of large momentum $\hbar\tau$ (see Fig. 1). One need only show how these processes simulate transitions caused by long-wavelength optical modes.

Energetic considerations have already been used to demonstrate that electron states of importance to free-carrier absorption in the ranges $\lambda \gtrsim 10\,\mu$, $T<450°\mathrm{K}$, lie rather close to the energy minima, i.e., near the boundaries of the Brillouin zone in Ge. The earlier argument is reinforced by noting that states near the elongated ends of the constant-energy ellipsoids are not nearly as important to absorption as lower mass states. It follows that τ, $\varepsilon(\tau,t)$, and $\omega(\tau,t)$ are nearly independent of (τ,t) for those short-wavelength modes which contribute most to absorption. The above-named quantities can therefore all be replaced by mean values to a good approximation, e.g.,

$$\omega(\tau,t)=\omega(|\mathbf{K}_f-\mathbf{K}_0-\boldsymbol{\kappa}|,\,t). \qquad \text{(B.6)}$$

In these cases, the product $\varepsilon(\tau,t)\cdot\mathbf{B}_n$ can be regarded as the deformation parameter. The resemblance to processes involving long-wavelength optical modes is thus established.

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Role of Traps in the Photoelectromagnetic and Photoconductive Effects*

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When carriers recombine through traps, the excess concentrations of mobile electrons and holes are not necessarily characterized by a single lifetime τ. Under the assumption that electrons and holes have separate lifetimes which in general are different, expressions for the steady-state photoelectromagnetic and photoconductive currents are obtained which show that in certain cases the photoelectromagnetic current is determined by a lifetime different from the one determining the photoconductive current. The results of the photoelectromagnetic and photoconductive measurements can be used to evaluate the parameters of any particular model which might be postulated for the recombination process. However, the theoretical treatment of the photoelectromagnetic and photoconductive effects presented here is independent of such models and so can be used as a method for testing their validity.

I. INTRODUCTION

In the course of recent photoelectromagnetic (PEM) and photoconductive (PC) measurements on InSb at 77°K performed at this laboratory, it was found that when the measurements are analyzed according to Kurnick and Zitter's[1] theoretical model, the carrier lifetime deduced from PEM data is much smaller than the lifetime deduced from PC data. This inequality is inconsistent with the theory, which in effect assumes an interband recombination process as opposed to recombination through traps, and therefore predicts that the same value for the lifetime will be obtained in both experiments. Rose[2] has pointed out, however, that if excess carriers are trapped at localized levels in the forbidden band for a significant time before recombining, then one expects the PEM lifetime to be smaller than the PC lifetime.

The purpose of the present paper is to generalize Kurnick and Zitter's model to include the effects of trapping. This will be done in a manner which is independent of any models and statistics of the trapping process itself, i.e., there is no need to mention concentrations of traps, their positions in the forbidden energy gap, etc. The theory will show how the PEM effect, in conjunction with the PC effect, can indicate whether trapping of excess carriers occurs, and, in fact, can provide data from which the parameters of a particular trap model may be inferred. This approach is particularly valuable in those cases where the carrier lifetimes are so short that they can be measured best, if at all, by the steady-state PEM and PC effects.

The equations derived here have been applied to PEM and PC measurements on p-type InSb from 77° to 300°K; the results are to be published shortly.

II. TRAPPING EFFECTS

Over considerable temperature ranges recombination of excess carriers by way of trapping levels in the forbidden energy gap is the predominant recombination mechanism in the bulk semiconductors Ge, Si, and InSb,[3] as well as in many thin-film semiconductors and photoconductive insulators.

For recombination through traps it is instructive to consider the following two limiting cases (which Rose[2] rigorously distinguishes in terms of the position of the Fermi level with respect to the trap level and the trap capture cross sections):

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[1] S. W. Kurnick and R. N. Zitter, J. Appl. Phys. **27**, 278 (1956).

