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Magnetoresistance in PbS, PbSe, and PbTe at 295°, 77.4°, and 4.2°K*

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Magnetoresistance measurements made on 14 single crystals of PbS, PbSe, and PbTe at 77.4°K conformed to the general phenomenological weak-field theory; i.e., the magnetoresistance was proportional to H^2, and varied sinusoidally with the angle between the sample current and magnetic field. Because of the extremely high mobilities in the samples at 4.2°K, weak-field theory was generally not applicable, and saturation effects and striking deviations from the sinusoidal behavior were observed at this temperature. The longitudinal magnetoresistance was generally larger (in some cases, 4 or 5 times larger) than the transverse at both 77.4° and 4.2°K. The results did not appear to be significantly different for *n*- and *p*-type material. The data generally favor a model with considerable mass anisotropy; in particular, the many-valley model having ellipsoids of revolution along the ⟨111⟩ directions, with a ratio of longitudinal to transverse effective mass of between 4 and 6, seems most appropriate for PbTe.

INTRODUCTION

MAGNETORESISTANCE measurements have been made on 14 single crystals of PbS, PbSe, and PbTe at 295°, 77.4°, and 4.2°K. The crystals were from a group on which Hall and resistivity measurements were reported earlier,[1] and were synthetic with the exception of one natural PbS sample. They had low resistivities, constant extrinsic carrier concentrations of the order of 10^{18} per cm^3, and room temperature mobilities in the range 500–1700 cm^2/v-sec. The mobilities increased rapidly with decreasing temperature, reaching values as high as 800 000 cm^2/v-sec at 4.2°K.

The lead salts crystallize in the cubic NaCl structure, and samples generally cleave along the planes formed by the cubic axes. All of the crystals measured were cleaved, and thus the sample current was invariably parallel to a cubic axis. The directions of the applied magnetic field were confined to the plane determined by the current axis and one of the other cubic axes. The sample dimensions were usually about 1×1×3 or 4 mm.

The simple dc apparatus used for these measurements and details of the experimental procedure have been described elsewhere.[2] The data at 77.4° and 4.2°K were obtained by directly immersing the sample in liquid nitrogen and liquid helium. The sample currents used were generally about 5 or 10 ma, and the magnetic field strengths ranged from 360 to 4300 gauss.

The experimental configuration used permitted the evaluation of the longitudinal magnetoresistance coefficient M_{100} and the transverse coefficient $M_{100}{}^{001}$ (the subscripts and superscripts are the current and magnetic field directions, respectively). We define these coefficients by the equation

$$\frac{\Delta\rho}{\rho_0}=M_{abc}{}^{def}\mu_H{}^2H^2/c'^2, \tag{1}$$

where $\Delta\rho/\rho_0$ is the fractional change in the zero-field resistivity, μ_H is the Hall mobility, H is the magnetic field strength, and the factor c' depends on the system of units used. We shall employ the usual mixed system in which H is in gauss and μ_H in cm^2/v-sec. In this case, $c'=10^8$.

It has been shown[3] that for any cubic crystal and for $\mu_H H \ll 10^8$,

$$\Delta\rho/\rho_0=\left[b+c\left(\sum_{j=1}^{3}\iota_j\eta_j\right)^2+d\sum_{j=1}^{3}\iota_j{}^2\eta_j{}^2\right]H^2, \tag{2}$$

where b, c, and d are constants, and the ι's and η's are the direction cosines of the current and magnetic field with respect to the crystal axes. Equation (2) reduces to

$$\Delta\rho/\rho_0=\left[b+(c+d)\cos^2\theta\right]H^2 \tag{3}$$

for our experimental configuration, where θ is the angle between current and magnetic field.

The galvanomagnetic properties of three cubically symmetric many-valley models have been derived for classical statistics and weak magnetic fields by Abeles and Meiboom[4] and Shibuya.[5] These models assume constant energy surfaces which are ellipsoids of revolution in k space oriented along the ⟨100⟩, ⟨110⟩, or ⟨111⟩ directions. Simple relations which do not depend on the degree of degeneracy exist between the constants of Eq. (2) or Eq. (3) for these models.[6] They are as

* This paper is a portion of a thesis submitted to the University of Maryland in partial fulfillment of the requirements for the Doctor of Philosophy degree in physics. Copies of the complete thesis are available from the U. S. Naval Ordnance Laboratory.

[1] R. S. Allgaier and W. W. Scanlon, Phys. Rev. **111**, 1029 (1958). For further details, see reference 2.

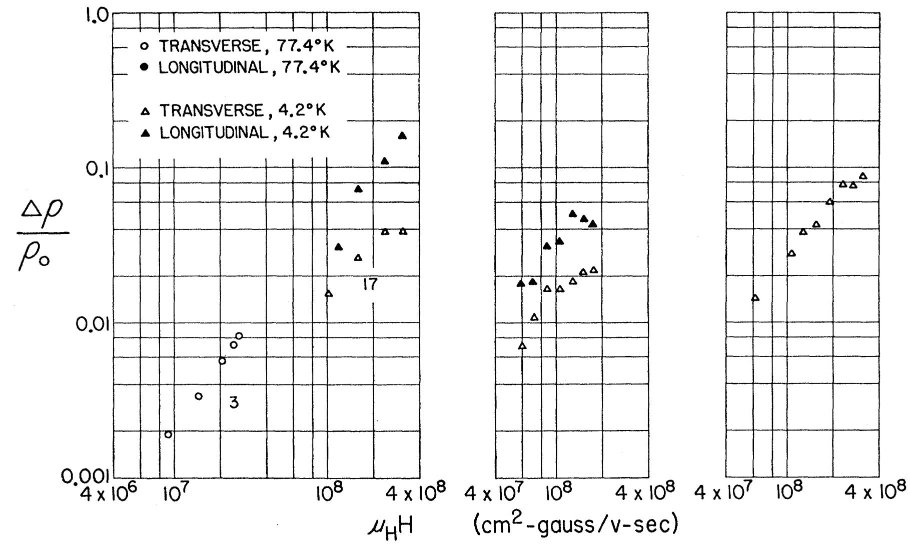
[2] R. S. Allgaier, Naval Ordnance Laboratory Report NAVORD-6037, 1958 (unpublished).

