Alpha-Decay Barrier Penetrabilities with an Exponential Nuclear Potential: Even-Even Nuclei*

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The real potential derived by optical-model analysis of data on elastic scattering of alpha particles is used for calculation of barrier penetrabilities for all known alpha decay groups of even-even nuclei. The barrier penetration factors were calculated by numerical integration in the WKB approximation taking into account centrifugal barrier effects, but ignoring noncentral interactions. Using these penetration factors and the experimental alpha half-lives, the reduced level widths δ^2 are calculated. Ratios of δ^2 values for ground and excited-state alpha groups are tabulated as a set of reduced hindrance factors.

INTRODUCTION

THEORETICAL calculations of barrier-penetration factors for alpha emission have traditionally been made by assuming an abrupt nuclear cutoff to the Coulombic potential at some "effective nuclear radius," although some attempts have been made to take into account the effects of a finite range to the nuclear potential.[1,2] Uncertainties regarding the nuclear potential for alpha particles have made it difficult to gain much knowledge of the absolute probabilities of alpha-particle formation by nuclei. It is important that one be able to separate the barrier penetrability from the intranuclear dynamic effects on alpha-decay rates. By using a nuclear potential derived from alpha-scattering information, we hope to have obtained such a fundamentally more significant treatment of alpha-decay data.

Recently there have been careful optical-model analyses of alpha-particle scattering data, and these analyses define the real potential in the nuclear surface region quite well. Originally, potentials of the Woods-Saxon form were used in the optical-model analysis.[3] There were some problems of nonuniqueness of fits and some apparent dependence of potentials on the alpha-particle bombarding energy (see discussion by Rasmussen[4]). Calculations of barrier-penetration factors for ground-state transitions of even-even alpha emitters have been made with the aforementioned nuclear potential.[4]

Igo has continued a careful study of the problem of optical-model analysis and has recently published a simple exponential expression for the real part of the alpha-nuclear potential valid in the surface region for $|V|\lesssim 10$ Mev[5]:

$$V(r)=-1100\exp\left\{-\left[\frac{r-1.17A^{\frac{1}{3}}}{0.574}\right]\right\}\text{ Mev},$$

where r is the distance in fermis (1 fermi$\equiv 10^{-13}$ cm) and A is the mass number. This expression gives a good fit for target elements from argon to lead and for bombarding energies between 18 and 48 Mev.

METHOD OF CALCULATION

It seems reasonable to expect that this potential should be nearly that experienced by alpha particles (3 to 8 Mev) emitted in alpha decay. Accordingly, we have used Igo's potential to calculate barrier penetration factors for most of the known alpha emitters. We have taken the natural logarithm of the penetration factor P to be equal to twice the WKB integral,

$$-\int_{R_i}^{R_0}\frac{(2M)^{\frac{1}{2}}}{\hbar}\left[V(r)+\frac{2Ze^2}{r}+\frac{\hbar^2}{2mr^2}l(l+1)-E\right]^{\frac{1}{2}}dr,$$

evaluated between the inner and outer classical turning points, where the integrand vanishes. Here M is the reduced mass of the alpha particle, Ze is the charge on the daughter nucleus, l is the orbital angular momentum of the emitted alpha, and E is the total decay energy that would be exhibited by the nucleus if stripped of its orbital electrons, i.e., alpha-particle energy plus recoil energy plus electron-screening corrections as given in Eq. (25.1) by Perlman and Rasmussen.[6]

The integrations were carried out numerically by the use of an IBM-650 digital computer. The outer turning point was found by solution of a quadratic equation and the inner turning point was found by a simple iterative procedure. The barrier integral was evaluated by a modified Simpson's-rule summation with the barrier region divided into 128 equal intervals. Simpson's rule was modified at the ends to better take into account the fact that the integrand is zero at the turning points and behaves there as $C|r-R_t|^{\frac{1}{2}}$. The Simpson's rule applied is

$$I_{128}=\tfrac{1}{3}\Delta r(3y_1+4y_2+2y_3+4y_4+2y_5+\cdots+2y_{123}+4y_{124}+2y_{125}+4y_{126}+3y_{127}).$$

The error introduced by using only 128 intervals is

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TABLE I. Ground-state transitions of even-even nuclei ($l=0$).

