Excitation of Spin Waves in an Antiferromagnet by a Uniform rf Field*

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It is possible to excite spin waves in an antiferromagnet by a uniform rf field, provided that spins on the surface of the specimen experience anisotropy interactions different from those acting on the spins in the interior. Modes with an odd number of half-wavelengths should be excited in a flat plate. The condition for different anisotropy interactions is worked out and proves to be a lenient condition. Experiments which would determine the exchange energy constant and the anisotropy field should be possible using sufficiently thin platelets of single crystals having parallel faces.

I. INTRODUCTION

KITTEL[1] has shown that under certain conditions exchange and magnetostatic spin waves may be excited in a ferromagnetic insulator by a uniform rf field. This excitation is in contrast to the White-Solt effect[2] produced by an inhomogeneous rf field. Recently, Seavey and Tannenwald[3] and Jarrett and Waring[4] have found these excitations in thin films and thin crystals, respectively. The application of the Kittel theory to antiferromagnets was suggested by Strandberg and Douglass,[5] and is the subject of this paper.

The local environment in antiferromagnets, as well as in ferromagnets, of a spin at the surface of a crystal is markedly different from that in the interior. Anisotropy interactions which would normally vanish in the interior because of symmetries no longer do so with the lower symmetry of the surface. It was shown by Kittel that the effect of the surface anisotropy is to pin the surface spins, leading to modes which interact with a uniform rf field. That is, if one thinks of a line of length L with the origin at one end, the modes will have the form $\sin(p\pi z/L)$, where p is an integer. The modes of odd p provide an instantaneous transverse moment which couples with a uniform rf field.

At first glance the situation in antiferromagnets might appear to be different. There is no net magnetization in the antiferromagnet, so that a $\sin(p\pi z/L)$ excitation would not appear to lead to any net moment. However, as Keffer and Kittel[6] point out, a linear combination of two modes exist at resonance. Both modes have a net transverse magnetization and rotate in opposite senses. For both resonance modes, the spins on the two sublattices precess in different sized circles, the difference in the amplitudes of the precession leading to the net magnetic moment. The sum of the moments of the two modes generates an oscillating moment perpendicular to the z axis. Along a line of spins, the phases of these oscillating moments will differ by an amount determined by the wave number of the spin wave. Thus, at any instant of time there will exist a net transverse moment which can interact with the rf field if and only if an odd number of half-wavelengths are excited. If there exists an even number of half-wavelengths, the instantaneous transverse moment will sum to zero and there will be no interaction with the rf field.

II. EVALUATION OF SURFACE ANISOTROPY STRENGTH

We first examine the question of the strength of the surface anisotropy required to fix the end spins. On an atomic model, the equations of motion for the end spin ($m=1$) and its nearest neighbor ($m=2$) in a line of N spins along the z axis is

$$\begin{aligned}\partial\mathbf{S}_1/\partial t&=(2J/\hbar)(\mathbf{S}_1\times\mathbf{S}_2)+\gamma\mathbf{S}_1\times(\mathbf{H}_A{}^{(1)}+\mathbf{H}_0),\\ \partial\mathbf{S}_2/\partial t&=(2J/\hbar)(\mathbf{S}_2\times\mathbf{S}_1+\mathbf{S}_2\times\mathbf{S}_3)\\ &\quad+\gamma\mathbf{S}_2\times(\mathbf{H}_A{}^{(2)}+\mathbf{H}_0),\end{aligned}\tag{1}$$

where J is the exchange integral, $\mathbf{H}_0$ the external static magnetic field, $\mathbf{H}_A{}^{(1)}$ the surface anisotropy field directed along the $+z$ axis for the end spin, and $\mathbf{H}_A{}^{(2)}$ the anisotropy field directed along the $-z$ axis for the second spin ($m=2$). We assume for simplicity that $\mathbf{H}_0$ is parallel to $\mathbf{H}_A{}^{(1)}$. We make the usual approximations for an antiferromagnet[7] $S_1{}^z=S$, $S_2{}^z=-S$. Defining[7a] $S_1{}^+=S_1{}^x+iS_1{}^y$ and $S_2{}^+=S_2{}^x+iS_2{}^y$ our equations of motion become

$$\begin{aligned}\partial S_1{}^+/\partial t&=i(2JS/\hbar)(S_1{}^++S_2{}^+)-iS_1{}^+\gamma(H_A{}^{(1)}+H_0),\\ \partial S_2{}^+/\partial t&=-i(2JS/\hbar)(2S_2{}^++S_1{}^++S_3{}^+)\\ &\quad+iS_2{}^+\gamma(H_A{}^{(2)}-H_0).\end{aligned}\tag{2}$$

