Fermi Interaction Caused by Intermediary Chiral Boson

YASUTAKA TANIKAWA,* *Institute for Advanced Study, Princeton, New Jersey*

AND

SATOSI WATANABE, *International Business Machines Research Laboratory, Ossining, New York*

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The Fermi interaction is interpreted as a nonlocal interaction resulting from a double Yukawa-type interaction in which the intervening boson has a definite chirality. The theory is quantizable and renormalizable, and in the "local" limit, the results agree with the usual (V–A) theory of the direct Fermi interaction. The general framework of the theory not only gives a basis for the existence of parity-nonconserving interactions, but also determines the allowed forms of such interactions. The nonlocal effect of the intervening boson propagator tends to give an improved agreement with experiments.

1. NATURE OF WEAK INTERACTIONS

PARITY nonconservation[1] is obviously a negative concept. One can indeed ask: Is there any positive theoretical reason for the existence of the so-called parity-nonconserving interactions? Is there any theoretical prescription to determine the forms of such interactions? Partial answers to these questions have been given by the two-component neutrino theory of Landau, Salam, Lee, and Yang,[2] the chirality invariant theory of Sudarshan and Marshak,[3] the two-component theory of all fermions of Feynman and Gell-Mann,[4] and the mass-reversal invariant theory of Sakurai.[5]

These theories, however, have serious shortcomings. The two-component neutrino theory cannot explain those parity-nonconserving interactions which do not involve neutrinos,[6] unless one introduces an additional assumption.[7] The Sudarshan-Marshak theory introduces the chirality invariance as an *ad hoc* principle to account for the $(V-A)$ theory which happened to agree with experimental data of weak interactions, and offers no justification to strong interactions. The Feynman-Gell-Mann theory has an inherent difficulty in quantizing the two-component field with a finite mass which satisfies the Klein-Gordon equation. In Sakurai's theory, the physical meaning of the mass-reversal transformation is not clear.[8] Furthermore, all these theories are invariably unrenormalizable. In spite of all these objections, however, one has to acknowledge that the phenomenological Hamiltonian used in these theories agrees with most of the experimental facts concerning beta decay, μ decay, and μ capture.[3,4,9]

The theory proposed in the present paper not only gives essentially the same results as the above-mentioned theories in the areas where these are successful, but also is free from the kind of objections enumerated in the foregoing paragraph. Namely, it gives a mathematical framework in which the parity-conserving and parity-nonconserving interactions have equal justification, giving a unified theoretical standpoint to determine the forms of both types of interactions. The theory is quantizable and renormalizable. Furthermore, the present theory, due to the nonlocal nature of the derived four-fermion interaction, seems to provide a better agreement with experiments in the areas where the other theories are not quite as successful, in particular, in relation to the μ decay and π decay. The present theory also incorporates the selection rule regarding muons proposed by Konopinski and Mahmoud, and also recently by Nishijima.[10] On the other hand, it should be admitted that the present theory is just as incapable as the other competing theories in explaining why the parity-conserving interactions have stronger coupling constants than the parity-nonconserving interactions.

In the current field theory, the space-time coordinates are not physical quantities, but parameters labeling the field strengths. As far as the proper Lorentz transformations (which are connected continuously to the identity transformation) are concerned, all tensors behave as the coordinate transformation dictates. However, once a reflection is involved, different kinds

* Present address: Kobe University, Kobe, Japan.

[1] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).

[2] T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957); A. Salam, Nuovo cimento **5**, 299 (1957); L. Landau, Nuclear Phys. **3**, 127 (1957).

[3] E. C. G. Sudarshan and R. E. Marshak, Phys. Rev. **109**, 1860 (1958).

[4] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1957).

[5] J. J. Sakurai, Nuovo cimento **7**, 649 (1958); see also Ouchi, Senba, and Yonezawa, Progr. Theoret. Phys. (Japan) **15**, 431 (1956); T. Ouchi, Progr. Theoret. Phys. (Japan) **17**, 743 (1957).

[6] The parity nonconservation in the neutrino-less decay of Λ^0 was proved by the experiments of Crawford, Cresti, Good, Gottstein, Lyman, Solmitz, Stevenson, and Ticho, Phys. Rev. **108**, 1102 (1957).

[7] T. D. Lee proposed that a pair consisting of a very heavy fermion S and a neutrino could serve as the central intelligence office for all fermion interactions including hyperons. This pair takes a role just as our heavy chiral bosons. T. D. Lee, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics, 1957* (Interscience Publishers, Inc., New York, 1957).

[8] S. Watanabe, Progr. Theoret. Phys. (Japan) **15**, 81 (1956).

[9] Goldhaber, Grodzins, and Sunyar, Phys. Rev. **102**, 1015 (1958); L. A. Page and M. Heinberg, Phys. Rev. **106**, 1220 (1957); Culligan, Frank, Holt, Kluyver, and Massam, Nature **180**, 751 (1957); Hermannsfeldt, Maxson, Stähelin, and Allen, Phys. Rev. **107**, 641 (1957).

[10] K. Nishijima, Phys. Rev. **108**, 907 (1957). E. J. Konopinski and H. M. Mahmoud, Phys. Rev. **92**, 1045 (1953).

of behaviors among tensors appear, which are not always in conformity with the coordinates. This is the origin of parity. Thus, parity is not just a property of parameters, but a physical quantity. If parity is a physical quantity, it may be quite natural to assume the existence of a quantity which is "complementary" to it. This leads to the concept of chirality. Once one admits chirality in one's consideration, the usual full Lorentz group becomes too narrow a frame to encompass all physical quantities. The tensorial quantities should then be represented in general by irreducible representations of the extended algebraic system including not only the Lorentz group but also parity and chirality operations. Some quantities may be in eigenstates of parity and some others in eigenstates of chirality. The chirality operation will mix two eigenstates of parity, and the parity operation will mix eigenstates of chirality. Whether a quantity is an eigenstate of parity or an eigenstate of chirality, or even a mixture of them it returns to its original value by the combined CTP transformation. In other words, the extended algebraic system is conceived within the domain of CTP invariance.

The basic hypotheses of the present formalism are the following: (1) The elementary interactions are renormalizable. (2) The Lagrangians are invariant either for a chirality operator or for the parity operator. These two alternatives are "complementary" to each other.[11] (3) Bosons are either in a chirality eigenstate or parity eigenstate. This assumption is permissible, since chirality as well as parity can be a constant of motion for a boson, *whether or not it is massless*. (4) The theory is invariant for time reversal T (of the Wigner type). This last assumption eliminates a certain arbitrariness left in the definition of chirality. The theory can also be made invariant for charge conjugation C, but we explain only the T-invariant theory here.

