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Excitations in Liquid Helium: Neutron Scattering Measurements*

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The energy-momentum spectrum of the excitations in liquid helium II has been measured in a neutron scattering experiment. At $T=1.1$°K, data were obtained in the momentum range $p/\hbar=0.55$ to 2.36 A^{-1}. The results bear a striking resemblance to the spectrum proposed by Landau in 1947. If a phonon spectrum of the form $E=v_1p$ is fitted to the point at $p/\hbar=0.55\ \text{A}^{-1}$, a value of 239 ± 5 meters/sec is obtained for v_1. The measured spectrum passes through a maximum of $E/k=13.92\pm0.10$°K at $p/\hbar=1.11\pm0.02\ \text{A}^{-1}$. There is a minimum at $p/\hbar=1.92\pm0.01\ \text{A}^{-1}$, which may be represented by Landau's roton expression, $E=\Delta+(p-p_0)^2/2\mu$, with $\Delta/k=8.65\pm0.04$°K, $p_0/\hbar=1.92\pm0.01\ \text{A}^{-1}$, $\mu=(0.16\pm0.01)m_{\text{He}}$. Above $p/\hbar=2.18\ \text{A}^{-1}$, the spectrum rises linearly with a slope corresponding to the velocity of first sound. Data were obtained in the region of the minimum at $T=1.6$°K and at $T=1.8$°K. The spectrum has the same general shape observed at $T=1.1$°K, shifted downward by 0.22°K at 1.6°K, and by 0.50°K at 1.8°K. The temperature variation may be represented by the empirical expression $\Delta/k=8.68-0.0084T^7$ °K. At the higher temperatures, the energy spread of the excitations was larger than the energy resolution of the apparatus. The observed full width at half maximum was ~1°K at $T=1.6$°K, and ~2°K at $T=1.8$°K.

I. INTRODUCTION

LANDAU[1] has developed a theory which describes liquid helium II at temperatures not too near the lambda point. He treats the liquid as a slightly excited quantum mechanical system, in which deviations from the ground state are described in terms of the normal modes, or "elementary excitations" of the system. Let $E(p)$ be the energy of an excitation of momentum p. If the energy of the ground state is taken as zero, then the energy of the system may be written as $E(p)$ times the number of excitations of momentum p, summed over all values of p. In this treatment, it is assumed that $E(p)$ does not depend on the number of excitations present. If the system is in equilibrium at some temperature T, the number of excitations is given by the appropriate statistical distribution law, and the thermodynamic properties of the liquid may be calculated provided $E(p)$ is known. In 1947, Landau proposed a spectrum of the form shown in Fig. 1. The parameters were adjusted to fit the available data on the specific heat and second sound velocity in helium II. An excellent review of the theory is given by Feynman.[2]

Landau's theory was very successful in explaining the observed properties of liquid helium II, and in predicting new properties (for example, the increase in the velocity of second sound at temperatures below 1°K).[3] It remained to be shown that the excitations in liquid helium do, in fact, possess the required energy spectrum.

Feynman[4] has succeeded in constructing a wave function for the excitations in liquid helium which leads to a spectrum of the form required by Landau's theory. Brueckner and Sawada[5] have obtained a similar spectrum by studying the properties of a dense Einstein-

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[1] L. Landau, J. Phys. U.S.S.R. **5**, 71 (1941); **8**, 1 (1944); **11**, 91 (1947); Phys. Rev. **70**, 356 (1941); **75**, 884 (1949).

[2] R. P. Feynman, in *Progress in Low Temperature Physics*, edited by C. J. Gorter (North-Holland Publishing Company, Amsterdam, 1955), Vol. 1, p. 17; Revs. Modern Phys. **29**, 205 (1957).

[3] Discussion and references are given in H. C. Kramers, in *Progress in Low Temperature Physics*, edited by C. J. Gorter (North-Holland Publishing Company, Amsterdam, 1957), Vol. 2, p. 59.

[4] R. P. Feynman, Phys. Rev. **91**, 1291, 1301 (1953); **94**, 262 (1954); *Progress in Low Temperature Physics*, edited by C. J. Gorter (North-Holland Publishing Company, Amsterdam, 1955), Vol. 1, p. 17.

[5] K. A. Brueckner and K. Sawada, Phys. Rev. **106**, 1117, 1128 (1957).

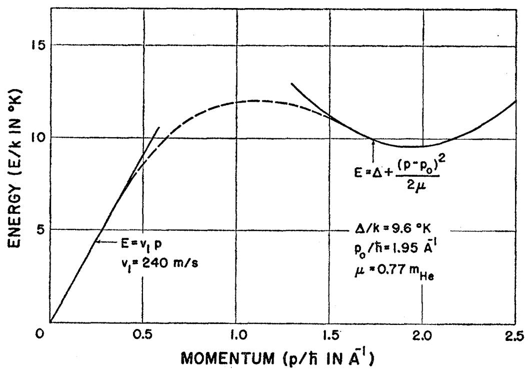

FIG. 1. Excitation spectrum proposed by Landau in 1947, based on the then available specific heat and second sound data. The spectrum was approximated by the two analytic functions shown as solid lines. The linear function represents excitations of long wavelength, or phonons. v_1 is the velocity of first sound. The parabolic function represents excitations of wavelength comparable to the average interatomic spacing, or rotons. Δ, p_0 and μ are adjustable constants, chosen to fit the thermodynamic data. k is Boltzmann's constant, and $\hbar$ is Planck's constant divided by 2π.

