Higher Order Corrections to the Allowed Beta Decay*

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The shape of the allowed beta spectrum and the directional correlations of the allowed beta ray and gamma or alpha ray have been investigated theoretically with an assumption of VA. We take into account the Coulomb field due to the daughter nucleus, the finite de Broglie wavelength effect, and the contribution of the second forbidden matrix elements, $\mathfrak{M}(r^2)$, $\mathfrak{M}(\boldsymbol{\alpha}\cdot\mathbf{r})$, $\mathfrak{M}(\boldsymbol{\sigma}r^2)$, $\mathfrak{M}((\boldsymbol{\sigma}\cdot\mathbf{r})\mathbf{r})$, $\mathfrak{M}(\gamma_5\mathbf{r})$, and $\mathfrak{M}(\boldsymbol{\alpha}\times\mathbf{r})$, simultaneously. Relations between coordinate-type and momentum-type matrix elements are given in the nonrelativistic approximation. In this case, $\mathfrak{M}(\boldsymbol{\alpha}\times\mathbf{r})/\mathfrak{M}(\boldsymbol{\sigma})=M^{-1}$. The correction factors for the beta spectra of B^{12} and N^{12} as well as their ratio have almost no energy dependence, since several corrections cancel each other. On the other hand, this ratio varies by 12% over the whole spectrum, if we adopt $\mathfrak{M}(\boldsymbol{\alpha}\times\mathbf{r})/\mathfrak{M}(\boldsymbol{\sigma})=M^{-1}(\mu_p-\mu_n)$ as given by Gell-Mann. The beta-alpha directional correlation of Li^8 is discussed also.

I. INTRODUCTION

IN an allowed transition, the ordinary theory gives the beta spectrum to be constant if we divide the beta intensity by the statistical factor. Furthermore, it gives isotropies of the directional correlations of beta rays and alpha or gamma rays. This comes from the fact that the ordinary theory assumes s-wave leptons only. However, when the electron energy becomes very large, there appear some contributions from p- and d-wave leptons. These contributions are mainly interferences between s waves and p or d waves. They are about $(v/c)(p\rho)$ and $(p\rho)^2$ times as large as that of the s waves, where v is the average velocity of nucleons inside the nucleus, p is the electron momentum in units of mc, and ρ is the nuclear radius in units of the electron Compton wavelength. We call these the contributions of the second forbidden transition to the allowed beta decay.

The above effect for the allowed beta spectrum was first calculated by Fujita and Yamada to determine the upper and lower limits of C_P/C_T from the beta spectra of He^6 and B^{12}.[1] These authors treated the correction factor for the beta spectrum by expanding L_0 in a power series in $(p\rho)$ up to the quadratic terms and taking into account the interferences between $\mathfrak{M}(\beta\boldsymbol{\sigma})$ and $\mathfrak{M}(\beta\boldsymbol{\sigma}r^2)$, $\mathfrak{M}((\beta\boldsymbol{\sigma}\cdot\mathbf{r})\mathbf{r})$, $\mathfrak{M}(\beta\gamma_5\mathbf{r})$, or $\mathfrak{M}(\beta\boldsymbol{\alpha}\times\mathbf{r})$. Directional correlations taking into account the same effect were given by Morita and Yamada and applied to the beta-alpha correlation of Li^8.[2] Zweifel also calculated the allowed beta spectrum with contribution from the second forbidden transition.[3] He assumed STP or VTP interactions.

Recently, Gell-Mann noted again the same effect in the beta spectrum, with momentum-type matrix elements, $\mathfrak{M}(\gamma_5\mathbf{r})$ and $\mathfrak{M}(\boldsymbol{\alpha}\times\mathbf{r})$, only.[4] He estimated the magnitude of $\mathfrak{M}(\boldsymbol{\alpha}\times\mathbf{r})$ also. Since his beta-decay interaction involves an additional term, which represents the beta decay through the meson cloud of the nucleon (e.g., $p\rightarrow\pi^++n$, $\pi^+\rightarrow\pi^0+e^++\nu$, and $n+\pi^0\rightarrow n'$), the magnitude of $\mathfrak{M}(\boldsymbol{\alpha}\times\mathbf{r})$ estimated by him is large compared with that given in the old theory; see Appendix. Here we call the theory of beta decay, which does not have the meson-current term as above, the "old theory." In order to test his theory, Gell-Mann considered the measurement of the allowed beta spectra of both charges. Comparing these two spectra, the effect due to the $\mathfrak{M}(\boldsymbol{\alpha}\times\mathbf{r})$ term may become twice as large, because $\mathrm{Re}(C_A{}^*C_V+C_A{}'^*C_V{}')\{\mathfrak{M}^*(\boldsymbol{\sigma})\mathfrak{M}(\boldsymbol{\alpha}\times\mathbf{r})\}$ changes its sign if one changes the sign of the charge of the electron. Its energy dependence on the electron is linear.

We know, however, several corrections to the allowed beta spectrum, which are proportional to the electron energy and change their sign if the sign of the electron charge is changed, even within the framework of the old theory of beta decay. Namely, there are Coulomb corrections to the electron wave functions as well as to the nuclear matrix elements. In particular, the Coulomb term in the finite de Broglie wavelength effect is important. Without making an estimate of these corrections, we cannot obtain any conclusion on the validity of Gell-Mann's theory of beta decay in comparison with experimental data.