Hypothesis (1) immediately excludes the direct four-fermion interaction. The allowed types are boson-boson interactions (such as electromagnetic interaction of pions) and boson–two-fermion (Yukawa-type) interactions in which the bosons can be a scalar or a vector. It should, however, be noted that it is rather an exceptional case that a vector Yukawa-interaction becomes renormalizable like the electromagnetic interaction of a charged fermion.[12] In fact, inclusion of a vector boson in the present formalism leads to a theory for which no guarantee exists for renormalizability. For this reason, the Yukawa-type interaction considered in the present paper will be limited only to the scalar type in the later sections. Hypothesis (3) requires that the boson be in a parity or chirality eigenstate. These considerations delimit the allowed types of interactions to a relatively small number, and all the well-established interactions are included in this frame. For instance, the electromagnetic interaction of a fermion and the pion-nucleon interaction are of the renormalizable Yukawa-type invariant for parity operation. The only types of allowed interactions which are not usually considered are the Yukawa-type interactions in which the intervening bosons are in chirality eigenstates. These are exactly the types required to derive the desired weak interactions of four spinors by elimination of boson fields. It is true that if a chiral boson is electrically charged, its electromagnetic interaction introduced in the usual fashion will become invariant for neither parity operation nor chirality operation, contradicting hypothesis (2). However, if there exist two kinds of chiral bosons with the same charge and opposite chiralities then their total electromagnetic interaction can become invariant for parity. Furthermore, the possibility of a chirality-conserving electromagnetic interaction seems not to be excludable, as its effect on the vacuum polarization can be expected to be very small. The idea of deriving the Fermi interaction from renormalizable Yukawa interactions was previously proposed by one of the authors (Y.T.).[13] The present paper may be considered as a revised version of the theory, formulated with an explicit use of the concept of chirality and adapted to the newer experimental facts. The exposition in the following sections will follow more or less an inductive, rather than a deductive, line of thinking; *viz.*, we shall first give the phenomenological Fermi interaction, to which the theory should reduce in the "local" limit. From there, we shall infer what kinds of chiral bosons should exist in nature. We shall then show conversely that the existence of such bosons, with the help of the conservation laws of charge, lepton numbers, and baryon numbers, leads uniquely to the desired forms of the Fermi interaction and to none other. Some consequences of this theory will then be discussed. Finally in the Appendix a detailed explanation of the definition and properties of chirality operators in tensor analysis will be given. A new version of the proof of the CTP-theorem is given, and the reason why the present theory does not violate this theorem is explained.

2. YUKAWA-TYPE INTERACTION AND ENSUING NONLOCAL PARITY-NONCONSERVING FERMI INTERACTION

The parity of a quantity in the present context is defined as being positive if it transforms purely according to the coordinate transformation for the proper Lorentz group and a space inversion, and as being negative if it not only transforms according to this rule but also changes its sign for a space inversion. This

[11] Y. Tanikawa and S. Watanabe, Phys. Rev. **110**, 289 (1958).

[12] See, in particular, Sheldon Glashow, thesis, Harvard University, 1958 (unpublished). The present authors thank Dr. S. Glashow and Dr. S. Okubo for informing them of the most recent results on the renormalizability of a vector field and for comments on the present theory.

[13] Y. Tanikawa, Phys. Rev. **108**, 1615 (1957), and references quoted therein.

definition includes the ordinary definition of the parity of a field. Chirality is defined as a quantity which anticommutes with the parity thus defined.[14] A more precise definition will be given in the appendix, but it is not needed for the moment. Hypothesis (2) of the preceding section, as can be seen in the appendix, amounts to the requirement that an interaction term be composed of two factors of the same parity or of the same chirality. In the case of a Yukawa-type interaction, the parity or chirality of the factors will be determined by that of the boson according to hypothesis (3). Thus, the nucleon source of the pion-nucleon interaction (negative parity) is interpreted as reflecting the negative parity of the pion-field. In view of the important roles played by the Yukawa interactions, it is only too natural to conjecture the existance of a boson in a chiral eigenstate giving rise to a Yukawa-type interaction in which the factor due to the spinors is also in the corresponding chirality eigenstate.

Therefore, the experimental fact that the weak interactions of four fermions are parity-nonconserving leads to an unambiguous conclusion that they have to be mediated by a Yukawa-type interaction in which the intervening boson is a scalar or a vector in a chiral eigenstate, implying that the factor due to spinors is also in a chiral eigenstate. Although we later limit ourselves to scalar bosons, we shall first include in our consideration also vector bosons, since it is of some interest to investigate the general relationship between the $V-A$ Fermi-interaction and those Yukawa-interactions which can reproduce such a Fermi-interaction. The basic interaction Hamiltonian will then have the form[15]:

$$H_s=g\bar{\psi}_1(1+e^{i\alpha}\gamma_5)\psi_2B+g\bar{\psi}_3(1+e^{i\alpha}\gamma_5)\psi_4B+\text{H.c.},\quad(2.1)$$

or

$$H_v=f\bar{\psi}_1\gamma_\mu(1+e^{i\beta}\gamma_5)\psi_2B_\mu+f\bar{\psi}_3\gamma_\mu(1+e^{i\beta}\gamma_5)\psi_4B_\mu+\text{H.c.},\quad(2.2)$$

where B (B_μ) is a *complex* scalar (vector) whose chirality is the same as its multiplier in the Hamiltonian. $\bar{\psi}=\psi^\dagger\gamma_4$, where $\psi^\dagger$ is the Hermitian conjugate of ψ. The abbreviation "H.c." means Hermitian conjugate. From the sole requirement that chirality is a quantity anticommuting with parity, the factor $e^{i\alpha}$ and $e^{i\beta}$ are not determined. A further requirement that the theory should be invariant for time-reversal, T, i.e., for PC, limits this arbitrariness to $e^{i\alpha}=\pm1$ and $e^{i\beta}=\pm1$. If the theory should be invariant for charge conjugation, C, i.e., for TP, then $e^{i\alpha}=\pm i$ and $e^{i\beta}=\pm i$. So as not to repeat similar deductions, we assume time-reversibility which seems not to contradict the experimental facts so far obtained.

$$e^{i\alpha}=\pm1,\quad e^{i\beta}=\pm1.\quad(2.3)$$

The S matrix of the second order in the coupling constants derived from (2.1) is (in the natural units $c=\hbar=1$)

$$S=(-i)^2\int\int T(K_s)\langle T[B(x)B^\dagger(x')]\rangle_0 d^4x d^4x'+\text{H.c.},\quad(2.4)$$

where K_s involves four spinors and depends on x and x'. T is Wick's chronological operator, and $\langle\ \ \rangle_0$ means the vacuum expectation value. In the "local" limit: $\langle T[B(x)B^\dagger(x')]\rangle_0\rightarrow-(i/m_B^2)\delta(x-x')$, which is obtained by $m_B\rightarrow\infty$, K_s will become proportional to the effective Hamiltonian of four-spinor interaction. m_B is the mass of the B particle. K_s in the local limit is given by

$$K_s=gg^*\bar{\psi}_1(1\pm\gamma_5)\psi_2\bar{\psi}_4(1\mp\gamma_5)\psi_3+\text{H.c.}\quad(2.5\text{a})$$

$$=\tfrac{1}{2}gg^*\bar{\psi}_1\gamma_\mu(1\mp\gamma_5)\psi_3\bar{\psi}_4\gamma_\mu(1\pm\gamma_5)\psi_2+\text{H.c.}\quad(2.5\text{b})$$

$$=-\tfrac{1}{2}gg^*\bar{\psi}_1\gamma_\mu(1\mp\gamma_5)\psi_3\bar{\psi}_2{}^c\gamma_\mu(1\mp\gamma_5)\psi_4{}^c+\text{H.c.}\quad(2.5\text{c})$$

(2.5a) is derived directly from (2.1) by eliminating the B field. (2.5b) is derived from (2.5a) by the use of the Pauli-Fierz relations,[16] whereas (2.5c) is derived from (2.5b) by expressing it in terms of charge-conjugate spinors (spinors being assumed to anticommute)

$$\psi^c=+C\bar{\psi}=-\bar{\psi}C,\quad\bar{\psi}^c=+C^{-1}\psi=-\psi C^{-1},\quad(2.6)$$

with

$$C^{-1}\gamma_\mu C=-\gamma_\mu{}^T,\quad(\mu=1,2,3,4),\qquad C^T=-C,\quad C^\dagger=C^{-1}.\quad(2.7)$$

Note that (2.5b) is of the $(V+A)$ type while (2.5c) is of the $(V-A)$ type. As is well known, a further application of the Pauli-Fierz relation on (2.5c) results only in an interchange of ψ_3 and $\psi_4{}^c$ in (2.5c) without any other modification. This can be seen more directly from (2.1) in which the interchange of ψ_3 and $\psi_4{}^c$ means nothing but writing the same quantity in terms of the charge conjugate fields.

