Magnetic Field Dependence of Ultrasonic Attenuation in Metals at Low Temperatures*

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A theory of the effect of a constant magnetic field on the behavior of ultrasonic attenuation in normal metals at low temperatures of the order of liquid helium temperatures is given. The ideas are of the same kind as those suggested by Pippard to account for the attenuation in the absence of an external field. The different geometries are specified by the directions of three vectors, the wave vector $\mathbf{q}$ of the acoustic wave, the direction of polarization $\mathbf{u}_0$, and the external magnetic field $\mathbf{H}_0$. The analysis shows that, for a transverse wave polarized in the direction of $\mathbf{H}_0$ (i.e., $\mathbf{u}_0$ and $\mathbf{H}_0$ are parallel and both are perpendicular to $\mathbf{q}$) the attenuation decreases as $|\mathbf{H}_0|^{-2}$ for large fields. When $\mathbf{u}_0$ and $\mathbf{H}_0$ are perpendicular and $\mathbf{q}$ is perpendicular to both, the attenuation increases as $|\mathbf{H}_0|^2$ for large $|\mathbf{H}_0|$. For a wave such that $\mathbf{u}_0$ and $\mathbf{q}$ are parallel and $\mathbf{H}_0$ is perpendicular to $\mathbf{q}$, the attenuation increases asymptotically to a constant value as $|\mathbf{H}_0|$ increases. The maxima and minima obtained experimentally by Morse and co-workers cannot be explained on this model. An absorption similar to that occurring in cyclotron resonance absorption is obtained in the attenuation of transverse waves ($\mathbf{u}_0$ perpendicular to $\mathbf{q}$) when $\mathbf{H}_0$ is parallel to $\mathbf{q}$.

I. INTRODUCTION

At low temperatures the main mechanism responsible for the attenuation of ultrasonic waves in metals is the scattering of conduction electrons by the ultrasonic phonons. The attenuation can be regarded as being the decrease, per unit distance travelled, in the number of phonons in the ultrasonic wave as it progresses in the metal. The phonon-electron scattering gives rise to a transfer of energy from the ultrasonic wave to the conduction electrons which, in turn, transfer their excess energy to the thermal phonons. Thus, there is an irreversible flow of energy from the acoustical phonons to the thermal phonons.

Early measurements of ultrasonic attenuation in both normal and superconducting metals have been performed by Bömmel[1] and MacKinnon.[2] In the region where the electron mean free path l is short as compared with the wavelength λ of the sound wave, the attenuation is proportional to the square of the ultrasonic frequency. However, the attenuation becomes proportional to the frequency when $l \gtrsim \lambda$. The attenuation also decreases with increasing temperature having a flat maximum at absolute zero. Recently,[3,4] it has been observed that the attenuation changes if the metal is in the presence of an external magnetic field $\mathbf{H}_0$. When $ql<1$, where $q=2\pi/\lambda$ is the propagation number of the sound wave, the attenuation decreases inversely as the first power of the magnetic field when $|\mathbf{H}_0|$ increases. For $ql>1$ the attenuation presents one or several maxima and minima as a function of $|\mathbf{H}_0|$ before decreasing as $1/|\mathbf{H}_0|$ for large fields. It has been suggested[3,5] that this effect is caused by electron resonances similar to those occurring in cyclotron resonance absorption. These experiments were all performed with $\mathbf{H}_0$ perpendicular to the direction of propagation of the wave.