In this paper, we will give the allowed beta spectrum including terms up to the order of $(p\rho)^2$. The contribution of the second forbidden transition to the allowed beta decay and the finite de Broglie wavelength effect[5] are then included simultaneously. Using the same procedure, the directional correlations of the allowed beta ray and alpha or gamma ray are given. In Sec. II, the parameters of the beta ray[6] are given. With them, the beta spectrum, beta-alpha, and beta-gamma directional

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[1] J. Fujita and M. Yamada, Progr. Theoret. Phys. (Kyoto) **10**, 518 (1953).

[2] M. Morita and M. Yamada, Progr. Theoret. Phys. (Kyoto) **13**, 114 (1955).

[3] P. F. Zweifel, Phys. Rev. **95**, 112 (1954). See also reference 8.

[4] M. Gell-Mann, Phys. Rev. **111**, 362 (1958).

[5] M. E. Rose and C. L. Perry, Phys. Rev. **90**, 479 (1953). Rose, Perry, and Dismuke, Oak Ridge National Laboratory Report ORNL-1459, 1953 (unpublished).

[6] M. Morita and R. S. Morita, Phys. Rev. **109**, 2048 (1958).

correlations are easily expressed. In Sec. III, the beta spectra of B^{12} and N^{12} are discussed in connection with Gell-Mann's work.[4] In Sec. IV, the beta-alpha directional correlation of Li^8 is discussed. A discussion of results is given in Sec. V. In the Appendix, relations between nuclear matrix elements, which contribute to the allowed beta decay from the second forbidden transition, are given in the nonrelativistic approximation.

II. FORMULAS

Since the calculation is straightforward, we write here the final results only. We assume a linear combination of vector and axial vector as the beta interaction. Formulas for a linear combination of scalar, tensor, and pseudoscalar will be obtained by inspection together with a rule described on page 2052 of reference 6. We use the same notation[6] and the units $\hbar=c=m=1$. Possible interferences between the allowed matrix elements and the proper second-forbidden ones, $\mathfrak{M}(R_{ij})$, $\mathfrak{M}(T_{ij})$, $\mathfrak{M}(A_{ij})$, and $\mathfrak{M}(S_{ijk})$, in the angular distributions are very small. They are omitted throughout this paper. Furthermore, formulas are given for the allowed transitions of beta decay only.

1. Parameters for Beta Ray: $b_{LL}^{(n)}$

$$b_{00}^{(0)}=(|C_V|^2+|C_V'|^2)[|\mathfrak{M}(1)|^2L_0+2\{\mathfrak{M}^*(1)\mathfrak{M}(r^2)\}(-\tfrac{1}{6}K^2L_0+\tfrac{1}{3}KN_0)+2\{i\mathfrak{M}^*(1)\mathfrak{M}(\boldsymbol{\alpha}\cdot\mathbf{r})\}(-\tfrac{1}{3}KL_0+N_0)]. \quad (1)$$

$$-3^{-\frac{1}{2}}b_{11}^{(0)}=(|C_A|^2+|C_A'|^2)[|\mathfrak{M}(\boldsymbol{\sigma})|^2L_0-2\{\mathfrak{M}^*(\boldsymbol{\sigma})\mathfrak{M}(\boldsymbol{\sigma}r^2)\}(\tfrac{1}{6}K^2L_0+\tfrac{1}{3}KN_0)+2\{\mathfrak{M}^*(\boldsymbol{\sigma})\mathfrak{M}((\boldsymbol{\sigma}\cdot\mathbf{r})\mathbf{r})\}\tfrac{2}{3}KN_0+2\{i\mathfrak{M}^*(\boldsymbol{\sigma})\mathfrak{M}(\gamma_5\mathbf{r})\}(-\tfrac{1}{3}KL_0+N_0)]+2\{\mathfrak{M}^*(\boldsymbol{\sigma})\mathfrak{M}(\boldsymbol{\alpha}\times\mathbf{r})\}\,\mathrm{Re}(C_A^*C_V+C_A'^*C_V')\times(\tfrac{1}{3}KL_0+N_0). \quad (2)$$

$$(\tfrac{2}{3})^{\frac{1}{2}}b_{11}^{(2)}=2(|C_A|^2+|C_A'|^2)[\{\mathfrak{M}^*(\boldsymbol{\sigma})\mathfrak{M}(\boldsymbol{\sigma}r^2)\}\times(\tfrac{1}{3}KL_{12}-N_{21})+\{\mathfrak{M}^*(\boldsymbol{\sigma})\mathfrak{M}((\boldsymbol{\sigma}\cdot\mathbf{r})\mathbf{r})\}(\tfrac{1}{3}KL_{12}+3N_{21})+\{i\mathfrak{M}^*(\boldsymbol{\sigma})\mathfrak{M}(\gamma_5\mathbf{r})\}2L_{12}]-2\{\mathfrak{M}^*(\boldsymbol{\sigma})\mathfrak{M}(\boldsymbol{\alpha}\times\mathbf{r})\}\times[\mathrm{Re}(C_A^*C_V+C_A'^*C_V')L_{12}+\mathrm{Im}(C_A^*C_V+C_A'^*C_V')H_{12}]. \quad (3)$$

