Optical Model Analysis of Elastic Scattering of 125-Mev K^+ Mesons in Nuclear Emulsions*

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The diffuse-surface optical model has been used to analyze the elastic scattering and interaction cross-section data obtained from 94 meters of positive K-meson track in nuclear emulsion between energies of 100 and 150 Mev. The analysis was carried out on the SWAC digital computer through the use of the existing proton code which required but slight modification, such as the use of the Klein-Gordon equation. A thorough investigation was made of the number of energy intervals and representative emulsion nuclei that were required for adequate accuracy in the computed averaged cross sections. This number was found to be one energy and two representative emulsion nuclei (heavy and light). The four parameters (R_0, the radius parameter; a, the edge thickness; V, the real part of the potential; and W, the imaginary part of the potential) were varied and the goodness of fit was tested by means of the χ^2 probability, there being nine degrees of freedom. An attractive real potential is fairly clearly ruled out and good fits were obtained for a repulsive real potential for a fairly wide range of physically acceptable geometrical parameters. If the R_0 and a are chosen from electron-scattering experiments, then $V=21\pm4$ Mev and $W=-11.0\pm1.5$ Mev. If R_0 is increased to 1.20×10^{-13} cm, then $V=14\pm2.5$ Mev and $W=-7\pm1$ Mev.

1. INTRODUCTION

THE remarkable success of the optical model in accounting for the elastic scattering of nucleons, α particles, and pions from complex nuclei, has led to its application to the scattering of K mesons by emulsion nuclei.[1,2] The results of such analyses are still somewhat ambiguous due to several factors: (1) participation of several types of target nuclei, (2) relatively large standard deviations in the experimentally obtained cross section, and (3) relatively small angular regions over which experimental cross sections are available. Still, the model is in fair agreement with the data when physically reasonable values are used for its parameters, and it appears worthwhile to carry out further analyses in the hope that this may help to determine the elementary K^+-meson-nucleon interaction.

The present analysis determines the parameters of the diffuse surface optical model for 100–150 Mev K^+ mesons elastically scattered by emulsion nuclei by fitting the differential elastic scattering cross section, $d\sigma/d\Omega$, and the total reaction cross section σ_R. The experimental cross sections were obtained from the analysis of interactions found in 94 meters of K^+-meson track in nuclear emulsion.[3] The reaction cross section averaged over emulsion nuclei was found to be $\sigma_R=299\pm26$ mb and the differential cross section will be indicated by the points on Figs. 4, 5, and 6. The theoretical values of the cross section were calculated from the phase shifts which were obtained by numerically integrating the Klein-Gordon equation for each angular momentum. The cross sections were then averaged over suitable energy and angular intervals and for the nuclear species representative of the emulsion. The parameters of the model, which include the real and imaginary potential depths, V and W, the half-falloff potential radius, $R=R_0A^{\frac{1}{3}}$, and the surface thickness a, were chosen to optimize the fit between experimental and theoretical cross sections.

The results of this analysis still leave a considerable amount of ambiguity inasmuch as a fairly wide range of the geometrical parameters R_0 and a are found to give acceptable fits to the experimental data. However, for a given set of these two geometrical parameters the potential depths are fairly well defined. It has also been confirmed that a repulsive potential is by far more likely than an attractive one.

2. EQUATIONS AND POTENTIALS

The theoretical cross sections were computed as usual from the phase shifts which are obtained by matching at large distances the logarithmic derivatives of the Coulomb functions with those of the radial solutions of the Klein-Gordon equation.

