Σ^+ from the reaction $\bar{K}^0+p \rightarrow \Sigma^+ + \pi^0$. Tracks 6 and 7 are consistent with either $K^+ \rightarrow \mu^+ + \nu$ or $\Sigma^+ \rightarrow \pi^+ + n$. However, the short lifetime of Track 6 strongly favors the Σ^+ interpretation—if track 6 is a K^+ it lived 1.9 $\times 10^{-3}$ mean lives, if a Σ^+, 0.6 mean lives. Finally we remark that in event A the neutral K lived 2.6 K_1^0 mean lives, and in event B, 3.0 K_1^0 mean lives.

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Decay of ${}_\Lambda H^3$ and the Spin Dependence of the Λ-Nucleon Interaction*

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A calculation is made of the fraction of the π-mesonic decays of the hypertriton ${}_\Lambda H^3$ which yield the two final products He^3 and π^-. This fraction is a function of the spin of ${}_\Lambda H^3$ and of the ratio p/s of the amplitudes for decay of the free Λ via the s- and p-wave channels. The results are compatible with the present experimental data. They indicate that probably $p/s \leqslant 1$, and that the spin of ${}_\Lambda H^3$ is $\frac{1}{2}$, which implies that the singlet Λ-nucleon interaction is more attractive than the triplet. These results are in agreement with those of a previous calculation by Dalitz for ${}_\Lambda H^4$.

ANALYSIS of the binding energies of the light hypernuclei shows that, in the absence of strong three-body forces, the Λ-nucleon interaction must be spin dependent. On this basis Dalitz and Downs[1] have been able to calculate the strengths of both the singlet and the triplet interactions for either interaction more attractive than the other, but neither the hypernuclear binding energies nor the radius and shape of the nuclear cores are known accurately enough to determine which of these interactions is in fact the more attractive. A more sensitive indication is provided by the probability for two-body mesonic decay of the light hypernuclei, since this depends strongly on the spin of the parent hypernucleus. Dalitz and Downs have made a qualitative estimate of this probability, and Dalitz[2] a detailed calculation, for the decay of ${}_\Lambda H^4$; Dalitz has also made a rough calculation for ${}_\Lambda H^3$. The present work gives a more exact calculation for the ${}_\Lambda H^3$ case.

Although data on the hypertriton are at present rather scarce, the greater simplicity of the three-body problem increases its importance. For $A>3$, it has been necessary to describe the hypernucleus as a Λ bound to a core nucleus which is not greatly distorted by the presence of the Λ. For $A=3$, on the other hand, calculations can be made with a wave function that takes into account correlations between each pair of particles. Because of this, we expect results obtained for ${}_\Lambda H^3$ to be eventually the most reliable.

The Λ decay interaction is $H=s+p\mathbf{q}\cdot\boldsymbol{\sigma}/q_\Lambda$, where $\mathbf{q}$ is the pion momentum and $\mathbf{q}_\Lambda$ its value for free Λ decay, and $\boldsymbol{\sigma}$ is the Λ spin vector. Since the spin of the product nucleus He^3 (we consider only π^- decay) is $\frac{1}{2}$, two-body decay can proceed only through the p-wave channel if ${}_\Lambda H^3$ has spin $\frac{3}{2}$, but through both the s and p channels if the ${}_\Lambda H^3$ spin is $\frac{1}{2}$. The proportion of π^- decays which yield two final products (${}_\Lambda H^3 \rightarrow He^3+\pi^-$) is given by the ratio of the two-body transition probability to the sum of the transition probabilities of all modes which give a π^-:

$$f=q\left|\int\bar{\psi}_{He^3}\bar{\psi}_{\pi^-}H\psi_{{}_\Lambda H^3}dV\right|^2\Big/\sum_n q_n\left|\int\bar{\psi}_n\bar{\psi}_{\pi^-}H\psi_{{}_\Lambda H^3}dV\right|^2.$$

The square of the matrix element in the numerator splits into two factors: (1) the square of the overlap integral between the space parts of the initial- and final-state wave functions, which represents the probability of the product nucleons sticking together in the He^3 configuration, and (2) a factor depending upon the ${}_\Lambda H^3$ spin which represents the proportion of suitably oriented initial spin states. The denominator is a more difficult matter, and Dalitz[2] has found it expedient to use two approximations: (1) Since the observed pion momenta all lie in a rather narrow range ($q=113$ Mev/c for the two-body and $\sim$85–100 Mev/c for the many-body decays[3]), we should be able to replace the q_n's by an average value q_{Av}. (2) Since the overlap with the ${}_\Lambda H^3$ space wave function can be expected to be small for final states of high internal energy among the

