First Excited State of $B¹¹$ and Spin-Flip Stripping*

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A theoretical treatment is presented of the anomalous angular distribution in the $B^{10}(d, \phi)B^{11}$ reaction to the first excited state of the final nucleus. The experimental distribution is characteristic of the orbital angular momentum of the captured neutron being 1. However, the known spins and parities of the target and final state nuclei do not admit this value. Sy coupling the spin of the stripped proton to the spin of the target nucleus $+$ neutron, an extra unit of angular momentum may be imparted to this system allowing the final state to be reached. The angular distribution from such a mechanism is calculated and fits rather well the experimental curves.

HE shell model predicts^{1,2} a spin of $\frac{1}{2}$ and odd parity for the first excited state of $B¹¹$. The experimental evidence to support these assignments has been given in detail byWilkinson' and seems to be conclusive. However, one is then left to explain the angular distribution of the $B^{10}(d,p)B^{11}$ reaction which leaves B^{11} in its first excited state. ' This experimental cross section is best fit with a Butler stripping curve for a neutron captured with an orbital angular momentum of unity. Since the ground state of B^{10} is $3(+)$, a final state of $\frac{1}{2}(-)$ cannot be reached with this value of the orbital angular momentum.

It was suggested' that this anomaly might be explained by a nucleon exchange process, so that the outgoing proton could come from the target nucleus and not solely from the deuteron as conventional stripping theory implies. The derivation of an expression for the angular distribution from such a process has been given.⁶ However, the assumptions made in this work are, as is pointed out in the paper, of a drastic nature. For example, it is assumed that there is no direct interaction between the two protons which exchange.

Numerical calculations based on this analysis have been made.⁷ In order to simplify the calculations it is necessary to assign definite shell-model states to the various nucleons involved and the final cross section is brought into agreement with the experimental curve only by adding to the exchange stripping cross section an isotropic background. This is taken to come from "compound nucleus" formation and any interference effects between the two processes are neglected.

A second suggestion has been made by Wilkinson' to explain the occurrence of the observed angular distribution. This is the idea that the stripped proton, suffering

I. INTRODUCTION α close collision with the system of neutron $+$ target nucleus, may Rip over its spin and thus deliver an extra unit of angular momentum to the system. This is unt of angular momentum to the system. This is
enough for a final state of $\frac{1}{2}(-)$ to be reached. It is the purpose of this work to calculate the angular distribution from such a mechanism.

II. THEORY

Our starting point is the expression for the cross section for definite spin states in a (d,p) reaction⁸:

$$
\sigma(\theta) = \frac{1}{4\pi^2 \hbar^4} \frac{k_p}{k_f} M_{pf} M_{di} \left| \int \chi_{j_f}^{m_f *}(\xi, \mathbf{r}_n, \sigma_n) \chi_{\mathbf{i}}^{\mu_p *}(\sigma_p) \right|
$$

× $\exp(-i\mathbf{k}_p \cdot \mathbf{r}_p') (V_p + V_{pn})$
× $\Psi(\xi, \mathbf{r}_n, \sigma_n, \mathbf{r}_p, \sigma_p) d\mathbf{r}_n d\mathbf{r}_p d\xi \Big|^2$. (1)

Here r_n and r_p are the neutron and proton coordinates measured from the center of mass of the target nucleus.

$$
\mathbf{r}_{p}'=\mathbf{r}_{p}-(M_{n}/M_{f})\mathbf{r}_{n}.
$$

 σ_p , σ_n refer to the spins of the proton and neutron. ξ represents the space and spin coordinates of all other nucleons. \mathbf{k}_p and \mathbf{k}_d are the wave vectors associated with the proton and deuteron motions. The indices i and f refer to the target and final nuclei, respectively.

$$
M_{pf} = M_p M_f/(M_p + M_f);
$$
 $M_{di} = M_d M_i/(M_d + M_i).$

 V_p is the interaction between the proton and target nucleus. V_{pn} is the interaction between the proton and neutron. Ψ is the state vector of the total scattering process and we shall make the approximation of replacing this by an incident plane wave of deuterons:

 $\Psi = \exp{\{\frac{1}{2}i\mathbf{k}_d \cdot (\mathbf{r}_n+\mathbf{r}_p)\}\phi_d(\left|\mathbf{r}_n-\mathbf{r}_p\right|)\chi_1^{md}(\mathbf{y}_d)\chi_{j_i^{m_i}}(\xi)}.$

It is well known' that this approximation gives good agreement with experiment for the normal stripping process. Thus, averaging over initial and summing over

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final spin states, we obtain

$$
\sigma(\theta) = \frac{1}{4\pi^2 \hbar^4} \frac{k_p}{k_d} M_{pf} M_{di} \sum_{m_d, m_i, m_p, m_f} \times \frac{1}{2(2j_i+1)} |I_{mfm_i}^{m_p m_d}|^2,
$$

where

$$
I_{mfm_i}^{m_p m_d} = \int \chi_{jf}^{mf} (\xi, \mathbf{r}_n, \mathbf{\sigma}_n) \chi_{ij}^{mf}(\mathbf{\sigma}_p)
$$

$$
I_{m_f m_i^{m_p m_d}} = \int \chi_{j_f^{m_f}} \times (\xi, \mathbf{r}_n, \mathbf{\sigma}_n) \chi_{\mathbf{i}}^{m_f} \times (\mathbf{\sigma}_p)
$$

$$
\times \exp(-i\mathbf{k}_p \cdot \mathbf{r}_p') (V_p + V_{pn})
$$

$$
\times \exp\{\frac{1}{2}i\mathbf{k}_d \cdot (\mathbf{r}_n + \mathbf{r}_p)\} \phi_d (\vert \mathbf{r}_n - \mathbf{r}_p \vert)
$$

$$
\times \chi_{\mathbf{1}}^{m_d}(\mathbf{\mu}_d) \chi_{\mathbf{i}}^{m_i}(\xi) d\mathbf{r}_n d\mathbf{r}_p d\xi. \quad (2)
$$

In conventional stripping theory one neglects the contribution from the interaction of the proton with the target nucleus or allows the proton to interact with some type of single-particle potential¹⁰ so what it is unable to transfer angular momentum to the other nucleons, this giving rise to the anomaly under consideration.

We shall neglect therefore the term in V_{pn} and parts of V_{p} of the above-mentioned kind since these are or v_p or the above-mentioned kind since these at incapable of leading to a final state of $\frac{1}{2}(-)$. Instead we shall consider that V_p includes a part which is capable of flipping the spin of the proton and giving up this unit of angular momentum to form the final nucleus. For simplicity, and since it is physically reasonable, we assume that this interaction takes place at the nuclear surface. Thus we write $V_p = f(\sigma_p, \xi) \delta(r_p - R)$, where R is the radius of the target nucleus.

We may then write

$$
I_{m_f m_i^{m_p m_d}} = \int \left[\int \chi_{j_f^{m_f *}}(\xi, \mathbf{r}_n, \sigma_n) \chi_{\frac{1}{2}}^{\mu_p *}(\sigma_p) \times f(\sigma_p, \xi) \chi_1^{m_d}(\mathbf{u}_d) \chi_{j_i^{m_i}}(\xi) r_n^2 dr_n d\xi \right] \qquad \text{Su}
$$

$$
\times \exp(-i\mathbf{k}_p \cdot \mathbf{r}_p') \delta(\mathbf{r}_p - R) \qquad \text{Su}
$$

$$
\times \exp\{i\frac{1}{2}\mathbf{k}_d \cdot (\mathbf{r}_n + \mathbf{r}_p)\} \phi_d(\|\mathbf{R}_n - \mathbf{r}_p\|) dr_p d\Omega_n, \qquad m_f,
$$

or, expanding the square brackets in states of given orbital angular momentum of the captured neutron,