The Klein-Gordon equation,

$$\nabla^2\psi+\frac{(E-V_T)^2-(mc^2)^2}{\hbar^2c^2}\psi=0,$$

reduces for $E\gg V$ to

$$\nabla^2\psi+k^2\left[1-\frac{V_T}{E_k}\left(\frac{1+\alpha}{1+\frac{1}{2}\alpha}\right)\right]\psi=0,$$

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[1] L. S. Osborne, Phys. Rev. **102**, 296 (1956); Cocconi, Puppi, Quareni, and Stanghellini, Nuovo cimento **5**, 172 (1957); Ceolin, Cresti, Dallaporta, Grilli, Guerriero, Merlin, Saladin, and Zago, Nuovo cimento **5**, 402 (1957); G. Costa and G. Patergnani, Nuovo cimento **5**, 448 (1957); Hoang, Kaplon, and Cester, Phys. Rev. **107**, 1698 (1957); D. Fouret-Davis, Phys. Rev. **106**, 816 (1957); Marchi, Quareni, Vignudelli, Dascola, and Mora, Nuovo cimento **5**, 1790 (1957); T. G. Lim and S. J. Bosgra, Nuovo cimento **8**, 340 (1958); Bhowmik, Evans, Nilsson, Prowse, Anderson, Keefe, Kerman, and Losty, Nuovo cimento **10**, 440 (1958).

[2] Igo, Ravenhall, Tiemann, Chupp, G. Goldhaber, S. Goldhaber, Lannutti, and Thaler, Phys. Rev. **109**, 2133 (1958).

[3] Oliver R. Price, thesis, University of California at Los Angeles, 1958 (unpublished); Price, Stork, and Ticho (to be published).

TABLE I. Energies and weights representative of the spread of K^+-meson energy.

	E (Mev)	Weight (%)
3-energy group	105	29
	125	43
	145	28
2-energy group	110	50
	140	50
1-energy group	125	100

and the radial equation for the Lth partial wave becomes

$$\frac{d^2\psi}{dr^2}+k^2\left[1-\frac{V_T}{E_k}\left(\frac{1+\alpha}{1+\frac{1}{2}\alpha}\right)-\frac{L(L+1)}{k^2r^2}\right]\psi=0, \quad (1)$$

where

$$k^2=(2mE_k/\hbar^2)(1+\tfrac{1}{2}\alpha),$$
$$\alpha=E_k/mc^2,$$
$$E_k=E-mc^2.$$

The potential is the same as that used in the proton scattering analysis[4]:

$$V_T=V_N+V_C,$$

where

$$V_N=(V+iW)/\{1+\exp[(r-R)/a]\},$$
$$V_C=(Ze^2/2R)(3-r^2/R^2) \quad \text{for} \quad r\leqslant R$$
$$=Ze^2/r \quad \text{for} \quad r\geqslant R,$$

and

$$R=R_0A^{\frac{1}{3}}.$$

V, W, R_0, and a constitute the four parameters of the diffuse-surface optical model.

Equation (1) was numerically integrated for each value of L which contributes to the cross sections. The numerical integrations were carried out by the Runge-Kutta method on the SWAC using a revised version of the proton scattering code.[4]

TABLE II. Composition of nuclear emulsion and representations used for computation.

	Element	A	Z	Weight (%)
Composition of nuclear emulsion	I	126.9	53	0.5
	Ag	107.9	47	21.4
	Br	79.9	35	20.9
	S	32.1	16	0.4
	O	16.0	8	22.7
	N	14.0	7	5.9
	C	12.0	6	28.1
3-representative target nuclei group		108	47	21.3
		80	35	21.3
		14	7	57.4
2-representative target nuclei group		94	41	42.6
		14	7	57.4

[4] Melkanoff, Nodvik, Saxon, and Woods, Phys. Rev. **106**, 793 (1957).

3. ANALYSIS

The theoretically computed cross sections must be suitably averaged before they can be compared with the experimental data. This must include an averaging of the differential elastic scattering cross section and of the total reaction cross section over the experimental energy spread and over the nuclear species which make up the emulsion. In addition, the computed differential elastic scattering cross section must be averaged over the angular regions of the experimental histograms.