* Supported by the joint program of the U. S. Atomic Energy Commission and the Office of Naval Research.
[1] R. H. Dalitz and B. W. Downs, Phys. Rev. **111**, 967 (1958).
[2] R. H. Dalitz, Phys. Rev. **112**, 605 (1958).
[3] Levi-Setti, Slater, and Telegdi, Nuovo cimento **10**, 68 (1958) and W. E. Slater (private communication). We are indebted to Dr. Slater for sending some unpublished data.

nucleons, extending the summation over all energies to give the completeness relation (remembering antisymmetry requirements) should be a fair approximation. From this point the calculation is straightforward.

With these methods, Dalitz derives the following expressions for the fraction of two-body decays.

If the $_\Lambda H^3$ has $J=\frac{3}{2}$,

$$f=\tfrac{1}{3}|p|^2\left(\frac{q}{q_\Lambda}\right)^2F^2(q)q\left\{|s|^2[1-\eta]+|p|^2\left(\frac{q_{Av}}{q_\Lambda}\right)^2\left[1+\tfrac{1}{3}\eta+\frac{1}{3}\left(\frac{q}{q_{Av}}\right)^3F^2(q)-\tfrac{1}{3}F^2(q_{Av})\right]\right\}^{-1}q_{Av}^{-1};\tag{1a}$$

if $J=\frac{1}{2}$ (and $T=0$),

$$f=\frac{3}{4}\left[|s|^2+\tfrac{1}{9}|p|^2\left(\frac{q}{q_\Lambda}\right)^2\right]F^2(q)q\left\{|s|^2\left[1+\tfrac{1}{2}\eta+\frac{3}{4}\left(\frac{q}{q_{Av}}\right)^2F^2(q)-\tfrac{3}{4}F^2(q_{Av})\right]\right.$$
$$\left.+|p|^2\left(\frac{q_{Av}}{q_\Lambda}\right)^2\left[1-\tfrac{5}{6}\eta+\frac{1}{12}\left(\frac{q}{q_{Av}}\right)^3F^2(q)-\frac{1}{12}F^2(q_{Av})\right]\right\}^{-1}q_{Av}^{-1}.\tag{1b}$$

The factors in the denominators containing the sticking probability F^2 are corrections for underestimate of the two-body mode; the η's arise from requiring antisymmetry in the two final protons. $F(q)$, the overlap integral, is given by

$$F(q)=\int\bar{\phi}_{He^3}(\Lambda np)\phi_{_\Lambda H^3}(\Lambda np)\times\exp[i\mathbf{q}\cdot(\mathbf{R}_\Lambda-\mathbf{R}_{c.m.})]d\mathbf{R}_\Lambda d\mathbf{R}_n d\mathbf{R}_p\tag{2}$$

($\mathbf{R}_\Lambda$ is the coordinate of the Λ particle before the decay and that of the product proton afterward), and the exchange integral η by

$$\eta=\int\bar{\phi}_{_\Lambda H^3}(\Lambda np)\phi_{_\Lambda H^3}(pn\Lambda)\times\exp[i\mathbf{q}_{Av}\cdot(\mathbf{R}_\Lambda-\mathbf{R}_p)]d\mathbf{R}_\Lambda d\mathbf{R}_n d\mathbf{R}_p.\tag{3}$$

For the He³ wave function we use

$$\phi_{He^3}=N\exp[-\alpha(r_{\Lambda p}+r_{pn}+r_{n\Lambda})],$$

with α taken as 0.41 fermi^{-1} to give the correct Coulomb energy with protons of rms charge radius of 0.72 fermi. For the $_\Lambda H^3$, we use the six-parameter wave function of Downs and Dalitz,[4]

$$\phi_{_\Lambda H^3}=N'[\exp(-ar_{\Lambda p})+x\exp(-br_{\Lambda p})]\times[\exp(-ar_{n\Lambda})+x\exp(-br_{n\Lambda})](e^{-a_3r_{pn}}+ye^{-b_3r_{pn}}),$$

obtained by a variational calculation for the strength of the Λ-nucleon interaction needed to give a Λ binding energy of 0.25 Mev with an interaction range corresponding to the exchange of two pions.[5] (The character of the Λ-nucleon interaction and the field-theoretic investigations concerning it are discussed in the paper of Dalitz and Downs[1]; in particular, the work of Lichtenberg and Ross[6] and Ferrari and Fonda[7] indicates that predominant two-pion exchange is most likely, and furthermore that the singlet interaction is stronger than the triplet.) The appropriate pion momenta are $q=113$ Mev/c, $q_\Lambda=101$ Mev/c, and the average over all the observed decays $q_{Av}=104$ Mev/c,[3] (the inclusion of several uncertain identifications does not affect this value). With these wave functions and momenta, we find (see the Appendix) $F(q)=0.61$, $F(q_{Av})=0.63$, and $\eta=0.31$.

