Decay of Hyperons and Mesons from the Universal Fermi Interaction*

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The decay of the hyperon, charged pion, and K meson except $K_{\pi 3}^+$ is investigated on the basis of the universal $V-A$ Fermi interaction together with the idea of the Gell-Mann tetrahedron, by treating the virtual baryon-antibaryon pair effect in a phenomenological way. It is shown that the decay rate of $K_{\pi 2}^+$ calculated by the parameters adjusted to the decay of π^+, $K_{\mu 3}^+$, and K_{e3}^+ is in agreement with experiment, thus suggesting a possible consistent picture of the model.

1. INTRODUCTION

It has been shown that the universal Fermi interaction with $V-A$ coupling scheme is remarkably successful in the weak processes mainly involving the nucleons and leptons.[1] The attempt to cover the wider area involving strange particles leads to the concept of the Gell-Mann tetrahedron.[2] Assuming that all weak processes are generated by the four-fermion interaction, we notice that the decay diagram of various reactions can be reduced to a small number of "structure elements" composed of the baryon-antibaryon loop with strongly coupled mesons (see Sec. 2).

This reminds us of the role of beta- and gamma-ray spectroscopy in the study of nuclear structure. When information of nuclear structure is desired from the data of β and γ decay, it has been found profitable to separate out the known factors due to Fermi or electromagnetic interaction and phase space and then "extract" the quantity directly connected with the unknown nuclear matrix element, which in turn leads to classification into groups for systematics of nuclear structure. The situation seems quite parallel for the decay of the elementary particles, where the "structure element" is the analog of the nuclear matrix element. The fact that there exists no reliable way of calculating the strongly interacting processes may correspond to the fact that there exists no absolutely exact way of computing the nuclear matrix element.

It seems interesting, therefore, to separate out the known characteristics of the Fermi interaction and the kinematical factors from the decay rate and pick up the quantity which represents the contribution of the dynamics of the strong interactions.

The purpose of the present paper is to investigate whether the universal $V-A$ Fermi interaction with the tetrahedron idea can explain decay processes at least qualitatively. In fact, the lifetimes of π, Λ, Σ, and Ξ are found to be expressed by a single, almost constant parameter, as it should be under reasonable approximation; and the transition probabilities of $K_{\mu 3}^+$, K_{e3}^+, and $K_{\pi 2}^+$ are described by another two parameters. There is no essential disagreement with the experiment within the framework of the present analysis. However, the difficulties connected with the π-e decay and leptonic decays of hyperons, and the branching ratio of the hyperon decay have not been touched.

2. TRANSITION RATE OF DECAY PROCESSES

According to the idea of the universal Fermi interaction and the tetrahedron concept, all weak interactions are ascribed to the prescribed four-fermion interaction, with unique coupling constant and unique coupling scheme $V-A$, and no other weak coupling such as the direct boson-fermion interaction[3] is introduced. The charged pions and K mesons are coupled strongly to the group of all possible baryon-antibaryon pairs provided only that isotopic spin, strangeness, and charge are conserved. These two groups and $(\mu\nu)$, $(e\nu)$ are coupled by weak interactions. Since the tetrahedron has no obvious counterpart for the neutral pairs, we simply assume that the group of the neutral pairs admissible by the dissociation of K^0, π^0, $\bar{K}^0$ have the weak interaction between the group. We do not assume any particular isotropic spin dependence for the weak interaction.

The Hamiltonian density of the weak interaction reads then

$$H=\sqrt{2}f\sum_{(AB)(CD)}\left(\bar{\psi}_D\frac{(1-\gamma_5)\gamma_\mu}{\sqrt{2}}\psi_C\right)\times\left(\bar{\psi}_B\frac{(1-\gamma_5)\gamma_\mu}{\sqrt{2}}\psi_A\right),\quad(2.1)$$

where f is the universal coupling constant 1.4×10^{-49} erg cm^3 and $(AB)(CD)$ denote the prescribed fermion pairs.

The strong interaction of the pion and K meson to the baryons are assumed to be charge independent,

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[1] E. C. G. Sudarashan and R. E. Marshak, Phys. Rev. **109**, 1860 (1958); R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958); J. J. Sakurai, Nuovo cimento **7**, 649 (1958).
[2] M. Gell-Mann and A. H. Rosenfeld, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Palo Alto, 1957), Vol. 7, p. 407.
[3] M. Gell-Mann, Phys. Rev. **111**, 362 (1958); a generalized weak boson-fermion interaction is discussed by M. Sugawara, Phys. Rev. (to be published).

and moreover, for the sake of definiteness, the global pion baryon interaction[4] is adopted. The K meson is assumed to have spin zero and the nomenclature scalar or pseudoscalar K meson should be understood in accordance with the hypothesis of global symmetry. For purpose of numerical estimates the pion-nucleon coupling constant in symmetric $ps(ps)$ theory is taken to be $g^2/4\pi=15$.