Atomic No.	Mass No.	Experimental data: α-particle energy with screening correction (Mev)	Partial half-life for α decay[a] (sec)	α group intensity (%)	R_i (fermis)	Calculated results: Barrier penetration factor[a] P	Reduced width δ^2 (Mev)
60	144	1.92	1.58 (23)	100	8.44	2.18 (−42)	0.0083
62	146	2.57	1.58 (15)	100	8.47	1.19 (−34)	0.0152
64	148	3.18	4.47 (9)	100	8.50	7.52 (−30)	0.0852
72	174	2.53	9.5 (22)	100	8.77	5.44 (−43)	0.0555
78	190	3.33	1.87 (18)	100	8.95	1.16 (−37)	0.013
78	192	2.63	3.17 (22)	100	8.95	3.04 (−46)	297
84	202	5.609	1.56 (5)	100	9.11	7.35 (−25)	0.0250
84	204	5.404	1.367 (6)	100	9.13	7.02 (−26)	0.0299
84	206	5.252	1.52 (7)	100	9.15	1.14 (−26)	0.0165
84	208	5.142	9.24 (7)	100	9.17	2.96 (−27)	0.0104
84	210	5.332	1.17 (7)	100	9.20	3.63 (−26)	0.00676
84	212	8.810	3.04 (−7)	100	9.35	1.32 (−13)	0.0714
84	214	7.714	1.636 (−4)	100	9.33	1.58 (−16)	0.111
84	216	6.808	1.58 (−1)	100	9.32	1.67 (−19)	0.109
84	218	6.032	1.827 (2)	100	9.32	1.31 (−22)	0.120
86	208	6.173	6.90 (3)	100	9.19	4.35 (−23)	0.00957
86	210	6.071	9.70 (3)	100	9.21	1.64 (−23)	0.0180
86	212	6.297	1.38 (3)	100	9.24	1.74 (−22)	0.0119
86	218	7.162	1.90 (−2)	99.8	9.34	4.67 (−19)	0.322
86	220	6.317	5.44 (1)	99.7	9.34	2.85 (−22)	0.184
86	222	5.521	3.31 (5)	100	9.33	5.38 (−26)	0.161
88	222	6.590	3.80 (1)	95	9.35	5.19 (−22)	0.138
88	224	5.717	3.15 (5)	94.8	9.34	5.91 (−26)	0.146
88	226	4.813	5.12 (10)	94.3	9.34	3.48 (−31)	0.152
90	226	6.367	1.853 (3)	79	9.37	8.40 (−24)	0.145
90	228	5.458	6.00 (7)	71	9.36	2.73 (−28)	0.124
90	230	4.718	2.528 (12)	74	9.37	6.61 (−33)	0.127
90	232	4.044	4.381 (17)	76	9.37	3.29 (−38)	0.151
92	228	6.709	6.943 (2)	75 est.	9.38	3.26 (−23)	0.0951
92	230	5.923	1.798 (6)	67.2	9.38	8.70 (−27)	0.123
92	232	5.357	2.321 (9)	68	9.39	7.51 (−30)	0.112
92	234	4.807	7.83 (12)	72	9.40	2.34 (−33)	0.113
92	236	4.538	7.53 (14)	75.3	9.41	2.71 (−35)	0.103
92	238	4.219	1.415 (17)	77	9.42	7.67 (−38)	0.203
94	234	6.230	5.40 (5)	75 est.	9.42	3.56 (−26)	0.112
94	236	5.803	8.50 (7)	68.9	9.43	2.65 (−28)	0.0876
94	238	5.535	2.822 (9)	72	9.44	9.30 (−30)	0.0786
94	240	5.202	2.073 (11)	75.5	9.46	9.75 (−32)	0.107
94	242	4.938	1.201 (13)	74	9.47	1.90 (−33)	0.0930
96	240	6.291	2.317 (6)	70	9.47	9.87 (−27)	0.0877
96	242	6.150	1.404 (7)	73.7	9.49	2.16 (−27)	0.0697
96	244	5.839	6.050 (8)	76.7	9.50	5.60 (−29)	0.0649
98	246	6.794	1.285 (5)	78	9.53	3.02 (−25)	0.0577
98	248	6.302	3.02 (7)	80	9.54	1.70 (−27)	0.0447
98	250	6.066	3.45 (8)	83	9.56	1.16 (−28)	0.0594
98	252	6.154	6.98 (7)	84.5	9.58	3.56 (−28)	0.0976
100	254	7.242	1.150 (4)	83	9.62	4.09 (−24)	0.0505

