Theory of Ultrasonic Cyclotron Resonance in Metals at Low Temperatures*

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The extent to which measurement of the propagation characteristics of circularly polarized acoustic shear waves as a function of a magnetic field applied parallel to the direction of propagation can reveal information about the band structure of metals is investigated theoretically. The interaction between the electrons and sound waves is treated classically and is viewed as taking place via an internal electric field and by scattering modified by the sound wave. The treatment is based on the combined solution of Boltzmann's and Maxwell's equations for a free-electron model. A simple physical picture involving cyclotron resonance is developed by means of which the treatment is extended to more general band models. The most striking results are obtained in case the mean free path is larger than the wavelength. Under these conditions, it is found that subject to certain restrictions, (1) absorption edges exist, the measurement of which can be related to the curvature of the Fermi-surface; (2) the shape of the Fermi-surface may be determinable from the shape of attenuation and dispersion curves within the absorption region; and (3) the cyclotron frequency may be measured. The results (2) and (3) may be dependent upon the simplifying assumption regarding the electron sound wave interaction.

I. INTRODUCTION

CYCLOTRON resonance experiments have been of great value in exploring the band structure of semiconductors. When applied to metals, however, such experiments are faced with great difficulties mainly associated with the metallic skin effects. It thus seems natural to inquire to what extent the internal electric field associated with a high-frequency sound wave can be used for such experiments.

The magnitude of these internal fields was calculated by Pippard[1] in his treatment of the attenuation of high-frequency sound waves in pure metals at low temperatures. His treatment demonstrates that the very large attenuations encountered under such conditions are in fact due to interaction with conduction electrons. On this basis, one would expect any magnetic field dependence of the energy absorption by the conduction electrons to be reflected directly in the attenuation. Such a strong magnetic field dependence of the attenuation was first found in experiments on tin by Bömmel.[2] The curve he obtained of the attenuation, A, *versus* the applied magnetic field, $\mathbf{H}$, for the case of $\mathbf{H}$ perpendicular to the sound wave propagation vector, $\mathbf{q}$, exhibits structure, which it has been suggested[3] is caused by a spatial resonance involving equality of the diameter of the electron orbit and one-half the sound wavelength. Similar results have since been obtained by others[4] and similarly interpreted.

In the present work, a theory is developed in more detail than reported previously[5] for a configuration more closely related to the ordinary cyclotron resonance situation. The situation considered involves a shear wave with $\mathbf{q}$ parallel to $\mathbf{H}$. (Their common direction is taken to be the z direction.) The lattice displacement, $\mathbf{S}$, and the resulting internal electric field, $\mathbf{E}$, are thus perpendicular to the magnetic field. Except when otherwise stated, the entire discussion concerns circularly polarized waves.

The main physical feature which characterizes the proposed experiment and distinguishes it from the usual cyclotron resonance experiment is the fact that the velocity of the electrons, $\mathbf{v}$, on the Fermi surface is much larger than the speed of sound, c_s. As a result of this, the time rate of change of electric field experienced by a conduction electron depends strongly on its velocity component in the direction of propagation of the sound wave; in fact the effective frequency, ω_e, is given by

$$\omega_e=\omega(v_z/c_s+1), \tag{1}$$

where ω is the applied frequency. It was shown by Pippard[1] that in the absence of a magnetic field, and for the electron mean free path, l, substantially larger than the sound wavelength, the attenuation of the sound wave is due to a group of electrons having a value of v_z in a narrow range such that the electric field to which they are subjected is always in a given direction, i.e., such that $\omega_e=0$. In the presence of a magnetic field the projection of electron orbits on the xy plane are closed and repeated at an angular frequency, ω_c, called the cyclotron frequency. For electrons describable by an effective mass, m^*, one has $\omega_c=eH/m^*c$. At a given value of magnetic field, there now will again be a group of electrons with a definite value of v_z which can gain energy continuously from the electric field, namely those for which

$$\omega_c=\omega_e=\omega(v_z/c_s+1), \tag{2}$$

unless the magnetic field is so large that Eq. (2) cannot be satisfied for any value of v_z on the Fermi surface. Thus attenuation of the sound wave may be expected