The decay processes which involve at least one strong interaction are represented in Figs. 1, 2, and 3 by the field-theoretic decay diagram in the lowest order. The term "structure" refers to the black-box loop due to a baryon-antibaryon pair. In fact, upon ignoring the higher order graphs which have strong interactions between loops and/or external lines, the black-box loop can involve arbitrary numbers of pion and K-meson lines inside. The decay processes consist only of combinations of the small number of "structure elements" shown in Fig. 4 plus the usual Fermi vertex, since the structure element is independent of the momenta and masses of the outgoing fermions.

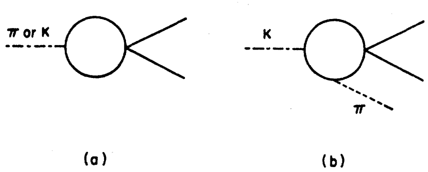


FIG. 1. Decay diagrams with simple structure.

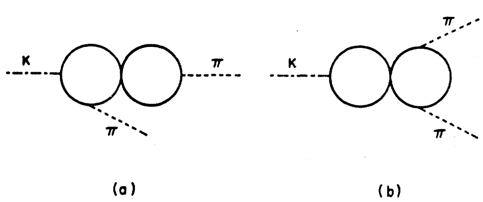


FIG. 2. Decay diagrams of $K_{\pi 2}{}^{+}$.

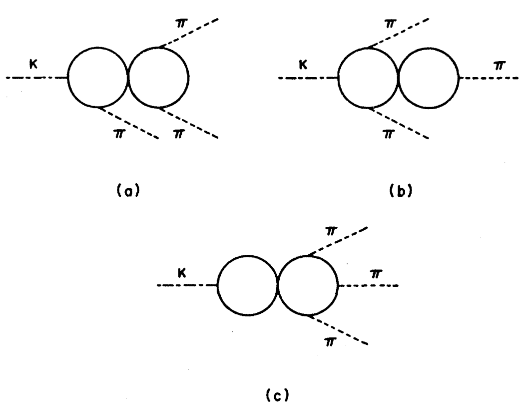


FIG. 3. Decay diagrams of $K_{\pi 3}{}^{+}$.

[4] M. Gell-Mann, Phys. Rev. **106**, 1296 (1957).

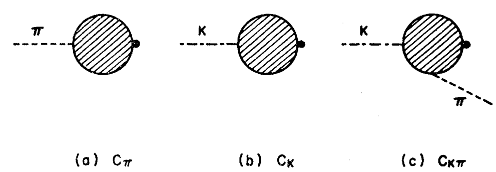


FIG. 4. Structure elements of decay processes. The dot on the loop denotes the Fermi interaction.

A. Two-Body Decay with Simple Structure

This process is characterized by the decay diagram of Fig. 1(a), and includes reactions $\pi^\pm\rightarrow\mu^\pm+\nu$, $\Lambda^0\rightarrow p+\pi^-$, $\Sigma^\pm\rightarrow n+\pi^\pm$, $\Xi^-\rightarrow\Lambda^0+\pi^-$, $K_{\mu 2}{}^{+}\rightarrow\mu^++\nu$.

Let us consider as an example of the case of pion decay. For reasons of Lorentz invariance we may conveniently write down the expression of the black-box loop in the form

$$\frac{g_\pi}{(2s_0)^{\frac{1}{2}}}\frac{1}{\sqrt{V}}M_0 s_\mu\sqrt{2}(2\pi)^2C_\pi\delta^4(s-k-p), \qquad (2.2)$$

where s, k, and p are the four-momenta of the pion, muon, and neutrino, respectively, V is the volume of quantization, M_0 is the nucleon mass, and C_π is a dimensionless phenomenological parameter which represents the contribution of the loop. It is important to realize that C_π is independent of the masses and the momenta of the external fermion lines regardless of any perturbation approximation. The numerical factor $\sqrt{2}(2\pi)^2$ is explicitly separated from C_π only in order to make comparison with perturbation calculations simple. The decay rate can be written

$$w(\pi^\pm\rightarrow\mu^\pm+\nu)=(fM_0{}^2)^2\frac{g_\pi{}^2}{4\pi}\frac{2}{(2\pi)^4}\left(\frac{m_\mu}{M_0}\right)^2 \times\left(\frac{m_\pi{}^2-m_\mu{}^2}{m_\pi{}^2}\right)^2 m_\pi|C_\pi(\pi^\pm\rightarrow\mu^\pm+\nu)|^2. \qquad (2.3)$$