(a) Averaging Over Energy Spread and Over Nuclear Species

The energy spread of the K^+ mesons is given in reference 1, and may be conveniently taken into account by averaging the cross section over various numbers of energy intervals as shown in Table I. Similarly the nuclear composition of the emulsion can also be taken into account by averaging the cross sections obtained with various "representative target nuclei" as shown in Table II. These two procedures were combined in so-called n-run groups of Table III; and the cross sections were obtained by averaging in the combined weights the results of n runs, each of which is carried out at a given energy and with a given "representative target nucleus." The same values of the optical model parameters were used in each of the n runs of an n-run group. This approximation is discussed later.

TABLE III. Representative energies and nuclear species used in computation for averaging the differential elastic scattering and total reaction cross section.

	E (Mev)	A	Z	Weight (%)
9-run group: 3-energy group and 3-representative target nuclei group (used in computation)	105	108	47	6.6
	105	80	35	6.6
	105	14	7	11.2
	125	108	47	9.8
	125	80	35	9.8
	125	14	7	26.4
	145	108	47	6.3
	145	80	35	6.3
	145	14	7	17.2
2-run group: 1-energy group and 2-representative target nuclei groups (used in computation)	125	94	41	42.6
	125	14	7	57.4

As a result of a number of trial computations with 9-run, 6-run, 4-run, and 2-run groups, it was found that the 2-run groups provided an adequate approximation to the much more realistic 9-run groups. This may be seen on Fig. 1 which shows $(d\sigma/d\Omega)\sin^4(\theta/2)$ *versus* θ for 2-run and 9-run groups for typical values of attractive and repulsive potentials.[5]

[5] The usual presentation of the results in the form $(d\sigma/d\Omega)/(d\sigma/d\Omega)_R$ *versus* θ [where $(d\sigma/d\Omega)_R$ is the Rutherford differential

The relatively narrow experimental energy spread thus allows the use of a single energy group and considerably reduces the amount of computation. It also makes it unnecessary to resort to q plots $[q=2k\sin(\theta/2)]$, as was done so successfully by Igo *et al.*,[2] which would have necessitated further revisions of the proton scattering code.

The bulk of the analysis was thus carried out with 2-run group computations and it was supplemented by twelve 9-run group computations after the region of best fit had been fairly precisely located in the parameter space.

(b) Averaging Over Angular Intervals

In order to take into account the angular interval grouping of the experimental data, the computed values of the differential elastic cross section were averaged over the experimental angular spreads. This was done by integrating the cross section according to Simpson's rule over the angular interval θ_{i-1} to θ_{i+1}. The experimental values of $[d\sigma(\theta_i)/d\Omega]_{\text{exp}}$ were then compared with the computed averaged value

$$\left\langle \frac{d\sigma(\theta_i)}{d\Omega} \right\rangle_{\text{Av}} = \frac{1}{6}\left[\frac{d\sigma(\theta_{i+1})}{d\Omega} + 4\frac{d\sigma(\theta_i)}{d\Omega} + \frac{d\sigma(\theta_{i-1})}{d\Omega} \right].$$

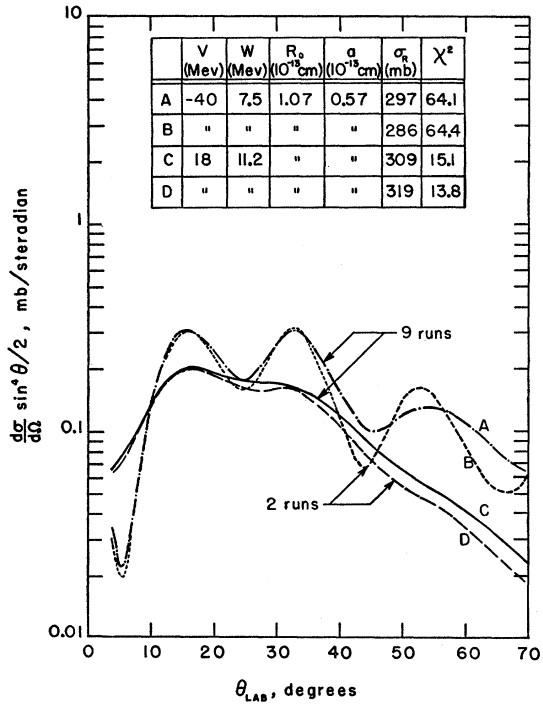

FIG. 1. Comparison of the results of 2-run and 9-run groups for typical values of attractive and repulsive potentials.

elastic scattering cross section] is not feasible here because of the averaging over energy intervals and/or representative target nuclei. However, the use of the factor $\sin^4(\theta/2)$ in the abscissa has a similar effect.