[2] A. Rose, *Proceedings of the Conference on Photoconductivity, Atlantic City, November* 4–6, edited by R. G. Breckenridge *et al.* (John Wiley and Sons, Inc., New York, 1956), p. 17.

[3] G. K. Wertheim, Phys. Rev. **104**, 662 (1956); R. A. Laff and H. Y. Fan, Bull. Am. Phys. Soc. Ser. II, 2, 347 (1957).

(a) Electrons in the conduction band, upon capture by empty traps, "immediately" combine with holes from the valence band and valence-band holes, upon being captured by traps, "immediately" unite with electrons from the conduction band.

(b) Excess carriers remain, on the average, for a "significant" length of time in traps before recombination occurs.

Both cases come under the heading of "recombination through traps," but case (b) is usually what is meant by the phrase "trapping of carriers" appearing in the literature. In (a), the traps are often referred to as "recombination centers."

In case (a), mobile electrons and holes, in the conduction and valence bands, respectively, decay at the same rate, and the population of occupied traps does not change. As a result, the photoconductive decay following a low-level light-pulse is exponential, and an injected pulse of carriers under an applied field will drift with a "normal" mobility. In short, these two effects will not distinguish qualitatively between case (a) and interband recombination. The reason is that in (a), carriers "almost" make interband transitions, pausing only briefly (i.e., for a time which is small compared with carrier lifetime) at the intermediate trap level. The difference between (a) and interband recombination is observed only through the dependence of lifetime on temperature and majority carrier concentration.

In case (b), mobile electrons and holes do *not* decay at the same rate, the photoconductive decay is a *sum* of exponential components, a considerable portion of excess carriers will be in trapped states, and the apparent mobility of a pulse of injected carriers drifting under an applied electric field will be abnormally small, as compared to the carrier mobility deduced from Hall and conductivity measurements. In addition, photocurrents will be strongly quenched by extra illumination.

One may inquire as to the consequences of the recombination processes (a) and (b) when steady-state PEM and PC currents are being measured. In the next section, it will be shown that in the case (a), both the PEM and PC effects will depend on a single bulk carrier lifetime. If it happens, however, that a considerable portion of excess carriers are trapped, as in case (b), the PEM and PC currents may each depend on a different lifetime parameter. The reason is that the PEM effect is basically a diffusion effect and so in extrinsic semiconductors is controlled by the diffusion of the minority carriers. In the PC effect, on the other hand, each carrier contributes independently to photoconductivity. Whenever excess majority carriers remain in conducting states long after most of the excess *minority* carriers have been trapped, the PEM effect "sees" only the minority carrier lifetime (characterizing the decay of excess minority carriers into traps), but the PC effect is dominated by the longer lifetime of the excess *majority* carriers. It should be mentioned that, as regards published data, only Sommers *et al.*,[4] in investigating the PEM-PC effects in CdS, distinguished between PEM and PC measurements of lifetimes.

In case (b), a corollary is that the so-called ratio lifetime, defined by Moss,[5] and by Kurnick and Zitter, will have a value different from either of the lifetimes observed in the PEM and PC effects.

III. THEORY

The theoretical treatment to be presented here follows that by Kurnick and Zitter (hereafter referred to as K-Z). For a slab-shaped sample, they obtained expressions for the PEM and PC short-circuit currents, but their derivation assumes an interband recombination process, as shown in their equations:

$$\nabla\cdot\mathbf{J}_e=-\nabla\cdot\mathbf{J}_h=-\frac{e}{\tau}\left(\frac{(n_0+n)(p_0+p)-n_0p_0}{n_0+p_0}\right). \quad (1)$$

Here $\mathbf{J}_e$ is the electron current density, $\mathbf{J}_h$ the hole current density, e the electronic charge, n_0, p_0 the equilibrium concentrations of electrons and holes, and n, p the excess carrier concentrations; mks units are used throughout.

For sufficiently small light intensities, the right-hand side of (1) reduces to

$$\frac{e}{\tau}\left(\frac{n+cp}{1+c}\right),$$

where $c=n_0/p_0$.

