resulting surface integral by the method of stationary phase. Neglecting terms which oscillate with R (the radius of the sphere) or decrease with increasing R, the result turns out to be

$$\Delta f = -i\mu\alpha Z\epsilon f, \tag{49}$$

in agreement with that obtained by the partial wave analysis. This also confirms Ravenhall's result that recoil effects do not contribute in Born approximation (note that the first term of H_{eff}' corresponds to second Born approximation in the electromagnetic interaction).

IV. CONCLUSIONS

To the extent that the finite mass of the electron or muon may be neglected, the dynamic recoil effect does not influence the cross section. To this approximation, recoil is taken into account by calculating with an energy $\mathcal{E} = E_{\mathrm{lab}}(1 - E_{\mathrm{lab}}/M)$, and by transforming calculated cross sections and angles from the center-of-momentum frame to the laboratory frame. Consideration of the finite mass of the electron or muon leads to a dynamic recoil effect on the cross section which is $\Delta\sigma/\sigma = (\alpha Z)(m/M)(m/E)\gamma$, where γ is an amplification factor (arising from the great cancellations in the partial wave sum) whose magnitude is very uncertain, but might be as great as 10^4 at the highest energies and greatest angles where electron scattering data are available.

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Alternative Method for Comparing Pion-Proton Scattering Data with Dispersion Equations*

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A method for comparing pion-proton scattering experiments with the predictions of the forward angle scattering dispersion equations is proposed, which allows the usual statistical measure (χ^2) of the agreement. A slight discrepancy is found between negative pion-proton data and the theory; however, the over-all agreement is considered satisfactory. Values of the coupling constant and S-wave zero-energy scattering lengths are determined. They are $f^2 = 0.08 \pm 0.01$, $a_1 = 0.193 \pm 0.050$, and $a_3 = -0.089 \pm 0.048$.

1. INTRODUCTION

SINCE the analysis of the pion-nucleon scattering by use of forward scattering dispersion equations was made by Puppi and Stanghellini,[1] several authors have discussed the lack of agreement between the theory and experiments.[2] It is desirable to make the comparison in a way more easily analyzed statistically than the Puppi-Stanghellini method. One such method is presented here,[3] and the results, which include a determination of the pion-nucleon coupling constant and the zero-energy S-wave scattering lengths, are reported.

2. METHOD OF ANALYSIS

One can write the forward scattering pion-proton dispersion equations as follows[4]:

$$\tfrac{1}{2}[D_+(1)+D_-(1)] \pm \tfrac{1}{2}\omega[D_+(1)-D_-(1)]$$
$$= D_\pm(\omega) - \frac{k^2}{4\pi^2}\,\mathrm{P.V.}\int_1^\infty \frac{d\omega'}{k'}\frac{\sigma_\pm^{\mathrm{tot}}(\omega')}{\omega'-\omega} - \frac{k^2}{4\pi^2}$$
$$\times \int_1^\infty \frac{d\omega'}{k'}\frac{\sigma_\mp^{\mathrm{tot}}(\omega')}{\omega'+\omega} \mp \frac{2k^2}{\omega\mp(1/2M)} f^2 \equiv J_\pm(\omega), \tag{1$\pm$}$$

where the notation and units are the same as in SS.[2] $D_\pm(\omega)$ is the real part of the $\pi^\pm$-p forward scattering amplitude in the laboratory system at pion energy ω. Define

$$C_1 = \tfrac{1}{2}[D_+(1)+D_-(1)],$$
$$C_2 = \tfrac{1}{2}[D_+(1)-D_-(1)]. \tag{2}$$

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[1] G. Puppi and A. Stanghellini, Nuovo cimento 5, 1305 (1957).
[2] H. J. Schnitzer and G. Salzman, Phys. Rev. 112, 1802 (1958). This will be referred to as SS. Additional references may be found in this paper.
[3] H. P. Noyes and D. N. Edwards (to be published) recast the comparison to facilitate statistical analysis. The analyses differ in that we use "experimental" cross sections, with no errors assigned to the integrals, while they use "theoretical" ones to evaluate the integrals, with an associated error. They find an f^2 for each energy while we require a fit to all energies with the same f^2.
[4] Goldberger, Miyazawa, and Oehme, Phys. Rev. 99, 986 (1955).

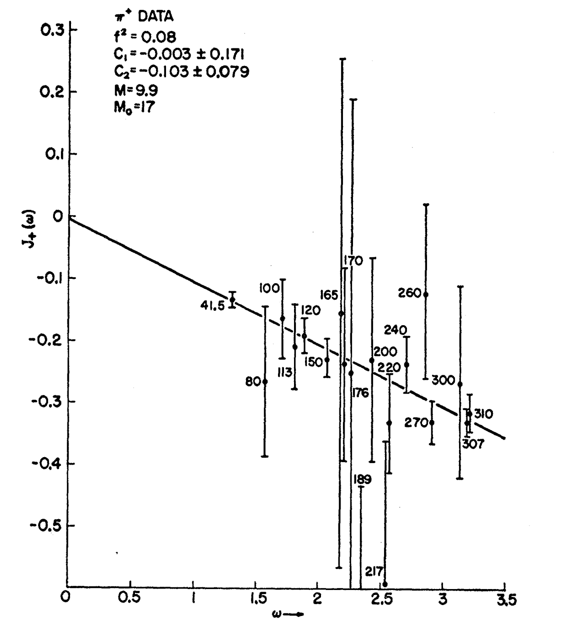

FIG. 1. Plot I of the π^+-p data, Eq. (3+) for $f^2=0.08$. The data should be consistent with a straight line to agree with the dispersion theory. The best fit is shown, with $M=9.9$ and $M_0=17$.