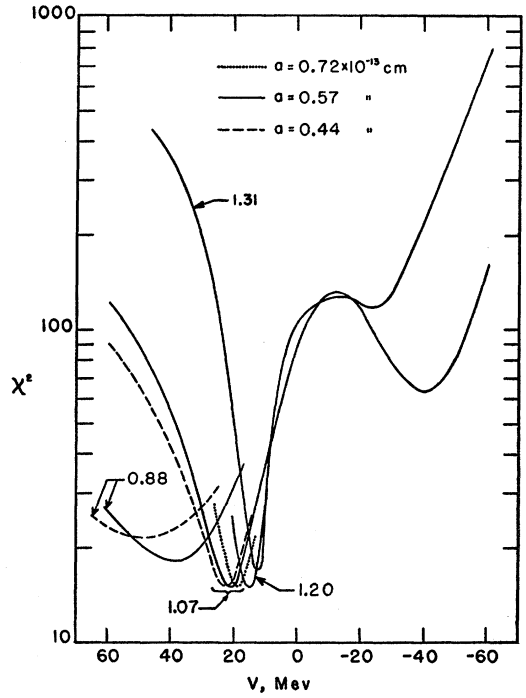

FIG. 2. χ^2 results of fitting the elastic scattering as a function of V for several sets of R_0 and a. The numbers labeling the curves are the values of R_0 in units of 10^{-13} cm. W is fixed by the reaction cross section.

The effect of this averaging procedure was found to be fairly important because of the steepness of the curve $d\sigma/d\Omega$ *versus* θ and the relative broadness of the experimental angular intervals.

The program also includes a relativistic center-of-mass to laboratory transformation of $d\sigma/d\Omega$ and θ since the experimental results are given in the laboratory system while the computed values are obtained in the center-of-mass system. The effect of transforming from laboratory to center-of-mass coordinates is appreciable only for the light elements, and thus contributes a small correction to the final results. The use of a relativistic transformation is hardly needed in the present analysis but was incorporated to allow for future analyses at higher energies of K^+ meson interactions in emulsions.

(c) Method of Data Fitting

The precision with which the computed values of the differential elastic scattering cross section fit the experimental data is estimated by calculating the value of χ^2 for each set of the parameters of the optical model:

$$\chi^2 = \sum_{i=1}^{12} \left\{ \frac{[\langle d\sigma(\theta_i)/d\Omega\rangle_{\text{Av}}]_{\text{exp}} - [\langle d\sigma(\theta_i)/d\Omega\rangle_{\text{Av}}]_{\text{calc}}}{\Delta[d\sigma(\theta_i)/d\Omega]_{\text{exp}}} \right\}^2,$$

where $[\langle d\sigma(\theta_i)/d\Omega\rangle_{\text{Av}}]_{\text{calc}}$ is the computed value of the differential scattering cross section averaged over

TABLE IV. Results of best fits to experimental data for various values of the geometrical parameters of the diffuse-surface optical model.

R_0 (10^{-13} cm)	a (10^{-13} cm)	V (Mev)	W (Mev)	σ_R (mb)	χ^2	P (%)	Number of averaged runs
0.88	0.44	50	35	294	21.5	1.0	2
0.88	0.57	40	24	302	18.4	3.0	2
1.07	0.44	22.5	12.9	297	15.2	8.2	2
1.07	0.57	20	11.2	309	15.1	8.4	2
1.07	0.72	18	8.9	302	15.5	7.6	2
1.20	0.57	15	7	300	15	8.8	2
1.31	0.57	12.5	5.1	301	17.1	4.6	2
1.07	0.57	20	11.2	308	14.2	11.0	9

energy, representative target nuclei, and angular spread and $\Delta[d\sigma(\theta_i)/d\Omega]_{\text{exp}}$ is the experimental standard derivation.