[3] G. L. Pearson and H. Suhl, Phys. Rev. **83**, 768 (1951).

[4] B. Abeles and S. Meiboom, Phys. Rev. **95**, 31 (1954).

[5] M. Shibuya, Phys. Rev. **95**, 1385 (1954).

[6] See reference 5 in which relations are given between the magnetoconductivity constants β, γ, and δ. It is easily shown that the same relations hold between the magnetoresistivity constants b, c, and d.



(a) n-type, Samples 3 and 17 (b) n-type, Sample 26 (c) p-type, Sample 35

FIG. 1. Transverse and longitudinal magnetoresistance in PbS as a function of the mobility-magnetic field strength product.

follows:

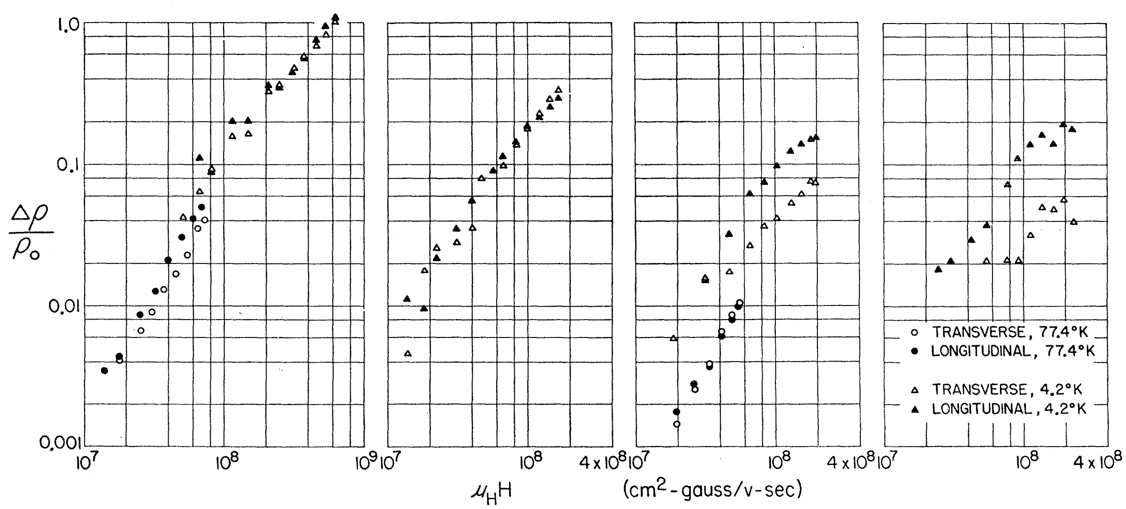
Model	Relation
⟨100⟩	$b+c+d=0$
⟨110⟩	$b+c-d=0$
⟨111⟩	$b+c \quad =0$

These theoretical relations may be used to select an appropriate cubic model by comparison with the experimentally evaluated constants.

The ellipsoids of the cubic models are characterized by a parameter K, the ratio of the longitudinal to transverse effective mass. For $K=1$, the cubic models all reduce to the simple isotropic band model for which, for lattice scattering and classical statistics, $M_{100}{}^{001}=0.27$ and $M_{100}=0$. The longitudinal coefficient $M_{100}=0$ for the ⟨100⟩ cubic model also.

PREVIOUS MAGNETORESISTANCE MEASUREMENTS ON THE LEAD SALTS

We have found only a few papers on magnetoresistance in the lead salts. Putley[7] published transverse magnetoresistance data for three synthetic PbSe specimens at 77.4°K, and transverse and longitudinal data on a single synthetic PbTe sample at 290°, 195°, 170°,



(a) n-type, Sample 27 (b) n-type, Sample 38 (c) p-type, Sample 24 (d) p-type, Sample 37

FIG. 2. Transverse and longitudinal magnetoresistance in PbSe as a function of the mobility-magnetic field strength product.

[7] E. H. Putley, Proc. Phys. Soc. (London) **B68**, 22 (1955).

