Consequences of Renormalizable Weak Interactions

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Consequences of the theory of renormalizable weak interactions caused by intermediary chiral bosons are further investigated with regard to the decay processes of the pion, K meson, and hyperons. It is shown that all those aspects of weak couplings which can be satisfactorily explained in terms of the local Fermi interaction with $V-A$ combination can be equally well reproduced by the present model. Furthermore, on account of the convergence of the present theory, the results of computations are less ambiguous than those obtained from the local Fermi interaction model. Finally some possible experimental tests of the theory, mainly by the use of the nonlocalizability of the lepton interactions inherent in this theory, are discussed.

I. INTRODUCTION

It is well known that the renormalization theory has been proven to be quite successful insofar as the electromagnetic and possibly the π-mesonic interactions are concerned. However, one of the striking features of the weak couplings, besides the fact that they are not invariant under space reflection and charge conjugation,[1] is that all the phenomenological forms of weak interactions suggested by recent experiments belong to unrenormalizable types. The situation may best be exhibited by the fact that the present experimental results seem to find their simplest and most unified phenomenological description in the so-called universal Fermi interactions of $V-A$ combination[2] which are known to be unrenormalizable in the current field theory. In fact, the Yukawa-type interactions like the $\pi\rightarrow\mu+\nu$ and $K\rightarrow\mu+\nu$ decay, for which renormalizable forms can be presumed, do not seem to play the role of a primary interaction. This is because the former would not explain the process[3] $\mu^-+p\rightarrow n+\nu$ and the latter could not be made responsible for the $K\rightarrow\mu+\nu+\pi$ decay.[4] Moreover, if we assume that the parity violation in the hyperon decays occurs through its extremum form, $1\pm\gamma_5$, which preserves the time-reflection invariance, the unrenormalizable derivative-type Yukawa interaction is favored by experiments as the effective Λ^0-decay coupling, since it predicts the angular distribution of the pion produced in the polarized Λ^0-decay consistent with recent experiments.[5] Under these significant circumstances, two distinct points of view may be conceivable. One is to accept this unrenormalizability as a fundamental feature[6] of the weak interactions; the other is, assuming that all basic primary interactions should have renormalizable forms, to speculate that all phenomenological weak reactions are caused by some unknown intermediary bosons.[7] If one takes the latter standpoint, one is naturally led to conjecture a possible relation between the parity nonconservation of weak interactions and the properties of the bosons which exist only as the medium underlying the weak reactions. Such attempts have recently been made by one of us (Y.T.) and Watanabe.[8] (Hereafter this paper will be referred to as I.)

So far, the consideration of renormalizable weak interactions might have been regarded as having only an academic interest since the lowest order perturbation would yield sufficiently correct predictions for weak interactions. However, as our knowledge about the nature of the weak interactions becomes more and more accurate with experimental progress, the choice between the renormalizable theory and the unrenormalizable theory cannot remain a purely academic problem. Thus we feel that it is a worthwhile task to investigate the consequences of the renormalizable theory in some detail. In this paper the consistency of the model proposed in I is further examined with regard to the higher order processes like the pion, K meson, and hyperon decays. It is shown that the theory reproduces almost all the qualitative successes of the direct Fermi interaction with $V-A$ combination. Further, the quantitative results are obviously more unambiguous in the renormalizable theory since the interactions in this theory are less singular than in the theory based on the primary Fermi interactions. In particular, in this theory all boson decays involving leptons are, in the lowest order, free from divergence and we need not have recourse to a complicated renormalization procedure at

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[1] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).

[2] E. C. G. Sudarshan and R. E. Marshak, *Proceedings of the Padua-Venice Conference on Mesons and Newly Discovered Particles, September, 1957* (to be published) and Phys. Rev. **109**, 1860 (1958); R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958); J. J. Sakurai, Nuovo cimento **7**, 649 (1958).

[3] Iwata, Ogawa, Okonogi, Sakita, and Oneda, Progr. Theoret. Phys. **13**, 19 (1955); S. Ogawa, Progr. Theoret. Phys. **13**, 367 (1955); J. L. Lopes, Phys. Rev. **109**, 509 (1958).

[4] d'Espagnat, Omnes, and Prentki, Nuclear Phys. **3**, 471 (1957); S. Oneda and S. Kamefuchi, Nuclear Phys. **4**, 301 (1957).

[5] F. S. Crawford *et al.*, Phys. Rev. **108**, 1102 (1957); F. Eisler *et al.*, Phys. Rev. **108**, 1353 (1957).

[6] For instance, see Umezawa, Konuma, and Nakagawa (to be published).

[7] H. Umezawa, Progr. Theoret. Phys. **7**, 551 (1952); S. Tanaka and M. Ito, Progr. Theoret. Phys. **9**, 169 (1953); Y. Tanikawa, *Proceedings of the International Conference of Theoretical Physics, Kyoto and Tokyo, 1953* (Science Council of Japan, Tokyo, 1954), p. 369 and Progr. Theoret. Phys. **10**, 232, 316 (1953); Y. Tanikawa, Phys. Rev. **108**, 1615 (1957).