As for the remaining parameter p/s of Eqs. (1), Dalitz[2] shows that the channel amplitudes s and p are approximately real, and that the up-down asymmetry coefficient for free Λ decay indicates $0.45\leqslant p/s\leqslant 2.25$. Thus we have as ranges for f: $J=\frac{3}{2}$, $0.04\leqslant f\leqslant 0.15$; $J=\frac{1}{2}$, $0.24\geqslant f\geqslant 0.10$.

The experimental data[3] include 9 two-body and perhaps 10 to 15 many-body π^- decays, giving $f_{exp}\cong 0.42$; this is about $1\frac{1}{2}$ or 2 standard deviations higher than the extreme calculated value of f, 0.24. Furthermore, the experimental value is more likely to increase than to decrease, since in general two-body decays are more likely to be overlooked.[8] A real disagreement may be present, but it would take more data to establish it.

Such a discrepancy might arise from the approximations involved in evaluating the denominators of Eqs. (1) (see reference 2); this effect would be difficult to evaluate. It might also arise from the wave functions themselves. Aside from the fact that a wave function from a variational calculation is always somewhat suspect, we may note that both the He³ and the $_\Lambda H^3$ functions fall off too rapidly at large separations (e.g., the wave function for He³ for one proton far out goes as $e^{-0.8r}$ instead of the correct $e^{-0.6r}/r$; that for $_\Lambda H^3$ for large separation goes as $e^{-0.2r}$ instead of the $e^{-0.1r}/r$ required by a binding energy 0.25 Mev). However, this in itself should make the sticking probability slightly too large rather than too small because the $\exp[i\mathbf{q}\cdot(\mathbf{R}_\Lambda-\mathbf{R}_{c.m.})]$ factor in the integral emphasizes

[4] B. W. Downs and R. H. Dalitz, Phys. Rev. (to be published).
[5] The values of the parameters used are: $a=0.11$, $b=0.80$, $a_3=0.38$, $b_3=1.14$, $x=1.69$, $y=2.14$; the final value of x of Downs and Dalitz is slightly smaller, but this makes no appreciable difference in the present work.
[6] D. B. Lichtenberg and M. Ross, Phys. Rev. **103**, 1131 (1956); and Phys. Rev. **109**, 2163 (1958).
[7] F. Ferrari and L. Fonda, Nuovo cimento **5**, 842 (1958).
[8] W. E. Slater (private communication).

the close-in behavior. A change in the rather uncertain value of the binding energy produces only small changes (in fact, too small) in the ${}_{\Lambda}H^3$ wave function parameters,[4] so that the sticking probability should not be much affected on this account.

In any case, we can say that the spin $\frac{1}{2}$ hypertriton is indicated, which implies that the singlet Λ-nucleon interaction is stronger than the triplet, in agreement with Dalitz *et al.*[1,2] and with the field-theoretic calculations.[6,7] In addition, p/s is probably $\leqslant 1$, also in agreement with Dalitz.

APPENDIX: EXPLICIT EVALUATION OF η AND $F(q)$

Making use of the triangular coordinates, η can be written immediately as

$$\eta = 8\pi^2 \int_0^\infty r_{n\Lambda} dr_{n\Lambda} \int_0^\infty r_{pn} dr_{pn} \int_{|r_{pn}-r_{n\Lambda}|}^{r_{pn}+r_{n\Lambda}} r_{\Lambda p} dr_{\Lambda p} \bar{\phi}_{\Lambda H^3}(\Lambda np) \phi_{\Lambda H^3}(pn\Lambda) \frac{\sin q_{Av} r_{\Lambda p}}{q_{Av} r_{\Lambda p}}. \tag{4}$$