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[7a] This approach was suggested by Nagamiya and de Gennes (private communication). For the case of a ferromagnet (reference 1), the corresponding method leads to the equation $(\partial S_1{}^+/\partial t)=(2JS/\hbar)i[a\partial S_1{}^+/\partial z+(a^2/2)\partial^2S_1{}^+/\partial z^2]-i\omega_1S_1{}^+$ which, using (5), gives for β/α at $z=0$ the value $\omega_e(ka)/[\omega+\omega_1+\omega_e(ka)^2/2]\approx\omega_e(ka)/2\omega_1\ll1$. This ratio should replace the values given by Eqs. (10) through (15) of reference 1.

We expand S_2^y in a Taylor series in a, where a is the lattice constant:

$$S_3^+ = S_1^+ + a\partial S_1^+/\partial z + (a^2/2)\partial^2 S_1^+/\partial z^2 + \cdots. \quad (3)$$

This expression is valid at low temperatures and for $(2JS/\hbar\gamma) \gg H_A^{(3)}$. We let $\omega_e = 2JS/\hbar$, $\omega_1 = \gamma(H_A^{(1)} + H_0)$, $\omega_2 = \gamma(H_A^{(2)} - H_0)$, and find

$$\begin{aligned} \partial S_1^+/\partial t &= i\omega_e(S_1^+ + S_2^+) - i\omega_1 S_1^+, \\ \partial S_2^+/\partial t &= -i\omega_e[2S_2^+ + 2S_1^+ + a\partial S_1^+/\partial z \\ &\quad + (a^2/2)\partial^2 S_1^+/\partial z^2] + i\omega_2 S_2^+. \end{aligned} \quad (4)$$

Taking as our solutions

$$\begin{aligned} S_1^+ &= e^{i\omega t}(\alpha_1 \sin kz + \beta_1 \cos kz), \\ S_2^+ &= e^{i\omega t}(\alpha_2 \sin kz + \beta_2 \cos kz), \end{aligned} \quad (5)$$

and inserting these into (4), our equations of motion become at $z = 0$

$$\beta_1(\omega + \omega_1 - \omega_e) - \omega_e \beta_2 = 0, \quad (6a)$$

$$\omega_e[2\beta_1 + \alpha_1(ak) - \beta_1(a^2/2)k^2] + \beta_2(\omega + 2\omega_e - \omega_2) = 0. \quad (6b)$$

Eliminating terms in β_2 we find for the ratio β_1/α_1

$$\frac{\beta_1}{\alpha_1} = \frac{\omega_e(ak)}{2\omega_e - \omega_e(ak)^2/2 + (\omega + 2\omega_e - \omega_2)(\omega + \omega_1 - \omega_e)/\omega_e}. \quad (7)$$

In an antiferromagnet $\omega^2 \sim \omega_A \omega_e$ where $\omega_A/\gamma = H_A$, the internal anisotropy field; $\omega_e \sim 10^{13}$, and $\omega_A \sim 10^{10}$. If we assume $ka \sim 10^{-3}$, then (7) reduces to

$$|\beta_1/\alpha_1| \cong \omega_e(ak)/\omega = (\omega_e/\omega_A)^{\frac{1}{2}}(ak) \ll 1, \quad (8)$$

and the ends behave as if they were fixed.

We find, using condition (8), that reflection symmetry in the Hamiltonian about the center of the line ($z = L/2$) results in

$$k = p\pi/L, \quad (9)$$

where p is any integer. For $p \sim 10$, $a/L \sim 10^{-4}$, we find $ka \sim 10^{-3}$. This justifies our assumptions in (8).