The theoretical treatments of ultrasonic attenuation in metals consist in calculations of the power loss by the ultrasonic wave to the electron cloud. In the region in which $ql \ll 1$ the attenuation is interpreted as arising from the viscosity of the electron gas.[6] The electrons travel between regions in the metal having different particle velocities, thus giving rise to shear stresses of the same kind as those which occur in the propagation of waves in a viscous fluid. The effect of an external magnetic field on the viscosity of the electron gas has been considered by Steinberg.[7] A magnetic field applied in the direction of polarization of a transverse wave tends to decrease the viscosity of the electron gas. In fact, the presence of the magnetic field shortens the range of the electrons in a plane perpendicular to $\mathbf{H}_0$, thus rendering the number of electrons that transfer momentum between two layers of the electron gas smaller than the same in the field-free case. Because the power loss by the ultrasonic wave is proportional to the viscosity, the attenuation is, in this case, a decreasing function of $\mathbf{H}_0$. When $ql>1$ the absorption occurs because of direct collisions between electrons and phonons. Kittel[8] has calculated the decrease in the number of ultrasonic phonons, caused by the collisions with conduction electrons, as the wave progresses in the metal. Pippard[9] has given a rather complete

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discussion of the problem in the absence of an external magnetic field. His argument is as follows. If the electron mean free path were infinite, the deformations of the lattice as the ultrasonic wave travels through the metal would produce adiabatic changes in the shape of the Fermi surface. However, the collisions tend to restore the Fermi surface to its original shape. But this process can never be complete, particularly if the relaxation time is long. Therefore, the total electronic energy will, on the average, be greater than its thermal equilibrium value. This excess energy is dissipated to the thermal phonons thus producing an attenuation of the acoustical wave. Pippard's treatment gives expressions for the attenuation for all values of ql.

In this paper we shall give a theory of the magnetic field dependence of ultrasonic attenuation in metals along similar lines of argument as those given by Pippard. A unified treatment based on the usual transport theory in the presence of a magnetic field $\mathbf{H}_0$ is given. The results of our work, in the case where the external field $\mathbf{H}_0$ tends to zero, coincide with those obtained by Pippard.[9]

We shall assume that the metal consists of a lattice of ions embedded in a uniform sea of conduction electrons, that there is one conduction electron per atom, and that the electrons can be properly described as a degenerate Fermi gas. The attenuation of sound waves on this model is caused by the following mechanism. When the wave propagates in the metal, the ions oscillate around their positions of stable equilibrium. The electrons will be dragged by the ions in their motion. However, there will always be a lag in the motion of ions and electrons, which will give rise to electric currents. These electric currents induce electromagnetic fields which are able to transfer energy to the conduction electrons. The power per unit volume W absorbed by the electrons can be calculated to determine the attenuation α by means of the relation

$$\alpha=2W/(\rho|\mathbf{u}|^2v_a), \tag{1}$$

where ρ is the density of the metal, v_a the velocity of sound, and $\mathbf{u}$ the particle velocity describing the sound wave.

In Sec. II we develop the method of calculation of the fields carried by the lattice and the power absorption W. The procedure consists in the simultaneous solution of the Maxwell equations governing the electromagnetic fields and the Boltzmann transport equation relating the electron currents to the fields. In Secs. III and IV we consider the special cases of shear and longitudinal acoustic waves, respectively. In both these cases the external magnetic field $\mathbf{H}_0$ is assumed to be perpendicular to the direction of propagation of the ultrasonic wave. The case in which the external magnetic field is parallel to the direction of propagation of the wave is treated in Sec. V. The results do not agree very well with experiment. A detailed account of the results and of the discrepancies with experimental information is given in Sec. VI. The most important difference from the experimental results is, perhaps, the inability of our theory to account for the maxima and minima that are observed when $ql>1$. Let us consider, to fix the ideas, the case when $\mathbf{H}_0$ is perpendicular to both the direction of polarization $\mathbf{u}_0$ of the wave and to the direction of propagation defined by the wave vector $\mathbf{q}$. Also assume a shear wave, i.e., that $\mathbf{u}_0$ is normal to $\mathbf{q}$. It is argued[5] that, if the magnetic field is properly chosen, the radius of the cyclotron orbit of a group of electrons on the Fermi surface may be such that the electrons are accelerated by the electric field associated with the acoustic wave, thus giving rise to a power absorption. However, if we consider a similar orbit which is equal to the previous one except that it is displaced in space, in the direction of propagation of the wave by half of the wavelength, then, the same amount of energy that the electronic cloud absorbed from the ultrasonic field by means of an electron in the first orbit is returned to it by an electron in the second orbit.