With the above wave function, we have

$$\eta = 8\pi^2 N'^2 \{ G(2a) + 2xG(a+b) + x^2 G(2b) \},$$

with

$$G(\beta) = H(\beta, a+a_3) + xH(\beta, b+a_3) + yH(\beta, a+b_3) + xyH(\beta, b+b_3),$$

$$H(\beta,\gamma) = I(\beta, \gamma, a+a_3) + xI(\beta, \gamma, b+a_3) + yI(\beta, \gamma, a+b_3) + xyI(\beta, \gamma, b+b_3),$$

$$\begin{aligned} I(\beta,\gamma,\delta) &= \frac{1}{q_{Av}} \operatorname{Im} \frac{\partial^2}{\partial\gamma\partial\delta} \int \exp\{-[(\beta - iq_{Av}) r_{\Lambda p} + \gamma r_{pn} + \delta r_{n\Lambda}]\} dr_{\Lambda p} dr_{pn} dr_{n\Lambda} \\ &= \frac{1}{q_{Av}} \operatorname{Im} \frac{\partial^2}{\partial\gamma\partial\delta} \frac{2}{(\beta+\gamma-iq_{Av})(\beta+\delta-iq_{Av})(\gamma+\delta)} \\ &= 8 \frac{(2\beta+\gamma+\delta)[(\beta+\gamma)(\beta+\delta) - a_{Av}^2][(\beta+\gamma+\delta)^2 + \gamma\delta - q_{Av}^2] - (\beta+\gamma+\delta)\{[(\beta+\gamma)(\beta+\delta) - a_{Av}^2]^2 - (2\beta+\gamma+\delta)^2 q_{Av}^2\}}{(\gamma+\delta)^2 \{[(\beta+\gamma)(\beta+\delta) - q_{Av}^2]^2 + (2\beta+\gamma+\delta)^2 q_{Av}^2\}^2}. \end{aligned}$$

To evaluate $F(q)$ we can write

$$F(q) = \int \bar{\phi}_{He^3}(\Lambda np) \phi_{\Lambda H^3}(\Lambda np) \exp(i\tfrac{2}{3}\mathbf{q}\cdot\boldsymbol{\varrho}) d\boldsymbol{\varrho} d\mathbf{r}_{pn}, \tag{5}$$

where

$$\boldsymbol{\varrho} = \tfrac{1}{2}(\mathbf{R}_n + \mathbf{R}_p) - \mathbf{R}_\Lambda.$$

Then using the wave functions gives

$$F(q) = NN'\{K(a,a_3,a) + 2xK(a,a_3,b) + 2xyK(a,b_3,b) + yK(a,b_3,a) + x^2K(b,a_3,b) + x^2yK(b,b_3,b)\},$$

with

$$\begin{aligned} &K(\beta-\alpha, \gamma-\alpha, \delta-\alpha) \\ &= K'(\beta,\gamma,\delta) = \int \exp[-(\beta r_{\Lambda p} + \gamma r_{pn} + \delta r_{n\Lambda})] \exp(i\tfrac{2}{3}\mathbf{q}\cdot\boldsymbol{\varrho}) d\boldsymbol{\varrho} d\mathbf{r}_{pn} \\ &= \frac{1}{(2\pi^2)^2} \frac{\partial^2}{\partial\beta\partial\delta} \int d\boldsymbol{\varrho} \exp(i\tfrac{2}{3}\mathbf{q}\cdot\boldsymbol{\varrho}) \int d\mathbf{r}_{pn} e^{-\gamma r_{pn}} \int d\mathbf{l} \exp[-i\mathbf{l}\cdot(\boldsymbol{\varrho} + \tfrac{1}{2}\mathbf{r}_{pn})][l^2+\beta^2]^{-1} \\ &\qquad \times \int d\mathbf{m} \exp[-i\mathbf{m}\cdot(\boldsymbol{\varrho} - \tfrac{1}{2}\mathbf{r}_{pn})][m^2+\delta^2]^{-1} \\ &= -8 \frac{\partial^3}{\partial\beta\partial\gamma\partial\delta} \int d\mathbf{l} (l^2+\beta^2)^{-1} \int d\mathbf{m} (m^2+\delta^2)^{-1} [\tfrac{1}{4}(\mathbf{m}-\mathbf{l})^2 + \gamma^2]^{-1} \delta(\tfrac{2}{3}\mathbf{q} - \mathbf{l} - \mathbf{m}) \\ &= -32 \frac{\partial^3}{\partial\beta\partial\gamma\partial\delta} \int \{ (l^2+\beta^2)[(\tfrac{2}{3}\mathbf{q} - 2\mathbf{l})^2 + 4\gamma^2][(\tfrac{2}{3}\mathbf{q} - \mathbf{l})^2 + \delta^2] \}^{-1} d\mathbf{l} \\ &= 8192\pi\beta\gamma\delta \int_0^\infty \frac{l^2}{(l^2+\beta^2)^2} [2l^2 + 4\gamma^2 - 2\delta^2 - (4/9)q^2]^{-2} \Bigg\{ \{(4/3)ql[2l + 4\gamma^2 - 2\delta^2 - (4/9)q^2]\}^{-1} \\ &\qquad \times \ln\left[\left(\frac{l^2+\delta^2+(4/9)q^2-(4/3)ql}{l^2+\delta^2+(4/9)q^2+(4/3)ql} \right) \left(\frac{4l^2+4\gamma^2+(4/9)q^2+(8/3)ql}{4l^2+4\gamma^2+(4/9)q^2-(8/3)ql} \right) \right] \\ &\qquad + 2\{[4l^2+4\gamma^2+(4/9)q^2]^2 - [(8/3)ql]^2\}^{-1} + \tfrac{1}{2}\{(l^2+\delta^2+(4/9)q^2]^2 - [(4/3)ql]^2\}^{-1} \Bigg\} dl, \end{aligned}$$

