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Analysis of the Reaction $C^{13}(He^3,\alpha)C^{12}$†

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The angular distributions of the alpha particles from the reaction $C^{13}(He^3,\alpha)C^{12}$ at 4.5 and 2.0 Mev have been analyzed from the point of view of nuclear stripping. By symmetrizing the wave function of the coplete Hamiltonian with respect to the exchange of alpha particles, the heavy-particle stripping amplitude is formally included. The experimental distributions change decidedly between 2.0 and 4.5 Mev, and the calculated curves employing a combination of pickup and α-stripping from C^{13} agree quite well with the experimental distributions. Interference between the two channels plays a major role in the final expressions.

I. INTRODUCTION

The theory of direct interactions[1,2] has been utilized successfully in recent years to analyze the angular distributions of particles from (d,n), (d,p), and inelastic scattering reactions. Reactions employing He^3 or H^3 as the bombarding particle are subject to analysis as direct interactions where both pickup[2] and stripping can occur in the same process.

The $C^{13}(He^3,\alpha)C^{12}$ reaction has been studied at 2 and 4.5 Mev by Holmgren, Greer, Johnston, and Wolicki.[3] Their results show marked variations in the angular distributions. At 4.5 Mev the angular distribution of the alpha particles oscillates rapidly, exhibiting two pronounced minima, one at 70°, the other at 145°, and three maxima at 25°, 115°, and approximately 180°. The two forward maxima are of approximately the same amplitude. The angular distribution corresponding to a bombarding energy of 2 Mev exhibits a forward maximum at 0°, a minimum at 55°, and a strong backward intensity. (See Figs. 1 and 2.)

The analysis in terms of the pickup of a neutron by the He^3 was inadequate to describe either angular distribution. It was suggested[4] that an analysis incorporating the process of the heavy-particle[5] stripping of an alpha particle from C^{13} with the process of pickup might account for the observed angular distribution. The following discussion formally incorporates these processes, and the results exhibit the characteristics found in the experimental angular distributions.

II. DEVELOPMENT OF THE DIFFERENTIAL CROSS SECTION

The general approach to be used in this development has been treated at length by Fulton and Owen.[6] The stripping of an alpha particle from the C^{13} nucleus can be incorporated formally by constructing the total wave function to be symmetric for the exchange of alpha particles. After the first-order terms which describe the direct process are separated from the higher-order terms, the symmetry can be exhibited in the initial state[6,7] or in the final state.[5,6] In the analysis of the (He^3,α) reactions the latter approach is the more convenient.

The total Hamiltonian for the reaction $C^{13}(He^3,\alpha)C^{12}$ can be written

$$\begin{aligned} H &= H_C+T_n+V_{nC}+H_H+V_{Hn}+V_{HC} \\ &= H_B+T_\alpha+V_{\alpha B}+H_H+V_{H\alpha}+V_{HB}. \end{aligned} \tag{1}$$

The subscript C refers to the C^{12} core for the neutron n in C^{13}, and the subscript B refers to the Be^9 core for an alpha in C^{13}. The subscript H will refer throughout to the incident He^3 nucleus. The total Hamiltonian H

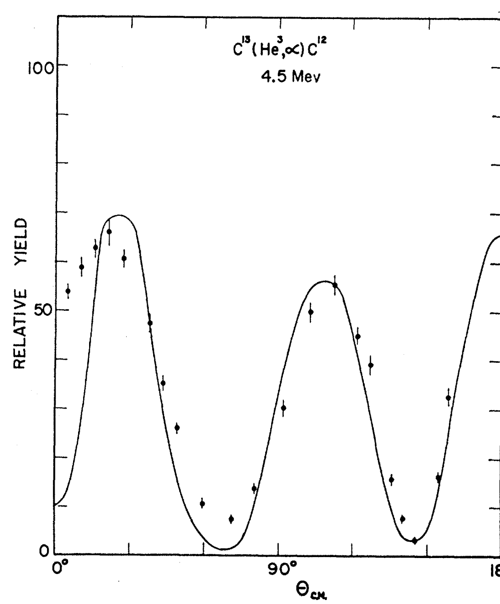

FIG. 1. Theoretical curve of Eq. (23) for the reaction $C^{13}(He^3,\alpha)C^{12}$ at a bombarding energy of 4.5 Mev in the lab system. The points are the data of Holmgren *et al.*, reference 3.