A few words should, perhaps, be said about the limitations of our treatment. We have assumed that the electrons have a constant relaxation time over the Fermi surface. This approximation is good for scattering by lattice imperfections. However, no such relaxation time for phonon-electron collisions can be defined at the low temperatures considered here. We think, nevertheless, that this limitation is not very important because, at low temperatures and for the usual crystals that can be obtained, impurity scattering is the dominant mechanism. Also the inherent approximations of the Boltzmann transport theory will be present in this theory. For magnetic fields such that the cyclotron resonance period is small compared with the relaxation time, the validity of the Boltzmann transport theory is questionable.

II. GENERAL THEORY

Consider a sample of metal in which a sound wave propagates in the direction of the wave vector $\mathbf{q}$. The wave may be characterized by the velocity $\mathbf{u}$ of the ions which can be expressed in the form

$$\mathbf{u}(\mathbf{r},t)=\mathbf{u}_0\exp(i\omega t-i\mathbf{q}\cdot\mathbf{r}), \tag{2}$$

where ω is the angular frequency and $\mathbf{u}_0$ is parallel to the direction of polarization of the wave. The variables t and $\mathbf{r}$ are the time and the position vector, respectively. The velocity of sound in the direction $\mathbf{q}$ is

$$v_a=\omega/|\mathbf{q}|. \tag{3}$$

While the wave propagates in the metal, the ions will experience oscillatory motions around their positions of stable equilibrium with velocities given by (2). The electrons will tend to screen the local electric charges. Thus, they will follow the ions in their motion. However,

there will always be a phase difference between the displacements of the two types of charges, thus creating local electric currents which have the same space-time periodicity as (2). These unbalanced local currents will produce a magnetic field $\mathbf{H}(\mathbf{r},t)$ which will, in turn, induce an electric field $\mathbf{E}(\mathbf{r},t)$. The field $\mathbf{E}$ is responsible for the motion of the electrons and will, therefore, be functionally related to the local currents. The fields $\mathbf{E}$ and $\mathbf{H}$ are obtained in terms of $\mathbf{u}$ by means of a self-consistent calculation which takes the screening property of the electrons into account. If the sound wave has a longitudinal component, i.e., if $\mathbf{u}\cdot\mathbf{q}\neq 0$, there will be other electric fields arising from local changes in the electron density n. In fact, the distortions of the lattice, produced by the propagation of a longitudinal wave, cause local changes in volume. When the time τ between lattice-electron collisions is much shorter than the period of the sound wave ($\omega\tau\ll 1$), the electrons have time to distribute themselves with densities in accordance with the local volume distortions in the crystal. These local charge distributions produce an electric field pointing in the direction of propagation of the wave. The electromagnetic fields carried by the lattice can be consistently calculated with the aid of Maxwell's equations.

As all the electromagnetic vectors in the lattice have the periodicity of $\mathbf{u}$, it follows, from Maxwell's equations, that

$$c\mathbf{q}\times\mathbf{E}=\omega\mathbf{H}, \tag{4}$$

and

$$c\mathbf{q}\times\mathbf{H}=4\pi i\mathbf{j}, \tag{5}$$

where c is the velocity of light and $\mathbf{j}$ is the total current density which is composed of the electronic and ionic current densities, i.e.,

$$\mathbf{j}=\mathbf{J}-Ne\mathbf{u}, \tag{6}$$

where $\mathbf{J}$ is the electron current density, N the number of atoms per unit volume in the crystal, and e the charge of the electron. In Eq. (5) we have neglected the displacement current. The electron current density $\mathbf{J}$ satisfies the equation of continuity

$$\mathbf{q}\cdot\mathbf{J}=e\omega N_1, \tag{7}$$

where N_1 is the departure of the electron density $n=N+N_1$ from its equilibrium value N. From Eqs. (5), (6), and (7) we obtain

$$N_1=N\mathbf{u}\cdot\mathbf{q}/\omega. \tag{8}$$

Equations (4), (5), and (6) permit us to determine $\mathbf{E}$, $\mathbf{H}$, and $\mathbf{J}$ in terms of $\mathbf{u}$ once we know the dependence of $\mathbf{J}$ on $\mathbf{E}$.