[8] Y. Tanikawa and S. Watanabe, preceding paper [Phys. Rev. **113**, 1344 (1959)]. See also J. Schwinger, Ann. Phys. **2**, 407 (1957).

all.[9] It is quite possible that the experimental data in the near future will require definitive, quantitative theoretical predictions for comparison. *The present theory necessarily exhibits nonlocalizability with respect to the lepton interactions, which may prove to be instrumental in testing the reality of the picture of weak interactions implied by the present standpoint.* Some of the circumstances in which such nonlocalizability may become observable are examined in the following (especially in Sec. VII).

II. THE BASIC INTERACTIONS

First we briefly summarize the basic interactions introduced in I to explain the weak reactions between nucleon, pion, and lepton systems. For details of the deduction, the reader is referred to I. We introduce neutral bosons in a chirality eigenstate and require that the basic interactions are invariant for the chirality operation.[10] The conservation of leptonic as well as baryonic number is also assumed. The interaction responsible for the β decay, $n \to p+e^- + \nu^c$, is

$$H_s = g\{\bar{p}(1-\gamma_5)e_-^c + \bar{n}(1-\gamma_5)\nu^c\}B + \text{H.c.} \quad (1)$$

The lepton number is defined by assigning $+1$ to the electron (e_-) and neutrino (ν). e_-^c, for instance, denotes the charge conjugate field of e_-, $e_-^c = C\bar{e}_-$, and describes an emission (annihilation) operator of an electron (positron).[11] Then the B meson is a chiral (spin zero) neutral boson with a baryonic number $+1$, leptonic number $+1$ and chirality $+1$. The antiparticle B^* (complex conjugate of B field) has opposite signs for these quantities. By emission and reabsorption of B particles, the interaction (1) leads approximately to the effective β coupling with $V-A$ combination:

$$\frac{g^2}{2(m_B^2 - m_n^2)} \bar{p}\gamma_\mu(1+\gamma_5)n\bar{e}_-\gamma_\mu(1+\gamma_5)\nu. \quad (2)$$

From the free β-decay rate, the coupling constants, F_V and F_A, of the direct Fermi interaction with $V-A$ combination are evaluated (in the units $\hbar = c = 1$) as

$$|F_V| = |F_A| = 1.01\times 10^{-5}(m_n)^{-2} = 1.8\times 10^{-7}(m_\pi)^{-2}. \quad (3)$$

Thus the coupling constant g and the mass of the B meson, m_B, are approximately related to the Fermi coupling constants by

$$\frac{|g|^2}{2(m_B^2 - m_n^2)} = |F_V| = 1.01\times 10^{-5}(m_n)^{-2}. \quad (4)$$

For the tentative value $m_B = 2300 m_e$, g is given by

$$|g|^2/4\pi = 7.4\times 10^{-7}. \quad (5)$$

[9] See, for instance, S. Weinberg, Phys. Rev. **106**, 1301 (1957).

[10] S. Watanabe, Nuovo cimento **6**, 187 (1957); S. Watanabe, Phys. Rev. **106**, 1306 (1957); Y. Tanikawa and S. Watanabe, Phys. Rev. **110**, 289 (1958); H. Umezawa and A. Visconti, Nuclear Phys. **4**, 224 (1957).

[11] $\bar{e}_- = (e_-)^\dagger\gamma_4$ and $C^{-1}\gamma_\mu C = -\gamma_\mu^T$, $C^T = -C$ and $C^\dagger = C^{-1}$.

For the μ^- capture by nuclei, we introduce the following interaction mediated by the B'' meson:

$$g''\{\bar{p}(1-\gamma_5)\mu_+ + \bar{n}(1-\gamma_5)\nu\}B'' + \text{H.c.} \quad (6)$$

Here we take the μ^+ as a particle (lepton number $+1$) and so this neutral (spin zero chiral boson B'' with chirality $+1$ is characterized by a baryon number $+1$ and a lepton number -1. It should be noted that this formalism requires a four-component theory for the neutrino field. To make a distinction between the ν spinor before which the factor $(1+\gamma_5)$ stands as in (2) and the ν spinor before which the factor $(1-\gamma_5)$ stands as in (6), the latter has been denoted by ω^c in I. In the present paper, we shall use a single symbol ν, assuming this to have four components. These two alternative ways of description are effectively equivalent. The interaction (6) describes the μ^- capture process as $\mu^- + p \to n + \nu_L^c$, where ν_L^c describes a left-handed antineutrino. The reason why we should not replace e_- just by μ_- in the interaction (1) is that the interactions of B with e^- as well as μ^- leads to an unwanted reaction $\mu^- + p \to p + e^-$. The coupling constants g'' of (6) and $m_{B''}$ are related to the coupling constants, $E_V = E_A$, of direct Fermi interactions with $V-A$ combination for the $\mu^- + p \to n + \nu$ process by the following approximate relation:

$$\frac{|g''|^2}{2\{m_{B''}^2 - (m_n - m_\mu)^2\}} = |E_V| = |E_A|. \quad (7)$$

The precise value of E_V is not yet known. It is, however, tempting to assume a symmetry between B and B'', $m_B \approx m_{B''}$ and the universality of coupling constants, $|g| \approx |g''|$, more or less in analogy, in spirit, to the assumption of the universality of Fermi interaction constants ($F_V = F_A = E_V = E_A$). At any rate, the interactions (1) and (6) have quite a similar character and the distinction between B and B'' may be only in the leptonic numbers.