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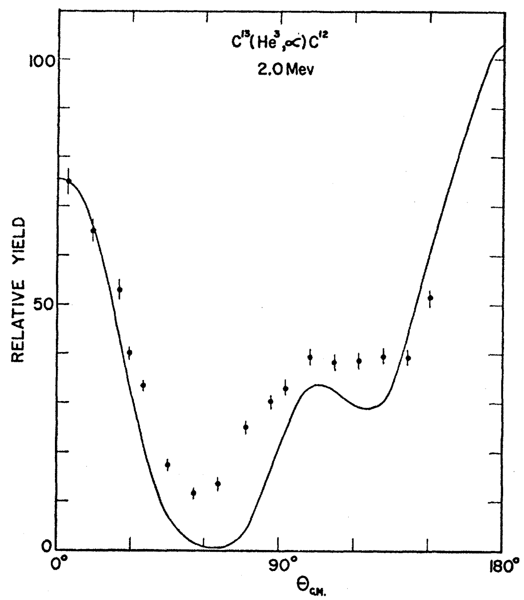

FIG. 2. Theoretical curve of Eq. (23) for the reaction $C^{13}(He^3,\alpha)C^{12}$ at a bombarding energy of 2.00 Mev. The points represent the data of Holmgren, reference 3.

can be split up in various ways to describe asymptotic conditions.

In order to describe the initial state for the pickup of n by H, we write

$$H_0^{(1)}=H_C+T_n+V_{nC}+H_H, \tag{2}$$

and

$$H_1^{(1)}=V_{Hn}+V_{HC}. \tag{3}$$

$H_0^{(1)}$ is the zeroth order Hamiltonian for the initial state, while $H_1^{(1)}$ is the corresponding initial-state perturbation.

In like manner for the pickup process the final state Hamiltonian and the corresponding perturbation are

$$H_0^{(2)}=H_C+T_n+H_H+V_{Hn}, \tag{4}$$

$$H_1^{(2)}=V_{nC}+V_{HC}. \tag{5}$$

The initial and final state eigenfunctions and Green's functions are defined by these operators:

$$(E-H_0^{(1)})\psi_i^{(1)}=0, \tag{6a}$$

$$(E-H_0^{(1)})G_1=1, \tag{6b}$$

$$(E-H_0^{(2)})\psi_f^{(2)}=0, \tag{6c}$$

$$(E-H_0^{(2)})G_2=1, \tag{6d}$$

$$(E-H)\Psi=0, \tag{6e}$$

$$(E-H)G=1. \tag{6f}$$

$$\psi_i^{(1)}=\exp(i\mathbf{k}_H\cdot\mathbf{R}_H)\times\exp\{-i\mathbf{k}_H\cdot[(M_n/M_I)\mathbf{R}_n+(M_C/M_I)\mathbf{R}_C]\}\times\zeta_{l(nC)}(\mathbf{R}_n-\mathbf{R}_C)\xi_{iC}\xi_{iH}; \tag{7}$$

$$\psi_f^{(2)}=\exp\{i\mathbf{k}_\alpha\cdot[(M_n/M_\alpha)\mathbf{R}_n+(M_H/M_\alpha)\mathbf{R}_H]\}\times\exp(-i\mathbf{k}_\alpha\cdot\mathbf{R}_C)\zeta_{l(nH)}(\mathbf{R}_n-\mathbf{R}_H)\xi_{fC}\xi_{fH}\xi_{f\alpha}. \tag{8}$$

The position vectors $\mathbf{R}_j$ are vectors relative to the center of mass of the complete system. The mass M_I refers to the mass of the C^{13} nucleus. The function ζ refers to the single-particle bound-state wave functions of the neutron in the initial and final states. The functions ξ refer to the internal wave functions of the remaining nucleons. As described in a previous paper,[6] the wave function of the complete Hamiltonian can be written

$$\Psi^{(2)}=G_2H_1^{(2)}(1+GH_1^{(1)})\psi_i^{(1)}. \tag{9}$$

In order to obtain Ψ in a form which emphasizes an alpha from the C^{13} as the outgoing alpha particle, we use a rearrangement of terms. The initial-state Hamiltonian and perturbation are

$$H_0^{(3)}=H_B+H_\alpha+H_H+V_{B\alpha}, \tag{10}$$

$$H_1^{(3)}=V_{\alpha H}+V_{HB}. \tag{11}$$

The subscript B refers to the Be^9 core of the alpha particle bound in C^{13}.