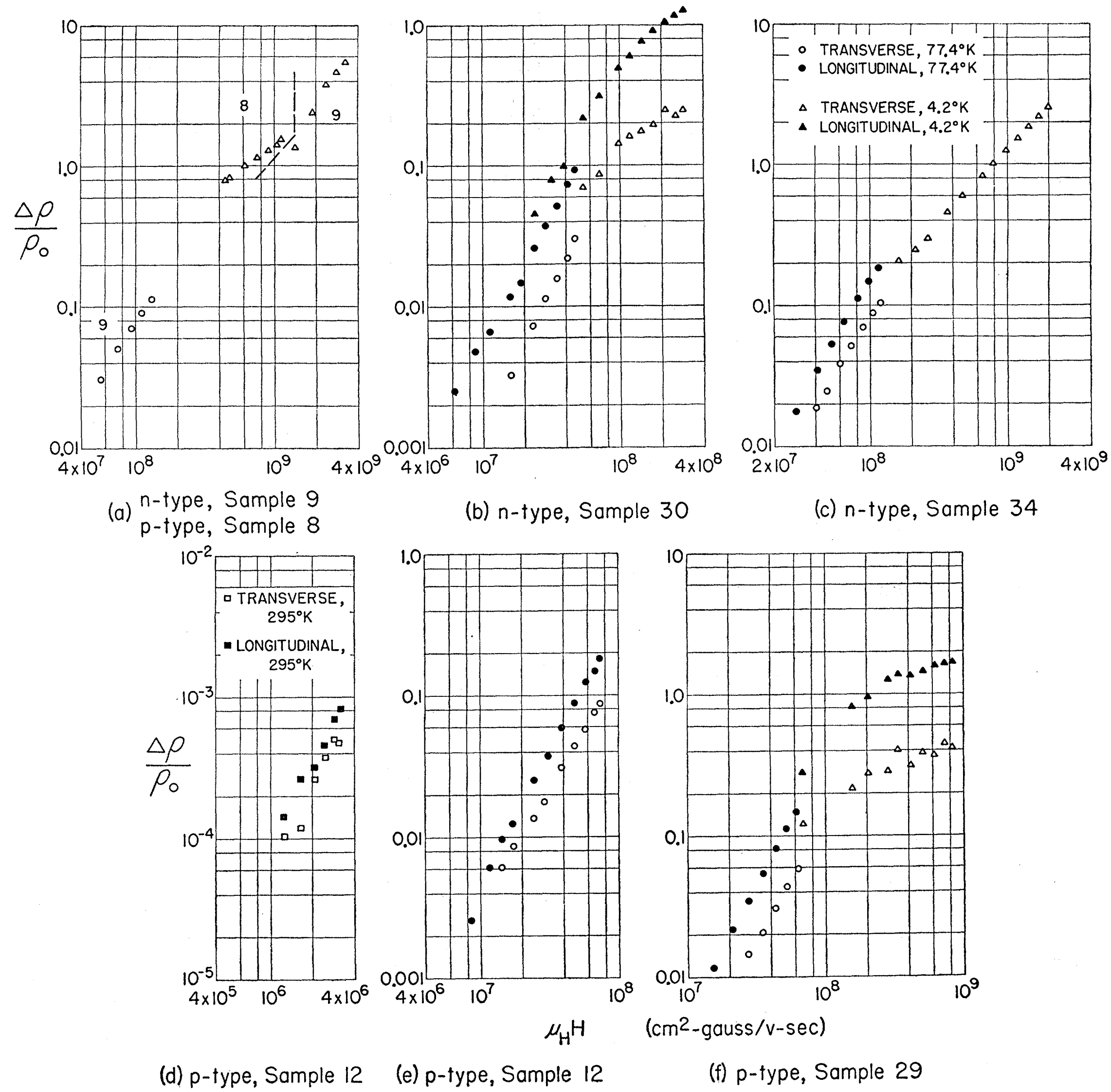


FIG. 3. Transverse and longitudinal magnetoresistance in PbTe as a function of the mobility-magnetic field strength product.

77.4°, and 20°K, using fields up to 10 000 gauss. As in the present work, the sample current was parallel to a cubic axis. Putley found that at temperatures down to 170°K, $\Delta\rho/\rho_0$ was proportional to H^2, but at lower temperatures the increase was less rapid at the higher field strengths. The coefficients M_{100} and M_{100}^{001} were roughly the same size, and were within a factor of 2 or 3 of the simple band transverse coefficient value of 0.27.

Irie[8] published magnetoresistance data at 93° and 200°K as a function of the angle between sample current and magnetic field for four specimens of natural PbS. Two samples having the sample current in the [110] direction showed only a slight variation in the magnetoresistance as a function of angle. The transverse coefficient in the two crystals measured with the current along a cubic axis was much larger than the longitudinal, and Irie concluded that the simple band or the ⟨100⟩ cubic model was appropriate for natural PbS. The values of the transverse coefficient M_{100}^{001} in Irie's data were as high as 7.

Shogenji and Uchiyama[9] recently published magnetoresistance data as a function of the angle between sample current and magnetic field on two crystals of PbTe at 90°K. The current was parallel to a cubic axis in one case and to a [110] direction in the other. They found that $b+c$ was much closer to zero than $b+c\pm d$, and hence concluded that the ⟨111⟩ model was appro-

[8] T. Irie, J. Phys. Soc. Japan **11**, 840 (1956).

[9] K. Shogenji and S. Uchiyama, J. Phys. Soc. Japan **12**, 1164 (1957).

priate. We calculate from their data that $M_{100}{}^{001}=0.11$ and $M_{100}=0.25$.

This review reveals that the total number of samples previously investigated was quite small, that no longitudinal data were obtained on PbSe, and that no data at all were obtained on synthetic PbS. In addition, magnetoresistance studied as a function of the angle between the sample current and magnetic field was limited to natural PbS and synthetic PbTe. Finally, with the exception of Putley's data at 20°K, the previous measurements were confined to temperatures at or above 77.4°K.

RESULTS

The transverse and longitudinal magnetoresistance for four PbS, four PbSe, and six PbTe samples as a function of the mobility-magnetic field strength product are shown in Figs. 1, 2, and 3. Both $\Delta\rho/\rho_0$ and $\mu_H H$ are plotted on logarithmic scales. The Hall mobilities of the samples at 77.4° and 4.2°K are listed in Table I. Because the mobilities were generally much higher at 4.2° than at 77.4°K, the $\mu_H H$ ranges covered by the data at the higher temperature were for the most part less than 10^8, while at the lower temperature they usually exceeded 10^8. The average values of the slopes of the transverse and longitudinal data, together with a rough average of the $\mu_H H$ range over which the slopes were measured, are listed in Table II.

The magnetoresistance at fixed magnetic field strength as a function of the angle between the sample current and magnetic field directions for two PbSe and five PbTe samples at 77.4°K and for one PbSe and two PbTe samples at 4.2°K is shown in Figs. 4 and 5. For the sake of brevity, this type of data will henceforth be referred to as "constant-field magnetoresistance data." The quantity plotted against angle is $(\Delta\rho/\rho_0)\times 10^{16}/\mu_H{}^2H^2$, which equals $M_{100}{}^{001}$ and M_{100} when the current and field are perpendicular and parallel, respectively. The $\mu_H H$ value at which each constant-field magnetoresistance curve was obtained is also indicated in Figs. 4 and 5.