In the vector boson case, the S matrix becomes

$$S=(-i)^2\int\int T(K_{v,\mu\nu})\langle T[B_\mu(x)B_\nu{}^\dagger(x')]\rangle_0 d^4x d^4x'+\text{H.c.},\quad(2.8)$$

where K_v will become proportional to the effective Hamiltonian in the local limit: $\langle T(B_\mu(x)B_\nu{}^\dagger(x'))\rangle_0\rightarrow$

[14] The parity conjugation operation introduced by Lee and Yang may be considered as a special chirality operation. However, the eigenstates of this operator were not considered by them [T. D. Lee and C. N. Yang, Phys. Rev. **102**, 290 (1956)]. For the concept of chirality, see also S. Watanabe, Phys. Rev. **106**, 1306 (1957); S. Watanabe, Nuovo cimento **6**, 187 (1957). See also references 11 and 23.

[15] The interaction constant $g(f)$ might have different values, e.g., g_{12} and g_{34} (f_{12} and f_{34}) for different pairs of $\bar{\psi}_1O\psi_2$ and $\bar{\psi}_3O\psi_4$. We put, however, $g_{12}=g_{34}=g$ ($f_{12}=f_{34}=f$) by a conjecture that $g(f)$ is a constant for any source of $B(B_\mu)$ as the electric coupling constant e is a constant for all charged fields.

[16] W. Pauli, *Zeeman Verhandelungen* (Haag, 1935), p. 31; M. Fierz, Z. Physik **104**, 553 (1957).

$-(i/m_{B\mu}^2)\delta_{\mu\nu}(x-x')$, $m_{B\mu}$ being the mass of the B_μ-particle. In this limit, one has

$$K_v=-ff^*\bar{\psi}_1\gamma_\mu(1\pm\gamma_5)\psi_2\bar{\psi}_4\gamma_\mu(1\pm\gamma_5)\psi_3+\text{H.c.} \quad (2.9\text{a})$$

$$=-ff^*\bar{\psi}_1\gamma_\mu(1\pm\gamma_5)\psi_3\bar{\psi}_4\gamma_\mu(1\pm\gamma_5)\psi_2+\text{H.c.} \quad (2.9\text{b})$$

$$=ff^*\bar{\psi}_1\gamma_\mu(1\pm\gamma_5)\psi_3\bar{\psi}_2{}^c\gamma_\mu(1\mp\gamma_5)\psi_4{}^c+\text{H.c.} \quad (2.9\text{c})$$

In this case, (2.9a) and (2.9b) are of the $(V-A)$ type and (2.9c) is of the $(V+A)$ type.

It can easily be seen from (2.1) and (2.2) that the charge difference between ψ_1 and ψ_2 must be equal to the charge difference between ψ_3 and ψ_4. Further, if B is neutral, then this difference must be zero. From the fact that always two charged fermions and two neutral fermions are involved in a Fermi-interaction, it follows that in the case of a neutral B, if ψ_1 and ψ_2 are charged (neutral), then ψ_3 and ψ_4 must be neutral (charged). If B is charged, then one of ψ_1 and ψ_2 (ψ_3 and ψ_4) must be charged and the other neutral.

3. BETA DECAY, μ DECAY, μ CAPTURE, AND PROPERTIES OF CHIRAL BOSONS

In the following, the neutral massless particle accompanying the negative beta-decay will be called, by definition, an antineutrino, ν^c. It is by now fairly well established that beta decay, μ decay, and μ capture are satisfactorily described by the following Fermi interactions or their equivalent[3,4,9]:

$$\beta \text{ decay:}\quad H_F=G\bar{p}\gamma_\mu(1+\gamma_5)n\bar{e}\gamma_\mu(1+\gamma_5)\nu+\text{H.c.}, \quad (3.1)$$

$$\mu \text{ decay:}\quad H_F=G\bar{\nu}\gamma_\mu(1+\gamma_5)\mu^-\bar{e}\gamma_\mu(1+\gamma_5)\nu+\text{H.c.}, \quad (3.2)$$

$$\mu \text{ capture:}\ H_F=G\bar{p}\gamma_\mu(1+\gamma_5)n\bar{\mu}^-\gamma_\mu(1+\gamma_5)\nu+\text{H.c.} \quad (3.3)$$

Since these formulas are unchanged by putting $\nu=(1+\gamma_5)\nu/2$, only two components are required to describe the neutrino field, whose "particle" has a negative helicity and "antiparticle" has a positive helicity. As far as the lepton number is concerned, if one assumes the lepton number of the negative electron to be positive, (3.1) implies that the neutrino ν has a positive lepton number, and (3.2) requires the negative muon to have a positive lepton number. According to this scheme, however, it is difficult to find any reason why there should not be such undesired processes as $\mu^-\to e^-+e^-+e^+$ and $\mu^-+p\to e^-+p$, for they satisfy lepton conservation. To forbid these processes, one need only interchange the lepton numbers of μ^+ and μ^-, and at the same time change accompanying neutrino to antineutrino, according to the suggestion made by Konopinski, Mahmoud, and Nishijima.[10] This means that $\bar{\nu}$ of (3.2) and ν of (3.3) should respectively be written as $\bar{\nu}^c$ and ν^c. This type of antineutrino ν^c accompanying the muon will have a negative helicity and the corresponding neutrino will have a positive helicity. These two will precisely occupy the components which are unused by the neutrino and antineutrino participating in the beta decay.

In accordance with our guiding idea that the helicities are a consequence of the underlying boson chirality, we should rather prefer to write simply ω for the muon-accompanying neutrino, and assume ω to have a negative lepton number, which automatically implies that μ^- has a negative lepton number. This will serve exactly the same purpose. Then, a further (equivalent) variant of interpretation suggests itself. One can deprive the μ and ω of the lepton numbers and assign a positive muon number to μ^- and ω. Then, the muon conservation will forbid the undesired processes. We shall adopt this mode of description in the following. (3.2) and (3.3) will then be written as

$$\mu \text{ decay:}\quad H_F=G\bar{\omega}\gamma_\mu(1+\gamma_5)\mu^-\bar{e}\gamma_\mu(1+\gamma_5)\nu+\text{H.c.}, \quad (3.4)$$

$$\mu \text{ capture:}\ H_F=G\bar{p}\gamma_\mu(1+\gamma_5)n\bar{\mu}^-\gamma_\mu(1+\gamma_5)\omega+\text{H.c.} \quad (3.5)$$

The difference between the conventional formulation and this formulation is that the former describes the μ decay, μ capture, and π decay as

$$\begin{aligned}&\mu^+\to e^++\nu_L+\nu_R{}^c;\quad \mu^-+p\to n+\nu_L;\\ &\pi^+\to\mu^++\nu_L\to(e^++\nu_L+\nu_R{}^c)+\nu_L,\end{aligned} \quad (3.6)$$

whereas the latter describes them as

$$\begin{aligned}&\mu^+\to e_++\nu_L+\omega_R{}^c;\quad \mu^-+p\to n+\omega_L;\\ &\pi^+\to\mu^++\omega_L\to(e^++\nu_L+\omega_R{}^c)+\omega_L,\end{aligned} \quad (3.7)$$

where the suffices R and L mean positive and negative helicities, respectively. The Konopinski-Mahmoud-Nishijima scheme can be obtained from (3.7) by replacing $\omega_R{}^c$ and ω_L by ν_R and $\nu_L{}^c$, respectively. These three interpretations can hardly be differentiated by the current experimental methods.