The final-state Hamiltonian emphasizing this process is

$$H_0^{(4)}=H_B+H_H+V_{HB}+H_\alpha, \tag{12}$$

$$H_1^{(4)}=V_{\alpha H}+V_{\alpha B}. \tag{13}$$

Note that the He^3 system is captured by the Be^9 core. Again initial- and final-state wave functions and Green's functions are defined:

$$(E-H_0^{(m)})\psi_k^{(m)}=0, \tag{14a}$$

$$(E-H_0^{(m)})G_m=1. \tag{14b}$$

The corresponding asymptotic functions are

$$\psi_i^{(3)}=\exp(i\mathbf{k}_H\cdot\mathbf{R}_H)\times\exp\{-i\mathbf{k}_H\cdot[(M_B/M_I)\mathbf{R}_B+(M_\alpha/M_I)\mathbf{R}_\alpha]\}\times\chi_{l(\alpha B)}(\mathbf{R}_\alpha-\mathbf{R}_B)\xi_{iB}\xi_{iH}; \tag{15}$$

and

$$\psi_f^{(4)}=\exp(i\mathbf{k}_\alpha\cdot\mathbf{R}_\alpha)\times\exp\{-i\mathbf{k}_\alpha\cdot[(M_H/M_F)\mathbf{R}_H+(M_B/M_F)\mathbf{R}_B]\}\times\chi_{l(HB)}(\mathbf{R}_H-\mathbf{R}_B)\xi_{fB}\xi_{fH}\xi_{f\alpha}. \tag{16}$$

The wave function Ψ of the complete Hamiltonian can then be written in a form which stresses a nuclear alpha particle as an outgoing term.

$$\Psi^{(4)}=G_4H_1^{(4)}(1+GH_1^{(3)})\psi_i^{(3)}. \tag{17}$$

It is recognized that actually there are three alphas in the C^{13} available for this process. Thus the operations of Eqs. (2) through (9) could be carried out again. By making use of the exchange properties of the individual wave functions, however, it is easily seen that the three α particles contribute equally. Thus the additional processes enter only as a multiplicative factor.

As yet no symmetry properties have been imposed upon Ψ to take into account the exchange of the various alpha particles. It should be remarked that the superscripts on Ψ do not denote different Ψ's but serve to identify the process stressed in a given development:

$$\Psi^{(2)}\equiv\Psi^{(4)}.$$

The rearranging of the leading terms,[8] however, proves convenient when we require the function to be symmetric under the exchange of alphas. By utilizing $\Psi^{(4)}$ as the exchange wave function, the alpha-stripping process is formally included when the projection of the symmetrized function Υ is taken on the function Ξ_f (defined below) with $\alpha(1)$ going out. We construct $\Upsilon(1,2)$ to be symmetric for the exchange of alpha particles 1 and 2. It is assumed that alpha 1 results from the pickup process, and alpha 2 results from the stripping process:

$$\Upsilon(1,2)=\Psi^{(2)}(1,2)+\Psi^{(4)}(2,1). \quad (18)$$

As described previously,[6] we are interested in the projection of $\Upsilon(1,2)$ on a particular final state Ξ_f, and wish to consider the coefficient of $\Xi_f r_{\alpha(1)}^{-1}\exp(ik_\alpha\cdot r_{\alpha(1)})$ as $\mathbf{r}_{\alpha(1)}$ is made large.

The scattering amplitude is

$$\mathcal{T}=\langle f^{(2)}|H_1^{(2)}(1+GH_1^{(1)})|i^{(1)}\rangle +\langle f^{(4)}|H_1^{(4)}(1+GH_1^{(3)})|i^{(3)}\rangle. \quad (19)$$

In Eq. (19) the superscripts 1, 2, 3, 4 refer to the eigenfunctions of a specific zeroth order Hamiltonian. It is assumed that direct interactions arise from the leading terms only. Even more specifically, we need retain only V_{nC} from $H_1^{(2)}$ and $V_{B\alpha}$ from $H_1^{(4)}$. The differential cross section for the direct interaction is thus

$$\frac{d\sigma}{d\Omega}=\frac{M_\alpha M_H M_I k_\alpha}{(2\pi\hbar^2)^2 M_F k_H}\sum_{\substack{\text{final}\\ \text{av initial}}}|\langle f^{(2)}|V_{nC}|i^{(1)}\rangle +\langle f^{(4)}|V_{\alpha B}|i^{(3)}\rangle|^2. \quad (20)$$