Similarly, the decay rate of the hyperon reads

$$w(Y\rightarrow N+\pi)=(fM_0{}^2)^2\frac{g_\pi{}^2}{4\pi}\frac{1}{(2\pi)^4}\left(\frac{M+M_0}{M_0}\right)^2 \times\left(\frac{M-M_0+m_\pi}{M}\right)^{\frac{3}{2}}\left(\frac{M-M_0-m_\pi}{M}\right)^{\frac{3}{2}} \times\left(\frac{M+M_0+m_\pi}{M}\right)^{\frac{1}{2}}\left(\frac{M+M_0-m_\pi}{M}\right)^{\frac{1}{2}} \times M|C_\pi(Y\rightarrow N+\pi)|^2, \qquad (2.4)$$

where M is the hyperon mass, and Y is either Λ or

Σ. The decay rate of Ξ^- reads

$$w(\Xi^-\to\Lambda^0+\pi^-)$$
$$=(fM_0^2)^2\frac{g_\pi^2}{4\pi}\frac{1}{(2\pi)^4}\left(\frac{M'+M}{M_0}\right)^2$$
$$\times\left(\frac{M'-M+m_\pi}{M'}\right)^{\frac{3}{2}}\left(\frac{M'-M-m_\pi}{M'}\right)^{\frac{3}{2}}$$
$$\times\left(\frac{M'+M+m_\pi}{M'}\right)^{\frac{1}{2}}\left(\frac{M'+M-m_\pi}{M'}\right)^{\frac{1}{2}}$$
$$\times M'|C_\pi(\Xi^-\to\Lambda^0+\pi^-)|^2,\quad(2.5)$$

where M and M' are the Λ mass and Ξ mass, respectively. Finally there results the expression

$$w(K_{\mu2}^+\to\mu^++\nu)=(Mf_0^2)^2\frac{g_K^2}{4\pi}\frac{2}{(2\pi)^4}\left(\frac{m_\mu}{M_0}\right)^2$$
$$\times\left(\frac{m_K^2-m_\mu^2}{m_K^2}\right)m_K|C_K(K_{\mu2}^+\to\mu^++\nu)|^2.\quad(2.6)$$

Inserting now the experimental values on the left-hand side, we find

$$\begin{aligned}|C_\pi(\pi^\pm\to\mu^\pm+\nu)|&=0.20,\\|C_\pi(\Lambda^0\to p+\pi^-)|&=0.42,\\|C_\pi(\Sigma^\pm\to n+\pi^\pm)|&=0.27,\\(g_K/g_\pi)|C_K(K_{\mu2}^+\to\mu^++\nu)|&=0.053.\end{aligned}\quad(2.7)$$

The C_π's are fairly constant as expected. The simplest way to interpret the difference between C_π and C_K is that the strong-coupling constant of the K-meson to the baryon is about, say, $\frac{1}{4}\sim\frac{1}{5}$ smaller than the pion-baryon coupling constant. This is in qualitative accordance with the K-meson coupling constant determined by the preliminary data of the photoproduction of K mesons in hydrogen[5] and the scattering of K mesons.[6] Another comment which can be made in perturbation approximation is that C_K is roughly proportional to the difference of the masses of the baryons which appear in the virtual baryon-antibaryon pair for a scalar K meson, while it is roughly proportional to the sum of the masses for a pseudoscalar K meson (see appendix).