Our next task is to interpret the formulas (3.1), (3.4), and (3.5) which are all of the $(V-A)$ type as corresponding to one of the three expressions (2.5c), (2.9a), and (2.9b) which are also of the $(V-A)$ type. First, with regard to the beta decay, one notices that if one identifies (3.1) with (2.9a) or (2.9b) it would imply a charged vector boson for which no renormalizability is guaranteed. Identification of (3.1) with (2.5c) leads to the basic Hamiltonian (2.1) with a neutral scalar boson:

$$H_s=g\bar{p}(1-\gamma_5)C\bar{e}B+g\bar{n}(1-\gamma_5)C\bar{\nu}B+\text{H.c.}, \quad (3.8)$$

where $G=|g|^2/2m_B{}^2$ in the local limit. (3.8) implies

$$B\rightleftarrows e^-+p,\quad B\rightleftarrows \nu_L+n. \quad (3.9)$$

Next, in the case of μ decay, (2.5c), (2.9a), and (2.9b) will lead respectively to a charged scalar, a charged vector, and a neutral vector. In order to adhere to a renormalizable theory, we shall choose (2.5c). This implies the basic Hamiltonian:

$$H_s=g\bar{\omega}(1-\gamma_5)C\bar{e}B'+g\bar{\mu}^-(1-\gamma_5)C\bar{\nu}B'+\text{H.c.} \quad (3.10)$$

In the local limit, we have $G=|g|^2/2m_{B'}{}^2$. (3.10)

TABLE I. Properties of intermediary bosons. The dagger means the "antiparticle," as defined in the text.

	B	$B^\dagger$	B'	$B'^\dagger$	B''	$B''^\dagger$
Electric charge	0	0	−1	+1	0	0
Lepton number	+1	−1	+1	−1	0	0
Muon number	0	0	+1	−1	+1	−1
Baryon number	+1	−1	0	0	+1	−1
Chirality	+1	−1	+1	−1	+1	−1

implies the transformation:

$$B' \rightleftarrows e^- + \omega_L; \quad B' \rightleftarrows \mu^- + \nu_L. \tag{3.11}$$

Finally for the μ capture, the assumption (2.5c) leads to a neutral scalar, while both (2.9a) and (2.9b) lead to a charged vector. As before, we have to choose (2.5c), which corresponds to the basic Hamiltonian:

$$H_s = g''\bar{p}(1-\gamma_5)C\bar{\mu}^- B'' + g''\bar{n}(1-\gamma_5)C\bar{\omega}B'' + \text{H.c.}, \tag{3.12}$$

which implies

$$B'' \rightleftarrows \mu^- + p; \quad B'' \rightleftarrows \omega_L + n. \tag{3.13}$$

In the local limit, one has $G = |g''|^2/2m_{B''}^2$. It should be noted that the nucleon stability can be guaranteed only by assuming that the B particle and B'' particle, though neutral, are represented by complex fields (thus differentiating particles from antiparticles) and that they have masses heavier than the nucleon.

The basic interactions of bosons, (3.9), (3.11), and (3.12) show that these bosons carry baryon numbers, muon numbers, and lepton numbers. Their values are listed in Table I. The "antiparticle" $B^\dagger(B'^\dagger, B''^\dagger)$ of the boson $B(B', B'')$ is to be considered as the quantum of the Hermitian conjugate field of $B(B', B'')$. As can be inferred from the fact that $[\bar{\psi}(1\pm\gamma_5)\varphi]^\dagger = \bar{\varphi}(1\mp\gamma_5)\psi$, the Hermitian conjugate of the B's will have opposite chiralities.

After having assigned these numbers to the bosons, one can examine all the possible Yukawa-type interactions allowed by conservation of electric charge, lepton number, muon number, and baryon number, assuming that the available fermions are nucleons, muons, electrons, neutrinos (ω particles), and their antiparticles. One immediately discovers that there can be no other interactions than those which have been already considered in (3.9), (3.11), and (3.13). This is not a trivial fact, and should be construed as of one the satisfactory features of the present theory. For instance, the B particle, being neutral and carrying positive baryon number and positive lepton number, can create a pair (e^-,p) or (ν,n), but none other, when it disappears.

The B particle bridges over the lepton and baryon families, the B' bridges over the muon and lepton families, and the B'' bridges over the baryon and muon families. The Konopinski-Mahmoud-Nishijima scheme can be obtained simply by equating the muon charge to the negative lepton charge in Table I. In this case also the conservation laws of electric charge, lepton number, and baryon number allow only the three basic reactions, (3.9), (3.11), (3.13), and none other (ω_L being identified as $\nu^c{}_L$). Thus, the uniqueness of the allowed interaction types is still upheld. The usual scheme implied by (3.1), (3.2), (3.3) is obtained by equating the muon charge to the lepton charge in Table I, and identifying ω_L with ν_L. In the case, B and B'' cannot be differentiated from each other, and the undesired processes cannot be forbidden. The electromagnetic interaction of the B' particle is discussed in appendix. The requirement that the electromagnetic interaction be parity-conserving leads to the assumption of the existence of a fourth chiral B particle which may play an unknown role in the Fermi interactions involving strange fermions. One could avoid this unidentified fourth particle either by identifying (3.4) with (2.9b) for the μ decay or by assuming a chirality-conserving electromagnetic interaction for the B' particle. In the first of these alternatives, the intermediary boson becomes a neutral vector for which renormalizability is no longer guaranteed although it is free from the complication due to the electromagnetism. As regards the second alternative, its possibility cannot be denied although it requires a further careful justification.

4. COUPLING CONSTANTS, MASSES OF BOSONS, MICHEL PARAMETER, $(\pi\text{-}e)/(\pi\text{-}\mu)$ DECAY RATIO, HYPERON INTERACTION

All experimental results which support the current $(V-A)$ theory support also the present theory.[2,3,4,9] In this section, we shall briefly discuss further comparison of the present theory with the experimental data.

The considerations of the foregoing sections were based on the local limit: $\langle T[B(x)B^\dagger(x')]\rangle_0 \to -(i/m_B^2) \times \delta(x-x')$ which amounts to assuming infinite masses of the intermediary bosons. Restoration of a finite mass of the boson not only has the effect of nonlocalizing the Fermi interaction but also leads to a different mass-dependent relation between the basic coupling constants, g, g', and g'', and the phenomenological Fermi interaction constant. In the case of beta decay, the unrenormalized Fermi constants, F_V and F_A, are approximately connected to the basic coupling constant g by

$$F_V = F_A = \tfrac{1}{2}(m_B^2 - m_N^2)^{-1}|g|^2, \tag{4.1}$$

where terms dependent on particle energy are neglected. From the conditions that there should be no term similar to the Fierz factor in the allowed beta decay[13] and that the nucleon should be a stable particle, it follows that $m_B \gtrsim 2300 m_e$ as a sufficient condition. To determine g and m_B separately, one will have to determine experimentally the nonlocal effect due to the factor $\langle T[B(x)B^\dagger(x')]\rangle_0$.