Bose gas composed of hard spheres. Similar results have been obtained by Lee, Huang, and Yang,[6] and by Tolmachev.[7]

Cohen and Feynman[8] have pointed out that the excitations in liquid helium may be studied by scattering long wavelength neutrons ($\lambda\gtrsim 4$ A) from the liquid. At temperatures below 2°K, the most important scattering process is one in which the neutron creates a single excitation in the liquid. In this case, conservation of energy and momentum requires that the energy and momentum of the created excitation be equal to the change in the energy and momentum of the neutron during the scattering. Measurements of the excitation spectrum of liquid helium by neutron scattering have been reported by Palevsky, Otnes, Larsson, Pauli, and Stedman[9] at Stockholm, by Henshaw[10] at Chalk River, and by the present authors[11] at Los Alamos. Although there is not complete agreement on the numerical values, the experiments do indicate that the scattering process involves the creation of single excitations in the liquid, and that the energy-momentum spectrum has the form predicted by Landau in 1947. This paper contains a complete description of the Los Alamos measurements mentioned above. The following paper[12] contains calculations of the thermodynamic properties of liquid helium based on the measured excitation spectrum.

[6] Lee, Huang, and Yang, Phys. Rev. **106**, 1135 (1957).
[7] V. V. Tolmachev, Repts. Acad. Sci. U.S.S.R. **101**, No. 6 (1955).
[8] M. Cohen and R. P. Feynman, Phys. Rev. **101, 13** (1957).
[9] Palevsky, Otnes, Larsson, Pauli, and Stedman, Phys. Rev. **108**, 1346 (1957); Palevsky, Otnes, and Larsson, Phys. Rev. **112**, 11 (1958).
[10] D. G. Henshaw, Phys. Rev. Letters **1**, 127 (1958).
[11] Yarnell, Arnold, Bendt, and Kerr, Phys. Rev. Letters **1**, 9 (1958).
[12] Bendt, Cowan, and Yarnell, following paper [Phys. Rev. **113**, 1386 (1959)].

II. EXPERIMENTAL METHOD

In the experiment proposed by Cohen and Feynman, a well-collimated beam of monoenergetic neutrons of wavelength $\lambda_i\gtrsim 4$ A is allowed to strike a target of liquid helium. The wavelength λ_f of neutrons scattered through an angle ϕ is then measured. By conservation of energy and momentum, the energy E and the momentum p of the excitations which are created are given by

$$E=h^2(\lambda_i^{-2}-\lambda_f^{-2})/2m, \tag{1}$$

$$p^2=h^2(\lambda_i^{-2}+\lambda_f^{-2}-2\lambda_i^{-1}\lambda_f^{-1}\cos\phi), \tag{2}$$

where m is the mass of a neutron, and h is Planck's constant. By varying λ_i or ϕ, the entire spectrum $E(p)$ may be traced out.

The only sufficiently intense source of neutrons with $\lambda\geq 4$ A is a thermal reactor. The neutron flux from such a reactor has a continuous distribution in wavelength given by the well-known Maxwell-Boltzmann distribution corresponding to the temperature of the reactor moderator (usually 300° to 500°K). It is possible to select neutrons of a given wavelength from the reactor distribution by the use of a suitable monochromator. However, it is difficult to obtain sufficient intensity at the desired resolution with such a device.

A polycrystalline filter may be used to remove from a neutron beam, by Bragg scattering, all neutrons with $\lambda<\lambda_c$, where λ_c is twice the maximum spacing between planes in the crystalline lattice. If the neutron absorption and incoherent scattering by the filter material are small, neutrons with $\lambda>\lambda_c$ are transmitted with little attenuation. Thus one obtains a sharp discontinuity in the neutron wavelength distribution at the cutoff wavelength λ_c. If neutrons having such a distribution are scattered from liquid helium at a fixed angle ϕ, and if the predominant scattering process involves the creation of single excitations, then the wavelength distribution of the scattered neutrons should also possess a sharp cutoff, but at a longer wavelength. If we identify λ_i with the cutoff wavelength in the incident neutron distribution, and λ_f with the cutoff in the scattered neutron distribution, we may use Eqs. (1) and (2) to calculate E and p of the excitations. The filter technique provides considerably greater intensity at a given resolution than that given by a monochromator. On the other hand, details of the line shape tend to be obscured by this method. Use of the filter technique permitted us to obtain, at practical counting rates, sufficient accuracy in the determination of $E(p)$ to permit calculation of the entropy of liquid helium with an error of the order of $\pm 5\%$. This accuracy is sufficient to allow a detailed comparison with values of the entropy obtained from specific heat measurements.

III. EXPERIMENTAL APPARATUS

A plan view of the apparatus is shown in Fig. 2. The Los Alamos Omega West Reactor was used as a source of thermal neutrons. With the reactor operating at a power of 800 kw, the thermal neutron flux at the inner end of the beam tube was 4×10^{12} neutrons/cm^2-sec.

Neutron wavelengths were measured by a crystal spectrometer. A crystal of natural rock salt (NaCl) was used in the (200) position as wavelength analyzer. The crystal was aligned with respect to the spectrometer arm by taking rocking curves in a double crystal setup. At each scattering angle ϕ, the spectrometer and its collimator were aligned optically by the use of a theodolite, and the spectrometer arm zero located by neutron counting. The value of the lattice space in rock salt is given as $a_0=5.63874\pm0.00002$ A.[13] In calculating neutron wavelength, we use the Bragg relation $\lambda=5.639 \sin\theta$, where θ is the angle between the (200) planes of the crystal and the axis of the spectrometer collimator. The spectrometer drive was designed to keep the angle between the direction of the spectrometer arm and the crystal planes also equal to θ.