III. THE PION DECAYS

According to the present standpoint, the $\pi^+ \to \mu^+ + \nu_L^c$ decay will occur through the interaction (6). The general form of the Feynman diagram for this process is given by Fig. 1. The black box represents an effective pion-nucleon interaction. The μ^+ and ν^c are emitted from different vertices (nonlocal) which are mediated by B'' field. In contrast to this, if we take the usual direct Fermi interaction, $\mu^- + p \to n + \nu$, the outgoing μ and ν are directly emitted from the same vertex (see Fig. 2) unless we include electromagnetic corrections. The matrix elements according to the direct Fermi interaction exhibit the well-known divergence.[12] It should be noted that in the present model, the propagator of the B'' meson effectively serves the purpose of a relativistic cutoff, and all the weak decay processes

[12] For the $V-A$ Fermi interaction, the divergence is logarithmic.

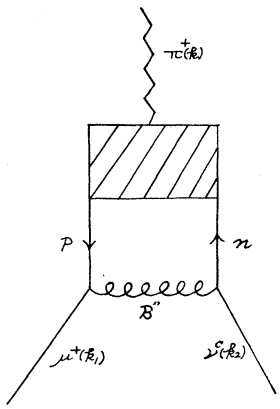

FIG. 1. Feynman diagram for the interaction (6).

of bosons involving leptons, like $\pi\to\mu+\nu$, $K\to\mu+\nu$, and $K\to e(\mu)+\nu+\pi$, etc., become always convergent. The matrix element of Fig. 1 takes the following form of an effective derivative-type Yukawa interaction:

$$\mathfrak{M}=f\bar{\mu}_+(k_1)(1+\gamma_5)(k\cdot\gamma)\nu(k_2)\varphi_\pi(k), \tag{8}$$

where

$$f=\left[\frac{G}{(4\pi)^{\frac{1}{2}}}\right]\left(\frac{|g''|^2}{4\pi}\right)\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\left[\frac{m_n}{m_{B''}{}^2-m_n{}^2}\times\left\{\frac{2m_{B''}{}^2}{m_{B''}{}^2-m_n{}^2}\ln\left(\frac{m_{B''}}{m_n}\right)-1\right\}+\mathcal{O}\right], \tag{9}$$

k_1 denotes the energy momentum four-vector of the μ meson so that $k_1{}^2=-m_\mu{}^2$ and $(k\cdot\gamma)$ means a scalar product constructed from k_α and γ_α, $\alpha=1, 2, 3, 4$. $\mathcal{O}$ in (9) represents the quantities which are smaller than the leading term of (9) by a factor of the order of $(-k_1{}^2)/M^2=m_\mu{}^2/M^2$ or of $-(k_1k_2)/M^2=(m_\pi{}^2-m_\mu{}^2)/2M^2$, where M is a parameter which satisfies $m_n{}^2<M^2<m_{B''}{}^2$. In deriving (8) and (9) we have written the contributions of black box of Fig. 1 as $G\gamma_5$ and regarded G as the coupling constant of the effective pion-nucleon interaction. It can be easily proved from general arguments that, even without taking the above procedure, the form of the matrix element always reduces to the one given by (8) using the equation of motion of the μ meson and neutrino. Strictly speaking, G, in this case, would depend on the mass of the μ meson. This, in fact, comes from the nonlocalizability of our lepton interactions and in the direct Fermi interaction model the black box of Fig. 2, which corresponds to f of (8), does not depend on the lepton mass. In this paper, however, we shall treat G as a mere constant. Putting G equal to the renormalized coupling constant of the pion-nucleon interaction[13] $(G^2/4\pi\approx 15)$ and using the value of $g''(\approx g)$ given by (5) (taking $m_{B''}\approx m_B\approx 2300m_e$), we get a mean life $\tau(\pi^+\to\mu^++\nu)\approx 2.7\times10^{-8}$ sec which is very close to the experimental value of the pion mean life 2.5×10^{-8} sec.[14] We have neglected the term $\mathcal{O}$ in this computation. Replacing the μ_+, ν^c, g'', and $m_{B''}$ by e_-, ν, g, and m_B, respectively, the matrix element for the process $\pi^+\to e^++\nu_L$ is obtained. The form of the matrix element $\mathfrak{M}$ given by (8) is the same as that given by the direct $V-A$ Fermi interaction except for the numerical factor f which is now convergent. The leading term of f is the same for both $\pi\to\mu$ and $\pi\to e$ decays if we assume $|g|=|g''|$ and $m_B=m_{B''}$, whereas the terms $\mathcal{O}$ now depend on the masses of leptons. This is also due to the nonlocal structure of our lepton interactions. However, this effect (under the approximations made here such as the neglect of the possible dependence of G on lepton masses and the neglect of the electromagnetic corrections) turns out to be too small to give an appreciable change in the well-known theoretical ratio of $\pi\to e$ to $\pi\to\mu$ decay, $R\sim1.36\times10^{-4}$, given by the universal $V-A$ Fermi interaction or the universal derivative-type Yukawa interaction.[2,3] That is, the inclusion of the terms $\mathcal{O}$ now gives the ratio[15]

$$R=W(\pi\to e)/W(\pi\to\mu)=1.36\times10^{-4}\times(1-a), \tag{10}$$

where $a=2\times10^{-3}$.