TABLE I. Hall mobilities of samples at 77.4° and 4.2°K.

Material and type	Sample number	Hall mobility (cm²/v-sec) 77.4°K	4.2°K
	3	6040	14 400
n PbS	17	11 000	68 500
	26	8750	40 200
p PbS	35	15 000	80 000
n PbSe	27	16 500	139 000
	38	12 700	38 200
p PbSe	24	12 800	44 100
	37	13 700	57 900
	9	31 600	800 000
n PbTe	30	10 700	66 500
	34	27 700	450 000
	8	16 200	256 000
p PbTe	12	16 200	...
	29	14 600	192 000

TABLE II. Slopes of the transverse and longitudinal magnetoresistance data as a function of the mobility-magnetic field strength product.

Material and type		Sample number	Long. or trans.	Temperature (°K)	Slope	Average $\mu_H H$
p	PbTe	12	*T*	295	1.9	0.02×10^8
p	PbTe	12	*L*	295	1.8	0.02
n	PbS	3	*T*	77.4	1.3	0.15
n	PbTe	30	*L*	77.4	1.8	0.20
n	PbTe	30	*T*	77.4	2.0	0.25
p	PbTe	29	*L*	77.4	1.9	0.30
p	PbTe	12	*L*	77.4	1.8	0.30
p	PbTe	12	*T*	77.4	1.6	0.30
p	PbSe	24	*T*	77.4	1.9	0.34
p	PbSe	24	*L*	77.4	1.7	0.34
n	PbSe	27	*L*	77.4	1.8	0.35
n	PbSe	27	*T*	77.4	1.7	0.35
p	PbTe	29	*T*	77.4	1.7	0.40
n	PbTe	34	*T*	77.4	1.8	0.60
n	PbTe	34	*L*	77.4	1.8	0.60
p	PbSe	37	*L*	4.2	1.3	0.80
n	PbTe	9	*T*	77.4	1.6	0.90
n	PbSe	38	*T*	4.2	1.2	1.00
n	PbSe	38	*L*	4.2	1.2	1.00
n	PbS	26	*L*	4.2	1.2	1.00
n	PbS	26	*T*	4.2	1.1	1.00
p	PbSe	24	*T*	4.2	1.0	1.00
p	PbSe	37	*T*	4.2	1.0	1.10
p	PbSe	24	*L*	4.2	0.9	1.10
n	PbTe	30	*L*	4.2	1.0	1.20
p	PbS	35	*T*	4.2	1.1	1.30
n	PbTe	30	*T*	4.2	0.5	1.60
n	PbS	17	*T*	4.2	0.8	1.60
n	PbS	17	*L*	4.2	1.7	1.90
n	PbSe	27	*T*	4.2	1.1	3.00
n	PbSe	27	*L*	4.2	1.1	3.00
p	PbTe	29	*T*	4.2	0.4	3.50
p	PbTe	29	*L*	4.2	0.3	3.50
n	PbTe	34	*T*	4.2	1.1	6.00
p	PbTe	8	*T*	4.2	0.7	7.50
n	PbTe	9	*T*	4.2	1.6	20.00

Table III lists values of the ratio of longitudinal to transverse magnetoresistance for the various samples, calculated from the constant-field and from the longitudinal and transverse data. In the latter case, ranges of ratio values are shown, since the ratio generally varied with $\mu_H H$.

DISCUSSION

Comparison with General Weak-Field Theory

The data at room temperature and at 77.4°K agreed quite well with the general weak-field theory. Equation (3) predicts that the transverse and longitudinal magnetoresistance should be proportional to H^2; Table II lists slope values for the $\log(\Delta\rho/\rho_0)-\log\mu_H H$ data between 1.7 and 2.0 in 13 of 15 cases for which the average of the measured $\mu_H H$ range was 6×10^7 or less. The constant-field data for the five PbTe samples at 77.4°K formed rather precise sine curves with extrema at the parallel and perpendicular configurations, as is also predicted by Eq. (3). The constant-field data for

