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Potential Fluctuations in the Transition Region to Superconductivity*

HANS MEISSNER
Johns Hopkins University, Baltimore, Maryland
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Gorter recently proposed a dynamical model to describe the state of a current-carrying wire, in the transition region to superconductivity. This model predicts in the transition region to superconductivity an alternating component of the potential difference across a wire carrying a strong direct current. The magnitude of this fluctuation should be of the order of a few percent of the average potential. Using a shielded transformer in the liquid helium and a battery-operated preamplifier, detection of this alternating component has been attempted. The noise output was somewhat larger when the sample was in the intermediate state than when it was in the superconducting state. However, instead of being a few percent of the average potential (as in the Gorter model), this additional alternating potential was only about one hundredth of one percent. It may be due partially to a slightly drifting bath temperature, since vibrations made it necessary to switch the pump off about 10 sec before the readings were taken.

I. INTRODUCTION

THE standard treatment of the transition to superconductivity of a current-carrying wire assumes a static distribution of superconducting domains embedded in a matrix of normal conducting material.[1] This theory is easily extended to include the effect of a superimposed longitudinal magnetic field,[2] which case is commonly called the "paramagnetic effect." In both cases there are small but significant differences between the experiments and the theoretical predictions.[2,3–7] Various proposals have been made to explain the difference between the experimental and theoretical values of the resistance.[8–10] So far none of these proposals has been completely satisfactory.

Gorter has recently proposed a completely different model.[11,12] Although the electric forces on a superconducting domain are very much smaller than the magnetic forces, he assumes that they are strong enough to align the domains in the direction of the electric field rather than the magnetic field.[13] This assumption seems, as Gorter himself remarks, to be in direct contradiction to powder patterns obtained by Shalnikov.[14] In the case of a current-carrying wire the domain structure is assumed to consist of alternately normal-conducting and superconducting concentric tubes. It is assumed that the Lorentz force of the magnetic field on the current through these tubes causes an inward motion of the boundaries, that is, causes the tubes to collapse. No reason is offered why the boundary should move under the influence of this Lorentz force, and there is some doubt that this will happen.[15] It is assumed that the potential difference along the sample is caused by an induced emf. As the superconducting tubes collapse toward the center of the wire, they leave a region of reduced magnetic field behind. Such a reduction is possible whenever supercooling occurs. If the field behind a superconducting collapsing tube becomes too low, a new superconducting tube is formed. It appears that in Gorter's model the new tube is not formed at the place of the minimum of the magnetic field, but at a larger radius, where the field is again critical. This is a feature which is very hard to understand. The model seems to become static if one supposes that the new tubes are formed at the place of the minimum of the magnetic field. Pippard[16] has calculated the rate of destruction of superconductivity by a current through a wire and also comes to the conclusion that the final state will be static.

The time which it takes for a tube section to travel from the outer surface of the wire to the center is determined (aside from the diameter of the wire) by the conductivity of the normal material. Each collapsing tube gives rise to a small fluctuation of the potential along the wire. Gorter develops this fluctuating potential into a Fourier series of the type

$$e=e_0[1+\sum a_k \cos(2\pi k\tau/\tau_f+\phi_k)], \tag{1}$$

where e_0 is the average potential, $\tau=t/\mu_0\sigma a^2$, t=time, σ=conductivity of normal material in mks units, a=radius of the wire, a_k=Fourier coefficients and ϕ_k=arbitrary phases. Estimates of the other constants have been made mainly for the case that the current I through the wire is 1.1 times the critical current:

* Supported by a Grant of the National Science Foundation.
[1] F. London, *Superfluids* (John Wiley and Sons, Inc., New York, 1950), Vol. 1, p. 120.
[2] Hans Meissner, Phys. Rev. **97**, 1627 (1955); Phys. Rev. **101**, 31 (1956); Phys. Rev. **103**, 39 (1956).
[3] James C. Thompson, Phys. Rev. **102**, 1004 (1956); J. Phys. Chem. Solids **1**, 61 (1956).
[4] R. B. Scott, J. Research Natl. Bur. Standards **41**, 581 (1948).
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[6] Hans Meissner, Phys. Rev. **109**, 668 (1958).
[7] H. Meissner and R. Zdanis, Phys. Rev. **109**, 681 (1958).
[8] C. G. Kuper, Phil. Mag. **43**, 1264 (1952).
[9] R. Hilsch, Z. Physik **149**, 1 (1957).
[10] Hans Meissner, Phys. Rev. **109**, 1479 (1958).
[11] C. J. Gorter, Physica **23**, 45 (1957).
[12] G. J. Gorter and M. L. Potters, Physica **24**, 169 (1958).
[13] In private correspondence Professor Gorter pointed out that he assumes that also the magnetic field tends to align the domains in the direction of the electric field. This may be true for the current-carrying wire in a transverse magnetic field, but not for a wire which is subject to a current only.
[14] A. I. Shalnikov, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 1071 (1957) [translation: Soviet Phys. JETP **6**, 827 (1958)].
[15] In the case of a current-carrying wire the assumption of the Lorentz force may not be necessary; see reference 16.
[16] A. B. Pippard, Phil. Mag. **41**, 243 (1950).