In the approximation that C_π is an unique constant for all processes, namely neglecting the virtual pion interaction between the loop and the external lines, the ratio of the partial decay rate reads

$$w(\pi^\pm\to\mu^\pm+\nu):w(\Lambda^0\to p+\pi^-)$$
$$:w(\Sigma^\pm\to n+\pi^\pm):w(\Xi^-\to\Lambda^0+\pi^-)$$
$$=2\left(\frac{m_\mu}{M_0}\right)^2\left(\frac{m_\pi^2-m_\mu^2}{m_\pi^2}\right)^2 m_\pi$$
$$:\left(\frac{M_\Lambda+M_0}{M_0}\right)^2\left(\frac{M_\Lambda-M_0+m_\pi}{M_\Lambda}\right)^{\frac{3}{2}}\left(\frac{M_\Lambda-M_0-m_\pi}{M_\Lambda}\right)^{\frac{3}{2}}$$
$$\times\left(\frac{M_\Lambda+M_0+m_\pi}{M_\Lambda}\right)^{\frac{1}{2}}\left(\frac{M_\Lambda+M_0-m_\pi}{M_\Lambda}\right)^{\frac{1}{2}}$$
$$:(\text{the same with } M_\Lambda\to M_\Sigma)$$
$$:\left(\frac{M_\Xi+M_\Lambda}{M_0}\right)^2\left(\frac{M_\Xi-M_\Lambda+m_\pi}{M_\Xi}\right)^{\frac{3}{2}}\left(\frac{M_\Xi-M_\Lambda-m_\pi}{M_\Xi}\right)^{\frac{3}{2}}$$
$$\times\left(\frac{M_\Xi+M_\Lambda+m_\pi}{M_\Xi}\right)^{\frac{1}{2}}\left(\frac{M_\Xi+M_\Lambda-m_\pi}{M_\Xi}\right)^{\frac{1}{2}}$$
$$=1:0.15\times10^2:0.94\times10^2:0.38\times10^2.$$

while the experimental ratio reads

$$1:0.60\times10^2:1.72\times10^2:?.$$

B. Three-Body Decay with Simple Structure

The process is featured by the decay diagram of Fig. 1(b), which includes the reactions $K_{\mu3}^+\to\mu^++\nu+\pi^0$ and $K_{e3}^+\to e^++\nu+\pi^0$. Again for reason of Lorentz invariance the black-box loop reads[7]

$$\frac{g_K}{(2s_0)^{\frac{1}{2}}}\frac{g_\pi}{(2t_0)^{\frac{1}{2}}}\left(\frac{1}{\sqrt{V}}\right)^2\sqrt{2}(2\pi)^2[F(k_\mu+p_\mu)+Gs_\mu]$$
$$\times\delta^4(s-k-p-t),\quad(2.9)$$

where s, k, p, and t are the four-momenta of the K-meson, muon (or electron), neutrino, and pion, respectively; $\sqrt{2}(2\pi)^2$ is just a numerical factor separated from F and G. The quantities F and G are in general relativistically invariant functions of the four-momenta, but dimensionless and of the same phase. We assume, however, that F and G are constant; in fact in lowest order perturbation theory F and G are found to be constants which depend on the masses of the K meson, pion, and baryons of the loop but are *independent* of the masses and momenta of the leptons within the accuracy of 10% or so. In other words, the structure element $C_{K\pi}$ is characterized by two constants F and G in this approximation. The integral over the final state can be done analytically,

[5] Silverman, Wilson, and Woodward, Phys. Rev. **108**, 501 (1957).

[6] P. T. Matthews and A. Salam, Phys. Rev. **110**, 569 (1958).

[7] F. Zachariasen, Phys. Rev. **110**, 1481 (1958).

yielding the decay rate

$$w(K_{\mu3}^+\to\mu^++\nu+\pi^0)$$

$$=(fM_0^2)^2\frac{g_K^2}{4\pi}\frac{g_\pi^2}{4\pi}\frac{1}{2(2\pi)^5}\left(\frac{m_K}{M_0}\right)^4 m_K\Bigg\{F^2\left(\frac{m_\mu}{m_K}\right)^2 \times[-\alpha\beta I_{-2}+(1+2\alpha\beta)I_{-1}-(2+\alpha\beta)I_0+I_1] +4FG\left(\frac{m_\mu}{m_K}\right)^2[I_{-1}-2I_0+I_1] +G^2[-\alpha\beta I_{-1}+(1+2\alpha\beta)I_0-(2+\alpha\beta)I_1+I_2]\Bigg\},\quad(2.10)$$

where

$$\alpha=(m_\pi+m_\mu)/m_\pi=0.491,$$

$$\beta=(m_\pi-m_\mu)/m_\pi=0.064,$$

$$I_{-2}=\int_{\alpha^2}^1\frac{[(x-\alpha^2)(x-\beta^2)]^{\frac{1}{2}}}{x^2}dx$$