Expressions (7) and (15) for the initial state and (8) and (16) for the final state are not in the form of total angular momentum wave functions. They result, however, from a partial expansion of the angular parts of the total angular momentum functions. The initial-state wave function can be written as

$$\Psi_i=\exp(i\mathbf{k}_H\cdot\mathbf{R}_H)\xi_{iH}\exp(i\mathbf{k}_i\cdot\mathbf{R}_i)\psi_i(j_i,\mu_i,\mathbf{R}_n,\cdots), \quad (21)$$

where $\psi_i(j_i,\mu_i,\mathbf{R}_n,\cdots)$ is the total internal wave function of C^{13} with total angular momentum $\mathbf{j}_i$ with projection μ_i, and the final state wave function as

$$\Psi_f=\exp(i\mathbf{k}_\alpha\cdot\mathbf{R}_\alpha)\xi_{f\alpha}\exp(i\mathbf{k}_C\cdot\mathbf{R}_C)\psi_C(j_C,\mu_C,\mathbf{R}_H,\cdots), \quad (22)$$

where $\Psi_C(j_C,\mu_C,\mathbf{R}_H,\cdots)$ is the total internal wave function of C^{12}, with total angular momentum $\mathbf{j}_C$ with projection μ_C.

[8] A similar approach was carried out in reference 6 for (d,n) reactions where antisymmetrization was required.

The angular part of the matrix element for either process in (20) is evaluated by expanding (21) in terms of (22) with plane-wave and Clebsch-Gordan expansions. For the pickup channel, $\psi_i(j_i,\mu_i,\mathbf{R}_n,\cdots)$ in (21) is expanded in terms of the set of product functions $\zeta_{l(nC)}(\mathbf{R}_n-\mathbf{R}_C)$ representing the C^{13} nucleus as a neutron plus C^{12} core. Thus (21) becomes

$$\Psi_i=\sum_{\mu_C}\langle j_iJ_{nC}j_C|\mu_i\mu_i-\mu_C\mu_C\rangle\psi_i^{(1)}, \quad (23)$$

where J_{nC} is the total angular momentum of the neutron relative to C^{12} and $\langle j_iJ_{nC}j_C|\mu_i\mu_i-\mu_C\mu_C\rangle$ is the Clebsch-Gordan coefficient corresponding to the vector addition $\mathbf{j}_i=\mathbf{J}_{nC}+\mathbf{j}_C$. The angular functions in $\psi_i^{(1)}$ are then further expanded in terms of those in $\psi_f^{(2)}$ and the product function $\zeta_{l(nH)}(\mathbf{R}_n-\mathbf{R}_H)\xi_{fH}$ in $\psi_f^{(2)}$ representing the alpha particle as He^3 plus the neutron is written as an expansion in the set of its total angular momentum functions. Thus (8) becomes

$$\psi_f^{(2)}=\sum_{\mu_H}\langle j_\alpha j_HJ_{nH}|\mu_\alpha\mu_H\mu_\alpha-\mu_H\rangle\Psi_f, \quad (24)$$

corresponding to the vector addition $\mathbf{j}_\alpha=\mathbf{j}_H+\mathbf{J}_{nH}$, where $\mathbf{j}_\alpha$ is the spin of the alpha particle with projection μ_α and $\mathbf{J}_{nH}$ is the total angular momentum of the neutron relative to He^3.

The plane-wave propagation of the neutron away from the C^{13} nucleus serves to define its direction of propagation as a preferred axis of quantization for expansion (23). Similarly, the direction of its plane-wave propagation toward He^3 defines a preferred axis for expansion (24). It is found, however, that the second plane wave propagates with orbital angular momentum $l_{nH}=0$, and thus the angular functions for the pickup channel can be expanded with the axis of quantization defined by the propagation of the neutron away from from C^{13}. The coefficients in these expansions give the angular part of the matrix element for this channel.