$I/I_c=1.1$. The value of the constants depends on the degree of super cooling, that is, on the ratio q_r of the reduced magnetic field to the critical field. τ_f is the (reduced) time between successive formations of the superconducting tubes. For $I/I_c=1.1$ and $q_r=0.9488$, Gorter obtains $\tau_f=0.2481$ and $a_1=0.018$, $a_2=0.005$ and $a_3=0.002$, while for $I/I_c=1.1$ and $q_r=0.7683$ he obtains $\tau_f=0.3917$ and $a_1=0.048$, $a_2=0.016$ and $a_3=0.008$. These amplitudes are an upper limit, since the periodic motions might be out of phase in different parts of a long wire.

The frequency of the lowest mode is given by

$$f=1/\mu_0\sigma a^2\tau_f. \tag{2}$$

Although this model certainly presents some new and brilliant ideas, the discussion above casts some doubt on its validity. Nevertheless, any calculation of the frequency of a possible fluctuation of the potential difference along the wire will always lead to an equation similar to Eq. (2) (see reference 16). Since so far nobody seems to have searched for such a fluctuation, it was found worth while to investigate the constancy of the potential in the frequency range given by Eq. (2). If it is found that the potential along the wire fluctuates in this frequency range much less than 1% of its mean value, it will be fairly safe to assume that such fluctuations do not play an important role in the intermediate state.

II. EXPERIMENTAL ARRANGEMENT

Most of the experimental arrangement is identical with the one described in references 6 and 7. In addition to the usual potentiometer and galvanometer connected to the potential leads of the sample, the primary of a special transformer was connected to the same potential leads. The transformer[17] was encased in a lead (Pb) shield and placed in the liquid helium bath. A battery-operated cathode follower (6C4) was placed directly on top of the cryostat, but was not always used. Its output was fed into a battery-operated preamplifier. The output of the preamplifier was displayed on an oscilloscope, and measured with a vacuum tube-voltmeter. Theoretically the sensitivity can be calculated from the turn's ratio of the transformer, the gain of the amplifier, and the sensitivity of the oscilloscope. However, in order to omit any uncertainties, the system was calibrated by superimposing a small alternating current on the direct curret.

The alternating component of the current gives rise to the following signals in the pickup transformer:

(a) A potential difference due to the resistance of the sample. This potential difference was usually negligibly small, even when the sample was fully normal conducting.

(b) Pickup in the loop formed by the sample and the potential leads. The pickup in the loop of length l and width D corresponds to a mutual inductance

$$M=(\mu_0 l/2\pi)\ln(D/a), \tag{3}$$

where a is the radius of the sample. The over-all voltage gain of amplifier and transformer as function of the frequency shows a rather sharp resonance peak at the high-frequency end. While such a peak usually is undesirable, it increases the gain at the high-frequency end so that even the second harmonic [see Eq. (1)] should be easily observable in spite of its lower expected amplitude.

The alternating calibration current was usually 1 ma, while the sample current was of the order of 20 amp. Comparision of these two numbers shows immediately that it is of utmost importance to keep the sample current extremely constant. A bothersome source of noise was vibration of the loop formed by the sample and the potential leads, causing an emf across the transformer. Since it was not feasible to mount the whole cryostat vibration-free, it was necessary to shut off the pump about 10 seconds before the readings were taken. During this time the bath temperature drifted slowly.

TABLE I. Data on the samples.

Sample No.	Diameter mm	Length mm	Residual resistance ohm	Residual resistivity[a] ohm cm	Frequency from Eq. (2) cps
Sn XVIII[b]	0.357	41.5	2.2×10^{-6}	3.88×10^{-10}	240–380
Sn XXIV[c]	0.325	59.4	1.2×10^{-5}	1.68×10^{-9}	1270–2000

[a] The contribution of boundary scattering has been subtracted.
[b] Single crystal; angle between crystal axis and wire axis is 79°.
[c] Polycrystalline extruded wire.