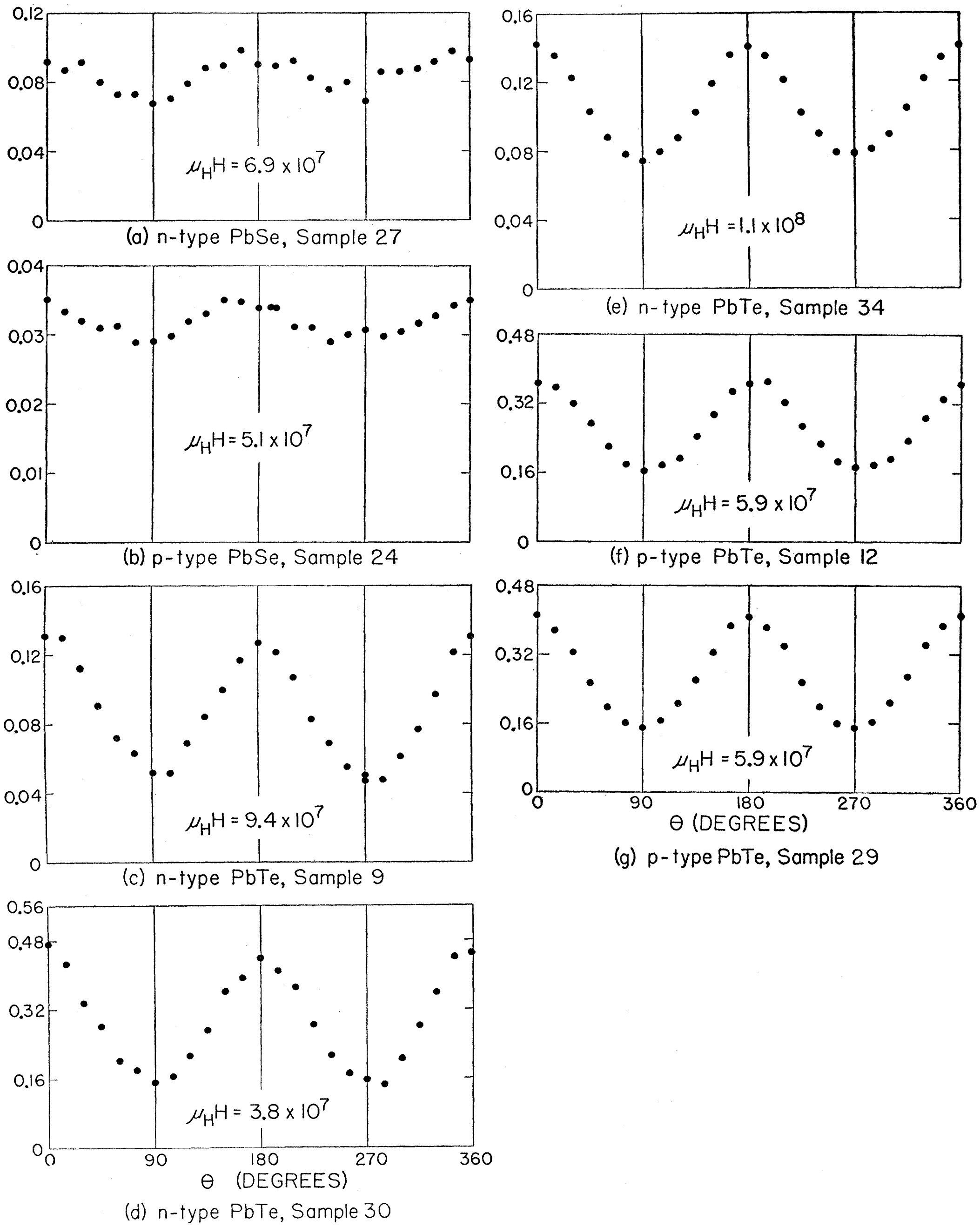


FIG. 4. Magnetoresistance at 77.4°K as a function of the angle θ between the sample current and magnetic field.

the two PbSe samples at 77.4°K were less precisely sinusoidal.

Saturation effects in the longitudinal and transverse magnetoresistance data at higher $\mu_H H$ values are quite evident in Figs. 1, 2, and 3, in the portions of the data obtained at 4.2°K. Table II shows that for $\mu_H H \geq 10^8$, the slopes generally decreased with increasing $\mu_H H$, reaching values as low as 0.3. However, there was a relative lack of saturation in some samples in the large-field region which may have been caused by inhomogeneities.[10] Despite the saturation effects and the moderate magnetic field strengths used, changes in the resistivity as large as 5.6 times the zero-field resistivity were measured at 4.2°K.

Deviations from the sinusoidal behavior predicted by weak-field theory occurred in the constant-field magnetoresistance data obtained at 4.2°K. This was not

[10] C. Herring (private communication).

surprising, since the measurements were made at $\mu_H H$ values exceeding 10^8. Samples 27 and 29 had extremely low resistivities at this temperature, and the magnetoresistance curves show considerable experimental scatter. A more precise curve was obtained for Sample 34, for which the most pronounced deviations from sinusoidal behavior occurred. The entire curve is symmetrical about directions which differ slightly from the parallel and perpendicular configurations, but this resulted from the rather inaccurate measurement of the absolute angle between current and field, although the measurement of relative angles was accurate to a fraction of a degree.

If Sample 34 contained inhomogeneities, they were not serious enough to cause the data to deviate from the symmetry imposed by the crystal lattice. The mean free path of the carriers (less than 0.01 mm) was too short, and the carrier concentration (5×10^{17} per cm^3) was too large for surface effects to be important. Peculiar effects due to the shorting effect of the contact area of the resistivity probes have been reported for high-mobility *n*-type InSb.[11] If this caused the effects in Sample 34, then it is surprising that the same general effect was not observed also in Samples 27 and 29, since the probe configuration was the same in all three cases. The shorting effect may be unimportant in these samples because of their very low resistivities (10^{-5}–10^{-6} ohm-cm) at 4.2°K.

There do not seem to be any differences in the results for *n*- and for *p*-type material. This is in marked contrast to the situation, for example, in germanium,[3] silicon,[12] and indium antimonide.[11,13] The average magnitudes of M_{100} and M_{100}^{001} were about 0.32 and 0.13 at 77.4°K, agreeing reasonably well with the values reported by Putley[7] and Shogenji and Uchiyama.[9]

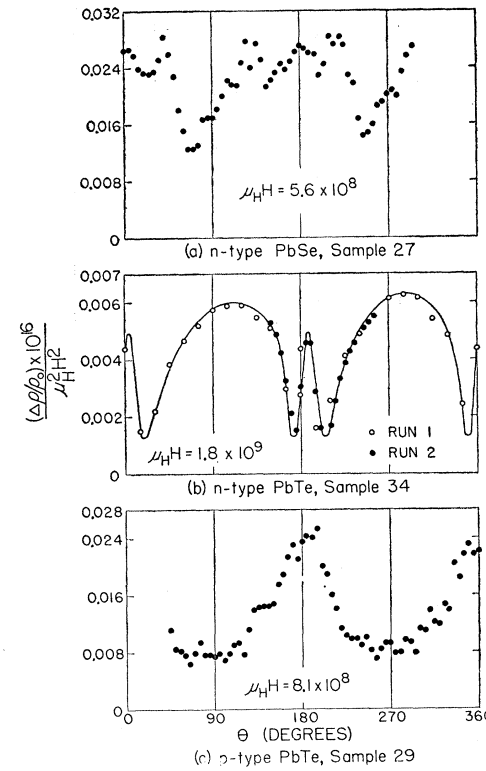


FIG. 5. Magnetoresistance at 4.2°K as a function of the angle θ between the sample current and magnetic field.