The assumption $m_B \simeq 2300 m_e$ and the experimental value, $F_V=F_A=1.01\times10^{-5}(m_N)^{-2}$, give[17]

$$|g|^2/4\pi=7.4\times10^{-7}. \quad (4.2)$$

For the μ capture, one obtains

$$F_V''=F_A''=\tfrac{1}{2}[m_{B''}^2-(m_N-m_\mu)^2]^{-1}|g''|^2. \quad (4.3)$$

If one assumes $m_{B''}\simeq m_B\simeq 2300m_e$, the experimental value of the rate of μ capture suggests that one can put $g''\simeq g$. Concurrently with $\pi^+\to\mu^++\omega_L$ (3.7), there will also be, due to (3.1), the process

$$\pi^+\to e^++\nu_L. \quad (4.4)$$

Except for the nonlocal effect, the present theory differs from the two-component neutrino theory only in the name of the neutrino accompanying the π decay, allowing the same kinematical consideration as in the latter theory. Thus, the ratio of π-e decay to π-μ decay is given by

$$[(\pi\text{-}e)/(\pi\text{-}\mu)]\simeq(m_e/m_\mu)^2[1-(m_\mu/m_\pi)^2]^{-2} \simeq 13.6\times10^{-5}. \quad (4.5)$$

The nonlocal effect due to $\langle T[B''(x)B''^\dagger(x')]\rangle_0$ and $\langle T[B(x)B^\dagger(x')]\rangle_0$ should not appreciably change this result insofar as $m_{B''}\simeq m_B$. A more detailed discussion of this problem is given by Oneda and Tanikawa.[18] For the μ decay, one has

$$F_V'=F_A'=\tfrac{1}{2}(m_{B'}^2-m_\mu^2)^{-1}|g'|^2. \quad (4.6)$$

The experimental value of the μ-decay lifetime gives

$$|g'|^2/4\pi\simeq(|g|^2/4\pi)(m_{B'}^2-m_\mu^2)/(m_B^2-m_N^2). \quad (4.7)$$

The nonlocal effect of the present theory tends to increase the Michel parameter (which is $\frac{3}{4}$ in the local limit) to a value closer to the experimental value. Adaptation of Lee and Yang's theory about the nonlocal effect on the ρ value[19] to the present case yields

$$\rho=0.75+0.27(m_\mu/m_{B'})^2, \quad (4.8)$$

which gives $\rho=0.79$ when $m_{B'}$ is about $2.6m_\mu$.[20] $m_{B'}$ is required by the present theory only to be larger than m_μ. The value $m_{B'}\simeq 2.6m_\mu$ and the value of m_B used in (4.2) give, in virtue of (4.7),

$$|g'|^2/|g|^2=10^{-1}. \quad (4.9)$$

It is interesting to note that this ratio is approximately equal to the ratio of the two strong interaction constants for K production and π production,

$$|g'^2|/|g^2|\simeq g_K^2/g_\pi^2. \quad (4.10)$$

This may be construed as a kind of fine structure existing within each family of strong and weak interactions. It is, however, to be admitted that the evaluation of the numerical values of the coupling constants g and g' depends on the evaluation of m_B and $m_{B'}$ which is by no means conclusive as of the present.

The fundamental weak interactions of (3.8) and (3.12) could be extended to cover also the weak interactions of strange particles. The simplest assumption is that the interaction terms,

$$g\bar{\Lambda}^0(1-\gamma_5)C\bar{\nu}B+\text{H.c.} \quad (4.11)$$

and

$$g''\bar{\Lambda}^0(1-\gamma_5)C\bar{\omega}B''+\text{H.c.}, \quad (4.12)$$

should be added in (3.8) and (3.12), respectively. One can show that these interactions are responsible for the decay of Λ^0 and K, such as

$$\Lambda^0\to p+\pi^-,\ n+\pi^0, \quad (4.13)$$

and

$$K\to 2\pi,\ 3\pi,\ \text{etc.} \quad (4.14)$$

The related problem is discussed in the following paper by Oneda and Tanikawa.[18]

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APPENDIX. CHIRALITY IN TENSOR CALCULUS AND *CTP* THEOREM

We consider the full group G of congruent transformations in the Minkowski space

$$x_\mu'=a_{\mu\nu}x_\nu,\quad (\mu,\nu=1,2,3,4) \quad (A.1)$$

which leave $x_\mu x_\mu$ invariant. Three quantities, σ, σ_t, σ_s, are defined by

$$\begin{aligned}&\sigma=\det(a_{\mu\nu}),\\&\sigma_t=\text{sign of } a_{44},\\&\sigma_s=\text{sign of }\det(a_{\alpha\beta}),\quad(\alpha,\beta=1,2,3)\end{aligned} \quad (A.2)$$

which satisfy

$$\sigma\sigma_t\sigma_s=1. \quad (A.3)$$

Each of the three groups: $\sigma=(+1,-1)$, $\sigma_t=(+1,-1)$, $\sigma_s=(+1,-1)$, is homomorphic to the group G. More generally, any representation of the group Σ consisting of two elements, I and Z, satisfying

$$\begin{aligned}&\Sigma=I,\quad\text{for}\quad\sigma=+1,\quad\text{(rotations)},\\&\Sigma=Z,\quad\text{for}\quad\sigma=-1,\quad\text{(inversions)},\\&I^2=Z^2=I,\quad Z=ZI=IZ,\end{aligned} \quad (A.4)$$

is a nonfaithful representation of G. In a similar fashion, one can introduce $\Sigma_t=(I,Z_t)$ and $\Sigma_s=(I,Z_s)$, corresponding to $\sigma_t=\pm1$ and $\sigma_s=\pm1$. However, we limit

[17] We define the Fermi constants $F_V(=F_A)=F$ by the expression $F\bar{p}\gamma_\mu(1+\gamma_5)n\bar{e}\gamma_\mu(1+\gamma_5)\nu$.

[18] S. Oneda and Y. Tanikawa (Phys. Rev.), following paper.

[19] T. D. Lee and C. N. Yang, Phys. Rev. **108**, 1612 (1957).

[20] This ρ value seems to be in agreement with the recent experimental values including the "local" radiative correction. The "nonlocal" effect on the radiative correction will not be very large.

our discussion only to Σ here, since the entire derivation which follows can easily be adapted to the other two. It should, however, be noted that the use of Σ_t and Σ_s in the same way as Σ is used in the following will lead to a formalism which departs from the framework of *CTP* invariance.

Let us write symbolically the transformation rules of "regular" tensors of any rank r as

$$t' = At, \tag{A.5}$$

where A is just an r-fold juxtaposition of $A_{\mu\nu}$ of (A.1). There are three other irreducible representations of the same rank σA, $\sigma_t A$, and $\sigma_s A$. The quantities transforming according to these three representations have been named pseudotensors of the first, second, and third kind, respectively.[21] We shall write

$$t' = Bt \tag{A.6}$$

to express any one of the four transformation rules considered. A tensor or pseudotensor is said to belong to the "plus class" or "minus class" according as all of its components remain unchanged or change their signs by the total inversion of four coordinates (which is incidentally a rotation in the present terminology). It is easy to show[22] the following rule: regular tensors and first-kind pseudotensors of *even* ranks and second- and third-kind pseudotensors of *odd* ranks belong to the plus class, while all other alternatives belong to the minus class. The *CTP* theorem of the field theory, as will presently be shown, is a direct consequence of this rule. This rule also shows that the regular and first kinds (as well as the second and third kinds) can be mixed as far as the class is concerned. As will presently become clear, this is the reason why the chirality operators can be defined within the frame of *CTP*-invariant theories.

The chirality operator X is defined as an operator anticommuting with Z of (A.4)[23]:

$$[I,X]_- = 0, \quad [Z,X]_+ = 0, \quad X^2 = I. \tag{A.7}$$

If one introduces $Y = iXZ$, this Y also satisfies (A.7). Thus,

$$\begin{gathered} X^2 = Y^2 = Z^2 = I, \quad XYZ = iI, \\ [X,Y]_+ = [Y,Z]_+ = [Z,X]_+ = 0. \end{gathered} \tag{A.8}$$

It should be emphasized that the existence of one chirality operator implies the existence of two such operators.