$$=-[(1-\alpha^2)(1-\beta^2)]^{\frac{1}{2}}+2\ln[(1-\alpha^2)^{\frac{1}{2}}+(1-\beta^2)^{\frac{1}{2}}] -\frac{\alpha^2+\beta^2}{\alpha\beta}\ln[\alpha(1-\beta^2)^{\frac{1}{2}}+\beta(1-\alpha^2)^{\frac{1}{2}}] +\frac{(\alpha-\beta)^2}{2\alpha\beta}\ln(\alpha^2-\beta^2),$$

$$I_{-1}=\int_{\alpha^2}^1\frac{[(x-\alpha^2)(x-\beta^2)]^{\frac{1}{2}}}{x}dx$$

$$=[(1-\alpha^2)(1-\beta^2)]^{\frac{1}{2}}-(\alpha^2+\beta^2)\ln[(1-\alpha^2)^{\frac{1}{2}}+(1-\beta^2)^{\frac{1}{2}}] +2\alpha\beta\ln[\alpha(1-\beta^2)^{\frac{1}{2}}+\beta(1-\alpha^2)^{\frac{1}{2}}] +\tfrac{1}{2}(\alpha-\beta)^2\ln(\alpha^2-\beta^2),$$

$$I_0=\int_{\alpha^2}^1[(x-\alpha^2)(x-\beta^2)]^{\frac{1}{2}}dx$$

$$=\tfrac{1}{2}[1-\tfrac{1}{2}(\alpha^2+\beta^2)][(1-\alpha^2)(1-\beta^2)]^{\frac{1}{2}} -\tfrac{1}{4}(\alpha^2-\beta^2)^2\ln[(1-\alpha^2)^{\frac{1}{2}}+(1-\beta^2)^{\frac{1}{2}}] +\tfrac{1}{8}(\alpha^2-\beta^2)\ln(\alpha^2-\beta^2),$$

$$I_1=\int_{\alpha^2}^1 x[(x-\alpha^2)(x-\beta^2)]^{\frac{1}{2}}dx$$

$$=\tfrac{1}{3}[(1-\alpha^2)(1-\beta^2)]^{\frac{3}{2}}+\tfrac{1}{2}(\alpha^2+\beta^2)I_0,$$

$$I_2=\int_{\alpha^2}^1 x^2[(x-\alpha^2)(x-\beta^2)]^{\frac{1}{2}}dx$$

$$=\tfrac{1}{4}[(1-\alpha^2)(1-\beta^2)]^{\frac{3}{2}}+\tfrac{5}{8}(\alpha^2+\beta^2)I_1-\tfrac{1}{4}\alpha^2\beta^2I_0.$$

Similarly for K_{e3}^+ the decay rate is, omitting the term of the order $(m_e/m_K)^2\sim10^{-6}$,

$$w(K_{e3}^+\to e^++\nu+\pi^0)$$

$$=(fM_0^2)^2\frac{g_K^2}{4\pi}\frac{g_\pi^2}{4\pi}\frac{1}{(2\pi)^5}\left(\frac{m_K}{M_0}\right)^4 m_KG^2I,\quad(2.11)$$

TABLE I. The numerical values of the functions I_n in (2.10) and (2.11).

I_{-2}	I_{-1}	I_0	I_1	I_2	I
0.931	0.547	0.363	0.263	0.205	0.0238

where

$$I=(1/24)-\tfrac{1}{3}\mu^2+\tfrac{1}{3}\mu^6-(1/24)\mu^8-\mu^4\ln\mu,$$

$$\mu=m_\pi/m_K=0.278.$$

The numerical values of the function I_n are given in Table I.

The ratio of the partial decay rate becomes

$$\frac{w(K_{e3}^+)}{w(K_{\mu3}^+)}=\frac{2.38G^2}{0.18F^2+0.78FG+1.89G^2}.\quad(2.12)$$

The branching ratio of K_{e3}^+ to $K_{\mu3}^+$ is of order of 1 regardless of the details of the dynamics provided F and G are quantities of the same order. This value for the branching ratio is not inconsistent with the present experimental data.