From a phenomenological point of view, these equations express a proportionality, and define the constant τ, which has the following well-known property: the excess carrier concentrations n and p, following a light pulse, will decay with time according to the factor $\exp(-t/\tau)$.

In order to include trapping effects, Eq. (1) will be replaced by the following for the case of small light intensities:

$$\nabla\cdot\mathbf{J}_e=-\nabla\cdot\mathbf{J}_h=en/\tau_n=ep/\tau_p. \quad (2)$$

Just as in the treatment of K-Z, it is assumed that (2) applies to the bulk of the material in the steady state, and that the production of electron-hole pairs by light takes place only at the surface of the material. (The case of electron-hole pair production in the bulk will be treated later.) The quantities τ_n and τ_p defined by (2) are called the electron and hole lifetimes.

In the present treatment, the K-Z equation,

$$\nabla\cdot\mathbf{E}=(e/\kappa\kappa_0)(p-n),$$

must also be altered to read

$$\nabla\cdot\mathbf{E}=(e/\kappa\kappa_0)(p-n-n_T), \quad (3)$$

where n_T, the concentration of excess electrons in traps, may be negative, corresponding to hole trapping. κ and κ_0 are the dielectric constant and permittivity of free

[4] Sommers, Berry, and Sochard, Phys. Rev. **101**, 987 (1956).
[5] T. S. Moss, Proc. Phys. Soc. (London) **B66**, 993 (1953).

space. It happens that (3), which expresses a space-charge effect, it is not required at all in the derivation of expressions for the small-signal PEM and PC short-circuit currents. However, the interpretation of (3) is particularly instructive, for it can be shown that in the bulk of the material, more than a Debye-Hückel length away from the surface, $\nabla\cdot\mathbf{E}=0$, i.e.,

$$p=n+n_T.$$

This is the electrical neutrality condition for the bulk. On comparing it with (2), one has the relations

$$\frac{\tau_p}{\tau_n}=\frac{p}{n}=1+\frac{n_T}{n}.$$

This shows that electron and hole lifetimes will be equal, if, and only if, excess carrier trapping is negligible: $|n_T|/n\ll 1$. In what follows, it will be seen that τ_p and τ_n can be obtained from PEM and PC measurements, and so one can deduce from the above the fraction of excess carriers which are trapped.

In order to emphasize the roles of the PEM-PC effects in determining τ_p and τ_n, it will be assumed, for the sake of simplicity, that surface recombination for electrons and holes is negligible and that the sample thickness is much larger than the carrier diffusion lengths. The equations of K-Z, with (2) substituted for (1), can be solved by procedures similar to those already outlined by K-Z to give expressions for the PEM and PC short-circuit currents per unit sample width. The results are

$$i_{\mathrm{PEM}}=\left(1+\frac{1}{b}\right)\frac{eI\mu B(D\tau_{\mathrm{PEM}})^{\frac{1}{2}}(1+c)^{\frac{1}{2}}}{[1+\mu^2B^2+bc(1+\mu^2B^2/b^2)]^{\frac{1}{2}}}, \tag{4}$$

$$i_{\mathrm{PC}}=\left(1+\frac{1}{b}\right)eI\mu E_x\tau_{\mathrm{PC}}. \tag{5}$$

Here, I is the effective photon flux density, μ is the electron mobility, B is the magnetic flux density, D the electron diffusion constant ($\mu kT/e$), E_x is the electric field component along the sample length, $c=n_0/p_0$, and b is the ratio of electron to hole mobilities. τ_{PEM} and τ_{PC} are effective lifetimes, defined as

$$\tau_{\mathrm{PEM}}\equiv\frac{\tau_n+c\tau_p}{1+c}, \tag{6}$$

$$\tau_{\mathrm{PC}}\equiv\frac{\tau_n+\tau_p/b}{1+1/b}. \tag{7}$$

The reason for so defining τ_{PEM} and τ_{PC} is that (4) and (5) above are identical in form to the corresponding K-Z expressions for the photocurrents, except that τ_{PEM} and τ_{PC} here replace the "interband recombination lifetime" τ of K-Z. Moreover, for the case of negligible trapping of excess carriers, one has

$$\tau_n=\tau_p=\tau_{\mathrm{PEM}}=\tau_{\mathrm{PC}}=\tau,$$

that is, all the lifetimes reduce to a single, unique τ.