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[1] A. Pippard, Phil. Mag. **46**, 1104 (1955).
[2] H. E. Bömmel, Phys. Rev. **100**, 758 (1955).
[3] A. B. Pippard, Phil. Mag. **2**, 1147 (1957).
[4] Morse, Bohm, and Gavenda, Bull. Am. Phys. Soc. Ser. II, **3**, 44 (1958).
[5] T. Kjeldaas, Jr., Bull. Am. Phys. Soc. Ser. II, **3**, 180 (1958).

for values of magnetic field such that

$$\omega(-v_0/c_s+1)<\omega_c<\omega(v_0/c_s+1), \tag{3}$$

where v_0 is the Fermi velocity. Moreover, from the nature of the variation with magnetic field in this region, information about the distribution of electron states over the Fermi surface should be obtainable.

In Sec. III, we return to a demonstration of the utility of the absorption edges implied by Eq. (3) for determining the Fermi surface. In order to put the intuitive ideas discussed on a more substantial basis and to determine the sample purity required to obtain adequate resolution, we examine quantitatively in Sec. II the magnetic field dependence of the attenuation and dispersion of the sound waves for a free-electron model.

II. FREE-ELECTRON MODEL

The magnetic field dependence of the propagation characteristics is calculated by a procedure in which the electron-sound wave interaction is treated completely classically. The calculation proceeds in three steps:

(*a*) By means of the Boltzmann equation for the electron distribution function f, the electron current $\mathbf{j}$ is obtained as a function of the internal electric field, $\mathbf{E}$, the lattice displacement velocity $\mathbf{U}$ (or lattice current density $\mathbf{J}$), and the external magnetic field.

(*b*) Maxwell's equations are then used to eliminate the electron current, yielding a relation between $\mathbf{E}$ and $\mathbf{J}$.

(*c*) A force per unit mass, F_d, arises from the reaction of the electric field on the lattice and calls forth the attenuated and dispersion in question.

The results obtained reduce at zero field to those of Pippard,[1] who obtained his results by kinetic considerations. In using a Boltzmann equation, we are extending to a magnetic field a procedure used by Holstein[6] who rederived Pippard's results by this more generally understood technique.

(*a*) The distribution function f equals f_0+f_1, where f_0 is the Fermi distribution. The lattice displacement and quantities dependent upon it are taken to behave like $e^{i(qz-\omega t)}$, so that $\partial/\partial t\rightarrow -i\omega$, $\partial/\partial z\rightarrow iq$. The relaxation is taken to be by impurities with a characteristic time, τ, depending only on electron energy, and the collision term is written as

$$\left(\frac{\partial f}{\partial t}\right)_{\text{coll}}=-\left(\frac{f-f_0\mathbf{U}}{\tau}\right), \tag{4}$$

where $f_0\mathbf{U}$ is a Fermi distribution centered on the lattice velocity $\mathbf{U}$. One may make the expansion $f_0\mathbf{U}\sim f_0-m\mathbf{v}\cdot\mathbf{U}(\partial f_0/\partial\epsilon)$. In the region $ql\gg 1$, which is the one discussed in Sec. I, the more interesting physical results are independent of τ and of the postulate that the relaxation is to a Fermi distribution centered on the lattice velocity. Because the present interest is in effects independent of intensity, the Boltzmann equation is linearized and becomes

$$-i\omega f_1+v_z(iq)f_1-e\frac{\partial f_0}{\partial\epsilon}(E_xv_x+E_yv_y)$$
$$-\frac{eH}{mc}\left(v_y\frac{\partial f_1}{\partial v_x}-v_x\frac{\partial f_1}{\partial v_x}\right)=-\frac{1}{\tau}\left(f_1+m\mathbf{v}\cdot\mathbf{U}\frac{\partial f_0}{\partial\epsilon}\right), \tag{5}$$