The right-hand side of Eq. (1), $J_\pm(\omega)$, is evaluated as in SS. For each experimental determination of $D_+(\omega)$ or $D_-(\omega)$, we have a value of $J_+(\omega)$ or $J_-(\omega)$, and an associated error. We consider only that part of the error in $J_\pm(\omega)$ due to $D_\pm(\omega)$.[3] The constants f^2, C_1, and C_2 are parameters to be determined. The π^+ and π^- data are represented by plotting $J_+(\omega)$ and $J_-(\omega)$ *versus* ω, respectively, which must satisfy

$$C_1\pm\omega C_2=J_\pm(\omega). \qquad (3\pm)$$

Therefore, as a consequence of crossing symmetry and of the dispersion equations, the π^+ and π^- experimental data should be represented by straight lines with common intercept C_1 at $\omega=0$, and equal and opposite slopes, $\pm C_2$. It is convenient to reflect the π^- data about $\omega=0$ so that all the data determine one straight line with slope C_2. For this we write

$$C_1+\omega C_2=\begin{cases} J_+(\omega) & \text{for } \omega\geqslant 1 \\ J_-(-\omega) & \text{for } \omega\leqslant -1. \end{cases} \qquad (3c)$$

We will call the plots of Eqs. (3+), (3−), and (3c) plots (I), (II), and (III), respectively. For any value of f^2, (I), (II), or (III) determines a pair of numbers (C_1, C_2) by a least-squares fit. If $\chi^2(f^2)$ is the weighted sum of the squares of the least-squares errors for a given value of f^2, then the best value of f^2 is obtained at M, the minimum of $\chi^2(f^2)$. The expected value for a satisfactory fit of the data by a straight line, is denoted by M_0. The dispersion equations imply the following:

(a) Each of the plots, (I), (II), (III), is consistent with a straight line, i.e., $M\leqslant M_0$.

(b) The same f^2 is determined by the minimum of each χ^2.

(c) With this f^2 each plot determines the same pair (C_1, C_2).

The assumption of charge independence relates C_1 and C_2 to the zero-energy S-wave scattering lengths,

$$a_1=(k_b/k)_{\omega=1}(C_1-2C_2),$$
$$a_3=(k_b/k)_{\omega=1}(C_1+C_2). \qquad (4)$$

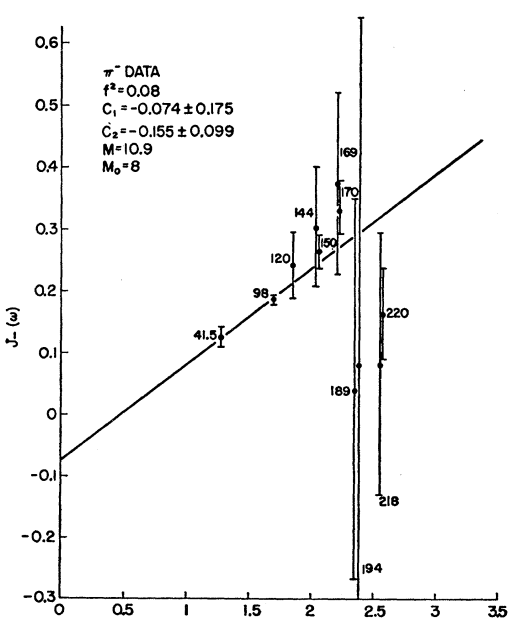

FIG. 2. Plot II of the π^--p data, Eq. (3−), for $f^2=0.08$.

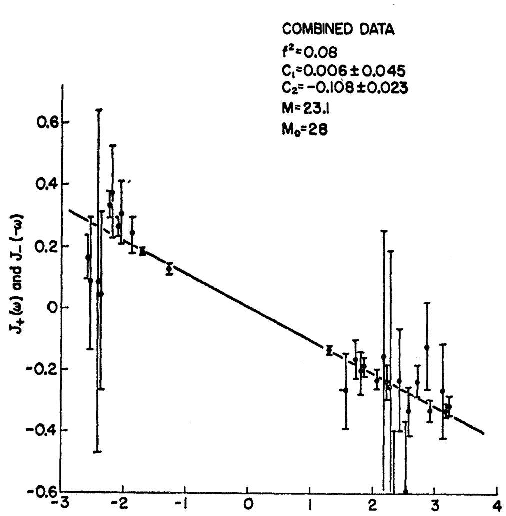

FIG. 3. Plot III of the combined data, Eq. (3c), for $f^2=0.08$.