III. EVALUATION OF THE OSCILLATOR STRENGTH

We now consider the magnitude of the excitation of the spin wave modes by the uniform rf field $H_x = h_0 \sin\omega t$. We employ a semiclassical theory of antiferromagnetic spin waves[7] and let the z axis be the direction of sublattice magnetization. We label the sublattices A and B, and take

$$\begin{aligned} \mathbf{S}_A &= (\epsilon_A^x, \epsilon_A^y, S), \\ \mathbf{S}_B &= (\epsilon_B^x, \epsilon_B^y, -S), \end{aligned} \quad (10)$$

where S is the classical atomic spin and ϵ_A^x, ϵ_A^y, ϵ_B^x, ϵ_B^y are the time-varying components of $\mathbf{S}_A$ and $\mathbf{S}_B$. We consider a one-dimensional line of spins and assume that the end spins are effectively pinned down by the surface anisotropy. Our solutions will be of the form

$$\begin{aligned} \epsilon_A^x &= \textstyle\sum_p \epsilon_A^{0x}(p) \sin k_p z \sin\omega t, \\ \epsilon_A^y &= \textstyle\sum_p \epsilon_A^{0y}(p) \sin k_p z \cos\omega t, \\ \epsilon_B^x &= \textstyle\sum_p \epsilon_B^{0x}(p) \sin k_p z \sin\omega t, \\ \epsilon_B^y &= \textstyle\sum_p \epsilon_B^{0y}(p) \sin k_p z \cos\omega t, \end{aligned} \quad (11)$$

where $k_p = p\pi/L$. The equations of motion[7] differ from (1) as H_A is used here for the internal anisotropy field and $H_0 = 0$. We solve the equations of motion for $H_0 \neq 0$ in the Appendix. The presence of a magnetic field serves only to bring the resonance into the observable microwave range and unnecessarily complicates our solution. The equations of motion take the form

$$\begin{aligned} \partial\epsilon_A^x/\partial t &= -(4JS/\hbar)\epsilon_A^y \\ &\quad - (2JS/\hbar)(\textstyle\sum_B \epsilon_B^y) + \gamma\epsilon_A^y H_A, \end{aligned} \quad (12a)$$

$$\begin{aligned} \partial\epsilon_A^y/\partial t &= (2JS/\hbar)(\textstyle\sum_B \epsilon_B^x) \\ &\quad + (4JS/\hbar)\epsilon_A^x + \gamma S H_x - \gamma\epsilon_A^x H_A, \end{aligned} \quad (12b)$$

$$\begin{aligned} \partial\epsilon_B^x/\partial t &= (4JS/\hbar)\epsilon_B^y \\ &\quad + (2JS/\hbar)(\textstyle\sum_A \epsilon_A^y) - \gamma\epsilon_B^y H_A, \end{aligned} \quad (12c)$$

$$\begin{aligned} \partial\epsilon_B^y/\partial t &= -(2JS/\hbar)(\textstyle\sum_A \epsilon_A^x) \\ &\quad - (4JS/\hbar)\epsilon_B^x - \gamma S H_x + \gamma\epsilon_B^x H_A. \end{aligned} \quad (12d)$$

The sums go over nearest neighbors only so that

$$\begin{aligned} \textstyle\sum_B \epsilon_B^x &= \textstyle\sum_B \sum_p \epsilon_B^{0x}(p) \sin k_p z \sin\omega t \\ &= 2\textstyle\sum_p \epsilon_B^{0x}(p)\gamma_p \sin k_p z \sin\omega t, \end{aligned} \quad (13)$$

where

$$\gamma_p = \cos k_p a. \quad (14)$$

Equation (12b) becomes

$$\begin{aligned} -\omega\textstyle\sum_p \epsilon_A^{0y}(p) \sin\omega t \sin k_p z \\ = (4JS/\hbar)\textstyle\sum_p \epsilon_B^{0x}\gamma_p \sin k_p z \sin\omega t \\ + (4JS/\hbar)\textstyle\sum_p \epsilon_A^{0x} \sin\omega t \sin k_p z + \gamma S H_x \\ - \gamma H_A \textstyle\sum_p \epsilon_A^{0x} \sin\omega t \sin k_p z. \end{aligned} \quad (15)$$