Adjusting this branching ratio to the experimental value[2] 1, we obtain

$$F/G=-4.9\quad\text{or}\quad 0.56,\quad(2.13)$$

while inserting the experimental decay rate to the left-hand side of (2.11) we get

$$(g_K/g_\pi)|G|=0.033.\quad(2.14)$$

C. Decay of $K_{\pi2}^+$

The decay diagrams of the process $K_{\pi2}^+\to\pi^++\pi^0$ are shown in Fig. 2. Fortunately, the diagram of Fig. 2(b) does not contribute to this process in our approximation, since the final two-pion state necessarily has total isotopic spin 2 because of statistics, while the virtual baryon-antibaryon pair can only have the total isotopic spin 0 or 1. The last statement is trivial if one adopts Gell-Mann's $Y-Z$ formalism for global symmetry.[4] The argument fails for $K_{\pi2}^0$ decay, where the two pions can be in the state of total isotopic spin 0.

Hence we are able to check the consistency of the idea by calculating the decay rate of $K_{\pi2}^+$ from the diagram of Fig. 2(a), where the structure elements are only C_π and $C_{K\pi}$ and both have been already determined to fit the experimental data. The decay rate reads

$$w(K_{\pi2}^+\to\pi^++\pi^0)$$

$$=(fM_0^2)^2\frac{g_K^2}{4\pi}\left(\frac{g_\pi^2}{4\pi}\right)^2\frac{4}{(2\pi)^6}\left(\frac{m_K}{M_0}\right)^2 m_K \times\left[1-4\left(\frac{m_\pi}{m_K}\right)^2\right]^{\frac{1}{2}}\left|G+2F\left(\frac{m_\pi}{m_K}\right)^2\right|^2|C_\pi|^2,\quad(2.15)$$

and predicts

$$w(K_{\pi 2}^+)=1.6\times 10^7\ \text{sec}^{-1} \quad \text{and} \quad 3.2\times 10^8\ \text{sec}^{-1}, \qquad (2.16)$$

for $F/G=-4.9$ and 0.56, respectively, taking $|C_\pi|=0.25$ and $|G|=(g_\pi/g_K)0.033$. The first number is in agreement with the experimental value 2.09×10^7 sec^{-1}, if one allows for the crudeness of the approximations. The ratio of the decay rates of $K_{\pi 2}^+$ and K_{e3}^+ is independent of $g_K^2/4\pi$, and can be written

$$\frac{w(K_{\pi 2}^+)}{w(K_{e3}^+)}=\frac{g_\pi^2}{4\pi}\frac{2}{\pi}\left(\frac{M_0}{m_K}\right)^2 \times\frac{(1-4\mu^2)^{\frac{1}{2}}}{[(1/24)-\frac{1}{3}\mu^2+\frac{1}{3}\mu^6-(1/24)\mu^8-\mu^4\ln\mu]} \times\left[1+2\mu^2\left(\frac{F}{G}\right)\right]^2|C_\pi|^2 =4.4 \quad \text{and} \quad 86, \qquad (2.17)$$

for $F/G=-4.9$ and 0.56, respectively, while the experimental ratio is 6.1. In fact in lowest order perturbation theory $F/G<0$, and $|F|>|G|$, with reasonable cutoff momentum.

3. SUMMARY AND DISCUSSION

The decay rates of pions, hyperons, and $K_{\mu 2}^+$ are expressed by a single phenomenological parameter. If the higher order corrections due to the strong interaction between the virtual loop and the external lines are omitted, these phenomenological parameters characterizing each reaction are expected to be constant independent of the decay mode. It is found that this is the case within a factor of 2. Also, if the K-meson-baryon coupling constant is assumed to be about $\frac{1}{4}$ of the pion-baryon coupling constant, then we have a consistent picture of the pseudoscalar K meson as far as $K_{\mu 2}^+$ decay is concerned. The small variation of this quasi-constant may be attributed either to the effect of higher order corrections so far omitted or to some sort of isotopic spin dependence of the weak interaction anticipated by the isotopic spin selection rules (Sec. 2, A).

The decays of K_{e3}^+, and $K_{\mu 3}^+$ are characterized by two phenomenological parameters which are roughly constant, say, within a factor of order unity. The ratio of the decay rate of K_{e3}^+ and $K_{\mu 3}^+$ is found to be of order of 1, and is actually insensitive to the details of the dynamics. This is consistent with experiment. Unfortunately, we cannot test the reliability of the constancy of these parameters because no more reactions of the same sort are available (Sec. 2, B).

In our approximation, the decay rate of $K_{\pi 2}^+$ is expressed by the phenomenological parameters found previously and serves as a consistency check of the whole picture and the approximation. The decay rate predicted is in agreement with the experimental value (Sec. 2, C).