One observes from (6) that in extrinsic material τ_{PEM} is just the lifetime of the minority carrier. This corresponds to the well-known fact that minority carriers control the carrier diffusion process in extrinsic semiconductors. On the other hand, (7) expresses the fact that both carriers contribute independently to photoconductivity, whether the material is intrinsic, p-type, or n-type.

Another feature to be noted is that the symbols μ and b refer to carrier mobilities in the conduction and valence bands, as inferred from Hall and conductivity data, and not to the apparent "trapping mobilities" sometimes discussed in the literature. As shown by K-Z, both μ and b can be obtained for the same sample from Hall and conductivity data, and from PEM measurements at magnetic fields large enough to influence the denominator of (4).

An important implication of the above equations is that τ_{PEM} and τ_{PC} are not, in general, identical; specifically, when trapping of excess carriers occurs, one expects $\tau_{\mathrm{PEM}}\neq\tau_{\mathrm{PC}}$. As a result, the so-called ratio lifetime, defined by Moss[5] and K-Z as

$$\tau_r\equiv(\tau_{\mathrm{PC}})^2/\tau_{\mathrm{PEM}},$$

is, in general, a quadratic function of τ_n and τ_p, and so has no simple interpretation, with the exception that when $\tau_n=\tau_p$, then τ_r becomes identical with all the other lifetimes. The ratio lifetime, which can be determined without a knowledge of the photon flux density incident on the sample, has been measured in InSb,[1] InAs,[6] and PbS,[5,7] but τ_r alone has little meaning unless $\tau_{\mathrm{PEM}}=\tau_{\mathrm{PC}}$.

The expressions above for the PEM and PC currents are derived from the transport equations of K-Z and the recombination relations (2). The entire theoretical treatment is phenomenological, and applies whether the recombination process takes place between bands or through traps; in short, the treatment is independent of recombination models and so can serve as a method for testing their validity.

In practice, values of electron and hole lifetimes as obtained from PEM-PC measurements can be compared with the predictions of a particular recombination model to decide whether the model is appropriate, and if so, to determine the parameter values of the model. If it is found that τ_n differs considerably from τ_p, it must be concluded that trapping of excess carriers, and not interband recombination, is the dominant process. For the model of a single level of traps, Shockley and Read[8] have derived expressions for τ_n and τ_p in terms of such parameters as trap concentration, position of the trap level in the forbidden energy gap, and the mean capture cross sections of the traps; Okada,[9] Landsberg,[10]

[6] J. R. Dixon, Phys. Rev. **107**, 374 (1957).
[7] W. W. Scanlon, Phys. Rev. **106**, 718 (1957).
[8] W. Shockley and W. T. Read, Jr., Phys. Rev. **87**, 835 (1952).
[9] J. Okada, J. Phys. Soc. Japan **12**, 1338 (1957).
[10] P. T. Landsberg, Proc. Phys. Soc. (London) **B70**, 283 (1957).

and Sah and Shockley[11] have indicated how τ_n and τ_p can be obtained when there is more than one trap level in the forbidden gap.

At surfaces where surface recombination is important, one may put, for the components of the carrier current densities perpendicular to that surface,

$$J_{e1}=-J_{h1}=ens_n=eps_p,$$

in analogy with (2); these boundary conditions define the surface recombination velocities s_n and s_p, which, of course, may or may not be equal.