The point relation $\mathbf{J}=\sigma\mathbf{E}$, where σ is the conductivity of the metal, is not valid in the problem at hand because of the presence of the magnetic field, and because, in the region of interest to us, the electron mean free path is comparable to distances in which the electric field $\mathbf{E}$ experiences radical changes in magnitude. Thus, Ohm's law is not an appropriate approximation here. The dependence of $\mathbf{J}$ on $\mathbf{E}$ is found by considering the transport problem in detail.

Let $(1/4\pi^3)f(\mathbf{r},\mathbf{k},t)d\mathbf{r}d\mathbf{k}$ be the number of electrons in an element of volume $d\mathbf{r}d\mathbf{k}$ around the point $(\mathbf{r},\mathbf{k})$ in μ space at the time t. Here $\mathbf{k}$ is the wave vector of the electron. The time rate of change of the distribution function f, coming from the drift of the electrons and the presence of the fields, is

$$-\mathbf{v}\cdot\nabla f-\frac{e}{\hbar}\left[\mathbf{E}+\frac{1}{c}\mathbf{v}\times(\mathbf{H}_0+\mathbf{H})\right]\cdot\nabla_k f,$$

where $v=\hbar\mathbf{k}/m$ is the velocity of the electron in the state $\mathbf{k}$ and m its mass. We now take into account the collisions of the electrons with lattice imperfections and thermal phonons by assuming the existence of a relaxation time τ which is a function of the electron energy $\epsilon=\hbar^2k^2/2m$ alone. The electron distribution function f will relax towards its local equilibrium value $\bar{f}$ with the characteristic time τ (here, as before, it is assumed that $\omega\tau\ll 1$; this assumption will be retained throughout this paper). The time rate of change of f arising from the collisions is $-(f-\bar{f})/\tau$. Then, the steady-state distribution function f must satisfy the equation

$$\frac{\partial f}{\partial t}+\mathbf{v}\cdot\nabla f+\frac{e}{\hbar}\left[\mathbf{E}+\frac{1}{c}\mathbf{v}\times(\mathbf{H}_0+\mathbf{H})\right]\cdot\nabla_k f+\frac{f-\bar{f}}{\tau}=0. \tag{9}$$

The local equilibrium value $\bar{f}$ of the distribution function is not equal to the Fermi function $f_0(\epsilon)$, because the electron density $n=N+N_1$ is not constant throughout the crystal, and because the lattice as a whole is moving locally with a velocity $\mathbf{u}$. The function $\bar{f}$ is given by

$$\bar{f}(\mathbf{r},t)=\{\exp[(\epsilon'-\epsilon_0')/kT]+1\}^{-1}, \tag{10}$$

where ϵ_0' is the local value of the Fermi energy, and ϵ' is the kinetic energy of the electron relative to the moving lattice. Here k is the Boltzmann constant and T the absolute temperature. To first order in $\mathbf{u}$, we have

$$\epsilon'=\epsilon-m\mathbf{v}\cdot\mathbf{u}, \tag{11}$$

and

$$\epsilon_0'=\epsilon_0+\tfrac{2}{3}\epsilon_0(N_1/N). \tag{12}$$

In Eq. (12) ϵ_0 is the Fermi energy of the unstrained metal. Thence,

$$\bar{f}=f_0(\epsilon-m\mathbf{v}\cdot\mathbf{u}-\tfrac{2}{3}\epsilon_0N_1/N)$$

$$=f_0(\epsilon)-\frac{df_0}{d\epsilon}(m\mathbf{v}\cdot\mathbf{u}+\tfrac{2}{3}\epsilon_0N_1/N). \tag{13}$$