The neutron detector used with the spectrometer consisted of two 5-cm diameter by 10-cm active length brass-walled counters filled to a pressure of 30 cm Hg with $B^{10}F_3$ (90% B^{10}). Neutrons from the analyzing crystal entered the detector through a channel 7.6 cm square. The neutron channel and detector system were surrounded by shielding of cadmium, paraffin, and lithium fluoride.

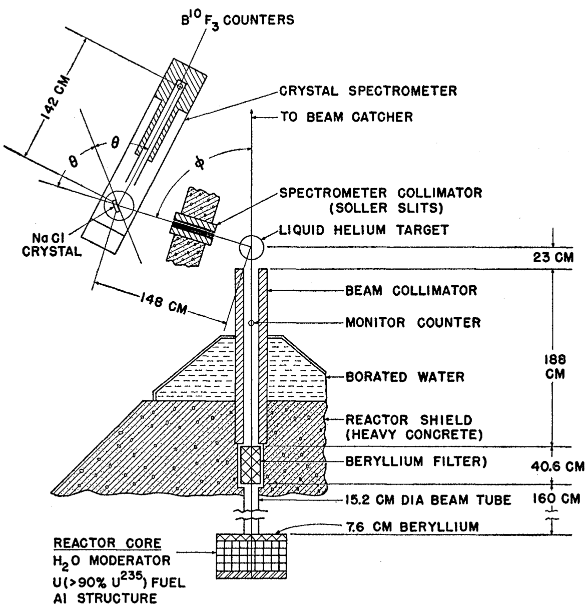

FIG. 2. Plan view of the apparatus. The beryllium filter was cooled by a jacket filled with liquid nitrogen, not shown. The beam tube was evacuated up to the inner end of the beam collimator, to provide insulation for the cold filter. The vacuum seal was a thin copper window, 0.03 cm thick. The beam collimator aperture was 15.2 cm wide and 7.6 cm high. The Soller slit collimator consisted of 21 vertical channels each 0.318 cm wide and 7.6 cm high, separated by 20 phosphor bronze septa each 0.051 cm thick. Distances between parts of the apparatus in a typical setup are indicated on the drawing. Additional shielding, not shown, around the target and spectrometer collimator, was used when data were being taken.

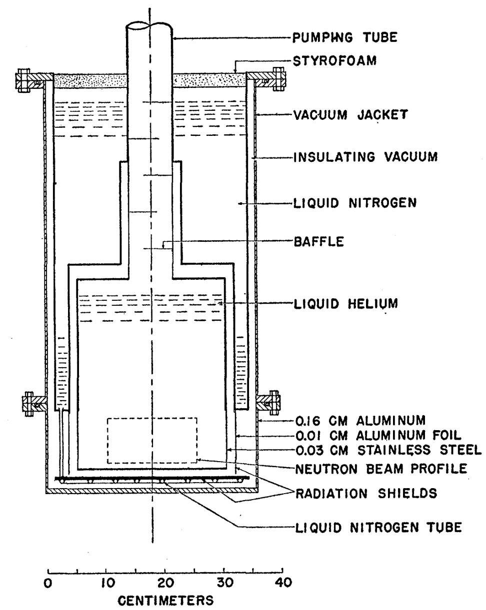

FIG. 3. Cross section of the liquid helium cryostat. The assembly has cylindrical symmetry about the indicated center line. The inner vessel containing helium is 25.4 cm in diameter, and 32.3 cm high. The lower 9 cm of this vessel serves as the target region, the remainder being the helium reservoir. Below the helium vessel is a copper plate, cooled by a coil of copper tubing through which liquid nitrogen flowed by convection. Mercury and oil manometers were used to measure the vapor pressure of the helium. The pressure above the helium surface was controlled by a mechanical forepump having a displacement of 103 liters per second, working through a diaphragm-type pressure regulator. The cryostat was mounted on a stand provided with leveling screws and casters, to allow easy removal from the target position.

A thin-walled fission counter, 2 cm in diameter and 2.5 cm long, containing 12 μg U^{235}, was placed in the beam collimator to monitor the incident neutron flux.

Four electronic counting channels, each consisting of a preamplifier, linear amplifier, and scaler, were provided. During normal operation, two channels were connected to the monitor counter, and two to the spectrometer detector. The use of dual channels for each counting system provided a convenient check of the electronic equipment. One of the channels connected to the monitor was arranged to turn off all scalers when a preset number of monitor counts had been accumulated.

Details of the liquid helium target are given in Fig. 3. The outside of the helium container, and the inside of the radiation shield, were covered with aluminum foil, to reduce the emissivity of these surfaces. When the cryostat was in good condition, the heat leak into the helium region was 0.07 watt, and the minimum temperature obtainable was 1.08°K. During part of the experiment, the heat leak increased to $\sim$0.15 watt, probably due to a small piece of aluminum foil becoming

[13] *American Institute of Physics Handbook*, edited by D. E. Gray (McGraw-Hill Book Company, Inc., New York, 1957), p. 2–53.

detached and forming a conduction path from the helium region to the radiation shield. When this condition prevailed, the minimum temperature was in the neighborhood of 1.15°K. The size of the target was chosen to be sufficiently large that the incident neutron beam did not strike any portion of the target structure which was "seen" by the spectrometer collimator. This resulted in a negligible counting rate when the empty cryostat was placed in the target position.