We multiply both sides of (15) by $\sin k_m z$ and integrate over z between 0 and L. We find

$$\begin{aligned} -\omega\epsilon_A^{0y} &= (4JS/\hbar)(\gamma_m \epsilon_B^{0x} + \epsilon_A^{0x}) \\ &\quad - \gamma H_A \epsilon_A^{0x} + 4\gamma S h_0/m\pi, \end{aligned} \quad (16)$$

for m odd. If m be even, the last term on the right vanishes and no excitations will take place. Similar results hold for (12a, c, d) so that

$$\begin{aligned} \omega\epsilon_A^{0x} &= -(4JS/\hbar)(\epsilon_A^{0y} + \gamma_m \epsilon_B^{0y}) + \gamma H_A \epsilon_A^{0y}; \\ -\omega\epsilon_A^{0y} &= (4JS/\hbar)(\epsilon_A^{0x} + \gamma_m \epsilon_B^{0x}) \\ &\quad - \gamma H_A \epsilon_A^{0x} + 4\gamma S h_0/m\pi; \\ \omega\epsilon_B^{0x} &= (4JS/\hbar)(\epsilon_B^{0y} + \gamma_m \epsilon_A^{0y}) - \gamma H_A \epsilon_B^{0y}; \\ -\omega\epsilon_B^{0y} &= -(4JS/\hbar)(\epsilon_B^{0x} + \gamma_m \epsilon_A^{0x}) \\ &\quad + \gamma H_A \epsilon_B^{0x} - 4\gamma S h_0/m\pi. \end{aligned} \quad (17)$$

We solve (17) for $\epsilon_A{}^{0x}$ and find

$$\epsilon_A{}^{0x}=\frac{4Sh_0}{m\pi}\frac{[H_A+H_e(1-\gamma_m)]}{[H_e{}^2(1-\gamma_m{}^2)+H_A(H_A+2H_e)-(\omega/\gamma)^2]}, \quad (18)$$

where $H_e=-4JS/\hbar\gamma$. The selection rule for excitation by a uniform rf field is that m, the number of half-wave-lengths, must be odd. We see, as in the ferromagnetic case, that the oscillator strength $\epsilon_A{}^{0x}$ decreases inversely with m. At resonance a comparison can be made between the amplitude (18) and the corresponding amplitude for ferromagnetic excitations. For $m\sim 1$ to 10, and $(a/L)\sim 10^{-4}$, one obtains $H_A\gg H_e(1-\gamma_m)$. Using Eq. (26) of reference 1 we find, noting $\omega_p=DSk_p{}^2+\omega_0\sim\omega_0$ under these conditions, that

$$\begin{aligned}\epsilon_{A\,\mathrm{ferro}}{}^{0x}/\epsilon_{A\,\mathrm{antiferro}}{}^{0x}&=(4\gamma S\omega_0h_0)/(H_A4\gamma^2Sh_0)\\&=H_0/H_A\sim 1,\end{aligned} \quad (19)$$

where H_0 is the applied field in the ferromagnetic case. Hence, the effect should have the same oscillator strength in antiferromagnetics as it has in ferromagnetics. To calculate the separation of spin wave modes, we note that at resonance

$$\begin{aligned}(\omega/\gamma)^2&=H_e{}^2(1-\gamma_m{}^2)+H_A(H_A+2H_e)\\&=H_e{}^2\sin^2(m\pi a/L)+H_A(H_A+2H_e);\end{aligned} \quad (20)$$

for $a/L\sim 10^{-4}$, $H_e\sim 10^6$, and $H_A\sim 10^3$, we have

$$\omega/\gamma\approx(2H_AH_e)^{\frac{1}{2}}[1+(H_e/4H_A)\pi^2m^2(a/L)^2]. \quad (21)$$

Thus,

$$\Delta\omega=\omega_{m+1}-\omega_{m-1}=\gamma\sqrt{2}(H_e{}^{\frac{3}{2}}/H_A{}^{\frac{1}{2}})\pi^2(a/L)^2m.$$