TABLE I. Summary of results.

	I π^+ data	II π^- data	III Combined data
f^2	$0.08_{-0.01}^{+0.02}$	$0.08_{-0.02}^{+0.01}$	0.08 ± 0.01
M	9.9	10.8	23.1
M_0	17	8	28
C_1	-0.003 ± 0.171	-0.074 ± 0.176	0.006 ± 0.045
C_2	-0.103 ± 0.079	-0.155 ± 0.099	-0.108 ± 0.023

The values of a_1 and a_3 so obtained may then be compared with those obtained from phase-shift analyses.

3. RESULTS

The results of the analysis are shown in Figs. 1–4. Inclusion of the 307- and 333-Mev values of $D_-(\omega)$ from Korenchenko and Zinov[5] causes condition (a) to be violated in plots II and III. Also, there is no minimum in $\chi^2(f^2)$, which decreases monotonically as f^2 is decreased. Since these two values of $D_-(\omega)$ are highly sensitive to D waves, as described in SS, they are omitted here. With their omission, a consistent analysis is obtained. This corroborates the less quantitative but similar conclusion in SS regarding these data.[6,7] The results are summarized in Table I. The error for f^2 is estimated from the width of the minimum of $\chi^2(f^2)$.

The S-wave scattering lengths from the constants (C_1, C_2) of III are[8]

$$a_1=0.193\pm0.050,$$

$$a_3=-0.089\pm0.048.$$

These are consistent with Orear's[9] values:

$$a_1=0.165\pm0.012,$$

$$a_3=-0.105\pm0.010.$$

Also we find $a_1-a_3=0.272\pm0.060$, which is in agreement with measurements of the Panofsky ratio.[10]

[5] S. M. Korenchenko and V. G. Zinov, Joint Institute for Nuclear Research, Dubna, U.S.S.R. (to be published).

[6] H-Y. Chiu and J. Hamilton, Phys. Rev. Letters **1**, 146 (1958), also reach the same conclusion regarding these data.

[7] H. J. Schnitzer and G. Salzman, Bull. Am. Phys. Soc. Ser. II, 2, 353 (1957) and SS D-waves are not included in analyzing data below 300 Mev, although they are included in reference 6.

[8] The more recent value of $D_-^b=0.195\pm0.006$ at 98 Mev, J. R. Holt (private communication), will change the numerical results of plot II and III slightly, however the conclusions are unaltered.

[9] J. Orear, Phys. Rev. **96**, 176 (1954).

[10] L. Marshall, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics, 1957* (Interscience Publishers, Inc., New York, 1957), II-32.

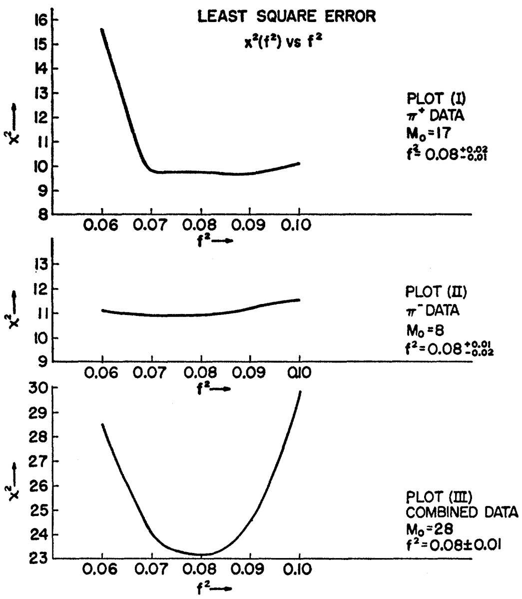

FIG. 4. A graph of the least-squares function $\chi^2(f^2)$ for various values of f^2. For each plot the best value of f^2 is estimated from the minimum of each curve, and the error estimated from the width of the minimum.

4. CONCLUSIONS

The π^- data are slightly inconsistent with a straight line; $M=10.8$ and $M_0=8$. This result agrees with SS and is regarded as not serious for the same reasons discussed there. Aside from this, we conclude that the pion-proton scattering data are consistent with dispersion theory, and we obtained $f^2=0.08\pm0.01$, $a_1=0.193\pm0.050$, and $a_3=-0.089\pm0.048$; values compatible with other determinations. Although this way of presenting the data gives a quantitive measure of the agreement between theory and experiment, it does not determine f^2, a_1, and a_3 precisely. Their accuracy would be improved by one or two measurements of $D_+(\omega)$ between 100 and 300 Mev and a measurement of $D_-(\omega)$ at $\sim$300 Mev as accurate as that of $D_-(\omega)$ at 98 Mev. Together with the low-energy Rochester values, these would give a better determination of the slope and intercept of the line in plot III.

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We are indebted to Professor G. Chew for suggesting a form of presentation amenable to statistical analysis, of which the method here given is a modification.