IV. MEASUREMENTS

When the spectrometer is set at a particular Bragg angle θ, a finite range of Bragg angles, centered at θ, is accepted. We may express the relative efficiency of detecting neutrons of wavelength corresponding to a Bragg angle θ', when the spectrometer is set at angle θ, by a normal distribution function:

$$f(\theta-\theta')=[\sigma(2\pi)^{\frac{1}{2}}]^{-1}\exp[(\theta-\theta')^2/2\sigma^2]. \tag{3}$$

Here σ corresponds to the standard deviation. The full width at half maximum is given by 2.3548σ. Let the neutron distribution entering the spectrometer be $g(\theta)$, where θ is the Bragg angle corresponding to the wavelength λ. Then the spectrometer counting rate as a function of angular setting is given by

$$I(\theta)=\int_0^\pi g(\theta')f(\theta-\theta')d\theta'. \tag{4}$$

If the incident neutron distribution has a cutoff at the Bragg angle θ_c, we may write in the neighborhood of θ_c,

$$\begin{aligned} g(\theta)&=0; \quad \theta<\theta_c,\\ g(\theta)&=I_0; \quad \theta\geq\theta_c, \end{aligned} \tag{5}$$

and the relative counting rate in the neighborhood of θ_c is then given by

$$I(\theta)=\frac{I_0}{\sigma(2\pi)^{\frac{1}{2}}}\int_{\theta_c}^\pi \exp[(\theta-\theta')^2/2\sigma^2]d\theta'. \tag{6}$$

If σ is not too large, the range of integration may be extended to infinity without changing the result.

Neutron distributions measured with the spectrometer were analyzed by fitting Eq. (6) to the data, after first subtracting the background. The information obtained for each neutron distribution was the Bragg angle of the cutoff, θ_c, and the full width at half maximum, $\delta\theta$.

The spectrometer was set up in the target position, and the neutron distribution in the incident beam was measured. The results are shown in Fig. 4. Data were taken on both sides of the arm zero position, to eliminate possible errors due to an inaccurate zero determination. Analysis of the cutoff by means of Eq. (6) gave a cutoff angle $\theta_c=44.49°$, corresponding to $\lambda_c=3.958\pm0.001$ A. A value of 0.6° was obtained for $\delta\theta$. The spread in the cutoff, $\delta\theta$, was taken as a measurement of the spectrometer resolution width, which we define as $\Delta\theta$. Other measurements taken at higher resolution gave the same wavelength for the cutoff. When the filter was cooled to the temperature of liquid nitrogen, the cutoff wavelength shifted to 3.952 ± 0.001 A, and the counting rate increased by a factor of ~7.

Kaufmann, Gordon, and Lillie[14] give as the best values of the lattice constants of beryllium at 25°C:

$$a_0=2.2854 \text{ A}, \quad c_0=3.5829 \text{ A}.$$

In a close-packed hexagonal crystal such as beryllium, the cutoff wavelength is given by $a_0\sqrt{3}=3.9584$ A. The maximum range of values of a_0 for different samples of beryllium, having different methods of preparation and degrees of purity, was ±0.0003 A. Using this figure as a measure of the maximum uncertainty in our knowledge of the lattice constants of beryllium, we have $\lambda_c=3.9584\pm0.0005$ A. This is in good agreement with our value of 3.958 ± 0.001 A for the cutoff wavelength in beryllium at room temperature. No data have been reported for the lattice constants of beryllium at the temperature of liquid nitrogen. Our value of 3.952 ± 0.001 A for the cutoff wavelength at this temperature corresponds to a lattice space $a_0=2.282\pm0.0006$ A.

Measurements were also made of the angular divergence of the incident neutron beam. The full width at half maximum was 2.0°. This angular divergence, together with the spread of 0.6° in the spectrometer resolution function, resulted in the acceptance, at a given scattering angle ϕ, of a range of scattering angles of full width at half maximum of 2.1°.

The coherent scattering cross section of vanadium is very much smaller than its incoherent scattering cross section.[15] The result is that a vanadium target will

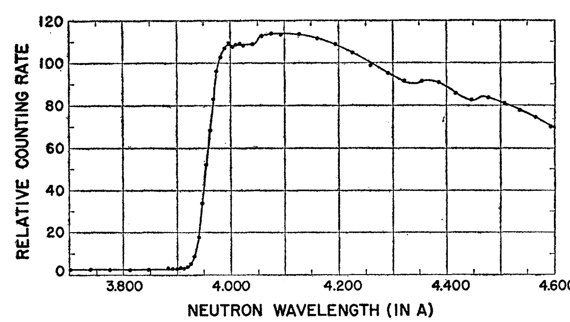

FIG. 4. The neutron wavelength distribution in the incident beam, with the beryllium filter at room temperature. The cutoff wavelength (half height of the initial steep rise) is 3.958 ± 0.001 A. The steepness of the cutoff corresponds to a full width at half maximum $\Delta\theta$ of 0.6° for the spectrometer resolution function. No evidence for partial transmission below the cutoff, characteristic of a filter of insufficient thickness, was seen. The small jump in the distribution at 4.05 A is due to the filtering action of the aluminum in the inner end of the beam tube.

[14] Kaufmann, Gordon, and Lillie, Trans. Am. Soc. Metals **42**, 785 (1950).