which may be rewritten

$$\frac{1-i\tau(\omega-v_zq)}{\tau}f_1-\omega_c\left(v_y\frac{\partial f_1}{\partial v_x}-v_x\frac{\partial f_1}{\partial v_y}\right)=e\frac{\partial f_0}{\partial\epsilon}\mathbf{v}\cdot\boldsymbol{\Gamma}, \tag{6}$$

where $\boldsymbol{\Gamma}=\mathbf{E}-(m/e\tau)\mathbf{U}$. Making the standard postulate of Gans[7] that $f_1=v_xX_1(v,v_z,z,t)+v_yX_2(v,v_z,z,t)$ and going to circularly polarized components

$$X^{\pm}=X_1\pm iX_2,\quad \Gamma^{\pm}=\Gamma_x\pm i\Gamma_y,\quad \text{etc.},$$

one obtains the pair of equations

$$X^{\pm}=\frac{e\tau(\partial f_0/\partial\epsilon)\Gamma^{\pm}}{1-i\tau(\omega\pm\omega_c-v_zq)}. \tag{7}$$

Going to polar coordinates in velocity space, integrating over azimuth and magnitude, and using the relation between Fermi velocity and electron density, one obtains

$$j^{\pm}=\sigma_0G^{\pm}[E^{\pm}-(1/\sigma_0)J^{\pm}], \tag{8}$$

where

$$G^{\pm}=\frac{3}{4}\int_0^{\pi}\frac{d\theta\sin^3\theta}{1-i\tau(\omega\pm\omega_c-qv_0\cos\theta)}, \tag{9}$$

and

$$\sigma_0=ne^2\tau/m.$$

(*b*) Maxwell's equations become

$$E^{\pm}=\frac{4\pi i}{\omega}\left(\frac{c_s}{c}\right)^2(J^{\pm}\pm j^{\pm}). \tag{10}$$

Eliminating $j^{\pm}$ between Eqs. (8) and (10), one obtains

$$E^{\pm}=-\frac{J^{\pm}}{\sigma_0}\left(\frac{1}{G^{\pm}}-1\right)\beta^{\pm}, \tag{11}$$

where

$$\beta^{\pm}=\frac{1}{1+i(\omega/4\pi\sigma_0G^{\pm})(c/c_s)^2}.$$

(*c*) The force on the ions per unit mass is

$$F_d^{\pm}=-\frac{m}{\tau M}U^{\pm}\left(\frac{1}{G^{\pm}}-1\right)\beta^{\pm}\mp i\Omega_cU^{\pm}.$$

[6] T. Holstein, Westinghouse Research Memo 60-94698-3-M17, 1956 (unpublished).

[7] R. Gans, Ann. Physik **20**, 293 (1906).

Here the last term arises from the action of the external magnetic field on the ions, $\Omega_c=eH_z/Mc$, where M is the mass of the ion. Since $F_d^{\pm}$ is proportional to the ion velocity, the original assumption that the spatial dependence is of the form e^{iqz} is in fact satisfied. Strictly speaking, the force on the lattice caused by the electron impurity scattering should be included. This can easily be done; however, it turns out to be negligible mainly because it depends on the difference of the electron and ion current. In the region of interest these currents almost cancel ($\beta\sim1$); outside of this region, moreover, the electric field force is dominant. In the interest of simplicity of presentation we therefore drop this term from the beginning. Upon substitution in the sound wave equation

$$\partial^2S^{\pm}/\partial l^2=c_0^2\partial^2S^{\pm}/\partial z^2+F_d^{\pm},$$

one finds

$$q^{\pm}=q_0\left\{1\mp\frac{\Omega_c}{\omega}+i\frac{mv_0}{Mc_0}\left(\frac{1}{G^{\pm}}-1\right)\frac{\beta^{\pm}}{\alpha}\right\}^{\frac{1}{2}}, \tag{12}$$

where $\alpha=ql$. Direct evaluation of Eq. (9) yields

$$G^{+}=\frac{3}{2}\left\{\left[r\left(1+\frac{1}{\alpha^2}-\gamma^2\right)+p\frac{2\gamma}{\alpha}-\frac{1}{\alpha^2}\right]+i\left[p\left(1+\frac{1}{\alpha^2}-\gamma^2\right)-r\frac{2\gamma}{\alpha}+\frac{\gamma}{\alpha}\right]\right\}, \tag{13}$$