This expansion is justified if $(m\mathbf{v}\cdot\mathbf{u})/kT$ and $[\frac{2}{3}\epsilon_0(N_1/N)]/kT$ are negligible as compared with unity. This is indeed the case for all practical purposes. For an

input power of 0.01 watt/cm^2 the strain in the metal is of the order of 10^{-6}. The copper and an ultrasonic frequency of 50 Mc/sec we estimate $u\approx 0.7$ cm/sec and $N_1\approx 10^{-6}N$. Thus, for a temperature as low as 1°K, $(m\mathbf{u}\cdot\mathbf{v})/kT\approx 10^{-4}$ and $[\frac{2}{3}\epsilon_0(N_1/N)]/kT\approx 10^{-2}$.

The deviation of the electron distribution function from its thermal equilibrium value f_0 is small as compared with f_0 and is proportional to the field $\mathbf{E}$. We shall, as is customary, assume a solution of (9) of the form

$$f=f_0+f_1, \tag{14}$$

where f_1 has the periodicity of $\mathbf{u}$. We linearize the equation resulting from (14) and (9) in the usual manner to obtain

$$(1+i\omega\tau-i\mathbf{q}\cdot\mathbf{v}\tau)f_1-\frac{e\tau}{mc}\mathbf{H}_0\cdot(\mathbf{k}\times\nabla_k f_1) = -\frac{df_0}{d\epsilon}\left[e\tau\mathbf{v}\cdot\left(\mathbf{E}+\frac{m}{e\tau}\mathbf{u}\right)+\tfrac{2}{3}\epsilon_0\frac{N_1}{N}\right]. \tag{15}$$

In this expression we have neglected $\mathbf{H}$, being of the order of 10^{-2} oersted. In the region of interest, when $|\mathbf{H}_0|\approx 500$ gauss or more, $\mathbf{H}$ produces a tilting of the cyclotron orbit by an angle of the order of 10^{-4} radian. It is now convenient to take polar coordinates (k,θ,φ) in $\mathbf{k}$ space with the polar axis parallel to the external magnetic field $\mathbf{H}_0$. Here k is the radius vector in $\mathbf{k}$ space and θ and φ are the polar angles of $\mathbf{k}$. After some transformations, (15) becomes

$$\frac{\partial f_1}{\partial\varphi}+\frac{1+i\omega\tau-i\mathbf{q}\cdot\mathbf{v}\tau}{\omega_c\tau}f_1 = -\frac{1}{\omega_c\tau}\frac{df_0}{d\epsilon}\left[e\tau\mathbf{v}\cdot\left(\mathbf{E}+\frac{m}{e\tau}\mathbf{u}\right)+\tfrac{2}{3}\epsilon_0\frac{N_1}{N}\right], \tag{16}$$

where $\omega_c=-eH_0/mc$ is the cyclotron resonance frequency of the electrons. Equation (16) can be solved exactly by a procedure invented by Chambers.[10] The solution is

$$f_1=-\frac{df_0}{d\epsilon}\frac{1}{\omega_c\tau}\frac{1}{e^{2\pi\gamma}-1}\times\int_\varphi^{2\pi+\varphi}d\varphi'\left[e\tau\mathbf{v}'\cdot\left(\mathbf{E}+\frac{m}{e\tau}\mathbf{u}\right)+\tfrac{2}{3}\epsilon_0\frac{N_1}{N}\right]\times\exp\left[-\int_{\varphi'}^{\varphi}\frac{1+i\omega\tau-i\mathbf{q}\cdot\mathbf{v}''\tau}{\omega_c\tau}d\varphi''\right], \tag{17}$$

with

$$\gamma=\frac{1}{2\pi}\int_0^{2\pi}\frac{1+i\omega\tau-i\mathbf{q}\cdot\mathbf{v}\tau}{\omega_c\tau}d\varphi. \tag{18}$$

The quantities $\mathbf{v}'$ and $\mathbf{v}''$ are the electron velocities that have the azimuthal angles φ' and φ'', respectively.