In addition to fitting the angular distribution, a given set of parameters must also yield agreement with the experimental reaction cross section. Now the reaction cross section might be taken into account by adding another term to the χ^2 summation, *viz.*:

$$\left\{\frac{(\bar{\sigma}_R)_{\text{exp}}-(\bar{\sigma}_R)_{\text{calc}}}{\Delta(\bar{\sigma}_R)_{\text{exp}}}\right\}^2.$$

In this case χ^2 would be characterized by nine degrees of freedom: 13 terms less 4 free parameters. Although such a procedure is intrinsically possible, it puts an integrated quantity on the same footing as the individual points of an angular distribution. Furthermore, it was noticed during preliminary computations that a variation in W produced considerable effect on σ_R though relatively little effect on $d\sigma/d\Omega$. It was therefore decided to use the reaction cross section to pin down the parameter W for given values of the other three parameters and thus considerably reduce the required amount of computation. Note that this procedure still yields 9 degrees of freedom: 12 terms less 3 free parameters (W is no longer a free parameter); the only difference is the implicit assumption that χ^2 for 13 terms is automatically minimized by choosing W to fit the reaction cross section for given values of V, R_0, and a. Further computation later showed that this assumption was not always justified; nonetheless the procedure was still followed in order to keep the total required computation within practical limits.

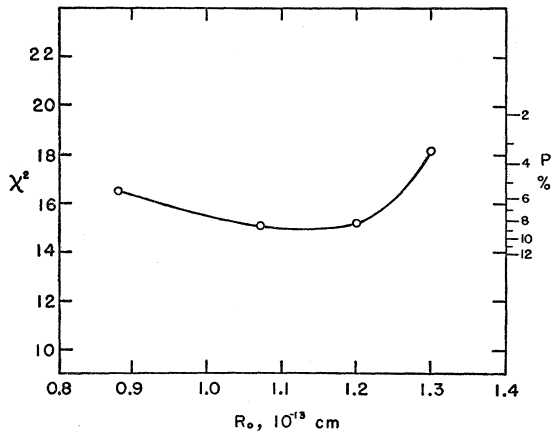

FIG. 3. Minimum χ^2 values and the χ^2 probability as a function of R_0 for $a=0.57\times10^{-13}$ cm.

After some exploratory runs it became clear that the data could almost equally well be fitted with a large number of sets of parameters, and it was therefore decided to explore systematically some physically meaningful values of the geometrical parameters R_0 and a. Thus for a given value of R_0 and a, a series of runs were carried out over a fairly wide range of values of V:

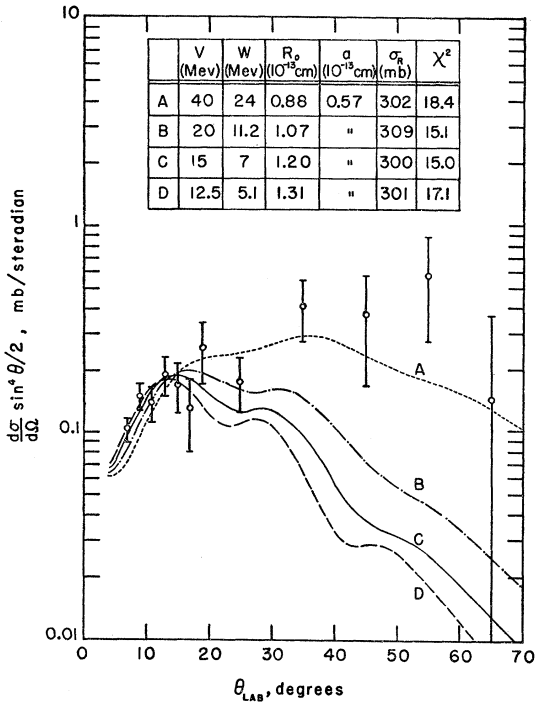

FIG. 4. Comparison of the best-fit calculations with the experimental data for several values of R_0 and for $a=0.57\times10^{-13}$ cm.