TABLE III. Experimental ratios of longitudinal to transverse magnetoresistance, M_{100}/M_{100}^{001}.

Material and type	Sample number	Temperature (°K)	Ratio: From angular data	Ratio: From longitudinal and transverse data
n PbS	17	4.2		1.6–4.1
	26	4.2		2.3
n PbSe	27	77.4	1.4	1.3–1.4
		4.2	1.5	1.0–1.3
	38	4.2		0.9–1.0
p PbSe	24	77.4	1.2	0.9–1.2
		4.2		2.1–2.4
	37	4.2		2.7–3.4
n PbTe	9	77.4	2.6	
	30	77.4	3.0	3.0–4.0
		4.2		3.0–5.0
	34	77.4	1.9	1.8–1.9
		4.2	0.8	
p PbTe	12	295		1.4–1.6
		77.4	2.2	1.5–2.1
	29	77.4	2.7	2.4–2.7
		4.2	3.0	3.9–4.4

Comparison with Specific Models

The experimental values of M_{100} were as large or larger than M_{100}^{001} in almost all our samples. This fact eliminates the simple band or the ⟨100⟩ cubic model, and thus disagrees with Irie's conclusions for natural PbS.[8] However, we do not have very precise nor extensive data for PbS, and we feel that further measurements must be obtained before a definite choice of models can be made.

To determine whether the ⟨110⟩ or ⟨111⟩ models may be appropriate for PbSe and PbTe, we must examine the theoretical predictions for these models in greater detail. Because the sample current was always parallel to a cubic axis of the crystal, we cannot evaluate the three magnetoresistivity constants of Eqs. (2) and (3), but only $b=\mu_H^2 M_{100}^{001}$ and $b+c+d=\mu_H^2 M_{100}$. It will be more convenient to discuss the theoretical predictions in terms of these two coefficients.

[11] H. P. R. Frederikse and W. R. Hosler, Phys. Rev. **108**, 1136 (1957).

[12] G. L. Pearson and C. Herring, Physica **20**, 975 (1954).

[13] H. P. R. Frederikse and W. R. Hosler, Phys. Rev. **108**, 1146 (1957).

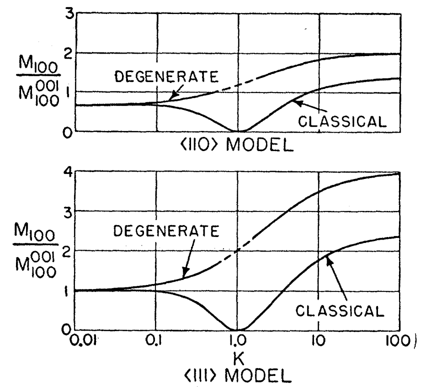


FIG. 6. Theoretical magnetoresistance ratio M_{100}/M_{100}^{001} as a function of the mass ratio K, for the ⟨110⟩ and ⟨111⟩ cubic models in weak magnetic fields, for completely degenerate statistics, and for classical statistics with the scattering law $\tau=lE^{-\frac{1}{2}}$. The dashed lines indicate unreliable portions of the curves for completely degenerate statistics. See last two paragraphs of section, "Comparison with Specific Models."

The Abeles and Meiboom[4] and Shibuya[5] theories were derived for the case of classical statistics. Shibuya considered the scattering law $\tau=lE^{-\frac{1}{2}}$, where τ is the mean free time, l is constant for a given temperature, and E is the energy, for all three cubic models, while Abeles and Meiboom investigated this law only for the ⟨111⟩ model, and the law $\tau=lE^{\frac{3}{2}}$ for the ⟨100⟩ model. Our samples had degeneracy temperatures of the order of 200°K, so that classical statistics are a rather poor approximation at 77.4°K, and the crystals are highly degenerate at 4.2°K.

For small fields, any statistics, and the scattering law $\tau=lE^n$, the longitudinal and transverse magnetoresistance coefficients in any direction for any given cubic model may be expressed in the form

$$M_{abc}=AF_{abc}(K), \tag{4}$$

$$M_{abc}{}^{def}=AF_{abc}{}^{def}(K)-1, \tag{5}$$

where A depends only on the statistics and the scattering law, and the $F(K)$ depend only on K, the ratio of effective masses characterizing the energy ellipsoids. Further details on these equations, and a summary of expressions for some specific cases, are given in Appendix A. It turns out to be particularly useful to work with the ratio M_{100}/M_{100}^{001}. This ratio is shown in Fig. 6, as a function of K, for the ⟨110⟩ and ⟨111⟩ cubic models, for two cases: classical statistics with $\tau=lE^{-\frac{1}{2}}$, and completely degenerate statistics (in this case, τ may be any function of E). The highest value of this ratio always occurs for the case of completely degenerate statistics, or for the mathematically equivalent case of classical statistics *with a constant scattering time* (see Appendix A).

Whether the ⟨110⟩ or ⟨111⟩ model is appropriate, it is clear from Fig. 6 and the high experimental values of M_{100}/M_{100}^{001} listed in Table III that a value of K greater than one is favored. The majority of the experimental ratio values for PbSe could be satisfied by either the ⟨110⟩ or the ⟨111⟩ model. However, the experimental ratio exceeds 2, the maximum theoretical value for the ⟨110⟩ model, for four of the five PbTe samples; thus the ⟨111⟩ model is favored for PbTe, in agreement with the conclusion of Shogenji and Uchiyama.[9] The experimental ratios in the PbTe crystals tend to be larger at 4.2°K than at 77.4°K. This agrees with the theory in that larger values are predicted for degenerate statistics, which are more appropriate at the lower temperature.