The measurements were thus made at an almost constant temperature. The temperature was chosen somewhat below the lambda-point of the helium, so that bubbling would not cause any noise. The sample current was adjusted to values of 0.9, 1.0, and 1.1 of the critical current I_c. At each value of the current the output voltage of the preamplifier was measured and the trace on the oscilloscope inspected as well as photographed.

The samples investigated were both made from tin. They are the same samples used for the resistance measurements reported in reference 6. Their data are given in Table I.

III. RESULTS

The results are summarized in Table II and shown in Figs. 1–3. Figure 1 shows the resistance and the output of the preamplifier as a function of the sample current at a temperature of 1.95°K for sample Sn XVIII. The galvanometer deflection was measured while the pump was running, whereas the pump was shut off 10 seconds before the output of the preamplifier was measured.

[17] The author is indebted to the Arnold Engineering Company for supplying the core free of charge.

Figure 1 shows that the critical current at this temperature is about 18.5 amp. The output of the preamplifier begins to increase at a current somewhat smaller than 18.5 amp. This might be due to the slight increase in temperature, when the pump was shut off during these measurements. The data are summarized in Table II. Between 16 and 20 amp the output of the preamplifier increases by about 20 mv, which, with a gain of 4.35×10^6, corresponds to an increase of 4.6×10^{-9} v of the alternating potential at the sample. This is a fraction of 0.015% of the average potential across the sample ($V_{dc}=3.1\times10^{-5}$ v).

Figure 2 shows the same graphs as Fig. 1, but for the polycrystalline sample Sn XXIV. The increase in the output of the preamplifier for this sample is very much larger than for sample Sn XVIII. Table II, however,

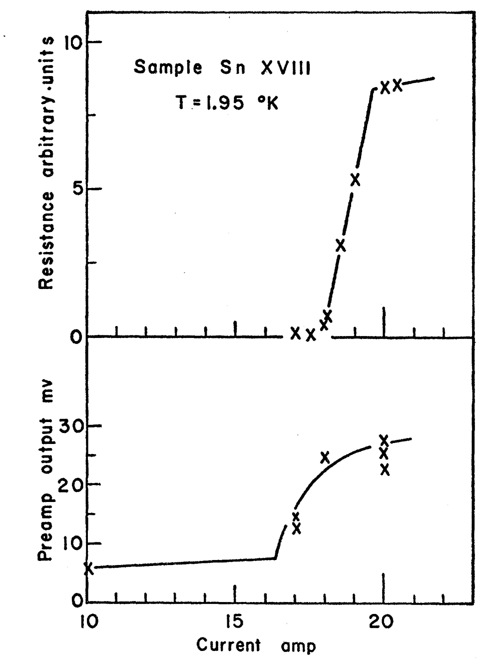


FIG. 1. Resistance of sample Sn XVIII (single crystal) and alternating component of the potential in the transition region.

shows that the ratio of the increase in the fluctuating voltage is still about the same percentage of the increase in the average voltage.

Figure 3 shows negatives of the oscilloscope traces.

The frequency of the additional noise was, for sample Sn XVIII, of the order of 600 cps as compared to the expected frequency of 200–300 cps. For sample Sn XXIV, a dominant portion of the noise had a frequency of about 7.5 kcps, which is almost precisely at the peak of the resonance of the transformer. The frequency has to be compared with an expected value of 1.25 to 2.0 kcps. In the computations for Table II, the midfrequency value of the gain of 3.5×10^6 has been used rather than the peak value of 25×10^6. The true

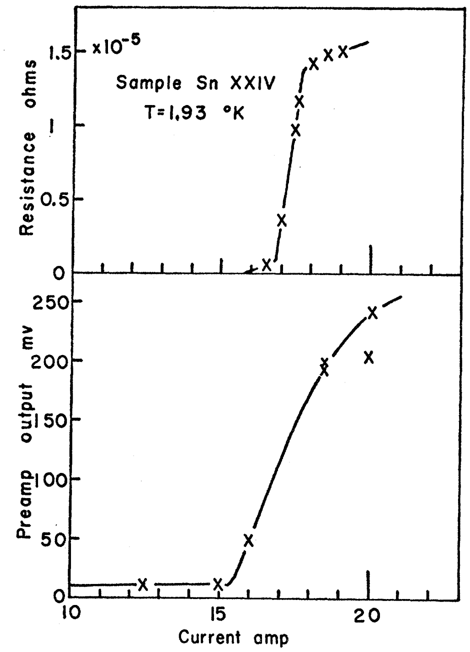


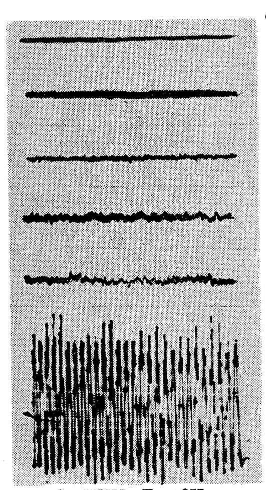
FIG. 2. Resistance of polycrystalline sample Sn XXIV and alternating component of the potential in the transition region.