Next we discuss the radiative process

$$\pi^+(p_\pi)\to e^+(k_1)+\nu(k_2)+\gamma(k).$$

There are essentially three different diagrams in which the initial pion, the intermediate charged particles (in the lowest order, the proton), and the outgoing electron emit a photon with energy-momentum four-vector k. The inclusion of all contributions from these diagrams is essential for deriving the following result. Neglecting the terms of order $m_\pi{}^2/M^2$ $(m_n{}^2<M^2<m_B{}^2)$ we get, in

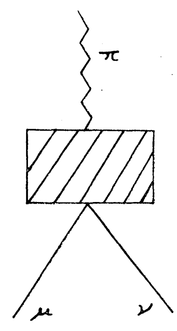

FIG. 2. Feynman diagram for the usual direct Fermi interaction,
$\mu^-+p\to n+\nu$.

[13] This is not strictly correct but should be permissible for the present purpose.

[14] It is interesting to note that for the $V-A$ Fermi interaction we get nearly the same result by introducing a Feynman convergence factor $K^2/(q^2+K^2)$ (q is an internal momentum) with the cutoff chosen also to be of the order $K\approx m_n$ [S. B. Treiman and H. W. Wyld, Phys. Rev. **101**, 1552 (1956)]. Using dispersion theory, it has been shown that the direct $V-A$ Fermi interaction, $\mu^-+p\to n+\nu$, together with the pion nucleon interaction could account for the probability of the pion decay [M. L. Goldberger and S. B. Treiman (to be published)]. In this connection it may be interesting to apply a similar technique to the present model.

[15] It would be interesting to examine whether the corrections which are neglected change the ratio R appreciably, but this is beyond the scope of the present paper.

lowest order perturbation theory,

$$\mathfrak{M}=ieG\frac{|g|^2}{M^2}\frac{\sqrt{2}}{24(2\pi)^2}\frac{m_n}{M^2}\bar{e}_+(k_1)(1+\gamma_5)\{3(k\cdot\gamma)(k_2\cdot\epsilon)-3(\epsilon\cdot\gamma)(k_2\cdot k)+2(p_\pi\cdot k)(\epsilon\cdot\gamma)\}\nu^c(k_2)\varphi_\pi, \quad (11)$$

where ϵ_α denotes the polarization four-vector of the photon and $m_n^2<M^2<m_B^2$. We have neglected the electron mass. For the sake of comparison we give the corresponding results obtained from the direct axial vector Fermi β-decay interaction,[14]

$$\mathfrak{M}=ieGF_A\frac{\sqrt{2}}{6(2\pi)^2}m_n\frac{(p_\pi k)}{m_n^2}\bar{e}(k_1)\gamma_5(\epsilon\cdot\gamma)\nu(k_2)\varphi_\pi. \quad (12)$$

Note that new factors of the form such as $(k\cdot\gamma)(k_2\cdot\epsilon)-(\epsilon\cdot\gamma)(k_2\cdot k)$ could appear in (11) which were absent in (12). This also comes from the nonlocalizability of our weak interactions. Namely, in deriving (11), three vectors, say, p_π, k, and k_2, could be treated as independent in constructing the matrix elements; whereas in (12), only two independent vectors, say, p_π and k, need be considered as long as we take a first order perturbation with regard to the weak vertex. Such a nonlocal effect may be appreciable, in particular, for the $\pi\rightarrow\mu+\nu+\gamma$ decay, which may become instrumental as a possible test of the present theory. The ratio of $\pi\rightarrow e+\nu+\gamma$ to $\pi\rightarrow\mu+\nu$ decay computed from (9) and (11) would be of the order 10^{-7}–10^{-8} which may be consistent with the present experiments.[16] Thus it may be said that the present model is not inconsistent with the pion decay phenomena.

IV. THE K-MESON DECAYS INTO LEPTONS

(1) Inclusion of Strange Particles

In order to include the decays of strange particles in the present theoretical formalism, we have to extend the interactions (1) and (6). The fact that the intermediate chiral bosons B and B'' are neutral makes it rather hard to forbid many unwanted processes. At the moment, we should be satisfied by a minimum extension of the theory which is still sufficient to derive all the known strange particle decays consistent with the change of strangeness by one, $|\Delta S|=1$. Thus we add to (1),

$$g_1\bar{\Lambda}_0(1-\gamma_5)\nu^cB+\text{H.c.}, \quad (1')$$

and to (6)

$$g_2\bar{\Lambda}_0(1-\gamma_5)\nu B''+\text{H.c.} \quad (6')$$

It may also be harmless to replace the Λ^0 by Σ^0 or by a linear combination of the Λ^0 and Σ^0.[17] A direct conse-

[16] Cassels, Rigby, Wetherell, and Wormald, Proc. Phys. Soc. (London) **A70**, 729 (1957).

[17] The inclusion of such terms as $\{\bar{\Sigma}_+(1-\gamma_5)e_-^c+\bar{\Sigma}_-(1-\gamma_5)\mu_-\}B$ +H.c. and $\{\bar{\Sigma}_-(1-\gamma_5)e_-+\bar{\Sigma}_+(1-\gamma_5)\mu_+\}B''$+H.c. may give rise to non-neutrino processes such as $K^+\rightarrow\pi^++\mu^+(e^+)+\mu^-(e^-)$, $K\rightarrow\pi^++\mu^+(e^+)+e^-(\mu^-)$, and $\Sigma^+\rightarrow p+e^+(\mu^+)+e^-(\mu^-)$, etc., unless we devise some special combinations of interactions which would give destructive interferences for these unwanted decay processes.