Some of the experimental values of M_{100}/M_{100}^{001} for PbTe at 4.2°K even exceed 4, the maximum theoretical value for the ⟨111⟩ model. This would seem to weaken the choice of this model, but actually, as we will now show, it strengthens it. The weak-field theory is not really applicable to most of our data because the condition $\mu_H H \ll 10^8$ was not usually satisfied. In particular, the data at 4.2°K were almost always obtained at $\mu_H H$ values larger than 10^8, and in two cases, larger than 10^9. Because of the equivalence of the constant τ and the degenerate statistics cases, we may apply to our data the analysis of the ⟨111⟩ model by Gold and Roth,[14] which is valid for any magnetic field strength. From their paper we derive that

$$\frac{M_{100}}{M_{100}^{001}}=\frac{2(2K+1)}{(K+2)}\times\left\{\frac{1+[3K(K+2)/(2K+1)^2]^2(\mu H/10^8)^2}{1+[3/(2K+1)][3K/(2K+1)]^2(\mu H/10^8)^2}\right\}, \tag{6}$$

in which we have used the conductivity mobility μ, rather than the Hall mobility μ_H, to avoid needless complexity.[15]

Equation (6) becomes larger than the corresponding weak field theory result when $\mu H \gtrsim 10^8$, particularly

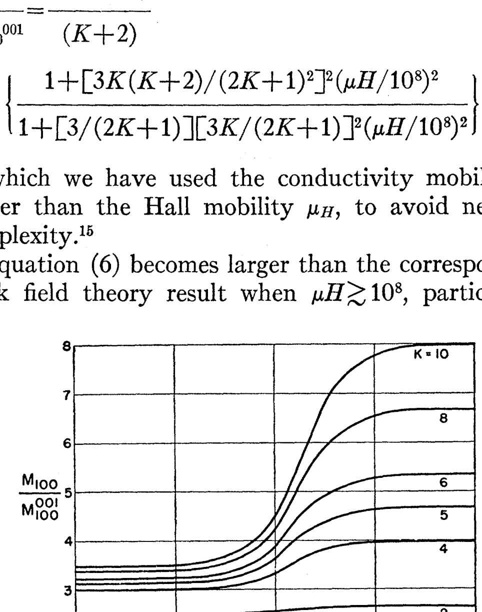


FIG. 7. Theoretical magnetoresistance ratio M_{100}/M_{100}^{001} as a function of μH, for the ⟨111⟩ cubic model and completely degenerate statistics, for several values of the mass ratio K.

[14] L. Gold and L. M. Roth, Phys. Rev. **107**, 358 (1957).

[15] We should point out that our Eq. (6) does not directly result from combining Gold and Roth's Eqs. (6) and (7), since their $\omega\tau(=\mu H)$ is expressed in terms of the transverse mass only, while our μH uses the mass defined by $3/m=2/m_t+1/m_l$.

when $K>1$. This is illustrated in Fig. 7, for several values of K ranging from 2 to 10. Most of the PbTe data are consistent with a K value between 4 and 6. The agreement between experiment and the theory for the ⟨111⟩ model with $K>1$ is thus improved by considering the more exact expression. On the other hand, we can deduce that an extension of the weak-field theory for the ⟨110⟩ model to larger μH would make this model less suitable. This follows from an examination of the saturation values of the coefficients which were derived by Shibuya[5] for classical statistics and the scattering law $\tau=lE^{-\frac{1}{2}}$, and which differ from the constant-τ case, for all three cubic models, only by the factor $9\pi/32$.[16] We find that for completely degenerate statistics, the saturation value of $M_{100}/M_{100}{}^{001}$ for the ⟨110⟩ model is

$$\frac{M_{100}}{M_{100}{}^{001}}=\frac{18(K+1)}{(K+5)(2K+3)}. \tag{7}$$

This expression, in contrast to that for the ⟨111⟩ model, is smaller than the weak-field value when $K>1$. It decreases from 1.10 at $K=2$ to 0.57 at $K=10$, and furthermore is never larger than 1.23 for any value of K.

We have been using the approximation of completely degenerate statistics. In Appendix B we consider the effect of applying highly degenerate statistics to the cubic models, and show that there should be a significant change in the theoretical results only when K lies within a certain range of values in the neighborhood of unity. This range of values depends on the degree of degeneracy of the samples under consideration, and in our case, K would have to lie between about 0.75 and 1.25. Our analysis has indicated considerably more mass anisotropy than this, and therefore the better approximation is not needed. The fact that there is a portion of the theoretical result which is not reliable has been suggested by the dashed lines in Fig. 6.

The analysis of the highly degenerate case shows that the ratio $M_{100}/M_{100}{}^{001}$ (as well as the ratio of any longitudinal to transverse coefficient) vanishes when $K=1$. This occurs since the transverse coefficient is no longer zero at $K=1$ in the higher approximation, while the longitudinal coefficient remains strictly zero for any statistics. The high experimental values of $M_{100}/M_{100}{}^{001}$ occurring for degenerate statistics thus confirm our conclusion that the effective mass is anisotropic.

SUMMARY AND CONCLUSIONS

The distinguishing features of the magnetoresistance data presented in this paper are the similarity of the results for n- and p-type material, the large ratios of longitudinal to transverse magnetoresistance, and the striking angular effects and large fractional changes in resistance at 4.2°K. The data favor a model with considerable mass anisotropy (prolate ellipsoids). The cubic model with ellipsoids along the ⟨111⟩ directions appears to be the most appropriate, at least for PbTe, for both the conduction and valence band. This choice is strengthened through the application of the theoretical treatment by Gold and Roth for arbitrary magnetic field strengths, over that derived only from the consideration of the weak-field theories of Abeles and Meiboom and Shibuya. Comparison of the Gold-Roth theory with the experimental data for PbTe suggests a value between 4 and 6 for K, the ratio of longitudinal to transverse effective mass.