In the above equations, L_0, etc., are the following combinations of electron wave functions:

$$L_0=(2p^2F)^{-1}[g_{-1}^2+f_1^2]=\tfrac{1}{2}(1+\gamma)-(5/3)\alpha Z\rho W-\tfrac{1}{3}\alpha(Z\rho/W)-\tfrac{1}{3}p^2\rho^2\rightarrow 1. \quad (4)$$

$$N_0=(2p^2F)^{-1}[f_{-1}g_{-1}-f_1g_1]=-(p^2/3W)\gamma-(\alpha Z/2\rho)+\tfrac{2}{3}(\alpha Z)^2W-(p^2/9W)(\alpha Z)^2+(11/18)p^2\alpha Z\rho\rightarrow-(p^2/3W)-(\alpha Z/2\rho). \quad (5)$$

$$L_{12}=(2p^2F)^{-1}\rho^{-1}[g_{-1}f_2\cos(\delta_{-1}-\delta_2)-f_1g_{-2}\cos(\delta_1-\delta_{-2})]\rightarrow-p^2/3W. \quad (6)$$

$$H_{12}=(2p^2F)^{-1}\rho^{-1}[g_{-1}f_2\sin(\delta_{-1}-\delta_2)-f_1g_{-2}\sin(\delta_1-\delta_{-2})]\rightarrow-\tfrac{1}{4}p\alpha Z. \quad (7)$$

$$N_{21}=(2p^2F)^{-1}\rho^{-2}[g_{-1}g_2\cos(\delta_{-1}-\delta_2)+f_1f_{-2}\cos(\delta_1-\delta_{-2})]\rightarrow-(1/15)p^2-(p^2/6W)(\alpha Z/2\rho). \quad (8)$$

The expressions for L_0 and N_0 involve the finite de Broglie wavelength effect. The arrow in each equation indicates the approximations $(\alpha Z)^2\ll 1$ and $(p\rho)\ll 1$. In Eqs. (1) through (8), the formulas are given for a negatron decay. In order to obtain the corresponding formulas for a positron decay, the following substitutions should be performed: $Z\rightarrow -Z$, $C_V\rightarrow C_V^*$, $C_V'\rightarrow -C_V'^*$, $C_A\rightarrow -C_A^*$, and $C_A'\rightarrow C_A'^*$.

Previous calculations by Fujita and Yamada[1,7] (*STP* assumption) agree with Eq. (2), if the necessary replacement of symbols is performed, while Zweifel's calculations[3] disagree with Eq. (2) in many points.[8] Equation (1) is consistent with Zweifel's calculations.[3] Equation (3) is also consistent with reference 2, if we put $Z=0$.[9] If we assume the nuclear matrix elements as the phenomenological parameters, which may be determined in accordance with experimental data, Eq. (2) involves the beta spectrum given by Gell-Mann[4] as a special case.

2. Intensity of Beta Ray

The intensity of the beta ray in the decay scheme $j\xrightarrow{\beta} j_1$ is given by

$$P(W)dW=(2\pi^3)^{-1}[(2j_1+1)/(2j+1)]\times F(\pm Z,W)pWK^2CdW \quad \text{for} \quad e^{\mp},$$

with

$$C=b_{00}^{(0)}-3^{-\frac{1}{2}}b_{11}^{(0)}. \quad (9)$$

Here C is called the correction factor for the beta spectrum. The energy variation of C also depends on the nuclear matrix elements. Now we assume[10]:

$$\mathfrak{M}(r^2)\approx\tfrac{3}{5}\rho^2\mathfrak{M}(1), \quad (10)$$

$$i\mathfrak{M}(\boldsymbol{\alpha}\cdot\mathbf{r})\approx\mp(\Lambda\alpha Z/4\rho)\mathfrak{M}(r^2) \quad \text{for} \quad e^{\mp}. \quad (11)$$

$$\mathfrak{M}(\boldsymbol{\sigma}r^2)\approx\tfrac{3}{5}\rho^2\mathfrak{M}(\boldsymbol{\sigma}), \quad (12)$$

$$\mathfrak{M}((\boldsymbol{\sigma}\cdot\mathbf{r})\mathbf{r})\approx\tfrac{3}{5}\eta\rho^2\mathfrak{M}(\boldsymbol{\sigma}), \quad (13)$$

$$i\mathfrak{M}(\gamma_5\mathbf{r})\approx(1/2M)\mathfrak{M}(\boldsymbol{\sigma})\mp(\Lambda\alpha Z/4\rho)\mathfrak{M}((\boldsymbol{\sigma}\cdot\mathbf{r})\mathbf{r}) \quad \text{for} \quad e^{\mp}. \quad (14)$$

$$\mathfrak{M}(\boldsymbol{\alpha}\times\mathbf{r})\approx M^{-1}\mathfrak{M}(\boldsymbol{\sigma}). \quad (15)$$

Here M is the rest mass of a nucleon. Λ is nearly one,

[7] There is a misprint in reference 1, which was noticed by Yamada at the time of its publication. In Eq. (6), the sign of $\int\beta\boldsymbol{\sigma}^*\int(\beta\boldsymbol{\sigma}\cdot\mathbf{r})\mathbf{r}$ should be reversed.

[8] We do not know whether Zweifel's calculation is based on a different approximation.

[9] There are two misprints in reference 2, which were known at the time of its publication. In the fifth line of Eq. (1), $-p^2/5$ should be read as $-p^2/30$; and in the seventh line of this equation, the front sign is minus instead of plus.

[10] As is known from their derivation, Eqs. (10) through (15) may involve an error of a few tens of percent. However, this does not change our further discussion. In the case of the high *ft* value, a special consideration is necessary; see Sec. IV.

except for light nuclei where Λ is very large; see Eq. (A6). Equations (10), (12), and (13) follow from the assumption of uniform density of nuclear matter. In Eq. (13), η will be given, e.g., by the single-particle shell model. Equations (11), (14), and (15) are given in the nonrelativistic approximation in Appendix.