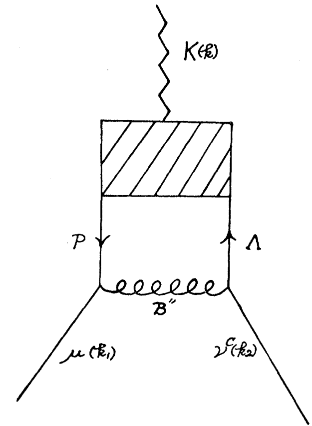

FIG. 3. Feynman diagram for the decay $K^+\rightarrow\mu^+(k_1)+\nu_L{}^c(k_2)$.

quence of the above restriction by which only the Λ^0 or possibly Σ^0 is responsible for the basic weak interactions (1′) and (6′) is that there will appear a great asymmetry in some of the decay processes. For instance, the decay modes $\Sigma^+\rightarrow n+e^+(\mu^+)+\nu(\nu^c)$ and $K^+\rightarrow\pi^++\pi^++e^-(\mu^-)+\nu^c(\nu)$ will not occur, whereas the decay modes $\Sigma^-\rightarrow n+e^-(\mu^-)+\nu^c(\nu)$ and $K^+\rightarrow\pi^++\pi^-+e^+(\mu^+)+\nu(\nu^c)$ will.[18] The existence of both interactions (1′) and (6′) is necessary in order to yield the decay modes $K\rightarrow\mu+\nu+\pi$ and $K\rightarrow e+\nu+\pi$, simultaneously.

(2) The $K\rightarrow\mu+\nu$ Decay

Analogously to the $\pi\rightarrow\mu+\nu$ decay, the decay $K^+\rightarrow\mu^+(k_1)+\nu^c{}_L(k_2)$ occurs as a $g''g_2$ process (Fig. 3). Expanding the matrix element in terms of $k_1^2/M^2=-m_\mu^2/M^2$ and $(k_1k_2)/M^2=-(m_K^2-m_\mu^2)/2M^2$ $(m_n^2<M^2<m_{B''}^2)$ (this is not so good an approximation as in the case of π decay but would probably be sufficient for the arguments of the order of magnitude), we get a convergent result

$$\mathfrak{M}\approx f'\bar{\mu}_+(k_1)(1+\gamma_5)(k\cdot\gamma)\nu(k_2)\varphi_K(k),$$

where

$$f'=\left[\frac{G_k}{(4\pi)^{\frac{1}{2}}}\right]\left(\frac{g_2g''}{4\pi}\right)\left(\frac{1}{\pi}\right)^{\frac{1}{2}}\times\left(\frac{m_n}{M^2}\right)\frac{1}{6}\left\{2\left(\frac{m_\Lambda}{m_n}\right)+1\right\}. \quad (13)$$

For the K-Λ-N interaction we have assumed

$$G_K(\bar{\Lambda}_0\gamma_5nK_0{}^*+\bar{\Lambda}_0\gamma_5pK_+{}^*)+\text{H.c.} \quad (14)$$

For the K-Λ-N interaction of the scalar type,

$$G_K(\bar{\Lambda}_0nK_0{}^*+\bar{\Lambda}_0pK_+{}^*)+\text{H.c.} \quad (15)$$

we should replace m_Λ in (13) by $-m_\Lambda$.

Again the form of (13) is a derivative-type Yukawa

[18] S. Oneda and S. Kamefuchi, *Proceedings of the Padua-Venice Conference on Mesons and Newly Discovered Particles, September, 1957* (to be published), Appendix.

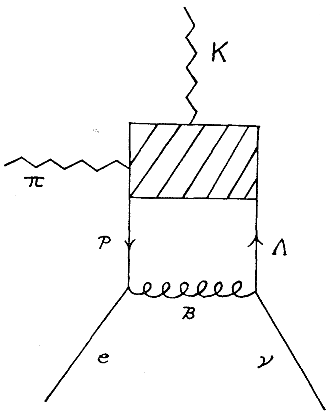

FIG. 4. Feynman diagram for the decay $K^+(p_K)\to e^+(k_1)+\nu(k_2)+\pi(k)$.

interaction, and thus the $K\to e+\nu$ decay, which could occur as a g_1g process, would be about 10^{-5} times less probable[3] than the $K\to\mu+\nu$ decay provided that $|g_1|\approx|g_2|$ and $|g|\approx|g''|$.

If we assume that $|g|=|g_1|=|g_2|$, the experimental value of the partial decay mean life, $\tau(K\to\mu+\nu)=2.08\times10^{-8}$ sec, would require, in the above approximation, $G_K^2/4\pi\approx1$ to 3 for the "γ_5" type interaction (14), and $G_K^2/4\pi\approx7$ to 17 for the "1" type interaction (15), according to the range of variation of the value of the parameter M.