For the heavy-particle stripping channel, expansions similar to (23) and (24) are made to express C^{13} as an alpha particle plus Be^9 core and to combine the Be^9 core and He^3 into the C^{12} nucleus. In this case Be^9 propagates toward He^3 with $l_{HB}=0$, and the axis of quantization for the angular expansions is defined by the direction of propagation of the alpha particle away from C^{13}. The same axis of quantization must be used for both matrix elements in (20), and it is convenient to transform the functions of the heavy-particle stripping channel to the pickup axis. This transformation introduces the associated Legendre function $P_{l(\alpha B)}^{m(\alpha B)}(\cos\beta)$ into the heavy-particle angular matrix element. The angle β is the angle between the two axes ($\mathbf{K}_1$ and $\mathbf{K}_2$, defined below), and is a function of the angle of the outgoing alpha particle. The angular matrix element thus contributes a non-numerical factor to the total matrix element.

III. THE ANALYSIS

The radial matrix elements appearing in the differential cross section presented in Eq. (20) can be considerably simplified by noting a simple relation between the potentials and the two initial-state Hamiltonians:

$$\langle f^{(2)}|V_{nC}|i^{(1)}\rangle = \langle f^{(2)}|T_{\text{c.m.}}+\epsilon_{nC}-T_C-T_n-T_H|i^{(1)}\rangle, \quad (25a)$$

and

$$\langle f^{(4)}|V_{\alpha B}|i^{(3)}\rangle = \langle f^{(4)}|T_{\text{c.m.}}+\epsilon_{\alpha B}-T_B-T_\alpha-T_H|i^{(3)}\rangle. \quad (25b)$$

Here ϵ_{nC} and $\epsilon_{\alpha B}$ represent the binding energies of the outer neutron in C^{13} and an alpha particle in C^{13}, respectively, $T_{\text{c.m.}}$ is the total kinetic energy in the center-of-mass system, and $T_j=(-\hbar^2/2M_j)\nabla_j^2$.

The procedure followed is to substitute the Fourier transforms of the bound-state functions into the matrix elements. These Fourier transforms will constitute the major components of the angular distribution. They are

$$\zeta_{l(nC)}(\mathbf{R}_n-\mathbf{R}_C)=(2\pi)^{-\frac{3}{2}}\int\cdots\int G_{l(nC)}(\mathbf{K}_1')\exp(i\mathbf{K}_1'\cdot[\mathbf{R}_n-\mathbf{R}_C])d\mathbf{K}_1',$$

$$\zeta_{l(nH)}(\mathbf{R}_n-\mathbf{R}_H)=(2\pi)^{-\frac{3}{2}}\int\cdots\int F_{l(nH)}(\mathbf{k}_1')\exp(i\mathbf{k}_1'\cdot[\mathbf{R}_n-\mathbf{R}_H])d\mathbf{k}_1',$$

$$\chi_{l(\alpha B)}(\mathbf{R}_\alpha-\mathbf{R}_B)=(2\pi)^{-\frac{3}{2}}\int\cdots\int G_{l(\alpha B)}(\mathbf{K}_2')\exp(i\mathbf{K}_2'\cdot[\mathbf{R}_\alpha-\mathbf{R}_B])d\mathbf{K}_2',$$

$$\chi_{l(HB)}(\mathbf{R}_H-\mathbf{R}_B)=(2\pi)^{-\frac{3}{2}}\int\cdots\int F_{l(HB)}(\mathbf{k}_2')\exp(i\mathbf{k}_2'\cdot[\mathbf{R}_H-\mathbf{R}_B])d\mathbf{k}_2'.$$

These transforms can now be inserted into the integrals of Eq. (25). The operators T_j can be applied to the exponential functions under the integral sign and the integrations over all spatial coordinates performed, giving

$$\langle f^{(2)}|V_{nC}|i^{(1)}\rangle=\Lambda_1\int\cdots\int d\mathbf{K}_1'\,d\mathbf{k}_1'\,F_{l(nH)}^*(\mathbf{k}_1')G_{l(nC)}(\mathbf{K}_1')$$
$$\times\left\{T_{\text{c.m.}}+\epsilon_{nC}-\tfrac{1}{2}\hbar^2\left[M_H^{-1}k_H^2+M_n^{-1}\left(\mathbf{K}_1'-\frac{M_n}{M_I}\mathbf{k}_H\right)^2+M_C^{-1}\left(\mathbf{K}_1'+\frac{M_C}{M_I}\mathbf{k}_H\right)^2\right]\right\}$$
$$\times\delta(\mathbf{K}_1'-[\mathbf{k}_\alpha-(M_C/M_I)\mathbf{k}_H])\delta(\mathbf{k}_1'-[(M_H/M_\alpha)\mathbf{k}_\alpha-\mathbf{k}_H]), \quad (26a)$$