[a] The number in parentheses is the power of 10 by which the preceding number is to be multiplied.

somewhat different for different alpha emitters, being greatest for the lowest energy cases. In a typical case, the ground-state transition of Cm^{242}, we have I_{32} (32 intervals) $=31.0526$, $I_{64}=31.0159$, and $I_{128}=31.0129$. The absolute error in I_{128} is probably less than $|I_{128}-I_{64}|$ or 0.003. Rounding errors in the computer at the eighth significant figure are probably two orders of magnitude less than this. Thus, the penetration factors calculated here should be accurate to about 1%, the error consistently given penetration factors that are on the low side. Using the experimental decay rate data, we calculate a reduced alpha emission width δ^2 from the following expression:

$$\lambda=\delta^2 P/h,$$

TABLE II. Excited state transitions.

Atomic No.	Mass No.	Experimental data: α-particle energy with screening correction (Mev)	Partial half-life for α decay[a] (sec)	α group intensity (%)	Spin and parity	Calculated results: Barrier penetration factor P	Reduced width δ_L^2 (Mev)
84	210	4.544	1.17 (7)	0.0012	2+	3.16 (−31)	0.00931
86	218	6.564	1.90 (−2)	0.2	2+	1.62 (−21)	0.186
86	220	5.782	5.44 (1)	0.3	2+	6.10 (−25)	0.259
86	222	5.020	3.31 (5)	0.07	2+	4.59 (−29)	0.132
88	222	6.268	3.8 (1)	4	2+	1.32 (−23)	0.229
		5.946		0.0094	2+	4.35 (−25)	0.0163
		5.801		0.032	1−	1.22 (−25)	0.186
		5.756		0.002	4+	1.46 (−26)	0.103
88	224	5.481	3.15 (5)	4.9	2+	1.97 (−27)	0.226
		5.186		0.01	2+	4.15 (−29)	0.0220
		5.076		0.01	1−	1.29 (−29)	0.0705
88	226	4.629	5.12 (10)	5.7	2+	1.10 (−32)	0.291
		4.376		0.014	2+	1.48 (−34)	0.0529
		4.219		0.0021	1−	1.22 (−35)	0.0966
90	226	6.258	1.853 (3)	19	2+	1.62 (−24)	0.181
		6.130		1.7	1−	5.91 (−25)	0.0445
		6.063		0.6	4+	5.92 (−26)	0.157
90	228	5.375	6.00 (7)	28	2+	5.40 (−29)	0.248
		5.245		0.4	1−	1.34 (−29)	0.0143
		5.209		0.2	4+	1.64 (−30)	0.0584
		5.174		0.03	3−	2.04 (−30)	0.00703
90	230	4.651	2.528 (12)	26	2+	1.28 (−33)	0.230
		4.512		0.2	4+	3.44 (−35)	0.0659
		4.469		0.03	1−	8.23 (−35)	0.00413
		4.404		0.001	3−	1.04 (−35)	0.00109
		4.309		8×10^{-6}	6+	1.24 (−37)	7.33×10^{-4}
		4.281		8×10^{-6}	5−	2.14 (−37)	4.24×10^{-4}
90	232	3.986	4.381 (17)	24	2+	5.70 (−39)	0.277
92	230	5.852	1.798 (6)	32.1	2+	2.24 (−27)	0.229
		5.701		0.4	4+	1.06 (−28)	0.0602
		5.695		0.3	1−	4.72 (−28)	0.0101
92	232	5.301	2.321 (9)	32	2+	2.05 (−30)	0.193
		5.174		0.32	4+	9.95 (−32)	0.0397
		5.036		0.01	1−	6.39 (−32)	0.00193
92	234	4.756	7.83 (12)	28	2+	5.98 (−34)	0.171
		4.64		0.3	4+	2.485 (−35)	0.0441
		4.311		2.5×10^{-5}	1−	3.29 (−37)	2.78×10^{-4}

where λ is the decay constant, and h is Planck's constant. This definition is equivalent to the previous definition of δ^2 applied to the model with the sharp-cutoff potential (see reference 6, pp. 149 to 151).

RESULTS—GROUND-STATE TRANSITIONS

Table I lists for even-even nuclei the data used, most of which are from Table I of reference 6, and three computed quantities of interest: R_i, the radius at which the alpha of the particular energy considered will enter the barrier; P, the penetration factor; and δ^2, the reduced emission width.