If B of (A.6) is a faithful representation of G, then ΣB is also one. Unless one uses a one-dimensional representation of Σ which is σ, the new representation is a reducible representation as far as G is concerned. However, it is not reducible for the enlarged algebraic system including X. Thus, we consider in the following the transformation of a quantity Q given by

$$\begin{aligned} Q \to Q' \equiv \Sigma BQ &= BQ, \quad \text{for rotations,} \\ &= ZBQ, \quad \text{for inversions.} \end{aligned} \tag{A.9}$$

Since $Z^2 = I$, there are two eigenvalues, $Z = \pm I$, and correspondingly two eigenvectors $W_\pm$. The transformation rule (*A*.9) for them is

$$W_+ \to BW_+, \quad W_- \to \sigma BW_- \quad \text{for all transformations,} \tag{A.10}$$

since, for $Z = -I$, Σ is equivalent to σ as can be seen from (A.4). Out of the four possibilities, $B = A$, σA, $\sigma_t A$, $\sigma_s A$, the first two (the last two) give the same transformation in (A.10) except that W_+ and W_- are interchanged. Note that $\sigma\sigma_t\sigma_s = 1$. Thus, without loss of generality, we can limit B to A and $\sigma_t A$. The choice of these two is particularly convenient since we have in either case $W_\pm \to \pm AW_\pm$ for space inversion, agreeing with the usual notion of parity. For $B = A$, W_+ is a regular tensor and W_- is a first-kind pseudotensor. For $B = \sigma_t A$, W_+ is a second-kind pseudotensor and W_- is a third-kind pseudotensor.

Let us write more generally

$$ZW_\pm = \pm W_\pm, \quad XU_\pm = \pm U_\pm, \quad YV_\pm = V_\pm. \tag{A.11}$$

Further, in order to fix our representation, let us determine the phase relation between W_+ and W_- by the condition $\frac{1}{2}(X + iY)W_- = W_+$, as is usually done in the theory of angular momenta. This allows us to write

$$U_\pm = (W_+ \pm W_-)/\sqrt{2}, \quad V_\pm = (W_+ \pm iW_-)/\sqrt{2}, \tag{A.12}$$

and

$$\begin{aligned} XW_\pm &= W_\mp, & XV_\pm &= \pm iV_\mp, \\ YU_\pm &= \mp iU_\mp, & YW_\pm &= \pm iW_\mp, \\ ZV_\pm &= V_\mp, & ZU_\pm &= U_\mp. \end{aligned} \tag{A.13}$$

A chirality change by parity operation and a parity change by chirality operation are explicitly expressed in (A.13). If Q in (A.9) is an eigenstate of a chirality, such as $U_\pm$ or $V_\pm$, one can see that the rotations do not change the chirality whereas the inversions do change the chirality. For instance,

$$X(ZBU_\pm) = -ZBXU_\pm = \mp(ZBU_\pm). \tag{A.14}$$

Let us now examine the concepts introduced above by concrete examples taken from spinor calculus. At first, we shall assume the mathematical definition of spinors, i.e., we shall assume that a spinor transforms as

$$\psi \to \psi' = S\psi, \quad \text{with} \quad \gamma_\mu A_{\mu\nu} = S\gamma_\nu S^{-1}. \tag{A.15}$$

It is well known that $\bar{\psi}\varphi$, $i\bar{\psi}\gamma_\mu\varphi$, $i\bar{\psi}\gamma_{\mu\nu}\varphi$ (where $2\gamma_{\mu\nu} \equiv \gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu$) are then second-kind pseudotensors, i.e., equivalent to W_+ in (A.10) with $B = \sigma_t A$. On the other

[21] S. Watanabe, Phys. Rev. **84**, 1008 (1951); also S. Watanabe, Sci. Papers Inst. Phys. Chem. Research (Tokyo) **39**, 157 (1941).

[22] S. Watanabe, Revs. Modern Phys. **27**, 26 (1955).

[23] X and Z were first introduced in the appendix of S. Watanabe, Phys. Rev. **84**, 1008 (1951). In particular, t and u in (A.51) there are chiral eigenstates, since they are interchanged by the parity operation. (A.50) there corresponds to (A.9) of the present paper.

hand, $i\bar{\psi}\gamma_5\varphi$, $i\bar{\psi}\gamma_\mu\gamma_5\varphi$, and $\bar{\psi}\gamma_{\mu\nu}\gamma_5\varphi$ are third-kind pseudotensors, i.e., equivalent to W_- in (A.10) with $B=\sigma_t A$. This fact, however, does not determine the phase relation between W_+ and W_-; therefore it does not lead to a unique definition of chiral eigenstates, although a chiral eigenstate must have a general form $\bar{\psi}O(1+e^{i\alpha}\gamma_5)\varphi$, with a yet undetermined α, where O is 1, $i\gamma_\mu$, or $i\gamma_{\mu\nu}$. To eliminate this ambiguity, one will have to invoke some physically observable fact. For instance, one can define V_+ by the requirement that the helicity of a φ particle be -1 when it is produced with an extreme relativistic speed through an interaction involving V_+ (more precisely through its Hermitian conjugate term). This implies that $V_+=\bar{\psi}O(1+\gamma_5)\varphi$, and this automatically determines the remaining five eigenstates in virtue of (A.12).

$$\begin{aligned}
&W_+:\ \bar{\psi}\varphi, && \bar{\psi}\gamma_\mu\varphi, && \bar{\psi}\gamma_{\mu\nu}\varphi;\\
&W_-:\ -i\bar{\psi}\gamma_5\varphi, && -i\bar{\psi}\gamma_\mu\gamma_5\varphi, && -i\bar{\psi}\gamma_{\mu\nu}\gamma_5\varphi;\\
&U_\pm:\ \bar{\psi}(1\mp i\gamma_5)\varphi, && \bar{\psi}\gamma_\mu(1\mp i\gamma_5)\varphi, && \bar{\psi}\gamma_{\mu\nu}(1\mp i\gamma_5)\varphi;\\
&V_\pm:\ \bar{\psi}(1\pm\gamma_5)\varphi, && \bar{\psi}\gamma_\mu(1\pm\gamma_5)\varphi, && \bar{\psi}\gamma_{\mu\nu}(1\pm\gamma_5)\varphi.
\end{aligned} \tag{A.16}$$

The present covenant about the eigenstates is such that in V_+ the factor $(1+\gamma_5)$ should stand before the "annihilated particle," which is φ in the expression (A.16). However, φ can be considered also as the creation operator of a φ^c particle, and $\bar{\psi}$ as the annihilation operator of a ψ^c particle. Therefore, one may doubt whether the definition thus agreed upon still holds if the same quantity is written in terms of $\bar{\varphi}^c$ and ψ^c instead of φ and $\bar{\psi}$ as in (A.16). This possible ambiguity actually does not exist in the cases of a scalar and a tensor, for $\bar{\psi}(1\pm\gamma_5)\varphi=\bar{\varphi}^c(1\pm\gamma_5)\psi^c$ and $\bar{\psi}\gamma_{\mu\nu}(1\pm\gamma_5)\varphi=-\bar{\varphi}^c\gamma_{\mu\nu}(1\pm\gamma_5)\psi^c$ in virtue of (2.6). In each case the same factor $(1\pm\gamma_5)$ is standing before an annihilation operator. However, in the case of a vector, one has $\bar{\psi}\gamma_\mu(1\pm\gamma_5)\varphi=-\bar{\varphi}^c\gamma_\mu(1\mp\gamma_5)\psi^c$, thus the chirality defined by the helicity of the annihilated particle changes in the second expression. To eliminate this ambiguity, one will have to resort to some kind of "charge," such as electric charge, lepton number, muon number, baryon number, etc. If φ carries a positive "charge," then φ^c will carry a negative "charge." In writing a tensor quantity in a form bilinear in spinors, one should then require that the positive "charge" be annihilated instead of the negative "charge" be created, or vice versa. By this agreement, the chirality value will be uniquely determined.