where

$$r=\frac{1}{2\alpha}\{\tan^{-1}[\alpha(1+\gamma)]+\tan^{-1}[\alpha(1-\gamma)]\},$$

$$p=\frac{1}{4\alpha}\ln\left(\frac{1+\alpha^2(1+\gamma)^2}{1+\alpha^2(1-\gamma)^2}\right),$$

and

$$\gamma=(c_s/v_0)(1+\omega_c/\omega),\quad G^{-}(H)=G^{+}(-H).$$

Figure 1 is a plot of the variation with γ of the real part of $(1/G^{+}-1)$ for several values of α. The most striking feature is the existence of an absorption edge for large α, occurring at $\gamma=1$. The value of the field at the absorption edge will be denoted by H_A. Under conditions of present experimental interest $\beta\sim1$. In the case of sodium, for example, for $\omega=10^8$ sec^{-1} a magnetic field of about 30 000 gauss would be required to change β by 1%; while the absorption edges would occur at about 2500 gauss.[8] According to Eq. (12), Fig. 1 is therefore a plot of the attenuation as a function of the magnetic field, provided the attenuation per wavelength is small.

In obtaining Eq. (13) we have taken $q=\omega/c_s$, which again is a good approximation only if the attenuation per wavelength is small. The maximum of $\mathrm{Re}(1/G^{\pm}-1)$

[8] At a certain large magnetic field $\sim10^6$ oersteds, the real part of the denominator of β goes through zero. According to our equations this results in a resonant attenuation for one of the polarizations.

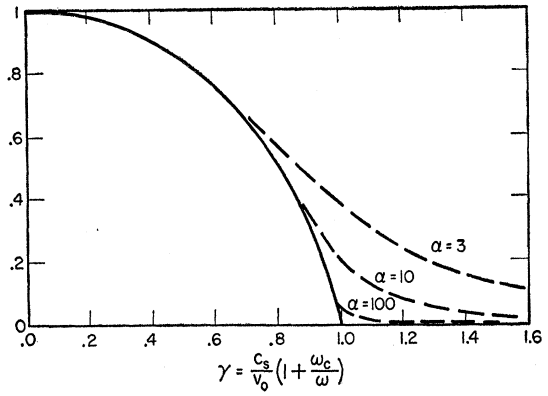

FIG. 1. Variation of $\mathrm{Re}(1/G^{\pm}-1)$ as a function of γ in the high-frequency region. This is equivalent to a plot of the attenuation *versus* magnetic field.

occurs at $\gamma=0$, and for this value of γ, $1/G^{\pm}\simeq4\alpha/3\pi$, for $\alpha\gg1$. The maximum attenuation per wavelength thus depends mainly on ion mass. In the case of sodium, $q^{\pm}\simeq q_0(1+i/470)$ under these conditions. Inspection of Eq. (9) indicates that if the imaginary part of q multiplied by l is substantially less than unity, no important corrections to Eq. (13) occur. Thus in Fig. 1 only the curve for $\alpha=100$ requires any modification in the case of sodium. Actually Eq. (9) may be integrated in terms of elementary transcendental functions for arbitrary q. From Eq. (12), q can then be determined by an iterative procedure in any case of interest. The validity of this correction is subject to some doubt, however, because when the attenuation per mean free path becomes large, surface conditions might be extraordinarily important. In particular, the effect of imposing surface boundary conditions on f_1 should be investigated to determine if there is anything "anomalous" occurring. Actually the main interest in the acoustic technique centers on the possibility of avoiding surface effects; therefore it is desirable to have $\alpha<100$.

The dispersion arising from the electron interaction is an odd function of γ, while the dispersion arising from the action of the external field on the ions is an odd function of H. The maximum change in c_s occurs in the vicinity of the absorption edges. From Eqs. (12) and (13), one obtains, for $\alpha\gg1$,

$$\mathrm{Max}\frac{\Delta c_s}{c_0}=-\frac{1}{2}\frac{mv_0}{Mc_0}\frac{1}{3},$$

which in the example of sodium equals 1/600. It should be noted that the change in speed for a (+)-polarized wave is very nearly oppositely equal to that for a (−)-polarized wave.