$$
I_{m_f m_i^{m_p m_d}} = \sum_{l_n, m_n} \left(\int \chi_{j_f}^{m_f *}(\xi, \mathbf{r}_n', \sigma_n) \chi_{\frac{1}{2}}^{m_f *}(\sigma_p) \times f(\sigma_p, \xi) \chi_1^{m_d}(\mathbf{u}_d) \chi_{j_i}^{m_i}(\xi) \, I_{l_n}^{m_n}(\Omega_n') d\mathbf{r}_n' d\xi \right) \times \left(\int \exp(-i\mathbf{k}_p \cdot \mathbf{r}_p') \delta(r_p - R) \times \exp\{\frac{1}{2} i\mathbf{k}_d \cdot (\mathbf{R}_n + \mathbf{r}_p)\} \times \phi_d(\left| \mathbf{R}_n - \mathbf{r}_p \right|) \, Y_{l_n}^{m_n *}(\Omega_n) d\mathbf{r}_p d\mathbf{r}_n \right). \tag{3}
$$

¹⁰ E.g., W. R. Cheston, Phys. Rev. 96, 1590 (1954).

FIG. 1. Comparison of theoretical and experimental cross sections for a deuteron energy of 6.2 Mev.

In the incident plane wave of deuterons we have substituted \mathbf{R}_n for \mathbf{r}_n , where \mathbf{R}_n is the value of \mathbf{r}_n on the nuclear surface. This follows Bhatia et al. by assuming that the stripping takes place at the nuclear surface.

This expression may be simplified by coupling the spin of the outgoing proton to the spin of the final nucleus:

$$
\chi_{j}j^{m} \chi^{*}(\xi, \mathbf{r}_{n}, \sigma_{n}) \chi_{\frac{1}{2}}\mu_{p}^{*}(\sigma_{p}) = \sum_{J, M} C_{j}j_{j} \chi^{Mm} \chi_{p} \Psi_{j}j_{j} \chi^{M}, \quad (4a)
$$

and by coupling together the spin of the deuteron, the spin of the target nucleus, and the orbital angular momentum of the neutron:

$$
\chi_{j_i^{m_i}}(\xi)\chi_1^{m_d}(\mathbf{u}_d)Y_{l_n^{m_n}}(\Omega_n)
$$

=
$$
\sum_{J',M'}\sum_{j,m}C_{J'j l_n}^{M'mm_n}C_{j j_i 1}^{m m_i m_d} \Psi_{J' j l_n}^{M'}.
$$
 (4b)

Substituting this into (3) and summing over the intermediate indices leads to

$$
\sum_{m_f,\mu_p,m_d,m_i} |I_{m_f m_i^{m_p m_d}}|^2
$$

$$
= \sum_{l_n} \left(\sum_{J,j} (2J+1) \left| \int \Psi_{Jj} \Psi_{Jj} M f(\sigma_p, \xi) \Psi_{Jj} l_n M d\mathbf{r}_n' d\xi \right|^2 \right) \times \frac{1}{2l_n+1} \sum_{m_n} |\alpha(l_n, m_n)|^2, \quad (5)
$$

where

$$
\alpha(l_n, m_n) = \int \exp(-i\mathbf{k}_p \cdot \mathbf{r}_p) \delta(r_p - R)
$$

$$
\times \exp\left\{\frac{i}{2}\mathbf{k}_d \cdot \left[\mathbf{r}_p + \mathbf{R}_n \left(\frac{M_i + 2M_n}{M_i + M_n}\right)\right]\right\}
$$

$$
\times \phi_d \left(\left|\mathbf{R}_n \frac{M_i}{M_i + M_n} - \mathbf{r}_p\right|\right) Y l_n^{m_n * (\Omega_n) d\Omega_n} dr_p.
$$

If the neutron is captured into a state of given orbital angular momentum, then only one term in the sum over l_n remains. We see that the factor

$$
\left|\int \Psi_{Jj_1\frac{1}{2}}M^*f(\sigma_p,\xi)\Psi_{Jj l_n}M d\mathbf{r}_n d\xi\right|^2
$$

gives us the strength of the interaction which flips the spin of the proton.