The next question pertains to how one should determine the chiralities of the Hermitian conjugates of the quantities listed in (A.16). For instance, the Hermitian conjugate of $\bar{\psi}(1\pm\gamma_5)\varphi$ is $\bar{\varphi}(1\mp\gamma_5)\psi$. In the first expression, the φ charge is annihilated, while in the second the φ charge is created. This is obviously a different problem from the one considered in the preceding paragraph. The agreement here must be so made that if φ and ψ carry the same charge the agreement would not lead to a self-contradiction. (The agreement of the preceding paragraph does not lead to such a self-contradiction even if φ and ψ carry the same charge.) This consideration leads to a simple rule for determination of the chirality of the Hermitian conjugates. One needs only compare the factors $(1\pm\gamma_5)$ and $(1\pm i\gamma_5)$ standing before the annihilation operator, when comparison is made between a couple of mutually Hermitian conjugate quantities. According to this rule, one concludes that in the scalar and tensor cases, the Y-chirality changes by Hermitian conjugation while the X chirality does not change. In the vector case, the X chirality changes while the Y chirality remains unchanged.

The above considerations about chirality refer only to a tensor quantity built from two spinors. The boson field quantities which appear in our theory are supposed to transform as one of the six quantities mentioned in (A.16), and to have chiralities as defined in these expressions.

Coming back to the basic definition, (A.8), the relation $XYZ=i$ shows that any linear combination Q of W_+ and W_- (U_+ and U_- or V_+ and V_-) returns to its original value multiplied by a phase factor i by a successive application of Z, Y, and X. Thus, any product $Q^\dagger(1)Q(2)$ is an invariant (including the phase factor) for XYZ. Any product of the type $Q(1)Q(2)$ changes its sign by XYZ but retains its absolute value. This situation is somewhat analogous to the CTP theorem. In the well-established parity-conserving cases, the terms appearing in the Lagrangian are of the types: $W_+^\dagger(1)W_+(1)$, $W_-^\dagger(1)W_-(1)$, (free Lagrangian), $W_+(1)W_+(2)$, (electromagnetic interaction of the spinor field), $W_-(1)W_-(2)$, (pion-nucleon interaction), $\{W_-^\dagger(1)\partial_\mu W_-(1)-[\partial_\mu W_-^\dagger(1)]W_-(1)\}W_{+\mu}(2)$, (electromagnetic interaction of pions). All these terms are characterized by the fact that they are invariant for the Z operation (including phase), and noninvariant for the X and Y operations. From the present standpoint of complete symmetry among the three operations, X, Y Z, the allowed terms must be characterized by their being invariant for one of the three operations and noninvariant for each of the remaining two. Thus, the allowed Yukawa-type interactions must be extended to include $U_+(1)U_+(2)$, $U_-(1)U_-(2)$, $V_+(1)V_+(2)$, $V_-(1)V_-(2)$, of which the first two are invariant for X and noninvariant for Y and Z, and the last two are invariant for Y and noninvariant for Z and X.

If a chiral boson field, say V_+, has an electromagnetic interaction, it will take the form $\{V_+^\dagger(1)\partial_\mu V_+(1)-[\partial_\mu V_+^\dagger(1)]V_+(1)\}W_{+\mu}$ which is noninvariant for any one of X, Y, Z. However, if there is another field $V_-(1)$ which has the opposite chirality to $V_+(1)$ but the same electric charge, then the total current due to $V_+(1)$ and $V_-(1)$ will have a positive parity, since $V_\pm(1)$ passes to $V_\mp(1)$ by parity operation, see (A.13). Thus, in order to make the electromagnetic interaction of the

B'-field parity-conserving, one is led to assume another chiral boson field, say B''' which forms a chiral doublet with B'. We cannot determine what leptonic, muonic, and baryonic charges (for that matter any charge other than electric charge) the B''' should carry. It is not excluded that such a chiral boson may have some role to play in a Fermi-interaction involving strange fermions. Another possibility is that the electromagnetic field can be decomposed into a pair of chiral fields, V_+ and V_-, and the V_+ part only may interact with the charged chiral B particle.

It is of interest to consider again the analogy of $Z/2$ to the z component of the spin of an electron. The four spin eigenstates of a two-electron system have then the following analogs:

$$W_+(1)W_+(2),\quad W_-(1)W_-(2),$$
$$W_+(1)W_-(2)+W_-(1)W_+(2),$$
$$W_+(1)W_-(2)-W_-(1)W_+(2).$$

The first two which are allowed in the present sense are characterized by the nonvanishing total z spin: $[Z(1)+Z(2)]/2$. The last two which are forbidden exhibit a spherical symmetry. The extension of this consideration to the x and y direction leads to the present criterion.

So far, we have not considered charge conjugation, which necessarily involves Hermitian conjugation. Time reversal of the Wigner type in field-theoretical definition also involves Hermitian conjugation. Let us write symbolically the operation of charge conjugation as

$$C:\quad Q\rightarrow Q'=e^{i\eta_C}CQ. \tag{A.17}$$

although CQ is not a linear operation. This C is different from the C of (2.6). Then, the field-theoretically redefined time reversal is given by

$$T:\quad Q\rightarrow Q'=e^{i\eta_T}CT_0Q, \tag{A.18}$$

where T_0 is the operator of time reversal defined from the mathematical point of view. The field-theoretical space inversion P is the same as its mathematical counterpart, P_0.

$$P:\quad Q\rightarrow Q'=e^{i\eta_P}P_0Q. \tag{A.19}$$

Combining (A.17), (A.18), and (A.19), one obtains

$$CTP:\quad Q\rightarrow Q'=e^{i(\eta_C+\eta_T+\eta_P)}T_0P_0Q, \tag{A.20}$$

by virtue of the commutability of C and T_0 and $C^2=1$. Formulas (A.17) through (A.20) are general rules applicable to any tensor quantities of any given rank and any given kind.[24]

Now according to the theorem mentioned before regarding plus and minus classes, we have $T_0P_0=(-1)^{r+1}$ for second and third kinds, and $T_0P_0=(-1)^r$ for regular and first kinds, where r is the rank of the tensor Q.[22] The "kind" here means the one defined by the "mathematical" transformation. From this one obtains immediately the CTP theorem[25,26]:

$$\begin{aligned} CTP:\quad Q\rightarrow Q'&=e^{i(\eta_C+\eta_T+\eta_P)}(-1)^{r+1}Q \\ &\qquad \text{for second and third kinds} \\ &=e^{i(\eta_C+\eta_T+\eta_P)}(-1)^{r}Q \\ &\qquad \text{for regular and first kinds.}\end{aligned} \tag{A.21}$$

It can easily be seen that this rule is internally consistent for multiplication of two tensor quantities of any ranks and any kinds with or without contraction. A scalar quantity, such as a term in the Lagrangian, obeys this rule with $r=0$. The eigenstates of X and Y are a mixture of second and third kinds or a mixture of regular and first kinds. Therefore the rule (A.21) still holds for them. It is now obvious that the use of Σ_t and Σ_s would violate the CTP theorem, since the chirality operators defined by them would mix the two lines of (A.21).