As a final application of the theory, consider the case of the steady-state PEM current when the light is absorbed in the bulk within a distance which is at least comparable to carrier diffusion lengths but which is still small compared to the thickness of the sample. For this purpose, it is convenient to define what might be called the "effective ambipolar magnetic diffusion length,"

$$L_D^*=\left[\frac{D\tau_{\mathrm{PEM}}(1+c)}{1+\mu^2B^2+bc(1+\mu^2B^2/b^2)}\right]^{\frac{1}{2}}. \tag{8}$$

With no magnetic field, L_D^* reduces to the diffusion length of the minority carrier in an extrinsic semiconductor. Also, for the case when light is strongly absorbed at the surface of the material, the PEM current, as given by Eq. (4), can be written as

$$i_{\mathrm{PEM}}=\left(1+\frac{1}{b}\right)eI\mu BL_D^*. \tag{9}$$

If light (assumed to be monochromatic, for simplicity) is absorbed in the bulk, however, the term

$$-eI\alpha\exp(-\alpha y)$$

must be added to each of the right-hand sides of the recombination relations (2), representing the volume generation of electron-hole pairs by the light. Here, α is the optical absorption coefficient and y is the coordinate in the direction perpendicular to the illuminated surface. The PEM current is then found to be given by

$$i_{\mathrm{PEM}}=\left(1+\frac{1}{b}\right)eI\mu BL_D^*\frac{\alpha L_D^*}{1+\alpha L_D^*}. \tag{10}$$

When $\alpha L_D^*\gg 1$, (10) reduces to the expression (9), as it should, since the physical meaning of $\alpha L_D^*\gg 1$ is that the absorption of light takes place within a distance much smaller than the effective diffusion length L_D^*, i.e., the light is absorbed "at the surface." In short, (9) and (4) are a special case of (10).

The purpose here in exhibiting (10) is to show more clearly how lifetimes enter in steady-state diffusion processes, namely, by way of the effective diffusion length L_D^*. It is L_D^* (evaluated at the appropriate magnetic field) which determines the steady-state behavior of the PEM effect, the p-n junction effect, and other diffusion phenomena.

[11] C. T. Sah and W. Shockley, Phys. Rev. **109**, 1103 (1958).

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General Impact Theory of Pressure Broadening*

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The work of two previous papers is extended and a theory of pressure broadening is developed which treats the perturbers quantum mechanically and allows for inelastic collisions, degeneracy, and overlapping lines. The impact approximation is used. It consists in assuming that it takes, on the average, many collisions to produce an appreciable disturbance in the wave function of the atom, and it results in an isolated line having a Lorentz shape. Validity criteria are given. When the approximation is valid, it is allowable to replace the exact, fluctuating interaction of the perturbers with the atom by a constant effective interaction. The effective interaction is expressed in terms of the one-perturber quantum mechanical transition amplitudes on and near the energy shell and its close relationship to the scattering matrix is stressed. The calculation of the line shape in terms of the effective interaction is the same as when the perturbers move on classical paths. Results are written explicitly for isolated lines. If the interaction of the perturbers with the final state can be neglected, the shift and width are proportional to the real and imaginary part of the forward elastic scattering amplitude, respectively. By the optical theorem, the width can also be written in terms of the total cross section. When the interaction in the final state cannot be neglected, the shift and width are still given in terms of the elastic scattering amplitudes, in a slightly more complicated fashion. Finally, rules are given for taking into account rotational degeneracy of the radiating states.

1. INTRODUCTION

IN two previous papers,[1] the theory of pressure broadening has been extended in two different directions. In I, it was shown how the motion of the perturbers can be treated quantum mechanically, and the width and shift of the line were expressed in terms of the quantum-mechanical scattering amplitudes. But

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[1] M. Baranger, Phys. Rev. **111**, 481 (1958), referred to as I.

M. Baranger, Phys. Rev. **111**, 494 (1958), referred to as II. Many of the results of II have also been obtained by A. C. Kolb and H. Griem, Phys. Rev. **111**, 514 (1958).