and

$$\langle f^{(4)}|V_{\alpha B}|i^{(3)}\rangle=\Lambda_2\int\cdots\int d\mathbf{K}_2'\,d\mathbf{k}_2'\,F_{l(HB)}^*(\mathbf{k}_2')G_{l(\alpha B)}(\mathbf{K}_2')$$
$$\times\left\{T_{\text{c.m.}}+\epsilon_{\alpha B}-\tfrac{1}{2}\hbar^2\left[M_H^{-1}k_H^2+M_\alpha^{-1}\left(\mathbf{K}_2'-\frac{M_\alpha}{M_I}\mathbf{k}_H\right)^2+M_B^{-1}\left(\mathbf{K}_2'+\frac{M_B}{M_I}\mathbf{k}_H\right)^2\right]\right\}$$
$$\times\delta(\mathbf{K}_2'-[\mathbf{k}_\alpha+(M_\alpha/M_I)\mathbf{k}_H])\delta(\mathbf{k}_2'-[\mathbf{k}_H+(M_H/M_F)\mathbf{k}_\alpha]), \quad (26b)$$

Integrating over the momentum coordinates, we obtain

$$\frac{d\sigma}{d\Omega}=\frac{M_\alpha M_H M_I k_\alpha}{(2\pi\hbar^2)^2 M_F k_H}|\Lambda_1|^2\sum_{\substack{\text{final}\\ \text{av initial}}}\left|C_1F_{l(nH)}^*(\mathbf{k}_1)G_{l(nC)}(\mathbf{K}_1)+\frac{\Lambda_2}{\Lambda_1}C_2F_{l(HB)}^*(\mathbf{k}_2)G_{l(\alpha B)}(\mathbf{K}_2)\right|^2, \quad (27)$$

where

$$C_1(T_0)=T_{\text{c.m.}}+\epsilon_{nC}-\frac{\hbar^2}{2M_n}(\mu_{Hn}^{-1}k_H^2+\mu_{nC}^{-1}k_\alpha^2-2\mathbf{k}_\alpha\cdot\mathbf{k}_H),$$

$$C_2(T_0)=T_{\text{c.m.}}+\epsilon_{B\alpha}-\frac{\hbar^2}{2M_n}(\mu_{\alpha B}^{-1}k_\alpha^2+\mu_{HB}^{-1}k_H^2-2\mu_{Bn}^{-1}\mathbf{k}_\alpha\cdot\mathbf{k}_H),$$

$$\mathbf{K}_1=\mathbf{k}_\alpha-(M_C/M_I)\mathbf{k}_H,$$

$$\mathbf{k}_1=(M_H/M_\alpha)\mathbf{k}_\alpha-\mathbf{k}_H,$$

$$\mathbf{K}_2=\mathbf{k}_\alpha+(M_\alpha/M_I)\mathbf{k}_H,$$

$$\mathbf{k}_2=\mathbf{k}_H+(M_H/M_F)\mathbf{k}_\alpha,$$

$$\Lambda_1=\int\cdots\int\xi_{fC}^*\xi_{fH}^*\xi_{f\alpha}^*\xi_{iC}\xi_{iH}d\tau,$$

$$\Lambda_2=\int\cdots\int\xi_{fB}^*\xi_{fH}^*\xi_{f\alpha}^*\xi_{iB}\xi_{iH}d\tau,$$

$$\mu_{ps}=\frac{M_pM_s}{(M_p+M_s)M_n}.$$

The vector arguments of the function in (27) indicate the angular matrix elements. When these are evaluated as discussed above and initial states averaged over and final states summed over, the differential cross section becomes

$$\frac{d\sigma}{d\Omega}=\frac{M_\alpha M_H M_I k_\alpha}{(2\pi\hbar^2)^2M_Fk_H}|\Lambda_1|^2\{|C_1F_{l(nH)}^*(k_1)G_{l(nC)}(K_1)|^2$$
$$+2(\Lambda_2/\Lambda_1)\cos\beta C_1C_2F_{l(nH)}^*(k_1)G_{l(nC)}(K_1)F_{l(HB)}^*(k_2)G_{l(\alpha B)}(K_2)+|(\Lambda_2/\Lambda_1)C_2F_{l(HB)}^*(k_2)G_{l(\alpha B)}(K_2)|^2\},\quad(28)$$

where $\cos\beta$ arises from the transformation from the heavy-particle stripping axis of quantization $\mathbf{K}_2$ to the pickup axis $\mathbf{K}_1$ and β is the angle between $\mathbf{K}_1$ and $\mathbf{K}_2$.