where we have made repeated use of the Fourier transform

$$e^{-Ax}=-\frac{1}{2\pi^2}\int \exp(-i\mathbf{l}\cdot\mathbf{x})\left\{\frac{\partial}{\partial A}(l^2+A^2)^{-1}\right\}d\mathbf{l}.$$

[Note that $K'(\beta,\gamma,\delta)=K'(\delta,\gamma,\beta)$, despite the unsymmetric appearance of β and δ in the last integral.] This last integral was integrated numerically with the electronic computer of the Cornell Computing Center to give the values quoted in the text.

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Direct-Interaction Model Calculation of High-Energy Proton-Carbon Scattering*

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An optical potential has been derived which includes, to first order in q^2, the effect of the nucleon-nucleon angular dependence. Nucleon-nucleon scattered amplitudes calculated from the 310-Mev nucleon-nucleon phase shifts have been put into the potential and the proton-carbon scattered amplitudes calculated from it by WKB approximation. Good agreement was obtained with the scattered amplitudes as derived directly from the 313-Mev proton-carbon data using an extension of the analysis employed previously by Bethe. The inclusion of the nucleon-nucleon angular dependence in the potential was found to be important in order to obtain the correct value of the imaginary part of the forward scattered amplitude and the correct proton-carbon angular dependence at moderately small angles. Phase shift solutions 1 and 6 of Stapp *et al.* were investigated and found to give essentially the same agreement with the differential cross section at small angles. Solution 6 was found to give a better fit to the polarization data than solution 1, but the significance of this is not clear.

I. INTRODUCTION

IN the direct-interaction model[1] of nuclear reactions one tries to understand the interaction of an incident particle with the nucleus in terms of the interaction of the particle with the individual nucleons composing the nucleus. At high energies the effect of binding is small and the particle-nucleus interaction can be related directly to the particle-nucleon interaction. In particular the optical potential for the elastic scattering of protons from nuclei at high energies can be simply related to the nucleon-nucleon scattering matrix.[2] Since the nucleon-nucleon scattering matrix is determined entirely by the nucleon-nucleon scattering phase shifts, a detailed analysis of the elastic scattering of protons from complex nuclei provides a good test for the direct-interaction model. Furthermore, such an analysis should provide a method for distinguishing between sets of possible nucleon-nucleon phase shifts.

Recently several authors have made such an analysis.[3,4] Bethe, in his analysis of the small-angle scattering and polarization of 313-Mev protons by carbon, showed that the proton-proton scattering phase shifts of Stapp *et al.*[5] and the neutron-proton phase shifts of Gammel and Thaler[6] were in quantitative agreement with the proton-carbon data. He was unable to distinguish between the five different phase shift solutions of Stapp, however; all solutions gave essentially the same agreement.

In his analysis of the experimental data Bethe found the value 8.6 f (1 fermi$=10^{-13}$ cm) for $g_{NI}(0)$, the imaginary part of the spin-independent scattered amplitude at 0°. This agreed with the value calculated from the optical potential derived from the nucleon-nucleon phase shifts. A more reliable value of $g_{NI}(0)$ can be obtained independently from the total neutron cross section at this energy. Since the cross section is nearly constant over a wide energy interval about 313 Mev, its value is known quite accurately. The cross-section data[7] give $g_{NI}(0)=9.45$ f.[8] This is in disagreement both with the proton-carbon data and with the direct-interaction model as calculated in B.

However, because of the over-all success of the

* Supported in part by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.

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