If we assume Gell-Mann's theory of beta decay,[4] then

$$\mathfrak{M}(\boldsymbol{\alpha}\times\mathbf{r})\approx M^{-1}(\mu_p-\mu_n)\mathfrak{M}(\boldsymbol{\sigma})\approx 4\lambda M^{-1}\mathfrak{M}(\boldsymbol{\sigma}), \quad (15')$$

with

$$C_A/C_V=C_A'/C_V'=-\lambda;\quad \lambda\approx 1.2.$$

In this case, $\mathfrak{M}(\gamma_5\mathbf{r})$ may have a different value from that given by Eq. (14). Since, however, we have no idea how to calculate it, we assume Eq. (14) for $\mathfrak{M}(\gamma_5\mathbf{r})$, hereafter.

Substituting Eqs. (4) through (8) and (10) through (15) into Eqs. (1) and (2), the correction factor for the allowed beta spectrum becomes

$$\begin{aligned}C=&(|C_V|^2+|C_V'|^2)|\mathfrak{M}(1)|^2\{\tfrac{1}{2}(1+\gamma)+(3/20)\Lambda(\alpha Z)^2\\ &\pm\alpha Z\rho[-\tfrac{1}{3}(5W+W^{-1})+\tfrac{1}{10}(\Lambda-2)K+\tfrac{1}{10}\Lambda(p^2/W)]\\ &-\rho^2[\tfrac{1}{3}p^2+\tfrac{1}{5}K^2+(2/15)(Kp^2/W)]\}\\ &+(|C_A|^2+|C_A'|^2)|\mathfrak{M}(\boldsymbol{\sigma})|^2\{\tfrac{1}{2}(1+\gamma)+(3/20)\Lambda\eta\\ &\times(\alpha Z)^2\pm\alpha Z\rho[-\tfrac{1}{3}(5W+W^{-1})+\tfrac{1}{10}(\Lambda\eta-4\eta+2)K\\ &+\tfrac{1}{10}\Lambda\eta(p^2/W)]-\rho^2[\tfrac{1}{3}p^2+\tfrac{1}{5}K^2+(2/15)(2\eta-1)\\ &\times(Kp^2/W)]+M^{-1}[\tfrac{1}{3}(-1\mp 2\lambda^{-1})K+\tfrac{1}{3}(-1\pm 2\lambda^{-1})\\ &\times(p^2/W)+(2\lambda^{-1}\mp 1)(\alpha Z/2\rho)]\}\quad\text{for } e^{\mp}. \quad (16)\end{aligned}$$

If we adopt Eq. (15′) instead of Eq. (15), the correction factor of the allowed beta spectrum becomes

$$C=[\text{Eq. (16) with all } 2\lambda^{-1} \text{ replaced by } 8]. \quad (16')$$

In Eqs. (16) and (16′), we have included the finite de Broglie wavelength effect only for L_0 of $\mathfrak{M}(\boldsymbol{\sigma})$. When we consider it for all of the terms in $b_{00}^{(0)}$ and $b_{11}^{(0)}$, the change of C is of the order of $C\times 10^{-4}$. As we can see from those formulas, the individual spectral shape may be different from that given by Gell-Mann.[4]

3. Directional Correlations

The directional correlations are always expressed by

$$W(\theta)=1+A_2P_2(\cos\theta). \quad (17)$$

(a) β—γ correlation in the decay scheme $j\xrightarrow{\beta} j_1\xrightarrow{\beta} j_2$.

$$A_2=[-(\tfrac{2}{3})^{\frac{1}{2}}b_{11}^{(2)}F_2(11jj_1)\mathfrak{F}_2(L_1L_1'j_2j_1)\div[C\mathfrak{G}_0(L_1j_2j_1)]. \quad (18)$$

(b) β—γ_2 correlation in the decay scheme $j\xrightarrow{\beta} j_1\xrightarrow{\gamma_1} j_2\xrightarrow{\gamma_2} j_3$, where γ_1 is unobserved.

$$A_2=[-(\tfrac{2}{3})^{\frac{1}{2}}b_{11}^{(2)}F_2(11jj_1)\mathfrak{G}_2(L_1j_2j_1)\mathfrak{F}_2(L_2L_2'j_3j_2)]\div[C\mathfrak{G}_0(L_1j_2j_1)\mathfrak{G}_0(L_2j_3j_2)]. \quad (19)$$

$\mathfrak{F}$ and $\mathfrak{G}$ are defined by

$$\mathfrak{F}_n(LL'j_aj_b)=\sum_{L,L'}(j_a\|L\|j_b)(j_a\|L'\|j_b)F_n(LL'j_aj_b), \quad (20)$$

$$\mathfrak{G}_n(Lj_aj_b)=\sum_L(j_a\|L\|j_b)^2(-)^{n+L-j_a-j_b}\times W(j_aj_aj_bj_b;nL)[(2j_a+1)(2j_b+1)]^{\frac{1}{2}}. \quad (21)$$

In particular,

$$\mathfrak{F}_0(LLj_aj_b)\equiv\mathfrak{G}_0(Lj_aj_b)\equiv\sum_L(j_a\|L\|j_b)^2.$$

(c) β—α correlation in the decay scheme $j\xrightarrow{\beta} j_1\xrightarrow{\alpha} j_2$, where only the Lth partial wave of the alpha ray is important.

$$A_2=[-(\tfrac{2}{3})^{\frac{1}{2}}b_{11}^{(2)}F_2(11jj_1)F_2(L_1L_1j_2j_1)L_1(L_1+1)]\div C[L_1(L_1+1)-3]. \quad (22)$$