For a plane polarized wave, this leads to a rotation of the plane of polarization by half the phase difference of its circularly polarized components. The effect is completely analogous to the Faraday effect, the direction of rotation being independent of whether the

direction of propagation is parallel or antiparallel to H. Thus, upon reflection back to an original transducer, the change in polarization angle is doubled. In the example of sodium for $\omega=10^8$ sec^{-1}, one finds in the case $H\sim H_A$ that in a path length of one centimeter a phase change of $\sim\pi/2$ takes place. The change in polarization is a function of H, being small for $|H|\ll|H_A|$, and for $|H|\gg|H_A|$. Depending upon the directional sensitivity at the transducer, extraneous "wiggles" may therefore be produced in curves of A *versus* H unless the change in the plane of polarization is specifically taken into account. Actually the attenuation of a (+)-polarized wave differs by a very slight amount from that for a (−)-polarized one, the fractional difference being c_s/v_0. Thus, for extremely long path lengths the description in terms of plane polarized waves with a rotating plane of polarization breaks down; the polarization becomes elliptical and eventually circular. In practice this effect may be neglected, since the amplitude is reduced by a huge factor before it becomes significant. The propagation characteristics of the (rotating) plane-polarized waves may be found from suitable combinations of Eq. (8). The attenuation now depends on the $\mathrm{Re}(1/G^+ + 1/G^- - 2)$ and is symmetric in H, its value is essentially the same as for a circularly polarized wave.[9] The dispersion depends on $\mathrm{Im}[1/G^+ + 1/G^-]$; thus the change in velocity with field is smaller than for circularly polarized waves, by a factor of $\sim c_s/v_0$. The maximum fractional change with field of the speed is thus $\sim 10^{-5}$.

Returning to the discussion of circularly polarized waves, one finds from Eq. (12) that by measuring the attenuation and the speed of sound waves as a function of magnetic field, it may be solved for

$$1\mp\frac{\Omega_c}{\omega}-\frac{mv_0}{Mc_0}\,\mathrm{Im}\left(\frac{1}{G^\pm}\right)\frac{\beta}{\alpha},$$

and for $(mv_0/Mc_0)\,\mathrm{Re}(1/G^\pm-1)(\beta/\alpha)$ as a function of magnetic field. From the latter quantity the magnetic field at which $\gamma=0$ can be determined, i.e., ω_c measured. It is possible, however, that this result is sensitive to the assumption that the electron-phonon interaction may be treated as taking place entirely via a macroscopic electric field. In any case, a very high precision of measurement would be required. In the next section an examination is made of the information obtainable from absorption edge measurements, which according to Eq. (3) occur at $\gamma=\pm 1$.

III

The preceding results require important qualifications and extensions before they can be applied to analysis of measurements on real metals. The most important point to settle is the question of the extent to which the electron-impressed phonon interaction can in fact be viewed as taking place via a macroscopic electric field. The answer depends on the details of the electron-phonon interaction and will vary from metal to metal. No attempt will be made in this paper to investigate this problem from a theoretical viewpoint. From the fact that the measured[10] attenuation in tin agrees within a factor of 3 with that predicted by Pippard,[1] it appears plausible to conclude that for most normal metals at least a substantial fraction of the interaction takes place via the type of field treated. On this basis one expects that the absorption edges of the entirely free-electron theory will appear in most cases at least as regions of rapid change in attenuation with field. Hereafter we shall assume that the entire interaction is of the electric field type and examine the consequences for somewhat more general band structures. The object is to obtain an understanding of the extent to which details of the field variation of A and c_s may be related to details of the Fermi surface, and to provide a procedure for predicting results based on more general band models for cases where the assumption is satisfied well enough. The present discussion will be limited to the "absorption region," i.e., $|H|<|H_A|$, and to the case $\alpha\gg 1$. Actually we wish to put an upper limit on α of about 100 to avoid any complications of the type discussed following Eq. (13). When these conditions are satisfied, $\mathrm{Re}(1/G)\gg 1$; thus collision drag effects may be neglected, and we may write instead of Eq. (8)