alternating potentials will be therefore smaller rather than bigger.

Observations have also been made of potential fluctuations in the normal-conducting state on sample Sn XXIV. It seems quite safe to assume that the very much smaller fluctuations in the normal-conducting state arise from bubbling of the liquid.

IV. DISCUSSION

There is a clear observation of an alternating component of the potential across a current-carrying wire in the transition region to superconductivity. The



Current	Cal. sign	V_{dc}
10	0	0
17	0	0
18	0	2.5×10^{-5}
20	0	3.1×10^{-5}
20	0	3.1×10^{-5}
20	1×10^{-7}	3.1×10^{-5}
amp	v	v

Sn XVII; $T=2$°K

FIG. 3. Oscilloscope traces for sample Sn XVIII (single crystal).

TABLE II. Experimental results.

Sample	Temperature °K	Sample current amp	$10^5 V_{dc}$ v	Preamp. output v	Gain	$10^9 V_{ac}$	$(V_{ac}/V_{dc})_{corr}$
Sn XVIII	1.95	10	0	0.006	4.35×10^6	1.4	
Sn XVIII	1.95	17	0	0.014	4.35×10^6	3.2	
Sn XVIII	1.95	18	2.5	0.025	4.35×10^6	5.8	
Sn XVIII	1.95	20	3.1	0.026	4.35×10^6	6.0	1.5×10^{-4}
Sn XXIV	1.93	10	0	0.015	3.6×10^6	4.2	
Sn XXIV	1.93	12.5	0	0.02	3.6×10^6	5.6	
Sn XXIV	1.93	15	0	0.02	3.6×10^6	5.6	
Sn XXIV	1.93	16	0	0.05	3.6×10^6	13.9	
Sn XXIV	1.93	17	17.5	0.20	3.6×10^6	56	
Sn XXIV	1.93	20	30	0.22	3.6×10^6	61	1.8×10^{-4}

magnitude of this component is certainly smaller than 0.018% of the average potential. The observed frequencies are somewhat higher than those calculated by Gorter. They are caused by fluctuations of the resistance rather than the current, since they are not present in the superconducting state.

Although the observed potentials are very much smaller than those predicted by the dynamic model, the observations here do not absolutely exclude the dynamic model, since, as Gorter remarked, the collapsing tubes might be out of phase in different parts of a long wire. If one assumes that they are in phase for a length of 3 wire diameters, one would obtain a reduction by a factor of 50 in the calculated values.

On the other hand, it is very easy to give other reasons for the observation of such fluctuations. They are probably not solely caused by the slow drift in temperature (which, even in the static model, requires rearrangement of the superconducting domains), since they were clearly present in sample Sn XXIV even when the pump was running, and the temperature rather constant. But of course thermal fluctuation will always be present and will always cause some disturbance of the superconducting domains. There exists also the possibility that the heat transfer from the wire to the bath, even in helium II, is not quite steady but fluctuates somewhat, giving rise to thermal fluctuations above the statistical value.

The observations described here do not throw very much light on the validity of the dynamic model proposed by Gorter. The potential fluctuations are of such a magnitude, that they could be explained by the static model as well as by the dynamic model.

V. ACKNOWLEDGMENTS

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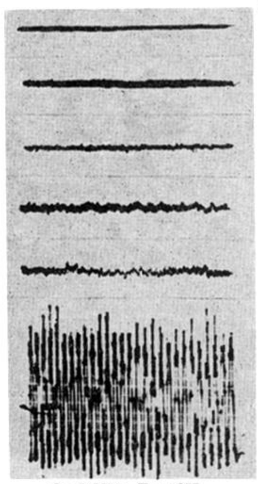

Current	Cal. sign	V_{dc}
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20	1×10^{-7}	3.1×10^{-5}
amp	v	v

Sn XVII; $T=2°K$

FIG. 3. Oscilloscope traces for sample Sn XVIII (single crystal).