Consequently if the K-Λ-N interaction is of the "γ_5" type (14), and if Gell-Mann and Schwinger's conjecture on global symmetry[19] of strong interactions are correct (G_K^2 is about ten times less that G^2), then the tentative assumption $|g''|=|g_2|$ seems to be consistent. Conversely, we might assume that $|g_1|^2$ and $|g_2|^2$ are about ten times less that $|g''|^2(\approx|g|^2)$ and that $|G|^2=|G_K|^2$ (universal strong "γ_5" interaction). In this case the $\Lambda^0\to p+e^-(\mu^-)+\nu$ mode would be about ten times less probable than the lifetimes predicted by the $V-A$ Fermi interaction. This might be favored by the recent experiments on the $\Lambda^0\to p+e^-(\mu^-)+\nu$ decay.[20]

(3) The $K\to e+\nu+\pi$ and $K\to \mu+\nu+\pi$ Decays

The $K^+(p_K)\to e^+(k_1)+\nu(k_2)+\pi(k)$ decay could also take place (see Fig. 4). The form of the matrix element turns out to be of the form

$$\mathfrak{M}=\mathcal{G}\bar{e}(k_1)(k\cdot\gamma)(1+\gamma_5)\nu(k_2)\varphi_K(p_K)\varphi_\pi(k),\quad(16)$$

where $\mathcal{G}$ is a numerical constant of the form $\mathcal{G}_0(1+\mathcal{O})$. We put the electron mass equal to zero. The form of (16) is the same as that given by the $V-A$ Fermi interaction except for the fact that the terms $\mathcal{O}$ of the order Q^2/M^2 (Q is an available kinetic energy of this decay process) depend not only on $(p_K\cdot k)$ but also on $(p_K\cdot k_1)$ and $(k\cdot k_1)$, whereas for the local $V-A$ Fermi interaction $\mathcal{O}$ depends only on $(p_K\cdot k)$ as long as we keep the lowest order for the weak vertex. This is useful for the possible detection of the nonlocalizability of lepton interactions in these decay processes (see Sec. VII for detailed discussions). The branching ratios of the $K\to e+\nu+\pi$ to $K\to\mu+\nu$ decay computed from (13) and (16) neglecting the terms $\mathcal{O}$ are of the order of roughly 20% for both the interactions (14) and (15), which are a bit larger than the experimental value ($\approx$10%) but may be satisfactory in view of our crude calculations. The frequency of $K\to\mu+\nu+\pi$ will be comparable with that of $K\to e+\nu+\pi$. Note that the interactions (1′) and (6′) predict $W(K_1^0\to e^\pm+\nu+\pi^\mp)=W(K_2^0\to e^\pm+\nu+\pi^\mp)=W(K^+\to e^++\nu+\pi^0)\times2$, etc., for these reactions.

V. TWO NEUTRINO DECAY PROCESSES OF THE K-MESON

In the present theory, the neutrino is always longitudinal so that the $K_1^0\to\nu+\bar\nu$ mode and the $K_2^0\to\nu+\bar\nu$ mode are always forbidden.[21] However, the $K^+\to\pi^++\nu+\bar\nu$ decay may take place through the interactions which we have introduced in this paper and its frequency would not be so different from other three-body decay modes of the K meson. Experimentally, the existence of this mode does not seem to be completely extablished at present.[22] If this decay mode is ruled out by future experiments, it may present a difficulty for the present model, and we would, at least, be forced to replace the interactions (1′) and (6′) by more complicated ones.

VI. THE HYPERON DECAYS AND K-MESON DECAYS INTO PIONS

(1) Hyperon Decays

The hyperon decays could also take place without introducing further types of interactions. That is, we may think of such channels as shown in Fig. 5 which occur through the intermediary of the combined effects of interactions (1),(1′) and (6),(6′). The black box again denotes the effective pion-nucleon interaction which we characterize as $G\gamma_5$. The general form of the matrix element given by Fig. 5 is as follows:

$$\mathfrak{M}=-f\bar{p}(q)(k\cdot\gamma)(1+\gamma_5)\Lambda_0(p)\varphi_\pi(k)$$
$$=-if\bar{p}(q)\{(m_\Lambda+m_n)\gamma_5+(m_\Lambda-m_n)\}\Lambda_0(p)\varphi_\pi(k).\quad(17)$$

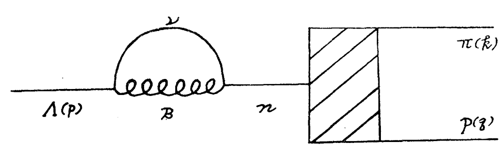

FIG. 5. Feynman decay for the hyperon decay through the intermediary of the combined effects of interactions (1),(1′) and (6),(6′).

[19] M. Gell-Mann and J. Schwinger, *Proceedings of the Seventh Rochester Conference on High-Energy Nuclear Physics, 1957* (Interscience Publishers, Inc., New York, 1957); M. Gell-Mann, Phys. Rev. **106**, 1296 (1957).

[20] Freden, Gilbert, and White, Bull. Am. Phys. Soc. Ser. II, **3**, 25 (1958).

[21] S. Oneda, Nuclear Phys. **3**, 598 (1957).

[22] See, for instance, M. Gell-Mann and A. H. Rosenfeld, in *Annual Review of Nuclear Science* (Annual Reviews, Inc., Palo Alto, 1957), Vol. 7, p. 407.

