Scattering of Polarized Electrons by Polarized Nucleons

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The differential cross section for the scattering of arbitrarily polarized charged spin one-half particles by arbitrarily polarized charged spin one-half particles with an anomalous magnetic moment has been calculated, including recoil effects.

SCATTERING of polarized electrons on polarized nucleons at high energies could be a useful tool to investigate the charge and magnetic moment structure of nucleons separately. Such experiments, either using the polarized electrons arising from beta decay, or in the form of double-scattering work, may perhaps be feasible in the not too distant future. It is therefore important to know as much as possible about the effects theoretically expected.

Rosenbluth[1] has calculated the cross section for the scattering of unpolarized electrons from unpolarized protons with anomalous magnetic moments. Bincer[2] has generalized this to the case when the fermions are longitudinally polarized. The cross section for the scattering of two identical spin one-half particles with arbitrary spin directions and with no anomalous magnetic moment has been calculated by Ford and Mullin.[3] Newton[4] has calculated the cross section for the scattering of longitudinally polarized electrons on nuclei having magnetic moments and arbitrary spin directions in the limit that the nuclei suffer no recoil.

We now extend the work of Bincer to the case when the fermions have arbitrary spin directions. Although throughout we refer only to the scattering of electrons by protons, the calculations are valid for the scattering of any two different spin one-half particles, one of which has only a Dirac magnetic moment while the other may have an anomalous magnetic moment. In the case of scattering from the neutron, all terms except those proportional to the square of the anomalous magnetic moment should be omitted from the expressions. The cross section was calculated in the first Born approximation using the usual trace methods. The spin directions of the fermions were included by the use of the unit four-vector[5]

$$s=\left(\boldsymbol{\xi}+\frac{\boldsymbol{\xi}\cdot\mathbf{p}}{m(E+m)}\mathbf{p},\quad i\frac{\boldsymbol{\xi}\cdot\mathbf{p}}{m}\right),$$

where $\boldsymbol{\xi}$ is the unit vector in the direction of the spin of the fermion in its own rest system. This is the direction we refer to as the direction of the spin of the particle.

The differential cross section for the scattering of polarized electrons (particle 1) from polarized protons (particle 2) with anomalous magnetic moment g_2 in units of the nuclear magneton [$(1+g_2)e_2/2m_2$ is its total magnetic moment] in an arbitrary reference frame is

$$\begin{aligned}d\sigma/d\Omega=&(e_1^2e_2^2/2b^2)|\mathbf{p}_1'|^2(a^2-m_1^2m_2^2)^{-\frac{1}{2}}\\&\times[|\mathbf{p}_1'|(E_1+E_2)-E_1'|\mathbf{p}_1+\mathbf{p}_2|\cos\theta_1']^{-1}\\&\times\{[2a(a-b)+bm_2^2](1-g_2^2b/2m_2^2)\\&+b(b+m_1^2)(1+g_2)^2+(1+g_2)m_1m_2\\&\times[2bs_1\cdot s_2+s_1\cdot ks_2\cdot k+(g_2b/m_2^2)\\&\qquad\times(bs_1\cdot s_2-s_1\cdot p_2s_2\cdot k)]\}.\end{aligned}$$

Here the unprimed quantities refer to the initial particles and the primed to the final. θ_1' is the angle between $\mathbf{p}_1$ and $\mathbf{p}_1'$, k is the four-momentum transfer $p_2'-p_2=p_1-p_1'$, a the invariant $p_1\cdot p_2=p_1'\cdot p_2'$, and $b=-\frac{1}{2}k^2$. By $p_1\cdot p_2$ is meant $\mathbf{p}_1\cdot\mathbf{p}_2-E_1E_2$. The charges e_1 and e_2 are in units such that for a unit point charge $e^2=\alpha$. If the charge and magnetic moment distributions of the proton are taken into account, e_2 and g_2 should be multiplied by $F_1(k^2)$ and $F_2(k^2)/F_1(k^2)$, respectively; $F_1(k^2)$ and $F_2(k^2)$ being the charge and anomalous magnetic moment form factors for the proton.[6]

In case particle 1 is initially unpolarized and its *final* spin direction is observed, s_1 is replaced by s_1' in the above expression and the entire expression is multiplied by the factor one-half.