It is to be noted that R_i is a function not only of mass number but also of energy. One sees, for example, a discontinuous increase of about 0.2 fermis for $Z=84$ in going across the 126-neutron shell, where the alpha energies increase discontinuously. If these calculations are to have fundamental significance as a calculation of the probability current impinging on the barrier, it is essential that the process of formation of alpha particles from their constitutent nucleons does not take place within the region of $r>R_i$. It is reasonable to suppose that alpha formation more readily occurs in the surface region than in the nuclear interior, since the low nucleon density in the surface means a small fermi momentum and less inhibition of nucleon clusters by the exclusion principle.

Electron-scattering experiments have shown that the charge density in Bi^{209} falls to half its central value at 6.47 fermis and to one-tenth at 7.82 fermis (see reference 4). R_i values for the polonium isotopes of about this mass number are $\sim$9.2. The R_i values obtained here with the Igo potential seem sufficiently larger than the size parameters of the nuclear charge

TABLE II.—*Continued*

Atomic No.	Mass No.	Experimental data: α-particle energy with screening correction (Mev)	Experimental data: Partial half-life for α decay[a] (sec)	α group intensity (%)	Spin parity	Calculated results: Barrier penetration factor P	Calculated results: Reduced width δ_L^2 (Mev)
92	236	4.49	7.53 (14)	27	2+	6.76 (−36)	0.152
		4.378		0.5	4+	2.51 (−37)	0.0759
92	238	4.172	1.415 (17)	23	2+	1.76 (−38)	0.265
		4.062		0.1	4+	5.27 (−40)	0.0384
94	236	5.756	8.50 (7)	30.9	2+	8.82 (−29)	0.118
		5.65		0.18	4+	6.90 (−30)	0.00880
		5.487		0.002	6+	1.23 (−31)	0.00550
94	238	5.492	2.822 (9)	28	2+	3.12 (−30)	0.0913
		5.394		0.095	4+	2.45 (−31)	0.00395
		5.243		0.004	6+	4.36 (−33)	0.00931
		5.044		7×10^{-6}	8+	1.74 (−35)	0.00409
		4.745		1.2×10^{-4}	0+	7.05 (−35)	0.0173
94	240	5.158	2.073 (11)	24.5	2+	3.03 (−32)	0.112
		5.054		0.1	4+	1.88 (−33)	0.00737
94	242	4.894	1.201 (13)	26	2+	5.57 (−34)	0.1103
96	242	6.106	1.404 (7)	26.3	2+	7.81 (−28)	0.0688
		6.005		0.035	4+	7.27 (−29)	9.83×10^{-4}
		5.851		0.006	6+	1.75 (−30)	0.00701
		5.645		3×10^{-5}	8+	1.02 (−32)	0.0602
		5.555		3.2×10^{-4}	1−	1.10 (−30)	5.95×10^{-4}
		5.24		1.4×10^{-4}	0+	1.53 (−32)	0.0186
		5.16		2×10^{-5}	2+	2.77 (−33)	0.0148
96	244	5.797	6.05 (8)	23.3	2+	1.98 (−29)	0.0557
		5.70		0.016	4+	1.75 (−30)	4.34×10^{-4}
		5.552		4×10^{-3}	6+	3.84 (−32)	0.00494
98	246	6.752	1.285 (5)	22	2+	1.20 (−25)	0.0409
		6.656		0.16	4+	1.41 (−26)	0.00253
		6.508		0.015	6+	4.83 (−28)	0.00692
98	250	6.023	3.45 (8)	17	2+	4.18 (−29)	0.0337
98	252	6.111	6.98 (7)	15.5	2+	1.30 (−28)	0.0494
		6.013		0.2	4+	1.24 (−29)	0.00666
100	254	7.202	1.15 (4)	17	2+	1.73 (−24)	0.0244
		7.102		0.4	4+	2.17 (−25)	0.00459

[a] The number in parentheses is the power of 10 by which the preceding number is to be multiplied.

density to give reasonable assurance that alpha formation does not appreciably occur within the potential barrier defined by the optical model potential. Values of P and δ^2 are given to three significant figures although in many cases, especially the rare earth examples, the experimental uncertainty in energy and half-life are such that only the order of magnitude of δ^2 is significant. The results for $_{78}Pt^{192}$ are so anomalous as to cast doubt on the experimental data.