$$j^\pm=\Sigma^\pm E^\pm. \tag{14}$$

The quantity $\Sigma^\pm$ will be called the transverse magnetoacoustic conductivity, and the basic task of the present section is to calculate it. The discussion will be limited to the case of $\beta\approx 1$, which means that $j^\pm\approx -J^\pm$, so that Eq. (14) becomes $E^\pm=-(1/\Sigma^\pm)J^\pm$. Taking account also of the force of the external magnetic field on the ions, one obtains in the same manner as before

$$(q^\pm)^2=q_0^2\left[1\mp\frac{Z\Omega_c}{\omega}\pm i\frac{Z}{M}\frac{ne^2}{\omega\Sigma^\pm}\right]. \tag{15}$$

Here Ze is the ionic charge, and the electron density n equals Z times the ion density ($\Omega_c=eH_z/Mc$ as before). From measurement of $A(H)$ and $c_s(H)$, $\Sigma^\pm(H)$ may be determined via Eq. (15). As will be indicated, the relationship of $\Sigma^\pm$ to the band structure can in some important cases be understood in detail by the intuitive methods of the introduction.

To explain these more fully, Eq. (9) is first rederived by this procedure. In the entirely free-electron case $\Sigma^\pm=\sigma_0 G^\pm$. For a group of free electrons, the field- and

[9] M. S. Steinberg [Phys. Rev. **110**, 772 (1958)] has independently calculated an attenuation coefficient for plane-polarized waves in a magnetic field. His value appears to be in agreement with ours.

[10] W. P. Mason and H. E. Bömmel, J. Acoust. Soc. Am. **28**, 930 (1956).

frequency-dependent conductivity is given by

$$\sigma^{\pm}=\frac{ne^2\tau}{m}\frac{1}{1+i(\omega\pm\omega_c)\tau}. \tag{16}$$

In the acoustic case the frequency observed by an electron is Doppler-shifted as given by Eq. (2); accordingly the conductivity due to a group of electrons, dn in number, having v_z between v_z and v_z+dv_z, is

$$d(\sigma_0 G^{\pm})=dn\frac{e^2\tau}{m}\frac{1}{1+i(\omega_e+\omega_c)\tau}.$$

From the relation $dn=\frac{3}{4}n(1-\xi^2)d\xi$, where $\xi=v_z/v_0$, and from Eq. (2), this may be rewritten

$$d(\sigma_0 G^{\pm})=\frac{3}{4}\frac{ne^2\tau}{m}\frac{(1-\xi^2)d\xi}{1+i(\omega v_0\xi/c_s+\omega\pm\omega_c)\tau}, \tag{17}$$

or

$$G^{\pm}=\frac{3}{4}\int_{-1}^{1}\frac{(1-\xi^2)d\xi}{1+i(qv_0\xi+\omega\pm\omega_c)\tau} \tag{18}$$

for $q=\omega/c_s$. Equation (18) is of course identical to Eq. (9).

In the region of magnetic field in which the equation $qv_0\xi+\omega\pm\omega_c=0$ has a solution, the solution is $\xi=\gamma$. The denominator has a strong minimum at this point for $\alpha\gg1$. In evaluating $\mathrm{Re}(G^{\pm})$ the reciprocal denominator may be treated as a δ-like function of constant area π/α, centered at $\xi=\gamma$, and of width $1/\alpha$. Thus $\mathrm{Re}(G^{\pm})\simeq(3\pi/4\alpha)(1-\gamma^2)$ for $1-|\gamma|>1/\alpha$, in agreement with Eq. (13). Now $\gamma\approx H/H_A$, thus

$$\mathrm{Re}(G^{\pm})\simeq\frac{3\pi}{4\alpha}\left(1-\frac{H^2}{H_A{}^2}\right)\quad\text{for } H^2<H_A{}^2. \tag{19}$$