In the center-of-mass system the differential cross section is

$$\begin{aligned}d\sigma/d\Omega=&e_1^2e_2^2[8p^4(E_1+E_2)^2\sin^4(\tfrac{1}{2}\theta)]^{-1}\\&\times\{[2a(a-b)+bm_2^2](1-g_2^2b/2m_2^2)\\&+b(b+m_1^2)(1+g_2)^2-2(1+g_2)p^2\sin^2(\tfrac{1}{2}\theta)\\&\times[2[E_1E_2\cos^2(\tfrac{1}{2}\theta)+p^2(1+g_2\sin^2(\tfrac{1}{2}\theta))]\\&\times\cos\lambda_1\cos\lambda_2+m_2\sin\theta(E_1-(E_1+E_2)\\&\times(g_2p^2/m_2^2))\cos\lambda_1\sin\lambda_2\cos\kappa_2+m_1E_2\sin\theta\\&\times\sin\lambda_1\cos\kappa_1\cos\lambda_2+2m_1m_2(\sin^2(\tfrac{1}{2}\theta)(1-g_2p^2/m_2^2)\\&\times\cos\kappa_1\cos\kappa_2+[1-(g_2p^2/m_2^2)\sin^2(\tfrac{1}{2}\theta)]\\&\qquad\times\sin\kappa_1\sin\kappa_2)\sin\lambda_1\sin\lambda_2]\}.\end{aligned}$$

[1] M. N. Rosenbluth, Phys. Rev. **79**, 615 (1950).
[2] A. M. Bincer, Phys. Rev. **107**, 1467 (1957).
[3] G. W. Ford and C. J. Mullin, Phys. Rev. **108**, 477 (1957); **110**, 1485(E) (1958). Also see A. Rączka and R. Rączka, Phys. Rev. **110**, 1469 (1958).
[4] R. G. Newton, Phys. Rev. **103**, 385 (1956); **109**, 2213 (1958); **110**, 1483 (1958).
[5] See H. A. Tolhoek, Revs. Modern Phys. **28**, 277 (1956).
[6] See, for example, Hofstadter, Bumiller, and Yearian, Revs. Modern Phys. **30**, 482 (1958), for the definition of the form factors. Notice that the Dirac moment has the same shape as the charge, so that the entire magnetic form factor is $(F_1+g_2F_2)$.

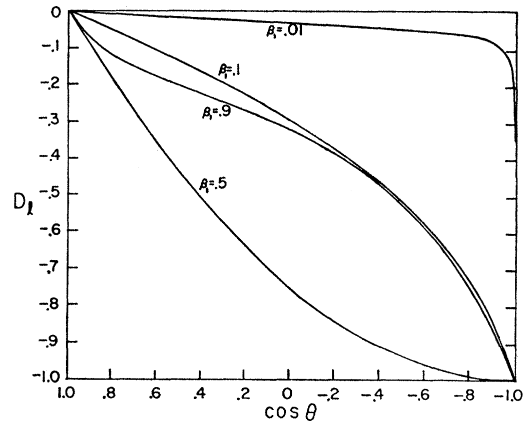

FIG. 1. The relative change of the electron-proton differential cross section due to reversal of the proton's spin direction. The cross sections are for longitudinally polarized electrons and protons in the center-of-mass system; β_1 is the velocity of the electron.

Here the initial direction of the electron is taken as the z axis and its final direction is in the $x-z$ plane. The scattering angle is θ and the polar and azimuthal angles of the spin direction of the electron are λ_1 and κ_1, respectively, and those of the proton are λ_2 and κ_2 (the azimuthal angles being measured from the x axis). In the center-of mass system a has the value $-(E_1E_2+p^2)$ and b the value $-2p^2\sin^2(\frac{1}{2}\theta)$, where p is the magnitude of the momentum of either particle.