For the particular case under consideration with $l_n=1$, $j_i=3$, and $j_f=\frac{1}{2}$, then only $J=1$ contributes to the sum over J and only $j=2$ to the sum over j. These restrictions could have been inserted earlier in Eqs. (4) .

III. NUMERICAL RESULTS

The angular distribution is determined by the factors $\alpha(1,m_n)$. These may be calculated by expanding the exponentials as sums in spherical harmonics, putting in a Yukawa shape for the deuteron wave function and doing the integrals numerically.

The results are shown in Figs. 1 and 2. The theoretical curves were normalized to give the best fit to the experimental data; however, the same normalization factor was used for both curves. The Butler curves are also shown for comparison. A pleasing feature is the reproduction of the high cross section at larger angles. It is probably possible to smear out the minima occurring in the higher energy theoretical curve by allowing the proton spin interaction to take place over a shell on the surface of the nucleus, i.e., by spreading out the 5-function interaction.

The one free parameter in determining the angular distribution was the radius of the target nucleus appearing in the α 's. This was chosen to be $R=5.45$ appearing in the α 's. This was chosen to be $R = 5.45$
 $\times 10^{-13}$ cm as compared to a value of $R = 6.0 \times 10^{-13}$ cm chosen⁶ to fit the $B^{10}(d,p)B^{11}$ (ground state) data with a Butler curve.

The occurrence of the $\delta(r_p - R)$ in the integral for $\alpha(1,m_n)$ will cut down contributions to this integral from high values of proton orbital angular momentum. This means that protons moving far away from the target nucleus suffer no interaction and cannot produce the final-state nucleus. This cutting out of the proton waves of high orbital angular momentum will reduce the dominance of the forward peak over the larger angle cross section given by the Butler theory.

IV. MISCELLANEOUS CONSIDERATIONS

Since we have shown that this type of mechanism will explain the particular angular distribution under consideration, one may ask whether this would also play a part in the cases where the reaction may proceed by normal stripping.

As the mechanism requires both the neutron and

Ed =8 Mev Butler curve
Spin-flip curve Experimental points units) FIG. 2. Compari-**(arbitrary** son of theoretical
and experimental experimental cross sections for a deuteron energy $\frac{1}{\sigma}$ 8 Mev. I I / , ~~i I $\overline{\alpha}$ $30⁶$ 60' g04 θ (C.M. angle)

proton to move close to the target nucleus, the effect will be largest where the Q value of the reaction is large as in the case considered. In the case where normal stripping takes place, the stripped protons move far away from the nucleus and one would expect spin-Rip stripping to make only a small contribution to the cross section. Even for the example now considered, the cross section is down in absolute magnitude by a factor of ten from a normal stripping curve. Thus any contribution from this process would not alter the angular distribution in a normal stripping curve, and hence would not interfere with the assignment of the l value to the captured nucleon.

The reduced neutron width as calculated by a straight fitting of a Butler curve to the experimental data should also be affected but little. However, a method of extracting reduced widths which depends upon comparing the experimental and theoretical partial cross sections for high values of the proton angular momentum¹¹ should still give a better value for the neutron width since protons of high angular momenta do not interact with the nucleus.

The polarization of the outgoing protons should be opposite in sign to the polarization in normal stripping opposite in sign to the polarization in normal strand this seems to be borne out experimentally.¹²

This type of process will be of importance when the normal stripping is inhibited. This will occur when the spin difference between the initial and final states is large and the parities are of such signs that a large l_n is required for normal stripping. In this case the contribution from normal stripping will be cut down by the centrifugal factor and by the fact that light nuclei on the shell model do not contain states of large orbital angular momentum. An investigation of the presently known properties of the levels of light nuclei reveals that there is another case in which spin-Rip may be important. This is the reaction $O^{18}(d,n)F^{19}$ (2.8-Mev

[&]quot;J.E. Bowcock, Proc. Phys. Soc. (London) A68, ⁵¹² (1955). "J. C. Hensel and W. C. Parkinson, Phys. Rev. 110, ¹²⁸ $(1958).$

level). The spin of O^{18} is $0(+)$ and the shell model gives $\frac{7}{2}(+)$ for the 2.8-Mev level; and indeed experiment $\frac{7}{2}(+)$ for the 2.8-Mev level; and indeed experiment¹ indicates that the spin is high, $\geq \frac{5}{2}$. If the level is $\frac{7}{2}$ and the reaction goes by normal stripping, the l value of the captured proton should be 4; whereas by a spin-Aip mechanism it could proceed by $l=2$. A level assignment of $\frac{5}{2}(+)$ would allow the reaction to go by $l=2$ in the normal stripping process. However, a polarization experiment might distinguish between the two mechanisms.