A generalization of Eq. (22) is obtained from Eq. (18) with the replacements[11]

$$\begin{aligned}&(L_1L_1'1\,-1|20)(j_2\|L_1\|j_1)(j_2\|L_1'\|j_1)\rightarrow\\ &\quad -(L_1L_1'00|20)(j_2\|L_1\|j_1)(j_2\|L_1'\|j_1)^*\quad\text{in }\mathfrak{F}_2,\\ &\quad (j_2\|L_1\|j_1)^2\rightarrow|(j_2\|L_1\|j_2)|^2\quad\text{in }\mathfrak{G}_0.\end{aligned}$$

In Eqs. (18), (19), and (22), we use the following approximations:

$$C=|\mathfrak{M}(1)|^2+\lambda^2|\mathfrak{M}(\boldsymbol{\sigma})|^2. \quad (23)$$

$$\begin{aligned}-(\tfrac{2}{3})^{\frac{1}{2}}b_{11}^{(2)}=&\lambda^2|\mathfrak{M}(\boldsymbol{\sigma})|^2(p^2/W)\{\mp\tfrac{1}{10}\alpha Z\rho(2\Lambda\eta-3\eta+1)\\ &+\rho^2[(2/15)(\eta+1)K+(2/25)(3\eta-1)W]\\ &+(2/3M)(\pm\lambda^{-1}+1)\}\quad\text{for } e^{\mp}. \quad (24)\end{aligned}$$

If we adopt Eq. (15′) instead of Eq. (15), we obtain

$$-(\tfrac{2}{3})^{\frac{1}{2}}b_{11}^{(2)}=[\text{Eq. (24) with } \lambda^{-1} \text{ replaced by } 4]. \quad (24')$$

Here the finite de Broglie wavelength effect is entirely neglected.

In C, the second forbidden matrix elements are omitted also. If we take into account these effects, A_2 changes by the order of $A_2\times 10^{-2}$.

Similar calculations for beta-gamma correlation have been done by Bernstein and Lewis,[12] and by Boehm, Soergel, and Stech.[13]

III. BETA SPECTRA OF B^{12} AND N^{12}

Recently Gell-Mann estimated that the ratio of the intensities of the negatron in the decay of B^{12} and the

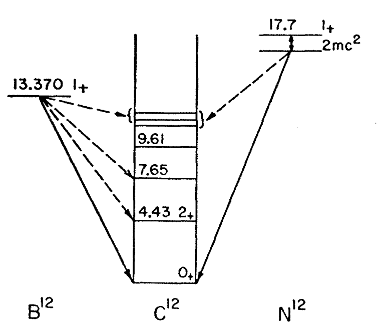

FIG. 1. Level diagram of B^{12}, C^{12}, and N^{12} in units of Mev. It is simplified from that of reference 14.

[11] M. Morita, Progr. Theoret. Phys. (Kyoto) **15**, 445 (1956), especially p. 456. M. Morita and R. S. Morita, Phys. Rev. **110**, 461 (1958), especially p. 464.

[12] After this work was completed, a similar calculation has been published, namely, J. Bernstein and R. R. Lewis, Phys. Rev. **112**, 232 (1958).

[13] Boehm, Soergel, and Stech, Phys. Rev. Letters **1**, 77 (1958).

positron in the decay of N^{12} varies by about 20% over the whole spectrum. In this section, we show its analysis based on our formulas.

The level diagram of B^{12}, C^{12}, and N^{12} is shown in Fig. 1.[14] In Eqs. (16) and (16′), $\eta=\frac{1}{2}$ for both e^- and e^+, if a nucleon goes from $p_{\frac{1}{2}}$ to $p_{\frac{3}{2}}$. Λ is 10.0 for e^- and -13.2 for e^+. Using these values and Eq. (16), the correction factors for beta spectrum (labeled C_- and C_+ for B^{12} and N^{12}, respectively) are almost constant in all energy regions. Consequently, C_-/C_+ in this approximation has no energy dependence. Several corrections cancel each other in this case. On the other hand, C_- and C_+ from Eq. (16′) have some energy dependence. They are shown in Fig. 2. The ratio, C_-/C_+, normalized by $[\mathfrak{M}(\boldsymbol{\sigma})$ of $e^-/\mathfrak{M}(\boldsymbol{\sigma})$ of $e^+]$ is given in Fig. 3. Its variation over the whole spectrum is 12%. This is the sum of the contributions of 16% from the $\mathfrak{M}(\boldsymbol{\alpha}\times\mathbf{r})$ term, -3% from the finite de Broglie wavelength effect, and -1% from the other matrix elements. Here, we have adopted Eq. (15′) as $\mathfrak{M}(\boldsymbol{\alpha}\times\mathbf{r})$. This corresponds to $a=2M^{-1}$ in the notation of reference 4. If we use $a=2.4M^{-1}$ in reference 4, we have a 20% effect due to the $\mathfrak{M}(\boldsymbol{\alpha}\times\mathbf{r})$ term instead of 16%. Equation (21) in reference 4 agrees with our Eq. (2), if we put $Z=0$ and neglect the finite de Broglie wavelength effect and coordinate-type matrix elements. When we take various values for coordinate-type matrix elements and $\mathfrak{M}(\gamma_5\mathbf{r})$, the C_-/C_+ does not differ appreciably from Fig. 3, because charge-dependent parts of these matrix elements are very small. (In this case, the energy variation of C_- or C_+ may change.)