The spin-independent term and the $\cos\lambda_1\cos\lambda_2$ term constitute the cross section obtained by Bincer. The remaining terms depend on the transverse polarization of the particles. For scattering at high energies, the only important spin-dependent terms are those depending on the longitudinal polarization of the electron, the two terms proportional to $\sin\lambda_1$ being of the order m_1/E_1 smaller than those proportional to $\cos\lambda_1$.

The coefficient of $\cos\lambda_1\cos\lambda_2$ divided by the spin-independent term gives a quantity of experimental interest:

$$D_l=\frac{d\sigma_+-d\sigma_-}{d\sigma_++d\sigma_-}.$$

The subscript of $d\sigma$ refers to the direction of the spin of the proton: $+$ in the z direction, $-$ in the $-z$ direction, u in the x direction, and d in the $-x$ direction. In the two special cases considered, the electron is polarized in the z direction, i.e., in the direction of its momentum. An analogous quantity,

$$D_t=\frac{d\sigma_u-d\sigma_d}{d\sigma_u+d\sigma_d},$$

is given by the coefficient of $\cos\lambda_1\sin\lambda_2\cos\kappa_2$ divided by the spin-independent term. Similar ratios can be readily constructed when the particles are polarized in other directions. D_l and D_t are shown in Figs. 1 and 2 for various electron velocities as functions of $\cos\theta$.

For electron energies much larger than its rest mass, the cross section in the rest frame of the proton is

$$\begin{aligned}d\sigma/d\Omega=&[e_1^2e_2^2/4E^2\sin^4(\tfrac{1}{2}\theta)][1+2(E_1/m_2)\sin^2(\tfrac{1}{2}\theta)]^{-2}\\&\times\{[1+(E_1/m_2)(2+g_2^2E_1/m_2)\sin^2(\tfrac{1}{2}\theta)]\\&\times\cos^2(\tfrac{1}{2}\theta)+2(E_1/m_2)^2\sin^4(\tfrac{1}{2}\theta)(1+g_2)^2\\&-(E_1/m_2)(1+g_2)\sin^2(\tfrac{1}{2}\theta)\cos\lambda_1[2[\cos^2(\tfrac{1}{2}\theta)\\&+(E_1/m_2)(1+g_2)\sin^2(\tfrac{1}{2}\theta)]\cos\lambda_2\\&+(1-g_2E_1/m_2)\sin\theta\sin\lambda_2\cos\kappa_2]\}.\end{aligned}$$

The meaning of the angles is the same as above. Since we have already specialized to high energies, only terms depending on the longitudinal polarization of the electron appear.

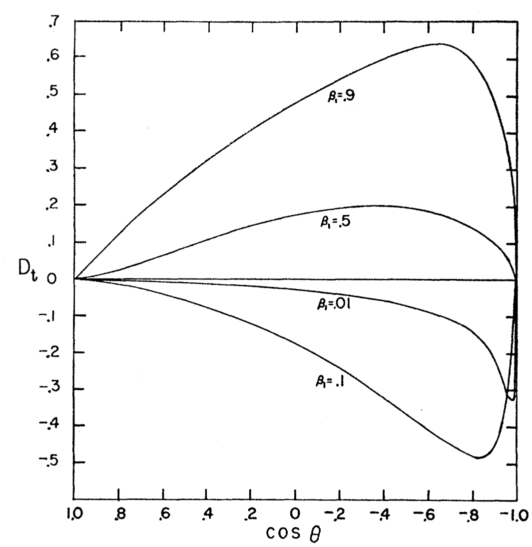

Fig. 2. The relative cross-section change for longitudinally polarized electrons and transversally polarized protons. The ratio is plotted for scattering in the plane of proton spin and initial momentum. For scattering in another plane, it must be multiplied by the cosine of the angle between proton spin and scattering plane.

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