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$Li⁶(n,t)He⁴ Cross Section for 12.5- to 18.3-Mev Neutrons$

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The cross section for the $Li^6(n,t)He^4$ reaction has been measured for 12.5- to 18.3-Mev neutrons. The neutrons were obtained from the $T(d,n)He^4$ reaction and their flux density was determined by counting the recoil He⁴ particles. A Li⁶I(Eu) scintillation crystal $1\frac{1}{2}$ inches in diameter by $\frac{1}{2}$ inch thick served as both the Li⁶ sample and the detector for the reaction products. The cross section is nearly linear from 34.3 mb at 12.5 Mev to 17.6 mb at 18.3 Mev. It is 28.1 ± 1.6 mb at 14.2 ± 0.2 Mev.

INTRODUCTION

VALUES of the cross section for the $Li^6(n,t)He^4$ re-
action have been reported up to 6.5 Mev¹ and at action have been reported up to 6.5 Mev' and at one higher neutron energy, $14.2 \text{ Mev.}^{2,3}$ The reaction in the high-energy region is of general interest as it leads to the formation of Li' at an excitation of about 21 Mev, and it is of some particular interest as the reaction of may be used within a $Li⁶I(Eu)$ scintillation crystal for the quantitative detection of neutrons. $4-7$ The positive ^Q value of 4.78 Mev and the absence of excited states of the products make this reaction unusually suitable for this purpose. The present investigation was undertaken to extend the cross-section measurements over the range of neutron energies from 12.5 to 18.3 Mev.

EXPERIMENTAL TECHNIQUES

The $Li⁶$ was contained in a $Li⁶I(Eu)$ crystal, which as a component of a scintillation spectrometer made it possible to record the pulse-height distribution of the reaction products with a 100-channel pulse-height analyzer. The apparatus arrangement is shown in Fig. 1.

The neutrons were monitored with either a KI crystal recoil alpha-particle counter or a $Li⁶I(Eu)$ scintillation counter, with the output pulses in each case being recorded with a 100-channel pulse-height analyzer.

Nearly monoenergetic neutrons were obtained from the $T(d,n)He⁴$ reaction, with the variation in energy being arrived at by varying the deuteron energy from 0.43 to 2.0 Mev and the angle of observation relative to the deuteron beam's axis from 0 to 150 degrees. The differential cross-section values of Bame and Perry' were used to correlate the data at different angles. The tritium target consisted of approximately 0.4 curie of H^3 in a 1-mg/cm² layer of zirconium, circular in form with a radius of 0.5 cm; the zirconium was evaporated onto a 0.25-mm-thick foil of platinum. The deuteron beam struck the target at a 45-degree angle, so the effective thickness of the tritium-bearing layer was 1.41 mg/cm'. The deuteron energy loss in passing through the zirconium layer varied from 342 to 155 kev and was taken into account in determining the average values of the deuteron energy which are mentioned above.

The time-integrated neutron flux density was determined in part of the experiment by the counting of the recoil alpha particles from the $T(d,n)He⁴$ reaction. An alpha-particle counter consisting of a 2-mm-thick KI crystal mounted on a DuMont 6291 photomultiplier tube was placed with the crystal in the vacuum system, as shown in the insert of Fig. 1. A circular aperture of 0.132 in. in diameter in a 0.010-in.-thick aluminum foil was placed at a distance of 11.92 cm from the target to limit the solid angle of acceptance to 6.21×10^{-4} stera-

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