Figure 1 is a semilogarithmic plot of δ^2 *vs* neutron number. For comparison with δ^2 calculated with other potentials, refer to Fig. 5 of reference 4 and the associated discussion. There are no important differences between the trends of δ^2 from Table I of this paper and the δ^2 values calculated with the earlier Igo-Thaler potential, as discussed in reference 4.

RESULTS—EXCITED-STATE TRANSITIONS

The extensive alpha-particle spectroscopic studies of the last few years have revealed many new transitions to excited states of even-even nuclei, and studies of associated gamma and electron radiations have made possible the definite spin assignments of many of these excited states. In other cases the systematic energy trends of excited states of even-even nuclei usually permit one to assign spins with confidence. (For an excited level populated by alpha decay from the ground state, 0+, of an even-even nucleus, the parity must be even if the spin is even, and odd if the spin is odd.)

Table II presents the results of the calculations on excited-state transitions. Table II is of the form of Table I except for an additional data column giving the assumed angular momentum l. The data are principally taken from Table I of reference 6, except for l values, which were not given there. Our l value assignments come from various publications, from inference from energy level systematics, and from private communications.[7]

The usual basis for discussion of rates of excited-state alpha transitions in even-even nuclei is the hindrance factor, F, the ratio of the rates of ground-state and of excited-state alpha intensities of the given nucleus multiplied by the ratio of barrier-penetration factors calculated by some prescription not taking into account any centrifugal barrier effects. Of more fundamental significance when angular momenta can be assigned to transitions, is the reduced hindrance factor, defined similarly to F except that the barrier penetrability prescription takes into account the centrifugal barrier effects. (See p. 181 of reference 6 for discussion of this terminology.)

We have calculated reduced hindrance factors as simply the ratio of δ^2 for the ground-state transition to $\delta_L{}^2$ for the excited-state. These ratios are summarized in Table III.

For the spherical nuclei (region of Pb^{208}) the calculated δ^2 values probably have fundamental significance in terms of the probability currents impinging on the barrier. For the spheroidal nuclei the interpretation is more complicated, and numerous publications have been devoted to the problems associated with this asphericity. For these spheroidal nuclei our calculations may serve as a basis for further analysis—a basis with somewhat more theoretical justification than presently published hindrance-factor values.

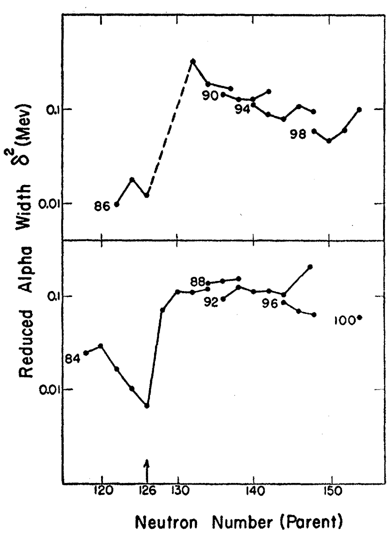

FIG. 1. Plot of reduced widths, δ^2 for ground-state alpha groups. Alternate even atomic numbers are plotted on different ordinate scales to avoid the overlapping of points. The break at 126 neutrons has long been noted. The break is less in ratio for this diffuse nuclear potential than for the sharp nuclear potential usually assumed. The δ^2 values for Rn^{218} and U^{238} are high in this as in other treatments.

TABLE III. Hindrance factors of excited-state transitions in even-even nuclei.