The Butler integrals F_l in general must be modified by a cutoff procedure in order to fit the experimental data. This cutoff procedure at present is supported only by phenomenological arguments relating to the absorption of the incoming wave at points inside of the target nucleus and to Coulomb and nuclear scattering distortions of the incident and outgoing wave.

Reducing the amplitude of ζ_l and χ_l for $r<R$ has the practical effect of increasing the relative amplitude of the momentum wave function at large momenta and causing the momentum wave function to oscillate more rapidly. These effects of course imply distortion, since all distortions of the wave function generally provide a relative change in the shape of the momentum distributions.

In the ordinary deuteron stripping problem the so-called continuous momentum functions G_l are associated with momentum transfers $\mathbf{K}$ which vary slowly over the angular distribution, with the result that changes in the integration of G_l have little effect on the final result.

In the analysis of the $C^{13}(He^3,\alpha)C^{12}$ reaction the momentum transfer $\mathbf{K}_1$ varies in magnitude by a factor of 2. In addition the $G_{l(nC)}$ function passes through zero once in the interval from 0° to 180°. Thus the $G_{l(nC)}(K_1)$ and to some extent $G_{l(\alpha B)}(K_2)$ are quite sensitive to variations in Θ.

The data could be fitted using integrations from $0\rightarrow\infty$ in the case of $G_{l(nC)}(K_1)$. However, a much more satisfactory fit is obtained when $G_{l(nC)}(K_1)$ is cut off in the same manner as used for the F_l functions.

The data were fitted with Eq. (23) using the cutoff radii R_1 for $F_{l(nH)}^*$, R_2 for $F_{l(HB)}^*$, a_1 for $G_{l(nC)}$, and a_2 for $G_{l(\alpha B)}$. The theoretical curves of Eq. (28) and the experimental points are shown in Figs. 1 and 2. Table I presents the parameters used for the curves shown (radii are in units of 10^{-13} cm).

TABLE I. The parameters used in the theoretical curves [Eq. (28)] of Figs. 1 and 2.

E_H	$l(nC)$	$l(nH)$	$l(\alpha B)$	$l(HB)$	a_1	R_1	a_2	R_2	Λ_2/Λ_1
2.0 Mev	1	0	1	0	4.61	5.00	4.27	5.50	0.50
4.5 Mev	1	0	1	0	4.15	4.80	4.88	6.10	1.75

IV. DISCUSSION AND CONCLUSIONS

The agreement between the curves obtained from Eq. (28) and the experimental data is rather striking when one considers the approximate nature of such a first-order calculation. Such agreement, however, may not be surprising when the fundamental amplitudes governing the angular distribution exhibit a rapid oscillation over the interval from zero degrees to 180 degrees.

The use of distorted waves in the main should tend to smear out the peaks and shift the maxima and minima. The freedom of the choice of the "cutoff radius" in large part takes the latter distortion into account.

Better fits to the data could have been achieved perhaps by additional variations in the parameters. It is well to emphasize that we should not expect much more than the general structure of the angular distribution.

Specific consideration of distortions has been omitted throughout. For example, the data at 2.00 Mev below the Coulomb barrier should have a sizeable correction arising from Coulomb effects. The fact that some variation in the parameters was required at this energy suggests the presence of distorted waves. The success of the fits suggests that the interpretation of the reaction $C^{13}(He^3,\alpha)$ at these energies as a direct interaction is consistent with experiment. A large part of both theoretical distributions is supplied by the interference between the "pickup" and "heavy-particle stripping" channels. Furthermore, the variations between the curves for the two energies are accounted for by the kinematics.

Satchler[9] has suggested that the magnitude of the exchange components provides a measure of the cluster parameters which arise in the many-body problem. Hence one could measure the relative cluster amplitudes, for example, by studying the heavy-particle components of (He^3,p), (He^3,d), (He^3,He^3), (He^3,α) reactions using a given target nucleus.

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[9] G. R. Satchler (private communication).