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Using (17) and

$$\mathbf{J}(\mathbf{r},t)=\frac{e}{4\pi^3}\int f_1(\mathbf{k},\mathbf{r},t)\,\mathbf{v}\,d\mathbf{k}, \tag{19}$$

we obtain the required relation between $\mathbf{J}$ and $\mathbf{E}$. This relation turns out to be

$$\mathbf{J}(\mathbf{r},t)=\frac{3}{4\pi}\frac{Ne^2}{m\omega_c}\int_0^\pi d\theta\sin\theta\int_0^{2\pi}d\varphi\frac{\mathbf{n}(\theta,\varphi)}{e^{2\pi\gamma}-1}\int_\varphi^{2\pi+\varphi}d\varphi'\times\left[\mathbf{n}(\theta,\varphi')\cdot\left(\mathbf{E}+\frac{m}{e\tau}\mathbf{u}\right)+\frac{1}{3}\frac{mv_0}{e\tau}\frac{N_1}{N}\right]\times\exp\left[-\int_{\varphi'}^{\varphi}\frac{1+i\omega\tau-i\mathbf{q}\cdot\mathbf{v}''\tau}{\omega_c\tau}d\varphi''\right]. \tag{20}$$

In Eq. (20), $\mathbf{n}(\theta,\varphi)$ is a unit vector in the direction of the electron velocity $\mathbf{v}$ having polar angle θ and azimuthal angle φ. To obtain (20) we have made use of the fact that, for temperatures much lower than the Fermi degeneracy temperature, $-df_0/d\epsilon$ behaves as the Dirac δ-function $\delta(\epsilon-\epsilon_0)$. Once we have obtained $\mathbf{J}$ and $\mathbf{E}$ in terms of $\mathbf{u}$, the power absorption per unit volume of the sample is given by

$$W=\tfrac{1}{2}\,\mathrm{Re}(\mathbf{J}^*\cdot\mathbf{E}). \tag{21}$$

We shall first consider the situation in which $\mathbf{H}_0$ is perpendicular to $\mathbf{q}$. This is also the important case from the experimental point of view as the results of Bömmel[3] and Morse *et al.*[4] correspond to this geometry. Let us take a system of Cartesian coordinates x, y, z with the z axis parallel to $\mathbf{H}_0$ and the y axis in the direction of $\mathbf{q}$. We shall assume that $\mathbf{u}$ is in an arbitrary direction. In this case Eq. (20) reads

$$\mathbf{J}(\mathbf{r},t)=\frac{3}{4\pi}\frac{Ne^2}{m\omega_c}\frac{1}{e^{2\pi\gamma}-1}\int_0^\pi d\theta\sin\theta\int_0^{2\pi}d\varphi\,\mathbf{n}(\theta,\varphi)\int_\varphi^{2\pi+\varphi}d\varphi'\times\left[\mathbf{n}(\theta,\varphi')\cdot\left(\mathbf{E}(\mathbf{r},t)+\frac{m}{e\tau}\mathbf{u}(\mathbf{r},t)\right)+\frac{1}{3}\frac{mv_0}{e\tau}\frac{N_1(\mathbf{r},t)}{N}\right]\times\exp[\gamma(\varphi'-\varphi)+ia\gamma\sin\theta(\cos\varphi'-\cos\varphi)]. \tag{22}$$

In (22), γ and a are defined by the relations

$$\gamma=(1+i\omega\tau)/\omega_c\tau, \tag{23}$$

$$a=ql/(1+i\omega\tau). \tag{24}$$

Equation (22) defines a conductivity tensor relating $\mathbf{J}$ to $\mathbf{E}$. It is convenient, to determine the components of this tensor, to introduce the expressions $J_\pm=J_x\pm iJ_y$. When this is done, the components of the conductivity tensor can easily be expressed in terms of integrals containing Bessel functions. The integrations can be

performed, in the form of a power series, to obtain

$$J_x=\sigma_{xx}\mathcal{E}_x+\sigma_{xy}\mathcal{E}_y+\tfrac{1}{3}\sigma_{xy}\frac{iamv_0}{e\tau}\frac{N_1}{N}, \tag{25}$$