By the same procedure which led to Eq. (19), $\Sigma^{\pm}$ may now be rederived for more general Fermi surfaces. We consider Fermi surfaces which consist of one or more surfaces of revolution about axes parallel to the k_z axis. This is somewhat more general than is necessary to include overlapping bands each with spherical energy surfaces. Consider the expression[11] for the conductivity,

$$\sigma_{ij}=\frac{e^2}{4\pi^3}\int_{E=\zeta}\frac{dS\,\tau v_i v_j}{\hbar v}. \tag{20}$$

Here dS is an element of area of the Fermi surface $E(\mathbf{k})=\zeta$, and $v_i=(1/\hbar)(\partial E/\partial k_i)$. Thus

$$\sigma_{xx}=\sigma_{yy}=\sigma_0=\frac{e^2}{8\pi^3\hbar}\int_{E=\zeta}dS\,\tau v(1-\xi^2),\quad\text{where } \xi=v_z/v.$$

The electrons on a plane perpendicular to the k_z axis have the same cyclotron frequency and the same value of v_z. The element of area is now taken to be that formed on the surface between two such planes infinitesimally separated. Accordingly one may write

$$\Sigma^{\pm}=\frac{e^2}{8\pi^3\hbar}\int_{E=\zeta}\frac{dS\,\tau v(1-\xi^2)}{1+i(qv_z+\omega\pm\omega_c)\tau}. \tag{21}$$

Integration over azimuth yields

$$\Sigma^{\pm}=\frac{e^2}{4\pi^2\hbar}\int_{-1}^{1}\frac{d\xi\,k_r k_\rho(1-\xi^2)^{\frac{1}{2}}v\tau}{1+i(qv\xi+\omega\pm\omega_c)\tau}, \tag{22}$$

or the sum of such expressions if more than one closed is involved. Here $k_r{}^2=k_x{}^2+k_y{}^2$, and k_ρ is the radius of curvature of the curve formed by the intersection of the Fermi surface and any plane containing the axis of revolution. In (22), the quantities k_r, k_ρ, v, ω_c, and τ are regarded as functions of ξ. Let ξ_0 be the value of ξ at which the imaginary part of the denominator vanishes. The integrand is sharply peaked at ξ_0, and

$$\int_{-\infty}^{\infty}\frac{d\xi}{1+(qv\xi+\omega\pm\omega_c)^2\tau^2}=\frac{\pi}{qv(\xi_0)\tau(\xi_0,H)}, \tag{23}$$

provided the variation of $v\tau$ may be neglected in a range of ξ equal to $1/qv\tau\ll1$. Thus

$$\mathrm{Re}(\Sigma^{\pm})=(e^2/4\pi q\hbar)k_r(\xi_0)k_\rho(\xi_0)(1-\xi_0{}^2)^{\frac{1}{2}}, \tag{24}$$

(or the sum of such expressions). Calculating ω_c by the Shockley[12] procedure, one obtains

$$\cot(\cos^{-1}\xi_0)=\frac{\xi_0}{(1-\xi_0{}^2)^{\frac{1}{2}}}=\frac{eH}{\hbar qc}\frac{1}{k_r}. \tag{25}$$

The main point to be made is that the variation with field in the absorption region of $\mathrm{Re}(\Sigma^{\pm})$ is independent of any moderate variation of τ with position on the Fermi surface or with magnetic field. The Fermi surface can be reconstructed from Eqs. (24) and (25) provided it consists of a single surface. The measurements yield

$$k_r k_\rho(1-\xi^2)^{\frac{1}{2}}=f(H)=f\left(\frac{\xi k_r}{(1-\xi^2)^{\frac{1}{2}}}\frac{\hbar qc}{e}\right).$$

Substituting $k_\rho=[(1-\xi^2)^{\frac{1}{2}}/\xi]dk_r/d\xi$, this becomes a first-order linear differential equation which may be solved for ξ as a function of k_r. Noting that

$$\xi/(1-\xi^2)^{\frac{1}{2}}=dk_r/dk_z,$$

it is seen that an additional quadrature will yield k_r as a function of k_z.