[15] D. J. Hughes and J. A. Harvey, *Neutron Cross Sections*, Brookhaven National Laboratory Report BNL-325 (Superin-

scatter neutrons isotropically and without change in energy. This property of vanadium was used to obtain an independent calibration of the spectrometer at most scattering angles. A sample consisting of 500 grams of granular vanadium in a flat, thin-walled aluminum box was placed in the target position, and the cutoff in the distribution of scattered neutrons was determined with the spectrometer. The observed value of the cutoff wavelength should coincide with the cutoff in the incident neutron distribution. The average of fourteen such measurements made at different scattering angles and with independent determinations of the arm zero setting was 3.952 A, which is the same as the value obtained for the cutoff in the incident beam. The rms deviation of the 14 measurements from the mean was ± 0.002 A, the greatest individual deviation being 0.004 A. This is equivalent to a rms deviation of $\pm 0.03°$ in the arm zero settings, assuming no error in the determination of the cutoff wavelengths. We consider the calibration using the vanadium scatterer to be slightly more accurate than that made by locating the arm zero, and, when possible, have adjusted the angular scale to place the measured cutoff at precisely 3.952 A ($\theta = 44.49°$). This method has the advantage that the calibration and actual measurements are made in the same way, so that possible systematic errors which might be introduced by our method of locating the cutoffs tend to be eliminated.

The helium temperature was determined by vapor pressure measurements made every half hour during a run. Vapor pressures were converted to temperatures by means of the T55E scale. The spectrometer was set at a fixed angle θ, and counts were taken until a preset number, usually 90 000, of monitor counts had been accumulated. The spectrometer was then moved to another angle, and the process repeated. The neutron distribution was first surveyed to find the approximate location of the cutoff, and then additional data were taken to allow precise determination of its location. From 4 to 7 individual runs of 90 000 monitor counts were taken at each spectrometer setting in the neighborhood of the cutoff.

A typical run of 90 000 monitor counts lasted about 8 minutes. During this time, approximately 50 background counts would be recorded, at any angular setting. The counting rate below the cutoff was essentially equal to the background as determined by turning the analyzing crystal 3° away from the Bragg angle. The number of counts at the top of the cutoff varied with the scattering angle, ϕ, due to a change in the helium cross section with angle, and because the distance between the target and the spectrometer was not the same at all angles, being smallest at $\phi = 90°$. The observed number of true counts at the top of the cutoff during a run of 90 000 monitor counts was 100 for $\phi = 20°$, rose to 300 at $\phi = 90°$, and fell to 200 at

tendent of Documents, U. S. Government Printing Office, Washington, D. C., 1955).

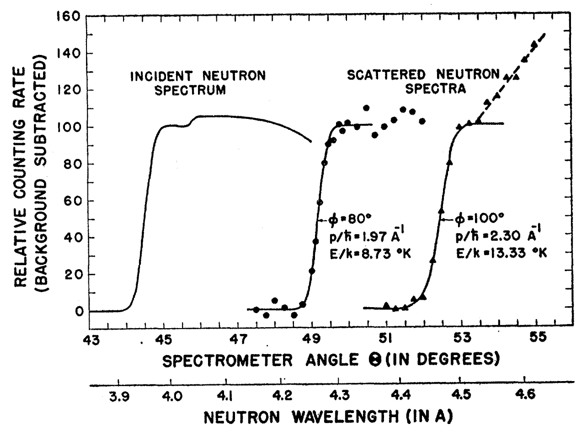

FIG. 5. Wavelength distribution of neutrons scattered from liquid helium at 80° and 100°. Background has been subtracted, and the distributions have been normalized to have the same height at the top of the cutoff. The incident neutron distribution is shown for comparison. The solid curves represent the theoretical cutoff shape given by Eq. (6), fitted to the experimental points. The calculated energy and momentum of the excitations which were created are also indicated. The distribution at $\phi = 80°$ is typical of those obtained at all angles $\leq 90°$. Above 90°, the form of the distribution changed gradually to the form indicated for $\phi = 100°$. Above 105°, the linear rise in the distribution obscured the cutoff, preventing extension of the measurements to higher angles.

$\phi = 105°$. Counting was continued until from 1000 to 3000 true counts at the top of the cutoff had been recorded. Determination of the cutoff at each angle and helium temperature required from 6 to 12 hours of counting time.

Typical scattered neutron distributions are shown in Fig. 5. At scattering angles greater than 90°, the shape of the scattered neutron distribution changed. A linear rise in the distribution was noted, which became progressively more prominent as the scattering angle increased. The $\phi = 100°$ distribution in Fig. 5 is typical of this effect, and illustrates the manner in which the data were analyzed in these cases. The largest angle at which a reliable measurement of the cutoff could be made was 105°. At larger angles, the linear rise in the distribution above the cutoff made location of the peak height very difficult. A survey of the neutrons scattered at 125° by liquid helium showed a distribution which rose above background at about 4.55 A, and continued to rise with approximately constant slope at least as far as 5.3 A, which was the largest wavelength which could be reached by the spectrometer at that angular setting. No indication of a sharp cutoff was seen.