We consider, for example, the case of Cr_2O_3 for which we have accurate values of H_A and H_e. From Foner,[8] $H_A=900$ and $H_e=2\times 10^6$ oersteds. This results in a $(2H_AH_e)^{\frac{1}{2}}$ resonance at 1.8 mm. From above, taking $(a/L)=5\times 10^{-4}$, we find

$$\Delta\omega/\gamma=\Delta H\approx 330\text{ m}\quad\text{oersteds.}$$

The line width for the $(2H_AH_e)^{\frac{1}{2}}$ resonance is narrow enough so that it should be possible in thin single crystals of Cr_2O_3 to resolve the spin wave structure. With thicker crystals, the apparent single line may be skewed by the spin wave structure.

The high frequencies required to reach resonance in an antiferromagnet make this effect experimentally a more difficult one to observe than in a ferromagnet. A pulsed-field technique might induce transient effects which would swamp out any resonance modes. Observation of this effect may only be possible when the state of the art is such that 1–2 mm microwaves will be obtainable in the laboratory.

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APPENDIX

We derive the oscillator strength $\epsilon_A{}^{0x}$ for $H_0\neq 0$. In this case, (17) becomes

$$\begin{aligned}\omega\epsilon_A{}^{0x}&=-(4JS/\hbar)(\epsilon_A{}^{0y}+\gamma_m\epsilon_B{}^{0y})\\&\qquad+\gamma H_A\epsilon_A{}^{0y}+\gamma H_0\epsilon_A{}^{0y};\\-\omega\epsilon_A{}^{0y}&=(4JS/\hbar)(\epsilon_A{}^{0x}+\gamma_m\epsilon_B{}^{0x})\\&\qquad-\gamma H_A\epsilon_A{}^{0x}-\gamma H_0\epsilon_A{}^{0x}+4\gamma Sh_0/m\pi;\\\omega\epsilon_B{}^{0x}&=(4JS/\hbar)(\epsilon_B{}^{0y}+\gamma_m\epsilon_A{}^{0y})\\&\qquad-\gamma H_A\epsilon_B{}^{0y}+\gamma H_0\epsilon_B{}^{0y};\\-\omega\epsilon_B{}^{0y}&=-(4JS/\hbar)(\epsilon_B{}^{0x}+\gamma_m\epsilon_A{}^{0x})\\&\qquad+\gamma H_A\epsilon_B{}^{0x}-\gamma H_0\epsilon_B{}^{0x}-4\gamma Sh_0/m\pi.\end{aligned} \quad (A1)$$

We solve for $\epsilon_A{}^{0x}$ and find

$$\begin{aligned}\epsilon_A{}^{0x}=(4\gamma Sh_0/m\pi)\{&-\omega^2\gamma[H_A+H_e(1-\gamma_m)+H_0]\\&+\gamma^3[H_A+H_e(1-\gamma_m)-H_0][H_e{}^2(1-\gamma_m{}^2)\\&+H_A(H_A+2H_e)+H_0{}^2]\}\\&\times\{\omega^2-[\gamma H_0\pm\gamma(H_A(H_A+2H_e)\\&\qquad+H_e{}^2(1-\gamma_m{}^2))^{\frac{1}{2}}]^2\}^{-2}.\end{aligned} \quad (A2)$$

Equation (A2) shows that the resonance is shifted from the free-field resonance by the factor γH_0. Expanding the denominator of (A2) for $m\sim 10$ and $H_A\ll H_e$, we find that resonance occurs at

$$\omega/\gamma\approx\pm H_0+(2H_AH_e)^{\frac{1}{2}}[1+(H_e/4H_A)\pi^2m^2(a/L)^2], \quad (A3)$$

which is to be compared with (21). In a resonance experiment at constant ω, we pass through a series of spin wave resonances at discrete static field intensities between 0 and ω/γ. For example, in order to sweep through the resonances corresponding to a given m in Cr_2O_3, one must vary H_0 from 0 [if $\omega/\gamma=(2H_AH_e)^{\frac{1}{2}}$] to $\approx 80m^2$ oersteds. As can be seen from (A2), the amplitudes will also vary slightly differently with ω than in (18) but this difference is unimportant.