Alpha emitter	Energy of final state (kev)	Reduced hindrance factor 2+ state	4+ state	6+ state	1− state	Other state
Po^{210}	804	0.726				
Rn^{218}	609	1.73				
Rn^{220}	545	0.711				
Rn^{222}	510	1.22				
Ra^{222}	324.6	0.603				
	650	8.52				
	800				0.695	
	850		1.34			
Ra^{224}	241	0.646				
	540	6.65				
	650				2.07	
Ra^{226}	187	0.522				
	450	2.88				
	610				1.57	
Th^{226}	111.1	0.802				
	242				3.27	
	309		0.928			
Th^{228}	84.47	0.502				
	217				8.66	
	253		2.13			
	289					17.7 (3 −)
Th^{230}	67.62	0.553				
	210		1.926			
	253				30.7	
	320					117 (3 −)
	416			173		
	445					299 (5 −)
Th^{232}	59	0.545				
U^{230}	72.13	0.538				
	226.4		2.04			
	230.4				12.1	
U^{232}	57.5	0.580				
	186.1		2.82			
	326				57.9	
U^{234}	52.4	0.658				
	170		2.50			
	505				407	
U^{236}	49	0.674				
	163		1.35			
U^{238}	48	0.767				
	160		5.30			
Pu^{234}	47	1.08				
Pu^{236}	47.5	0.741				
	156		9.86			
	321			16.0		
Pu^{238}	43.50	0.861				
	143.31		19.9			
	296.4			8.45		
	499					19.2 (8+)
	806					4.54 (0+)
Pu^{240}	45	0.958				
	151		14.5			
Pu^{242}	45	0.835				
Cm^{242}	44.11	1.013				
	146.0		70.9			
	303.7			9.94		
	514					11.6 (8+)
	605				118	
	~930					~3.8 (0+)
	~1010					~4.8 (2+)
Cm^{244}	42.88	1.16				
	141.8		150			
	292			13.1		
Cf^{246}	42.12	1.41				
	140		26.9			
	291			8.35		
Cf^{250}	44	1.76				
Cf^{252}	43.4	1.99				
	143		14.7			
Fm^{254}	42	2.07				
	140		11.0			

[7] I am especially indebted to Dr. F. Asaro and Dr. F. S. Stephens for communication of several of their unpublished spin assignments and other data.

TABLE IV. Reduced hindrance-factor comparison for Cm^{242} and Th^{230}.

Nucleus	Excited-state energy (kev)	Spin and parity	Hindrance factor[a]	Centrifugal barrier factor[a]	Reduced hindrance factor: Perlman and Rasmussen[a]	Reduced hindrance factor: This work
Cm^{242}	0	0+	(1)	(1)	(1)	(1)
	44	2+	1.7	1.6	1.1	1.01
	146	4+	390	4.9	80	71
	304	6+	350	29	12	10
	514	8+	5000	340	15	12
	605	1−	500	1.2	420	380[b]
	935	0+	20	1	20	18[b]
	1030	2+	45	1.6	28	24[b]
Th^{230}	0	0+	(1)	(1)	(1)	(1)
	68	2+	1.1	1.7	0.65	0.55
	210	4+	12	5.4	2.2	1.93
	253	1−	38	1.2	32	31
	320	3−	370	2.8	130	117
	416	6+	8200	40	205	173
	445	5−	4900	14	350	299

[a] See reference 6.
[b] These are the reduced hindrance factors that would be calculated using the intensity values used in reference 6. The entries in our Table III are based on newer revised experimental intensities and are different.

Let us compare our reduced hindrance factors for Cm^{242} and Th^{230} with results of earlier calculations. Hindrance factors and centrifugal-barrier factors have previously been given[6] for Cm^{242} and Th^{230}. The values of our Table III are to be compared with the quotient of hindrance factor and centrifugal-barrier factor. Table IV gives this comparison.

Our calculations seem to yield systematically somewhat lower (5 to 15%) values of the reduced hindrance factors than the older calculations. In part this difference may be due to the slightly greater influence of the centrifugal potential with the present diffuse-potential model, because the centrifugal potential not only raises but somewhat thickens the barrier by displacing the inner turning point inward. In order to assess the influence of the centrifugal potential by itself, calculations were run for hypothetical alpha groups of ${}_{88}Ra^{224}$ having identical energies to the ground-state transition but with l values of 2 and 4. The centrifugal potential reduces the barrier penetrability by factors of 1.708 and 5.917 for $l=2$ and 4, respectively. Values of the inner turning point (R_i) for $l=0$, 2 and 4 are 9.344, 9.333, and 9.308 fermis, respectively.

CONCLUDING REMARKS

It is outside the scope of this paper to go into details as to how these new results may modify earlier theoretical interpretations of alpha decay. The results here are mainly offered as a basis for future fundamental theoretical studies. It is worth noting that the ground-state transitions beyond the 126-neutron shell show δ^2 values of the order of 0.1 Mev, systematically falling off from maximum values for $Z=86$ to smaller values for the heavier nuclei. Rn^{218} and U^{238}, in these as in other calculations, show reduced widths abnormally large compared to their nearest neighbors. The nuclei with 126 or less neutrons show especially small reduced widths that are an order of magnitude less than the average of heavier nuclei (Po^{210} is especially small).

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