First it should be noted that this form is indeed equivalent to the derivative-type Yukawa interaction. It is known that this form of effective coupling is convenient[6,23] for explaining the observed angular distribution $1+\alpha\cos\theta$ ($\alpha\approx 0.5$) of the Λ^0-decay,[5] where θ is the angle between the direction of the decay pion and the direction of the Λ^0 polarization (in the Λ^0 rest system). For the Yukawa-type direct (renormalizable) interaction $\bar{p}(1+r\gamma_5)\Lambda_0\varphi_\pi+\text{H.c.}$, r must be as large as 10 in order to get such a large asymmetry factor. In Fig. 5, we discarded those corrections that are due to strong interactions between the initial hyperon and the black box or intermediate neutron or final proton. Under similar approximations, it has been pointed out[24] that the $V-A$ Fermi interaction could also reproduce the effective coupling of the form (17).

As a matter of fact, the numerical factor f of (17) contains a logarithmic divergence. We could separate the divergence by taking, for instance, the following renormalization procedure: There is no primary weak interaction of the form $h\bar{\Lambda}\gamma_\alpha p\partial_\alpha\varphi_\pi+\text{H.c.}$ where h is an "absolute" constant (that is, the renormalized constant h is zero). By this requirement, f turns out to be (taking $|g|=|g''|=|g_1|=|g_2|=|g|$ and $m_B=m_{B''}$)

$$f=\left(\frac{g^2}{4\pi}\right)\frac{G}{(4\pi)^{\frac{1}{2}}}\frac{1}{(2\pi)^{\frac{1}{2}}}\times\left[\frac{\frac{3}{2}m_\Lambda^2-m_B^2}{m_\Lambda^2-m_n^2}+\frac{(m_B^2-m_\Lambda^2)^2}{m_\Lambda^4}\ln\left(\frac{m_B^2}{m_B^2-m_\Lambda^2}\right)\right]. \quad (18)$$

Then the partial lifetime of the $\Lambda^0\rightarrow p+\pi^-$ decay is given by

$$\tau(\Lambda^0\rightarrow p+\pi^-)\approx 1.4\times 10^{-10}\text{ sec}, \quad (19)$$

or $\approx 14\times 10^{-10}$ sec [if we take $|g_1|^2=|g_2|^2=(1/10)|g|^2=(1/10)|g''|^2$] which is not very far removed from the observed value 4.3×10^{-10} sec.

In the present model concerning the hyperon decay and K-meson decay into pions, the source interaction of the transitions, $|\Delta S|=1$, is the transition between the Λ^0 and the neutron through the intermediary of the $B(B'')$ and neutrino fields. This amounts to stating that these decays satisfy the rule $|\Delta I|=\frac{1}{2}$ (I is isobaric spin) as long as we neglect the electromagnetic corrections. Thus the branching ratio of the Λ decay would be

$$W(\Lambda^0\rightarrow n+\pi^0)/W(\Lambda^0\rightarrow p+\pi^-)\approx\tfrac{1}{2}. \quad (20)$$

The $\Sigma^\pm\rightarrow n+\pi^\pm$ decays could take place in the lowest order, as gg_1 or $g''g_4$ processes,

$$\Sigma^\pm\xrightarrow[G]{}\Lambda^0+\pi^\pm\xrightarrow[gg_1(g''g_2)]{}n+\pi^\pm.$$

We assume that the Σ-Λ-π interaction is of the γ_5 type. The form of the effective Hamiltonian is proportional to

$$\bar{n}(q)\{(m_\Lambda+m_n)\gamma_5+(m_\Lambda-m_n)\}\Sigma(p)\varphi_\pi(k), \quad (21)$$

whereas that of the derivative-type Yukawa interaction is given by

$$\bar{n}(q)(k\cdot\gamma)(1+\gamma_5)\Sigma(p)\varphi_\pi(k)$$
$$=-i\bar{n}(q)\{(m_\Sigma+m_n)\gamma_5+(m_\Sigma-m_n)\}\Sigma(p)\varphi_\pi(k). \quad (22)$$

However, the asymmetry factor α of the $\Sigma^\pm\rightarrow n+\pi^\pm$ decays predicted by (21) is nearly the same as that by (22).

Note also that the cascade decay $\Xi\rightarrow\Lambda+\pi$ could occur whereas the $|\Delta S|=2$ transitions such as $\Xi\rightarrow N+\pi$ and $\Xi\rightarrow N+e+\nu$ would not take place, since our effective four-fermion interactions satisfy $|\Delta S|=0$ or 1.

(2) K-Meson Decays into Pions

The same intermediation of weak vertex could also be responsible for the K-meson decays into pions. The property of $|\Delta I|=\frac{1}{2}$ of the effective weak vertex would qualitatively account for the rarity of the mode $K^+\rightarrow\pi^++\pi^0$ compared with the mode $K^0\rightarrow\pi^++\pi^-$ and for the branching ratio of the τ^+-meson decays,

$$W(\tau^+\rightarrow\pi^++\pi^0+\pi^0)/W(\tau^+\rightarrow\pi^++\pi^-+\pi^-)\approx\tfrac{1}{4}.$$

It should not be difficult to explain the frequency ratio of the $\tau^+\rightarrow\pi^++\pi^-+\pi^+$ to $\theta^0\rightarrow\pi^++\pi^-$ because it is consistent with the argument based on the phase-space volume for these two decays.