Let us observe more closely the transformations (A.17), (A.18), and (A.19) in the special cases where Q is bilinear in spinors, i.e., of the form $\bar{\psi}O\varphi=\psi^\dagger\gamma_4O\varphi$. Let us understand in (A.17) that CQ means the quantity which is directly obtained by $\varphi\rightarrow\varphi^c$ and $\psi\rightarrow\psi^c$ where φ^c and ψ^c are defined by (2.6). (This implies $e^{i\eta_c}=1$ for $\varphi=\psi$.) Then, in general, $CQ=\pm Q^\dagger$ if O is one of the well-known six operators. We can add or eliminate the imaginary unit to or from O so that $CQ=+Q^\dagger$. W_+ and W_- in (A.16) turn out to satisfy precisely this condition, i.e., $CW_\pm=W_\pm{}^\dagger$. Disregarding in (A.17) the factor $e^{i\eta_c}$ which can be considered as an arbitrary gauge transformation for ψ and φ and which is a common factor to all the quantities, one then obtains

$$C:\quad W_\pm\rightarrow W_\pm{}^\dagger,\quad U_\pm\rightarrow U_\pm{}^\dagger,\quad V_\pm\rightarrow V_\mp{}^\dagger. \tag{A.22}$$

As we have seen before, the Hermitian conjugation itself entails the X-chirality change in the vector case, and Y-chirality change in the scalar and tensor cases. (A.22) means that in addition to this, charge conjugation entails the Y-chirality change. The results (A.22) are obtained in the case where Q is bilinear in spinors, but our basic assumption is that any boson field quantity that appears in our theory behaves like one of those Q's which are bilinear in spinors.

The results (A.22) lead to a useful conclusion. First, one has to note that if a quantity Q appears in a Lagrangian, it is necessary that its Hermitian conjugate $Q^\dagger$ also appears in the Lagrangian. If the chiral boson is in an X eigenstate, $U_\pm$, then $U_\pm$ as well as $U_\pm{}^\dagger$ will appear in a Lagrangian. The same is true for the quantity that multiplies $U_\pm$ in the Lagrangian. Now (A.22) shows that $U_\pm$ will pass to $U_\pm{}^\dagger$ by charge

[24] S. Watanabe, Revs. Modern Phys. **27**, 40 (1955). See, in particular, Sec. 4.

[25] For special case where Q is bilinear in spinors, see Table III in the paper quoted in reference 24. If the two spinors are the same, then $e^{i(\eta_C+\eta_T)}=-1$, $e^{i\eta_P}=1$. $\rho_{CP}\rho_{RPM}$ of the table is equal to $-(-1)^{r+1}$ which agrees with the first line of (A.21).

[26] W. Pauli, in *Niels Bohr and the Development of Physics*, edited by W. Pauli (Pergamon Press, Inc., London, 1955).

conjugation. Consequently, a theory that uses $U_\pm$ is a C-invariant (therefore TP-invariant) theory. Next, T_0 and P_0 defined by mathematical definition change both X chirality and Y chirality. (Incidentally, $CTP \approx T_0P_0$ thus conserves both chiralities.) Now, (A.22) shows that C results in Hermitian conjugation accompanied by Y-chirality change. Consequently $CP_0 = CP$ and $CT_0 = T$ will result in a simple passage from $V_\pm$ to its Hermitian conjugate $V_\pm{}^\dagger$. Thence, one concludes that a theory that uses $V_\pm$ is a T-invariant (therefore CP-invariant) theory. Obviously, it is possible to define a chirality so that its eigenstate has the form $\bar{\psi}O(1 \pm e^{i\alpha}\gamma_5)\varphi$ with an arbitrary real α. Such a theory in general will be only CTP invariant.

There are two apparently different prescriptions to perform T. One alternative, adopted by Pauli[26] and one of the authors (S.W.),[27] is as follows. When $Q = g\bar{\psi}O\varphi$ is given, one first takes $(g\bar{\psi}O\varphi)^T = g\varphi^T O^T \bar{\psi}^T$, and then applies $\varphi^T \rightarrow e^{i\alpha}\bar{\varphi}\gamma_4\gamma_5 C$ and $\bar{\psi}^T \rightarrow e^{-i\beta}C^{-1}\gamma_5\gamma_4\psi$. Thus,

$$T: \quad g\bar{\psi}O\varphi \rightarrow ge^{i(\alpha-\beta)}\bar{\varphi}\gamma_4\gamma_5(C^{-1}OC)^T\gamma_5\gamma_4\psi. \quad \text{(A.23)}$$

Another alternative, adopted by Lüders,[28] Lee, Oehme, and Yang,[29] can be derived from the first alternative in the following way. If there is $Q = g\bar{\psi}O\varphi$ in the Lagrangian, then there will also be its Hermitian conjugate $Q^\dagger = g^*\bar{\varphi}\gamma_4 O^\dagger \gamma_4 \psi$ in it. Applying (A.23) on $Q^\dagger$, one obtains $g^* e^{i(\beta-\alpha)}\bar{\psi}\gamma_4\gamma_5(C^{-1}\gamma_4 O^\dagger \gamma_4 C)^T \gamma_5\gamma_4\varphi$. Lüders proposes to compare this not to $Q^\dagger$ but to Q. Thus

$$T: \quad g\bar{\psi}O\varphi \rightarrow g^* e^{i(\beta-\alpha)}\bar{\psi}\gamma_4\gamma_5(C^{-1}\gamma_4 O^\dagger \gamma_4 C)^T\gamma_5\gamma_4\varphi. \quad \text{(A.24)}$$

Now, if one uses the representation in which $(\gamma_a{}^\dagger)^T = \gamma_a$ $(a = 1, 2, 3)$, $(\gamma_4{}^\dagger)^T = -\gamma_4$ and $(\gamma_5{}^\dagger)^T = -\gamma_5$, then $(C^{-1}\gamma_4 O^\dagger \gamma_4 C)^T$ can be obtained from O by replacing each γ_i $(i = 1, 2, 3, 4, 5)$ involved in O by $(\gamma_i{}^\dagger)^T$. Thus, (A.24) can be formally expressed as $\varphi \rightarrow e^{-i\alpha}\gamma_5\gamma_4\varphi$, $\bar{\psi} \rightarrow e^{i\beta}\bar{\psi}\gamma_4\gamma_5$, $\gamma_i \rightarrow (\gamma_i{}^\dagger)^T$, $g \rightarrow g^*$. The transformation rule (A.24) leads to a conclusion apparently contradictory to the result obtained in the foregoing with regard to the effect of T on the chirality eigenstate. For instance, we obtain

$$T: \quad \bar{\psi}(1+\gamma_5)\varphi \rightarrow \bar{\varphi}(1-\gamma_5)\psi, \quad \text{present formulation} \quad \text{(A.25)}$$

$$\bar{\psi}(1+\gamma_5)\varphi \rightarrow \bar{\psi}(1+\gamma_5)\varphi, \quad \text{Lüders.}$$

The present formalism implies a chirality change by T, while the Lüders formalism implies no chirality change by T. However, this paradox is obviously only a matter of convention. Indeed, if there is $\bar{\psi}(1+\gamma_5)\varphi$ in the Lagrangian, there will also be $\bar{\varphi}(1-\gamma_5)\psi$ in it. The difference lies only in pairing of two quantities when comparison is made. The physically observable results, of course, do not depend on this difference in interpretation. It is needless to say that our previous statement regarding the T invariance of the theory is independent of this difference in convention.

[27] See paper quoted in references 21 and 24.

[28] G. Lüders, Kgl. Danske Videnskab. Selskab. Mat.-fys. Medd. **28**, No. 5 (1954).

[29] Lee, Oehme, and Yang, Phys. Rev. **106**, 340 (1957).