication we note that a value $r^+\approx -5/K^2$, which gives an increase in $\sigma^+$ of about 3 mb in going from 0 to 110 Mev, would be given by $g_\Lambda{}^t/4\pi=g_\Sigma{}^t/4\sim 3.5$.

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[11] The fact that the physical processes involving $\Lambda$ or $\Sigma$ are not directly related to their respective coupling constants, makes it difficult to find definite evidence against such a great difference among their values. However, even a very rough analysis with dispersion relation techniques suggests that with such a difference in the coupling constants it would be difficult to explain the rather similar abundance of $\Lambda$ and $\Sigma$ in production and photoproduction processes.