-60 (20) 60 Mev, adjusting W to obtain agreement between the computed and experimental value of the reaction cross section. Further runs were carried out around the minima of χ^2 until they became quite well defined. Finally some 9-run groups were made at the minima in order to check the accuracy of the 2-run group results.

4. RESULTS

The results of the analysis are presented on Fig. 2 and in Table IV. The best value of the χ^2 probability was found to be 11%. While a more accurate model might give a higher value of P, the present results appear acceptable and attest to the adequacy of the

diffuse-surface optical model. It may be seen that a relatively large range of the parameters R_0 and a give a fairly good fit to the experimental data. This is also indicated on Fig. 3 where χ^2 and the χ^2 probability, P, are plotted as functions of R_0 for a given value of a, with V and W varied so as to optimize the fits as described above. The actual curves corresponding to the four points of Fig. 3 are shown on Fig. 4. Figure 5 shows some results in the more conventional form $d\sigma/d\Omega$. It appears that R_0 may be chosen between 0.8 and 1.3 if one allows the χ^2 probability to fall to one third of its maximum value. Similarly the surface thickness parameter, a, may be chosen anywhere between 0.71 and 0.44 to yield equally good fits to the experimental data. Regions of upper and lower limits

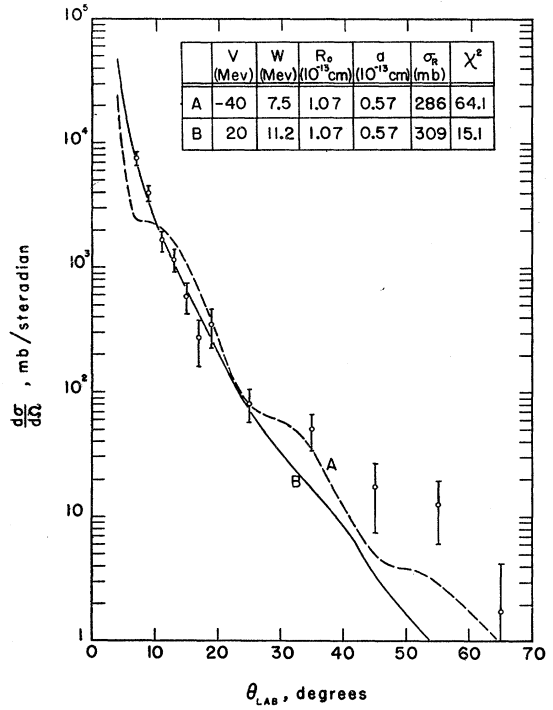

FIG. 5. Comparison of the best-fit attractive and repulsive potential calculations with the experimental data.

to a for a good fit were not explored. Typical curves showing the effects of varying a are presented on Fig. 6.

For given values of R_0 and a, the real potential depth V is fairly sharply defined while the imaginary potential depth is chosen to fit the reaction cross section. The behavior of these parameters as functions of R_0 for various values of a is indicated on Fig. 7.

In spite of the ambiguity in defining the best set of parameters it appears from Fig. 2 that a repulsive potential is definitely preferable to an attractive one as noted earlier.[2]

If the geometrical parameters R_0 and a are fixed by other information, such as electron scattering,[6] the

[6] Hahn, Ravenhall, and Hofstadter, Phys. Rev. **101**, 1131 (1956).