Figure 6 shows the distributions observed at a scattering angle of 72.5° for three different helium temperatures. The spectrometer was left in a fixed position during these measurements. It may be seen that, as the helium temperature is increased, the cutoff becomes wider, and shifts to shorter wavelengths. The width observed at $T = 1.1°K$ is that which is expected from the spectrometer resolution width $\Delta\theta = 0.6°$. The additional width at the higher temperatures is attributed

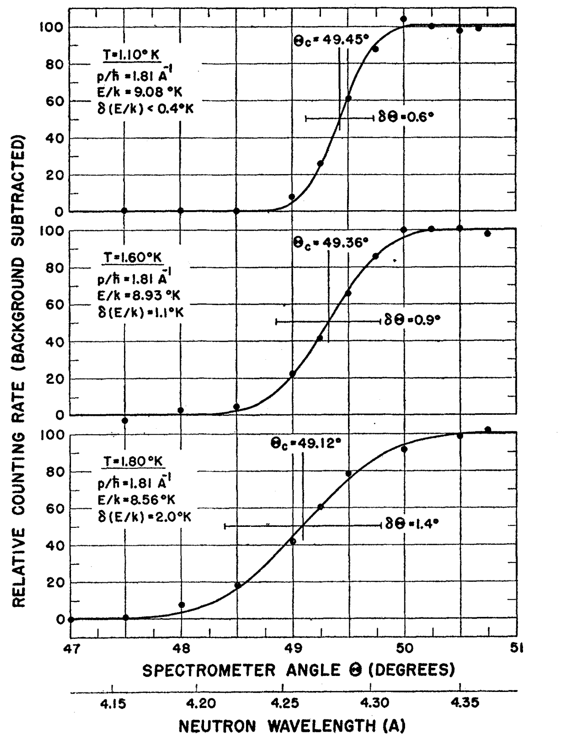

FIG. 6. Neutron distributions for 72.5° scattering as a function of helium temperature. The solid curves are calculated according to Eq. (6), using the indicated widths $\delta\theta$ and positions θ_c. The calculated values of excitation energy E and momentum p are shown at each temperature, together with the energy spread $\delta(E/k)$ inferred from the width of the cutoff. The spectrometer was left in a fixed position for these three runs, to improve the relative accuracy of the data.

to a natural energy spread of the excitations, which is given by the square root of the difference in the squares of the observed width $\delta\theta$ and the resolution width $\Delta\theta$.

A summary of the data is given in Table I. The scattering angle ϕ is considered to be known to $\pm 0.10°$.

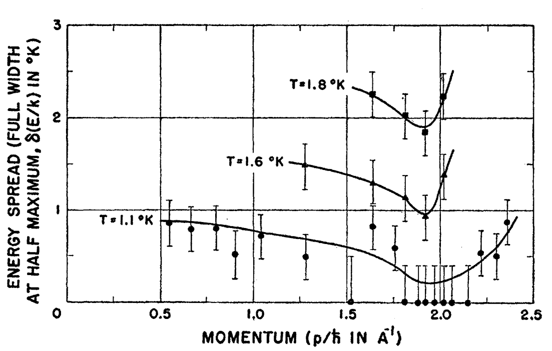

FIG. 7. Energy spread of the excitation spectrum of liquid helium at various momenta and temperatures, as inferred from the observed width of the cutoff in the scattered neutron distribution. Points plotted at zero energy spread represent measurements in which only an upper limit for the width, indicated by the height of the vertical bar, was obtained. The solid lines are included to indicate the trend of the data, and have no theoretical significance.

The spread in scattering angles accepted at a given setting was 2.1° full width at half maximum.

Due to the finite spread of scattering angles, each measurement represents the average energy for excitations having a certain range of momenta. The uncertainty stated for each value of the momentum indicates this range. The uncertainty in the position of the midpoint of the range is smaller by an order of magnitude. The stated error in the temperature measurements for each run indicates the range of the fluctuations in temperature during that run. The measured resolution of the spectrometer corresponds to a spread in wavelength of 0.04 A at 4 A, and 0.03 A at 4.5 A. We estimate that the error in locating the center of a cutoff is less than $\frac{1}{10}$ of the cutoff width. The stated errors for the cutoff wavelength represent the uncertainty in the difference between the indicated value and the cutoff for the incident neutron distribution, which is taken to be precisely 3.952 A. Since the cutoffs were measured

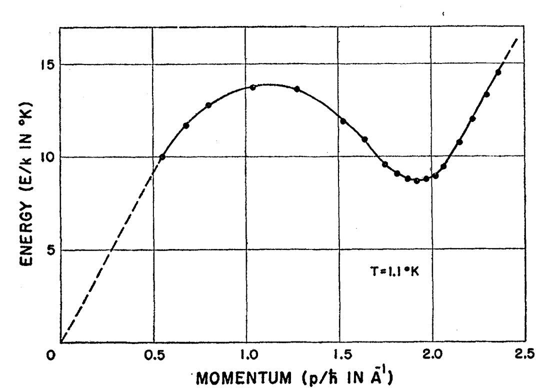

FIG. 8. The energy spectrum of the excitations in liquid helium at 1.1°K. The dashed line joining the origin and the first measured point has a slope corresponding to a first sound velocity of 239±5 meters/sec. The maximum occurs at $p/\hbar = 1.11 \pm 0.04$ A^{-1}, $E/k = 13.92 \pm 0.10$°K. The region of the minimum is shown in greater detail in Fig. 9.

relative to the cutoff in the incident neutron distribution in most cases, the particular value assigned for the latter quantity has a relatively small effect on the calculated excitation energy.