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APPENDIX A. EXPRESSIONS FOR MAGNETO-RESISTANCE COEFFICIENTS

We start from Eqs. (2), (4), and (5), which we repeat here for convenience:

$$\frac{\Delta\rho}{\rho_0}=[b+c(\sum_{j=1}^{3}\iota_j\eta_j)^2+d\sum_{j=1}^{3}\iota_j^2\eta_j^2]H^2, \tag{2}$$

$$M_{abc}=AF_{abc}(K), \tag{4}$$

$$M_{abc}{}^{def}=AF_{abc}{}^{def}(K)-1. \tag{5}$$

In Eqs. (4) and (5), the factor $A=G_1G_3/G_2{}^2$, where

$$G_p=\int E^{\frac{3}{2}}\tau^p(\partial f/\partial E)dE, \tag{8}$$

and f is the Fermi distribution function. Clearly, $A=1$ when τ is constant. When τ is a function of E, $A>1$ (τ must be defined so that the integrals do not diverge). For classical statistics and $\tau=lE^n$, A has the following values:

n	A
$-\frac{1}{2}$	$4/\pi=1.273$
$\frac{1}{2}$	$256/75\pi=1.086$
1	$9/7=1.286$
$\frac{3}{2}$	$32\,768/6615\pi=1.577$

In the limit of completely degenerate statistics, $(\partial f/\partial E)=-\delta(E_F)$ and $G_p=-E_F{}^{\frac{3}{2}}\tau^p(E_F)$, where E_F is the Fermi-level energy, and A reduces to unity in this case also.

Table IV lists the expressions for b, c, and d for the three cubic models. These expressions and Eq. (2) may be used to obtain the longitudinal and transverse magnetoresistance coefficients for any directions of sample current and magnetic field. Note from Eq. (2)

[16] L. Gold and L. M. Roth, Phys. Rev. 103, 61 (1956).

TABLE IV. Theoretical expressions for weak-field coefficients b, c, and d for cubic models.[a]

Model	b/μ_H^2	c/μ_H^2	d/μ_H^2
⟨100⟩	$A\left[\frac{(2K+1)(K^2+K+1)}{K(K+2)^2}\right]-1$	$-\left\{A\left[\frac{3K(2K+1)}{K(K+2)^2}\right]-1\right\}$	$-\left\{A\left[\frac{(2K+1)(K-1)^2}{K(K+2)^2}\right]\right\}$
⟨110⟩	$A\left[\frac{3(2K+1)(K+1)^2}{4K(K+2)^2}\right]-1$	$-\left\{A\left[\frac{(2K+1)(K^2+4K+1)}{2K(K+2)^2}\right]-1\right\}$	$A\left[\frac{(2K+1)(K-1)^2}{4K(K+2)^2}\right]$
⟨111⟩	$A\left[\frac{(2K+1)^2}{3K(K+2)}\right]-1$	$-\left\{A\left[\frac{(2K+1)^2}{3K(K+2)}\right]-1\right\}$	$A\left[\frac{2(2K+1)(K-1)^2}{3K(K+2)^2}\right]$

[a] $A=G_1G_3/G_2^2$; $K=$longitudinal mass/transverse mass.

that any longitudinal coefficient will always contain the sum $b+c$, and any transverse coefficient will contain b but not c. Consequently, the nature of the expressions for b, c, and d insures that the magnetoresistance coefficients will always have the form suggested in Eqs. (4) and (5).

From Eqs. (4) and (5) we have that

$$\frac{M_{abc}}{M_{abc}{}^{def}}=\frac{F_{abc}}{F_{abc}{}^{def}-1/A}, \tag{9}$$

and therefore this ratio has its maximum value for completely degenerate statistics or for a constant τ, and decreases with increasing $|n|$ in $\tau=lE^n$. It is interesting to note that for the case of completely degenerate statistics, Eq. (9) reduces to the simple form

$$\frac{M_{abc}}{M_{abc}{}^{def}}=\frac{rK+s}{tK+u}, \tag{10}$$

where r, s, t, and u are constants.

APPENDIX B. MAGNETORESISTANCE COEFFICIENTS FOR HIGHLY DEGENERATE STATISTICS

To apply highly degenerate, rather than completely degenerate, statistics to the cubic models, we must calculate the next higher approximation to the integrals G_p contained in the factor A (see Appendix A). Sommerfeld and Bethe's procedure[17] may be used to evaluate these integrals, and we obtain

$$G_p=E_F{}^{\frac{3}{2}+pn}\left[1+\frac{(2p+1)(2p+3)\pi^2}{24}\left(\frac{kT}{E_F}\right)^2\right]. \tag{11}$$

This result is accurate if $(kT/E_F)\ll 1$, and this same condition then leads to

$$A=1+\frac{\pi^2n^2}{3}\left(\frac{kT}{E_F}\right)^2=1+\Delta. \tag{12}$$

Equations (4) and (5) then become

$$M_{abc}=(1+\Delta)F_{abc}(K), \tag{13}$$

$$M_{abc}{}^{def}=(1+\Delta)F_{abc}{}^{def}(K)-1. \tag{14}$$

Since $\Delta\ll 1$, there is only a very slight effect on M_{abc}. However, $F_{abc}{}^{def}(K)\rightarrow 1$ as $K\rightarrow 1$. Let $F_{abc}{}^{def}-1=D$; then when $D\ll 1$,

$$M_{abc}{}^{def}\approx\Delta+D, \tag{15}$$

and therefore $M_{abc}{}^{def}$ will differ significantly from its less exact approximation only when D is comparable to or less than Δ.

The ratio $M_{abc}/M_{abc}{}^{def}$ will also differ in this circumstance from its value for completely degenerate statistics; it will in fact approach zero as $K\rightarrow 1$, since M_{abc} approaches zero while $M_{abc}{}^{def}$ does not. Thus the simple form for this ratio, Eq. (10), is not valid when $D\lesssim\Delta$.

[17] This procedure is described in F. Seitz, *Modern Theory of Solids* (McGraw-Hill Book Company, Inc., New York, 1940), pp. 147–149.