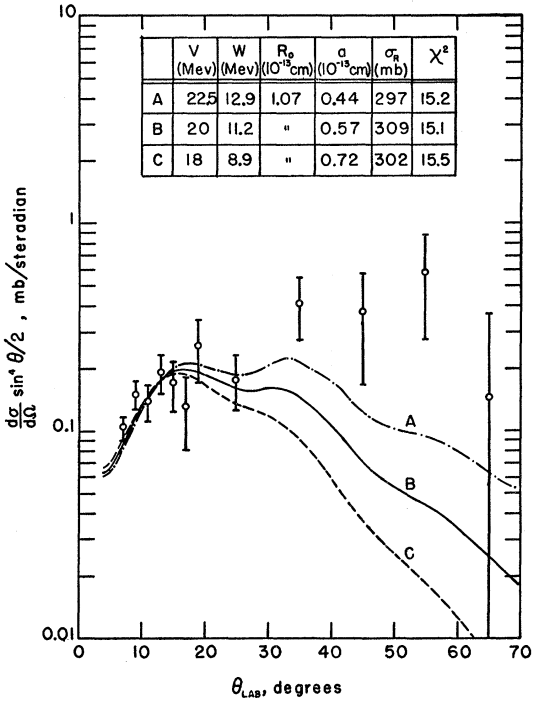

FIG. 6. Comparison of the best-fit calculations with the experimental data for several values of a and for $R_0=1.07\times10^{-13}$ cm.

potential depths are fairly well defined; thus for $R_0=1.07\times10^{-13}$ cm and $a=0.57\times10^{-13}$ cm, $V=21\pm4$ Mev and $W=11\pm1.5$ Mev, where the limits on V correspond to a threefold decrease in the χ^2 probability while the limits on W correspond to a change of one standard deviation in the reaction cross section.†

The results given above are based on several assumptions:

(1) The potential depths are sufficiently small to warrant neglecting terms of the order of $(V_T/E)^2$ in the original Klein-Gordon equation. This assumption seems justified by the results of the present analysis.

(2) The potential depths are independent of energy. While such an assumption is not strictly correct, comparison with the parameters of Igo *et al.*,[2] obtained at lower energy, show it to be an acceptable approximation.

(3) The potential depths are independent of mass numbers. The main variation is expected for light elements, but their contribution is small and this effect can hardly be expected to change the final results.

(4) Best fits may be obtained by fitting W to the reaction cross section for each set of values of V, R_0,

† *Note added in proof.*—Reanalysis of the correction for pseudo-elastic events as described in reference 3 has led to a lower reaction cross section $\sigma_R=276\pm26$ mb and a very slight increase in differential elastic cross section. The revised best-fit complex potentials for two geometries are:

10^{13} R_0, cm	10^{13} a, cm	V, Mev	W, Mev
1.07	0.57	23 ± 4	-9.7 ± 1.3
1.20	0.57	14 ± 3	-6.4 ± 0.9

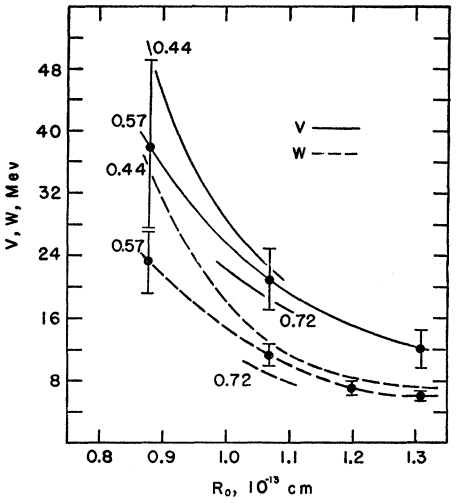

FIG. 7. Best-fit values of V and W as a function of R_0 for $a=0.44$, 0.57, and 0.75×10^{-13} cm.

and a. Although this is not strictly correct, we believe that the results would not be significantly changed if W were included in an over-all χ^2 fitting.

(5) The χ^2 distribution may be used even though some of the data follow a Poisson rather than a Gaussian distribution. At the larger angles intervals containing small numbers of events were used.

In conclusion the present experimental results admit of a fairly good fit by the diffuse-surface optical model of the nucleus, and although the parameters are not too well defined, they seem to lie in a physically reasonable region. For a given set of geometrical parameters, V and W are well determined. Furthermore a repulsive potential is definitely indicated. Further refinements of the model await more extensive and accurate data.

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