Determination of line shapes by the cutoff technique is at best difficult; we indicate the energy spread estimated from the width of the cutoff, but are forced to assign rather large uncertainties. No asymmetry in the shape of the cutoff was noted, indicating that the line shape was still approximately symmetrical at temperature up to $T = 1.8$°K. The only direct measurements of line shape have been made by Henshaw,[10] who finds a rather symmetric line at $T = 1.57$°K, and considerable asymmetry at $T = 2.08$°K. No data were obtained for intermediate temperatures, so our finding of symmetry up to $T = 1.8$°K cannot be directly confirmed. However, there is no apparent conflict between the two sets of measurements. The observed energy

TABLE I. Summary of experimental results. Energy spread is the full width at half-maximum inferred from the observed broadening of the cutoff in the scattered neutron distribution.

Scattering angle ϕ	Helium temperature T in °K	Cutoff wavelength λ_c in A	Momentum $p/\hbar$ in A^{-1}	Energy E/k in °K	Energy spread $\delta(E/k)$ in °K
20.0°	1.14±0.03	4.324±0.008	0.55±0.03	10.00±0.20	0.86±0.25
25.0°	1.14±0.03	4.397±0.006	0.67±0.02	11.68±0.15	0.79±0.25
30.0°	1.17±0.05	4.447±0.006	0.80±0.02	12.78±0.15	0.51±0.25
40.0°	1.13±0.01	4.493±0.006	1.04±0.02	13.75±0.15	0.72±0.25
50.0°[a]	1.15±0.03	4.487±0.006	1.28±0.02	13.64±0.15	0.49±0.25
50.0°	1.60±0.01	4.463±0.008	1.28±0.02	13.14±0.20	1.47±0.25
60.0°	1.14±0.01	4.406±0.006	1.52±0.02	11.88±0.15	<0.5
65.0°	1.08±0.01	4.362±0.004	1.64±0.02	10.89±0.10	0.82±0.25
65.0°	1.60±0.01	4.345±0.006	1.64±0.02	10.52±0.15	1.28±0.25
65.0°	1.80±0.01	4.340±0.008	1.64±0.02	10.38±0.20	2.25±0.25
70.0°[a]	1.14±0.02	4.305±0.004	1.76±0.02	9.56±0.10	0.59±0.25
72.5°	1.10±0.02	4.285±0.004	1.81±0.02	9.08±0.10	<0.4
72.5°	1.60±0.01	4.277±0.004	1.81±0.02	8.93±0.10	1.13±0.25
72.5°	1.80±0.01	4.263±0.004	1.81±0.02	8.56±0.10	2.02±0.25
75.0°[a]	1.13±0.01	4.273±0.004	1.87±0.02	8.80±0.10	<0.4
77.5°	1.08±0.01	4.266±0.004	1.92±0.02	8.64±0.10	<0.4
77.5°	1.60±0.01	4.256±0.004	1.92±0.02	8.40±0.10	0.92±0.25
77.5°	1.81±0.01	4.250±0.004	1.92±0.02	8.23±0.10	1.84±0.25
80.0°	1.13±0.01	4.270±0.004	1.97±0.02	8.73±0.10	<0.4
82.5°	1.08±0.01	4.280±0.004	2.02±0.02	8.97±0.10	<0.4
82.5°	1.60±0.01	4.273±0.004	2.02±0.02	8.79±0.10	1.37±0.25
82.5°	1.80±0.01	4.261±0.004	2.02±0.02	8.50±0.10	2.23±0.25
85.0°[a]	1.08±0.01	4.301±0.004	2.06±0.02	9.46±0.10	<0.4
90.0°	1.08±0.01	4.356±0.004	2.15±0.02	10.76±0.10	<0.4
95.0°[a]	1.20±0.03	4.413±0.006	2.22±0.02	12.03±0.15	0.53±0.25
100.0°[a]	1.22±0.01	4.472±0.006	2.30±0.02	13.33±0.15	0.50±0.25
105.0°	1.16±0.01	4.529±0.008	2.36±0.02	14.52±0.20	0.88±0.25

[a] Spectrometer calibration by arm zero method. In the other cases, the cutoff in the distribution of neutrons scattered from vanadium was used as the basic calibration.

spreads at various momenta and temperatures are shown in Fig. 7.

V. DISCUSSION

The measured excitation spectrum at $T=1.1$°K is shown in Fig. 8. The resemblance to Landau's proposed spectrum (Fig. 1) is striking. If we assume that the experimental point at 0.55 A^{-1} lies in the phonon region where $E=v_1 p$, then this point corresponds to a velocity of first sound $v_1=239\pm5$ meters/sec. Van Itterbeek, Forrez, and Teirlinck[16] give for the velocity of first sound at $T=1.1$°K the value 237.5±0.6 meters/sec.

In the neighborhood of the maximum, the experimental spectrum may be represented by the expression $E/k=13.92-11.5(p/\hbar-1.11)^2$ °K, where $p/\hbar$ is in A^{-1}.

In the region of the minimum, the experimental points may be represented by Landau's roton expression, $E=\Delta+(p-p_0)^2/2\mu$. A fit to the seven points nearest the minimum by the method of least squares gave the following values for the parameters:

$$\Delta/k=8.65\pm0.04\text{°K},$$
$$p_0/\hbar=1.92\pm0.01\ A^{-1},$$
$$\mu=(0.16\pm0.01)m_{\text{He}}.$$

The rms deviation of the seven experimental points from the fitted curve is ±0.04°K. A plot of the region of the minimum, showing the experimental points and the fitted curve, is given in Fig. 9.

[16] Van Itterbeek, Forrez, and Teirlinck, Physica **23**, 63, 905 (1957); A. Van Itterbeek and G. Forrez, Physica **20**, 133 (1954).