We have no way of testing the decays of the $K_{\pi 3}^+$ and neutral K mesons within the framework of this treatment.

We are tempted to say in conclusion that the universal Fermi interaction with $V-A$ coupling together with Gell-Mann's tetrahedron have the capability of explaining, at least, the decay of hyperons and charged mesons at large.

APPENDIX

For the purpose of reference, we give in this Appendix the results of a lowest order perturbation calculation of the dynamical quantities C_π, C_K, F, and G.

The constant C_π of the π-μ decay is calculated from the diagram of Fig. 1(a), and yields[8]

$$C_\pi=\sum_{\text{baryons}}\epsilon\frac{M}{M_0}\left\{\frac{1}{2}\ln\left(\frac{M^2+\lambda^2}{M^2}\right) +\frac{[4(M^2+\lambda^2)-m_\pi^2]^{\frac{1}{2}}}{m_\pi}\sin^{-1}\left[\frac{m_\pi}{2(M^2+\lambda^2)^{\frac{1}{2}}}\right] -\frac{(4M^2-m_\pi^2)^{\frac{1}{2}}}{m_\pi}\sin^{-1}\left(\frac{m_\pi}{M}\right)\right\}, \qquad (A.1)$$

where λ is the Feynman cutoff momentum, and ϵ is an isotopic spin factor whose value is given in Table II,

TABLE II. The factor ϵ in (A.1).

Vertex	ϵ
$(pn\pi^+)$	$\sqrt{2}$
$(\Sigma^+\Lambda^0\pi^+)$	1
$(\Lambda^0\Sigma^-\pi^+)$	1
$(\Sigma^+\Sigma^0\pi^+)$	-1
$(\Sigma^0\Sigma^-\pi^+)$	1
$(\Xi^0\Xi^-\pi^+)$	$\sqrt{2}$

and it depends on the type of baryon-antibaryon pairs in the loop. We neglect the mass difference between Λ and Σ throughout this paper. As the summation over baryons is additive, in order to fit the experimental lifetime of the pion, the cutoff momentum becomes unreasonably small, namely

$$\lambda\leq 0.2M_0. \qquad (A.2)$$

The constant C_K in the decay $K_{\mu 2}^+\rightarrow\mu^++\nu$ is more complicated because the masses of the baryon-antibaryon pairs in the closed loops are always different. We notice that C_K is approximately proportional to the mass difference of baryons for a scalar K meson,

[8] R. J. Finkelstein and M. Ruderman, Phys. Rev. 76, 1458 (1949).

and to the sum of the masses for a pseudoscalar K meson. We obtain

$$
\begin{aligned}
C_K=\sum_{\text{baryons}} \epsilon\frac{1}{8M_0}\Bigg((M'\mp M)\left(1-\frac{(M'\pm M)^2}{m_K^2}\right)\Bigg\{\left(1+\frac{M'^2-M^2}{m_K^2}\right)\ln\left(\frac{M'^2}{M'^2+\lambda^2}\right)\\
+\left(1-\frac{M'^2-M^2}{m_K^2}\right)\ln\left(\frac{M^2}{M^2+\lambda^2}\right)+\frac{2}{m_K}\left[2(M^2+M'^2)-m_K^2-\frac{(M^2-M'^2)^2}{m_K^2}\right]^{\frac{1}{2}}\left[\sin^{-1}\left(\frac{m_K+(M'^2-M^2)/m_K}{2M'}\right)\right.\\
\left.+\sin^{-1}\left(\frac{m_K-(M'^2-M^2)/m_K}{2M}\right)\right]-\frac{2}{m_K}\left[2(M^2+M'^2+2\lambda^2)-m_K^2-\frac{(M^2-M'^2)}{m_K^2}\right]^{\frac{1}{2}}\\
\times\left(\sin^{-1}\left[\frac{m_K+(M'^2-M^2)/m_K}{2(M'^2+\lambda^2)^{\frac{1}{2}}}\right]+\sin^{-1}\left[\frac{m_K-(M'^2-M^2)/m_K}{2(M^2+\lambda^2)^{\frac{1}{2}}}\right]\right)\Bigg\}+(M'\pm M)\frac{2}{m_K^2}\\
\times\left\{M'^2\ln\left(\frac{M'^2}{M'^2+\lambda^2}\right)-M^2\ln\left(\frac{M^2}{M^2+\lambda^2}\right)+\lambda^2\ln\left(\frac{M^2+\lambda^2}{M'^2+\lambda^2}\right)\right\}\Bigg), \quad \text{(A.3)}
\end{aligned}
$$

where ϵ is given in Table III, M is the mass of the baryon having the lower absolute value of strangeness, and M' is the mass of the baryon having the higher absolute value of strangeness in the combinations given in Table III. The upper sign in (A.3) is for a scalar K meson, and the lower sign is for a pseudoscalar K meson. For the sake of simplicity we take the coupling constants of the K meson to the baryons to be equal, though the mass difference among baryons is taken into account.