Since there is the negatron decay from B^{12} to the first excited state with 4.43 Mev of C^{12}, the possible energy region to test the energy variation of C_-/C_+ may be restricted to $27.2mc^2>W>18.5mc^2$. Then, we expect only 4% of the variation of C_-/C_+ from Eq. (16′) and none from Eq. (16). The lack of knowledge of the positron decays to the excited states of C^{12} brings further uncertainty in the experiment. However, future experiments with extra high precision are able to answer the difference between Eqs. (16) and (16′), namely between the old theory and the Gell-Mann's one.

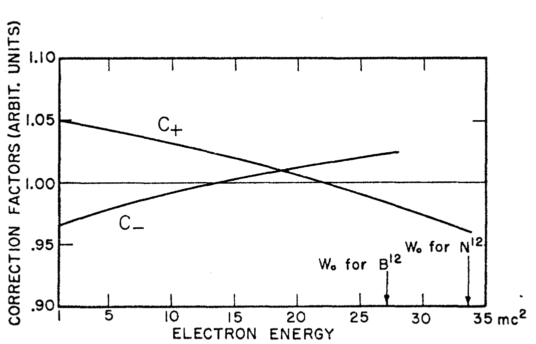

FIG. 2. Correction factors for beta spectra of B^{12}, C_-, and of N^{12}, C_+, given by Eq. (16′). They are almost constant if we use Eq. (16).

[14] F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. **27**, 77 (1955).

IV. BETA-ALPHA DIRECTIONAL CORRELATION OF Li^8

The beta decay of Li^8 leads primarily to the formation of the Be^8 nucleus in its first excited state, which decays into two alpha particles. The decay scheme is $2_+\rightarrow 2_+\rightarrow 0_+$. The spin of Li^8 was recently determined by an experiment on the beta-alpha-alpha directional correlation.[15] Experiments on the beta-alpha directional correlation were performed by several authors.[16] The latest experiment by Hanna *et al.* gives $W(\theta)=1+a\cos^2\theta$ with $a=0.01\pm0.03$ when it is averaged over a large part of the energy range.[16]

A theoretical investigation of this beta decay was performed by Morita and Yamada[2] taking into account the contribution of the second forbidden transition to the allowed one. They aimed to find a possible anisotropy of the directional correlation in the allowed beta decay, while the ordinary theory gives no anisotropy. They calculated the interference terms, $\{\mathfrak{M}^*(\beta\boldsymbol{\sigma})\mathfrak{M}(\beta\boldsymbol{\sigma}r^2)\}$, $\{\mathfrak{M}^*(\beta\boldsymbol{\sigma})\mathfrak{M}((\beta\boldsymbol{\sigma}\cdot\mathbf{r})\mathbf{r})\}$, and $\{\mathfrak{M}^*(\beta\boldsymbol{\sigma})\mathfrak{M}(\beta\boldsymbol{\alpha}\times\mathbf{r})\}$, with an assumption of tensor only. Formula was given in the approximation $Z=0$. In the numerical calculation, they assumed $\mathfrak{M}(\beta\boldsymbol{\sigma}r^2)\approx\mathfrak{M}((\beta\boldsymbol{\sigma}\cdot\mathbf{r})\mathbf{r})\approx\rho^2\mathfrak{M}(\beta\boldsymbol{\sigma})$ and $\mathfrak{M}(\beta\boldsymbol{\alpha}\times\mathbf{r})=0$. In the same approximation the result for VA is $a=0.005$ at $W=W_0$.

From Eqs. (22) through (24), the anisotropy coefficient, "a," of the beta-alpha directional correlation occurring in the expression

$$W(\theta)=1+a\cos^2\theta \tag{25}$$

is

$$a=\frac{p^2}{W}\left\{-\tfrac{3}{10}\alpha Z\rho(2\Lambda\eta-3\eta+1)+\rho^2\left[\tfrac{2}{5}(\eta+1)K+\frac{6}{25}(3\eta-1)W\right]+\frac{2}{M}(\lambda^{-1}+1)\right\}F_2(1122)F_2(2202). \tag{26}$$

If we adopt Eq. (24′) instead of Eq. (24), we obtain

$$a=[\text{Eq. (26) with replacing }\lambda^{-1}\text{ by }4]. \tag{26′}$$

Assuming $\eta=\frac{1}{5}$ and $\Lambda=12.7$, the "a" is evaluated at $W_0=26mc^2$. $a\approx0.01$ from Eq. (26), and $a\approx0.03$ from Eq. (26′).

There is a possibility that the estimated value of "a" from Eq. (26) increases about ten times. The reason is

[15] Lauterjung, Schimmer, and Maier-Leibnitz, Z. Physik **150**, 657 (1958); M. Morita, Phys. Rev. Letters **1**, 112 (1958). *Note added in proof.*—After this work was completed, two papers on the similar experiments have been published by Lauritsen, Barnes, Fowler, and Lauritsen, Phys. Rev. Letters **1**, 326 and 328 (1958).

[16] C. M. Class and S. S. Hanna, Phys. Rev. **89**, 877 (1953). D. St. P. Bunbury, Phys. Rev. **90**, 1121 (1954). Hanna, LaVier, and Class, Phys. Rev. **95**, 110 (1954). The data given in reference 15 may be useful.