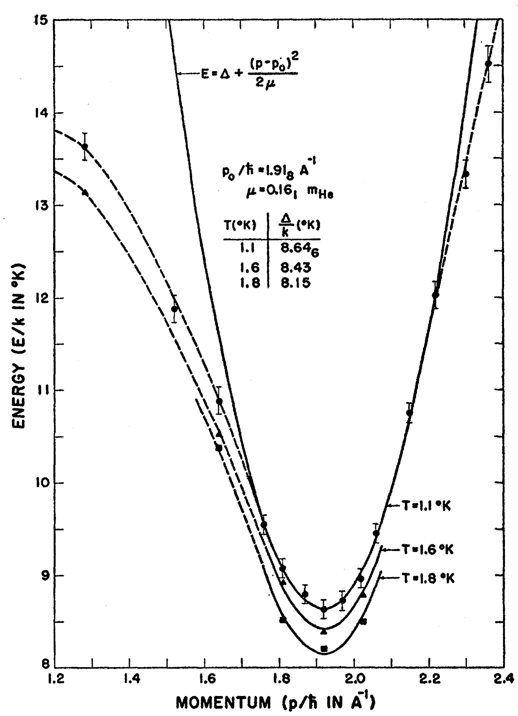

FIG. 9. Expanded plot of the region of the minimum, showing the temperature variation of the excitation spectrum. Errors are shown only on the 1.1° points, to avoid confusion. Relative errors between points at the same momentum are estimated to be approximately one half of the indicated absolute error. The solid curves were fitted to the data by the method of least squares, as described in Sec. V.

The slope of the Landau expression reaches the velocity of first sound at momentum p^*, where $p^*/\hbar = 2.16$ A^{-1}. For $p > p^*$, the excitation spectrum continues to rise with slope equal to the sound velocity, rather than rising quadratically. This behavior might be expected, since for momenta greater than p^*, an excitation of momentum p^* plus a phonon of momentum $p - p^*$ have a smaller total energy than an excitation of momentum p whose energy is given by a continuation of the roton curve. The change in the appearance of the wavelength distribution of the scattered neutrons, mentioned in Sec. IV and shown in Fig. 5, occurs for momenta greater than p^*. We feel that for an unambiguous analysis of the processes taking place at higher momenta, an experiment using monoenergetic incident neutrons should be performed.

At higher temperatures, the excitation energy for a given momentum decreased, and a noticeable energy spread was observed. This indicates that the density of excitations is great enough at these temperatures that the effect of interactions between the excitations can no longer be neglected. In the roton region, the data may be represented by the Landau expression fitted at $T = 1.1$°K, shifted downward by 0.22°K at $T = 1.60$°K, and by 0.50°K at $T = 1.80$°K.

The variation of the fitted values of Δ with T is given approximately by the empirical relation

$$\Delta/k = 8.68 - 0.0084T^7 \text{ °K}, \tag{7}$$

where T is in °K. This formula is suggested as a guide for interpolation between the measured values of the excitation energy at different temperatures; no particular theoretical significance is implied.

In the following paper,[12] a comparison is made between experimental determinations of the specific heat, entropy, second sound velocity, and normal fluid density of liquid helium, and values of these quantities calculated from the measured excitation spectrum.

VI. ACKNOWLEDGMENTS

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Excitations in Liquid Helium: Thermodynamic Calculations*

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The entropy, specific heat, normal fluid density, and velocity of second sound in liquid helium II have been calculated by applying statistical mechanics to the thermal excitations. The calculations were based on the energy-momentum relation obtained by neutron scattering measurements described by Yarnell, Arnold, Bendt, and Kerr, and were made on an IBM-704 electronic digital computer by numerical integrations over the observed excitation curve. A better approximation than Landau's has been obtained by extending Landau's theory to take account of the temperature dependence of the excitation curve. An expression of the form $E(p,T) = c - d(\rho_n/\rho)$ was used to interpolate the excitation energy between temperatures at which it was measured. Results between 0.2 and 1.8°K are not sensitive to the exact form of the interpolation expression. Agreement of the calculations with experimental measurements is as follows: entropy, ±3% in the temperature range 0.2 to 1.8°K; specific heat, ±4% between 0.2 and 1.7°K; second sound velocity, ±4% between 0.8 and 1.8°K, and ±2% between 1.0 and 1.7°K. The calculated normal fluid density ρ_n agrees with experimental values derived from second sound velocity and specific heat measurements within ±8% between 0.7 and 2.0°K, and within ±5% from 1.1 to 1.9°K. These values are, however, higher than those obtained from torsion pendulum measurements, which are 27% below the calculated value at 1.2°K. Also calculated as functions of temperature are the average effective mass (as defined by Landau) of excitations in four momentum intervals, and values of $-\kappa/\bar{B}$, the thermal conductivity κ divided by the average over momentum of the Khalatnikov nonequilibrium kinetic coefficient $-B$, and $\eta/\bar{C}$, the viscosity η divided by the average value of the Khalatnikov coefficient C.

I. INTRODUCTION

LANDAU[1] proposed that the thermodynamic properties of liquid helium II could be calculated by applying statistical mechanics to the ensemble of excitations, which may be treated as a gas. In Landau's treatment, interactions between the excitations are neglected. Experimentally, the effect of interactions is shown by a dependence of the excitation energy on the temperature. It is possible to extend Landau's theory to take account of the temperature dependence of the excitations, and this is done in Sec. II below. The

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[1] L. Landau, J. Phys. (U.S.S.R.) 5, 71 (1941); 11, 91 (1947).