TABLE III. The factor ϵ in (A.3).

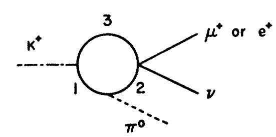
Vertex	ϵ
$(p\Lambda^0K^+)$	1
$(n\Sigma^-K^+)$	$\sqrt{2}$
$(p\Sigma^0K^+)$	1
$(\Lambda^0\Xi^-K^+)$	1
$(\Sigma^+\Xi^0K^+)$	$\sqrt{2}$
$(\Sigma^0\Xi^-K^+)$	-1

For the constants F and G, terms involving the pion and K-meson mass are consistently neglected with respect to the baryon mass. However, because of a fortuitous cancellation this approximation for F and G is estimated to be reliable within 10%.

TABLE IV. The factor ϵ in (A.4) and (A.5). The numbers 1, 2, and 3 specify the types of baryons appearing in the closed loop in the manner shown in Fig. 5. The upper and lower sign refer to a scalar and a pseudoscalar K-meson, respectively.

1	Λ^0	Σ^0	Σ^-	Ξ^0	Ξ^-	Ξ^-	p	p	n	Σ^+	Σ^0	Λ^0
2	Σ^0	Λ^0	Σ^-	Ξ^0	Ξ^-	Ξ^-	p	p	n	Σ^+	Λ^0	Σ^0
3	p	p	n	Σ^+	Λ^0	Σ^0	Σ^0	Λ^0	Σ^-	Ξ^0	Ξ^-	Ξ^-
ϵ	1	1	$-\sqrt{2}$	$\sqrt{2}$	-1	1	±1	±1	$\mp\sqrt{2}$	$\pm\sqrt{2}$	∓1	±1

FIG. 5. Decay diagram of $K_{\mu3}^+$ and K_{e3}^+.



The explicit expressions of F and G in this approximation read

$$
\begin{aligned}
F=\sum_{\text{baryons}}\frac{\epsilon}{4}\Bigg[-\frac{M'^3(M'\pm 2M)}{(M'^2-M^2)^2}\ln\left(\frac{M^2}{M'^2}\right)\\
-\ln\left(\frac{M^2+\lambda^2}{M^2}\right)-\left\{\frac{(M'^2+\lambda^2)(M'^2+\lambda^2\pm 2MM')}{(M'^2-M^2)^2}\right.\\
\left.-\frac{\lambda^2(M'^2+\lambda^2)}{(M'^2-M^2)^3}\right\}\ln\left(\frac{M'^2+\lambda^2}{M^2+\lambda^2}\right)\\
+\frac{\lambda^2(M^2-2M'^2-\lambda^2)}{(M'^2-M^2)}+\frac{3\lambda^2}{2(M'^2-M^2)}\Bigg] \quad \text{(A.4)}
\end{aligned}
$$

$$
\begin{aligned}
G=\sum_{\text{baryons}}\frac{\epsilon}{4}\Bigg[\frac{2M'^3(M'\pm M)}{(M'^2-M^2)^2}\ln\left(\frac{M^2}{M'^2}\right)\\
+2\ln\left(\frac{M^2+\lambda^2}{M^2}\right)+\left\{\frac{2(M'^2+\lambda^2)(M'^2+\lambda^2\pm MM')}{(M'^2-M^2)^2}\right.\\
\left.+\frac{\lambda^2(M'^2+\lambda^2)^2}{(M'^2-M^2)^3}\right\}\ln\left(\frac{M'^2+\lambda^2}{M^2+\lambda^2}\right)\\
+\frac{\lambda^2(M^2-2M'^2-\lambda^2)}{(M'^2-M^2)^2}-\frac{3\lambda^2}{2(M'^2-M^2)}\Bigg], \quad \text{(A.5)}
\end{aligned}
$$

where ϵ is tabulated in Table IV, M is the mass of the baryons labeled 1 and 2, and M' is that of the baryon labeled 3 in Fig. 5.