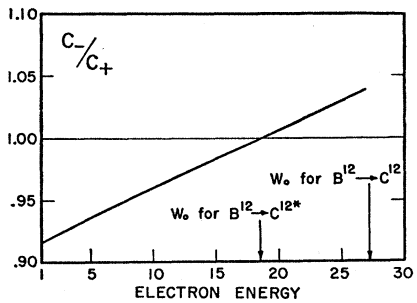

FIG. 3. The ratio of correction factors for beta spectra of B^{12} and N^{12}, C_-/C_+, given by Eq. (16′). It is almost constant if we use Eq. (16). The curve is normalized by [$\mathfrak{M}(\boldsymbol{\sigma})$ of $e^-/\mathfrak{M}(\boldsymbol{\sigma})$ of e^+].

as follows: The $\log ft$ of Li^8 is 5.6 and quite large.[17] This means that the $\mathfrak{M}(\boldsymbol{\sigma})$ is about ten times smaller than that of the favored transition. Then, the ratios between $\mathfrak{M}(\boldsymbol{\sigma})$ and the other matrix elements may increase about ten times. Consequently, if the experimental value of "a" is $\sim 3\%$ or larger, the effect may or may not be what Gell-Mann calculated. On the other hand, if it is very small, the estimated value of $\mathfrak{M}(\boldsymbol{\alpha}\times\mathbf{r})$ by him is improbable. In this connection, a precise measurement of the beta-alpha directional correlation of Li^8 is desirable. Furthermore, if the sign of "a" is positive, the spin of Li^8 is 2_+. The theoretical expression of "a" for the assumption of $1_+(3_+)$ of the spin of Li^8 is given from Eq. (26) or (26′) with a multiplication factor $-1(-2/7)$. This is another method of determining the nuclear spin of Li^8.

The situation is almost the same for the beta-gamma directional correlation. For example, the experimental data on F^{20} [13] are well explained with our theory.

V. CONCLUSION

The energy variation of C_-/C_+ depends mainly on the magnitude of the $\mathfrak{M}(\boldsymbol{\alpha}\times\mathbf{r})$ and partially on the finite de Broglie wavelength effect to L_0 of the $\mathfrak{M}(\boldsymbol{\sigma})$ term and on the Coulomb corrections. The latter two effects reduce the former one. Another determination of the value of $\mathfrak{M}(\boldsymbol{\alpha}\times\mathbf{r})$ is the beta-alpha or beta-gamma directional correlation, in which the beta decay has a small ft value.

We did not discuss the radiative corrections due to the virtual as well as real photons. Although these corrections have the same sign for both charges of electrons, their ratio has some energy dependence, because of the difference of the maximum energy.[18] Therefore, before doing comparison with experimental data and our formulas, it is necessary to subtract the effect due to the radiative corrections from the data. Since the finite-size correction to the beta spectrum is small in the low Z, we have not considered it. A possible effect of the strange particles[19] has been neglected also.

In the nth forbidden transition of beta decay, we expect also the finite de Broglie wavelength effect and the contribution from the $(n+2)$th forbidden transition. For example, the spectral shape in the unique forbidden transition with B_{ij} may deviate from the so-called a-shape (namely, energy dependence of K^2+p^2), when the maximum energy of the electron is very large. The negatron decay of N^{16} is the typical case.[20] Furthermore, in the beta-alpha and beta-gamma cascades, the directional-correlation functions may have a $\cos^4\theta$ term, if beta decay is caused by B_{ij} and the multipolarity of the gamma ray is quadrupole or higher. The ordinary theory gives no $\cos^4\theta$ term. In fact, such a calculation has been done by Kotani and Ross[21] in the case of the first forbidden transition.

Higher order corrections to the beta-neutrino correlation, beta-circularly polarized gamma correlation, and longitudinal polarization of the beta particles are at most 10% of the main asymmetries. Calculation for them will be published in a forthcoming paper.

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APPENDIX. RELATIONS BETWEEN NUCLEAR MATRIX ELEMENTS OF BETA DECAY

In this Appendix, we derive relations between nuclear matrix elements, which contribute to the allowed beta decay from the second forbidden transition, in the nonrelativistic approximation. We follow the similar calculations of the first forbidden matrix elements given by Ahrens and Feenberg,[22] and of the nth forbidden ones given by Yamada.[23] Especially, the latter is useful for our purpose. Substituting $\gamma_5\mathbf{r}$ for X in Eq. (15) of reference 23, we obtain

$$(\psi_f|\gamma_5\mathbf{r}|\psi_i) = -(2M)^{-1}(\chi_f|\gamma_5\mathbf{r}(\boldsymbol{\alpha}\cdot\mathbf{p})+(\boldsymbol{\alpha}\cdot\mathbf{p})\gamma_5\mathbf{r}|\chi_i)$$
$$= -(2M)^{-1}(\chi_f|2(\boldsymbol{\sigma}\cdot\mathbf{p})\mathbf{r}+i\boldsymbol{\sigma}|\chi_i). \qquad \text{(A1)}$$

Substituting $(\boldsymbol{\sigma}\cdot\mathbf{r})\mathbf{r}$ for X in Eq. (16) of reference 23, we obtain

$$(\psi_f|[H_0,(\boldsymbol{\sigma}\cdot\mathbf{r})\mathbf{r}]|\psi_i) = (2M)^{-1}(\chi_f|[p^2,(\boldsymbol{\sigma}\cdot\mathbf{r})\mathbf{r}]|\chi_i)$$
$$= -iM^{-1}(\chi_f|(\boldsymbol{\sigma}\cdot\mathbf{p})\mathbf{r}+(\boldsymbol{\sigma}\cdot\mathbf{r})\mathbf{p}|\chi_i), \qquad \text{(A2)}$$

with

$$H_0 = -\sum_k(\boldsymbol{\alpha}_k\cdot\mathbf{p}_k+M\beta_k).$$

[17] This is completely different from the case of B^{12} whose highest beta decay has $\log ft=4.2$, see reference 1.

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[20] The author would like to express his sincere thanks to Dr. L. J. Lidofsky for mentioning N^{16}.