A serious reservation must now be made in addition to the one made at the beginning of this section. Equation (22) has been derived by a method which is equivalent to a wave-packet-Boltzmann equation treatment.

[11] A. H. Wilson, *The Theory of Metals* (Cambridge University Press, Cambridge, 1953), second edition, Eq. (8·2·6).

[12] W. Shockley, Phys. Rev. **79**, 191 (1950).

The use of a Boltzmann equation treatment is questionable in the presence of a magnetic field, when $\omega_c\tau>1$. The transverse magnetoresistivity predicted by such techniques is incorrect for large fields.[13] This failure can conceivably be attributed[13] to a magnetic field dependence of τ however, and does not necessarily reflect on the applicability of the Boltzmann equation. It is possible that the main error from this source is simply the neglect of specific quantum effects.

The case of completely general Fermi surfaces is not treated. It is easy, however, to gain a general idea of the results to be expected on a wave-packet picture if the surface is not one of revolution. In this case v_z may vary along the orbit, and so may the rate at which the orbit is traversed. In general no electron on the surface can follow the direction of the electric field in detail. The internal electric field will not have a purely sinusoidal space dependence since its magnitude is almost exactly such as to cause the electron current to balance the positive-ion current. When all these nonsinusoidal effects are subjected to a spatial Fourier analysis, short-wavelength components are found. These lead to attenuation outside the fundamental absorption region, to a change in the details of the absorption inside this region, and to an increased diffuseness of the absorption edge.

The general tenor of these remarks indicates that detailed interpretation of $\Sigma^{\pm}$ will have value only when the deviation from a free-electron model is small or for gross band overlap. Reasonably well defined absorption edges probably exist somewhat more generally, and we now proceed to examine what one may hope to learn from their measurements.

At the absorption edge $\omega_c/v_z=\omega/c_s$, according to Eqs. (2) and (3). If now the Fermi surface is convex, and if v_z is an increasing function of k_z up to the surface, the measured value of ω_c/v_z is characteristic of a particular point on the Fermi surface, namely the point whose normal is parallel to the field. Here ω_c is the limiting cyclotron frequency as orbits in k space shrink down to the point in question. The quantity ω_c/v_z depends on the differential properties of the surface at this point, and it may be demonstrated that

$$K=\frac{c^2}{e^2H_A{}^2}\left(\frac{\omega_c}{v_z}\right)^2=\left(\frac{\hbar c\omega}{c_s eH_A}\right)^2. \tag{26}$$

Here K is the Gaussian curvature, i.e., the reciprocal of the products of the principal radii of curvature at the point in question. Perhaps the simplest method of deriving Eq. (26) is to calculate the cyclotron frequency by the Shockley[12] procedure for an arbitrary field direction for a general ellipsoidal energy surface, and to calculate K for this surface at the point where the normal is parallel to the field. An arbitrary surface is locally equivalent to an ellipsoid as far as second derivatives are concerned; therefore Eq. (26) follows generally from its validity for the ellipsoidal case.

If it were possible to make the measurements for a variety of crystalline directions, it would be possible to map out the Fermi surface in some detail. Unfortunately, circularly polarized waves can propagate only along a fourfold (or higher) axis (or one with accidental shear-wave degeneracy). Thus in a cubic crystal the measurement is generally limited to propagation in the [100] direction. In this case the principal radii of curvature are equal and the measurement of H_A yields the radius of curvature. If the value obtained differs only by a small amount from the value of the Fermi momentum on a spherical-band picture, an idea of the shape of the Fermi surface should be obtainable by fitting the discrepancy to the lowest order Kubic harmonic. Measurement of K in some additional directions should be possible with the judicious use of plane-polarized waves. The main precaution to observe is that the distance of propagation be sufficiently small so that the phase deviation of the (+)- and (−)-polarized components remains very small compared to λ. The modes of a sound wave propagating in an arbitrary direction are not pure shear modes; thus analysis of the magnetic effects on longitudinal propagation characteristic will also be required to interpret measurements with such waves.

[13] See, for example, R. G. Chambers, Can. J. Phys. **34**, 1396 (1956).

IV. ACKNOWLEDGMENTS

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