VII. CONCLUDING REMARKS

Introduction of the yet unobserved bosons may not be so appealing as to enjoy immediate acceptance. However, in view of the present state of relativistic field theory, we feel that it would not be meaningless to suspect that all primary interactions are renormalizable. We have seen that the present model could reproduce almost all the attractive qualitative features of the local Fermi interaction with $V-A$ combination[2] or of the derivative-type weak Yukawa interactions.[3,24] Moreover, reflecting the fact that primary interactions are all renormalizable, the theory is remarkably less singular than the direct Fermi interaction model, and we need not have recourse to an ambigious procedure such as the cutoff method. Direct detection of chiral bosons may not be possible, because they disintegrate into nucleons and leptons at once with partial lifetimes $\approx 6\times 10^{-18}$ sec and further their production rates will be too small to be observed. If the weak interactions are not at all renormalizable, multiple production of leptons may be anticipated at extremely high energy.[6] So careful experimental distinction is desired.

Before closing this paper, we should like to add some comments on the possibility of detection of the nonlocal effect of the present model. In the usual local theories, leptons always appear as a pair, and lepton vertices are

[23] A. Pais and S. B. Treiman, Phys. Rev. **109**, 1759 (1957), reference 3; J. J. Sakurai, Nuovo cimento **7**, 649 (1958).

[24] S. Oneda and A. Wakasa, Nuclear Phys. **1**, 445 (1956); see also Sakurai's paper in reference 23.

always localized as long as electromagnetic corrections are neglected. The present theory predicts that leptons are emitted from different vertices which are connected by a chiral boson of a finite mas. Thus, in effect, lepton interactions become always nonlocal. This presents a striking contrast to the other possible theories in which weak interactions are caused by the intermediary of some boson with zero baryonic and leptonic number. For instance, the $V-A$ Fermi interactions may be mediated by some heavy charged vector meson with vector coupling. In this case, the lepton vertex still remains localizable. The experimental values of the Michel parameter of μ-decay[25] and of the ratio of π-e to π-μ decay might be an indication of the nonlocalizability of the lepton interactions.

Let us, for instance, take the decay $K\rightarrow e(\mu)+\nu+\pi$. Since the maximum decay electron energy is about 200 Mev, this process should permit one to investigate distances of nonlocalizability of the order of 10^{-13} cm. If the electron-neutrino interaction is local and does not contain derivatives as is the case with the local Fermi interactions or with the intermediation of a charged vector meson with vector coupling, the unknown functions of the matrix elements which are determined by strong interactions depend on the pion energy. Thus the localizability of the electron-neutrino interaction could be checked by studying the angular or energy distribution of decay products for fixed values of the pion energy E_π. It should be noted that this conclusion does not depend on the localizability of strong interactions.[26] The angular distribution will take the following form:

$$W(E_\pi,\theta)=\frac{(m_K-E_\pi)^2(1-\chi^2)^2}{(1+\chi\cos\theta)^4}f(\cos\theta)|p_\pi|E_\pi dE_\pi\sin\theta d\theta, \quad (23)$$

where θ is the angle between the π-meson and electron momenta and $\chi=|p_\pi|/(m_K-E_\pi)$. As long as the lepton vertex is localized (usual Fermi-type interaction) the expression $f(\cos\theta)$ contains only polynomials of $\cos\theta$ up to second degree.[27] That is, it is of the form $f(\cos\theta)=a+b\cos\theta+c\cos\theta^2$. a, b, and c are constant if we fix E_π. Deviations from this distribution (appearance of the terms $\cos^n\theta$, $n>2$) would indicate the nonlocalizability of the electron-neutrino interaction. In fact, the terms Θ in (16) which contain the scalar product $(p_K\cdot k_1)$ or $(k\cdot k_1)$ explicitly show the nonlocalizability under consideration. They are not negligibly small (probably of the order of 25% of the leading term), and will allow their detection if the nonlocal lepton interactions are real. The $K\rightarrow\mu+\nu+\pi$ decay may also be as useful as the $K\rightarrow e+\nu+\pi$ decay since the terms proportional to m_μ would survive which are negligibly small in the case of the $K\rightarrow e+\nu+\pi$ decay. An analogous method would also be applied to the angular distribution of the μ meson in the decay mode $\pi\rightarrow\mu+\nu+\gamma$ with fixed photon energy. There, the angular correlation function $f(\theta)$ also contains only polynomials of $\cos\theta$ up to second degree as far as the lepton interaction is localizable.[18,28]

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[25] It should be noted that in the present model the explanation of the Michel parameter is possible. See also T. D. Lee and C. N. Yang, Phys. Rev. **108**, 1611 (1957); S. Bludman and A. Klein, Phys. Rev. **109**, 550 (1958); A. Sirlin (to be published).

[26] Bogolubov, Bilenky, and Logunov, *Proceedings of the Padua-Venice Conference on Mesons and Newly Discovered Particles, September, 1957* (to be published).

[27] A. Pais and S. B. Treiman, Phys. Rev. **105**, 1616 (1957). One of us (S.O.) wishes to express his thanks to Professor A. Pais for the discussion on this problem.

[28] S. Kamefuchi and S. Oneda, Nuclear Phys. **6**, 114 (1958).