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ψ and χ are the wave function of a nucleon and its large component, respectively. **p** is momentum vector of the nucleon in this Appendix. k is the nucleon number. Now we can prove easily the following equality, by using Eqs. (3) and (16) of reference 23.

$$(\chi_f|(\boldsymbol{\sigma}\cdot\mathbf{r})\mathbf{p}|\chi_i)=(\chi_f|(\boldsymbol{\sigma}\cdot\mathbf{p})\mathbf{r}|\chi_i). \quad \text{(A3)}$$

Inserting Eqs. (A2) and (A3) into (A1), the result is

$$\begin{aligned}(\psi_f|\gamma_5\mathbf{r}|\psi_i)&=-i(2M)^{-1}(\psi_f|\boldsymbol{\sigma}|\psi_i)-\tfrac{1}{2}i(\psi_f|[H_0,(\boldsymbol{\sigma}\cdot\mathbf{r})\mathbf{r}]|\psi_i)\\&=-i(2M)^{-1}(\psi_f|\boldsymbol{\sigma}|\psi_i)\\&\quad+i(\Lambda\alpha Z/4\rho)(\psi_f|(\boldsymbol{\sigma}\cdot\mathbf{r})\mathbf{r}|\psi_i). \quad \text{(A4)}\end{aligned}$$

Consequently,

$$i\mathfrak{M}(\gamma_5\mathbf{r})=(2M)^{-1}\mathfrak{M}(\boldsymbol{\sigma})\mp(\Lambda\alpha Z/4\rho)\mathfrak{M}((\boldsymbol{\sigma}\cdot\mathbf{r})\mathbf{r}) \quad \text{for } e^{\mp}. \quad \text{(A5)}$$

Here

$$\Lambda\approx 1\pm(W_i-W_f)A^{\frac{1}{3}}Z^{-1} \quad \text{for } e^{\mp}. \quad \text{(A6)}$$

W_i and W_f are energy eigenvalues of the initial and final nuclei, respectively.[24] Λ is nearly one, except for light nuclei. Similarly,

$$i\mathfrak{M}(\boldsymbol{\alpha}\cdot\mathbf{r})=\mp(\Lambda\alpha Z/4\rho)\mathfrak{M}(r^2), \quad \text{(A7)}$$

and

$$\mathfrak{M}(\boldsymbol{\alpha}\times\mathbf{r})=M^{-1}\mathfrak{M}(\boldsymbol{\sigma}). \quad \text{(A8)}$$

If we assume Gell-Mann's argument on $\mathfrak{M}(\boldsymbol{\alpha}\times\mathbf{r})$ [Eq. (26) of reference 4], the right-hand side of Eq. (A7) should be multiplied by $(\mu_p-\mu_n)$. Then,

$$\mathfrak{M}(\boldsymbol{\alpha}\times\mathbf{r})=M^{-1}(\mu_p-\mu_n)\mathfrak{M}(\boldsymbol{\sigma}). \quad \text{(A9)}$$

[24] See also Eqs. (18), (20), and (21) of reference 22.

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Ratio of Asymmetric to Symmetric Fission of U^{233} as a Function of Neutron Energy*†

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Radiochemical measurements of the relative yields of the fission products Mo^{99}, Ag^{111}, Ag^{113}, and Cd^{115} from the low-energy neutron-induced fission of U^{233} have been made at various neutron energies. The energies chosen were thermal, 1.8, 2.3, and 4.7 ev, the latter three corresponding to the peaks of previously reported resonances. It was found that the ratio of asymmetric to symmetric fission is larger at the 1.8- and 2.3-ev resonances than at thermal energies. At the 4.7-ev resonance however, this ratio is the same as at thermal energy, to within experimental uncertainties. In addition, it was found that the ratio for epi-cadmium neutrons differed from that for thermal neutrons. The results are consistent with Wheeler's prediction that the ratio of asymmetric to symmetric fission should depend upon the spin state of the fissioning nucleus.

INTRODUCTION

ONE of the features of present ideas[1,2] concerning slow-neutron-induced fission is the interesting prediction that the ratio of asymmetric to symmetric fission should be different for the two possible spin states of the compound system formed upon the addition of a neutron to the nucleus. According to the collective model[1] the rotational levels through which the compound nucleus must pass in order to fission may have considerably different fission thresholds for the two spin states, differing perhaps by as much as 1 Mev. A recent theoretical analysis[3,4] of the U^{233} fission cross section has indicated that a large fraction of the fission cross section at thermal energies arises from nuclei of one spin state of the compound nucleus, and that the prominent resonances at 1.8 and 2.3 ev are associated with the other spin state. In addition, the broad resonance at 4.7 ev is thought to belong to the same spin state as that which predominates at thermal energies. The slow-neutron-induced fission of U^{233} thus provides an ideal case for investigating Wheeler's ideas.

EXPERIMENTAL PROCEDURES

The U^{233} used in the present work was obtained in two lots from Oak Ridge National Laboratory. Analyses of samples of the two lots showed the following uranium isotopic composition:

Lot 1: U^{232}, <0.02%; U^{233}, 98.35%; U^{234}, 1.26%; U^{235}, 0.21%; U^{238}, 0.18%;

Lot 2: U^{232}, <0.02%; U^{233}, 97.90%; U^{234}, 1.78%; U^{235}, 0.05%; U^{238}, 0.28%.

* This work was done under the auspices of the U. S. Atomic Energy Commission.
† Preliminary results of this work were reported at the New York Meeting of the American Physical Society, January, 1958. See Bull. Am. Phys. Soc. Ser. II, 3, 6 (1958).
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