$$J_y=\sigma_{yx}\mathcal{E}_x+\sigma_{yy}\mathcal{E}_y+\tfrac{1}{3}\sigma_{yy}\frac{iamv_0}{e\tau}\frac{N_1}{N}, \tag{26}$$

$$J_z=\sigma_{zz}\mathcal{E}_z. \tag{27}$$

In Eqs. (25), (26), and (27) the following symbols are used

$$\mathfrak{E}(\mathbf{r},t)=\mathbf{E}(\mathbf{r},t)+\frac{m}{e\tau}\mathbf{u}(\mathbf{r},t), \tag{28}$$

$$\sigma_{xx}=\frac{3\sigma}{1+i\omega\tau}\sum_{r=0}^{\infty}\frac{(-1)^r a^{2r}}{(2r+1)(2r+3)}\left\{1-\frac{2r(r+1)^2}{\gamma^2}\right\}\times\prod_{n=0}^{r+1}\frac{1}{1+(n\gamma^{-1})^2}, \tag{29}$$

$$\sigma_{yy}=\frac{3\sigma}{1+i\omega\tau}\sum_{r=0}^{\infty}\frac{(-1)^r a^{2r}}{2r+3}\prod_{n=0}^{r+1}\frac{1}{1+(n\gamma^{-1})^2}, \tag{30}$$

$$\sigma_{xy}=-\sigma_{yx}=-\frac{3\sigma}{1+i\omega\tau}\frac{1}{\gamma}\sum_{r=0}^{\infty}\frac{(-1)^r a^{2r}(r+1)}{2r+3}\times\prod_{n=0}^{r+1}\frac{1}{1+(n\gamma^{-1})^2}, \tag{31}$$

$$\sigma_{zz}=\frac{3\sigma}{1+i\omega\tau}\sum_{r=0}^{\infty}\frac{(-1)^r a^{2r}}{(2r+1)(2r+3)}\prod_{n=0}^{r}\frac{1}{1+(n\gamma^{-1})^2}. \tag{32}$$

The Eqs. (29)–(32) are useful for the numerical computation of the coefficients of the conductivity tensor for large values of $\mathbf{H}_0$ (i.e., $|\gamma|\ll 1$) and for any value of a, and for $|a|\ll 1$ and any value of $\mathbf{H}_0$. However, it is important to know the behavior of the conductivity tensor for $|a|>1$ and fields $\mathbf{H}_0$ in the range in which $\omega_c\tau\sim 1$. Expressions for the components of the conductivity tensor developed in a powers series in a^{-1} can be found. The method to obtain this power series is outlined in the Appendix. The results of these expansions are

$$\sigma_{xx}=\frac{\sigma}{1+i\omega\tau}\left[\frac{3\pi}{4a}\coth(\pi\gamma)-\frac{3}{a^2}+\frac{3\pi}{4a^3}\coth(\pi\gamma)\left(1+\frac{3}{4\gamma^2}\right)-3\sum_{r=0}^{\infty}\frac{(-1)^r a^{-2r-4}}{(2r+1)(2r+3)}\left\{1+\frac{2(r+1)^2(r+2)}{\gamma^2}\right\}\times\prod_{n=0}^{r}\{1+(n\gamma^{-1})^2\}\right], \tag{33}$$

$$\sigma_{xy}=-\sigma_{yx}=\frac{\sigma}{1+i\omega\tau}\left[-\frac{3\pi}{4a^3}\frac{\coth(\pi\gamma)}{\gamma}+\frac{3}{\gamma}\sum_{r=0}^{\infty}\frac{(-1)^r a^{-2r-4}(r+1)}{2r+1}\prod_{n=0}^{r}\{1+(n\gamma^{-1})^2\}\right], \tag{34}$$

$$\sigma_{yy}=\frac{\sigma}{1+i\omega\tau}\left[-\frac{3\pi}{2a^3}\coth(\pi\gamma)+\frac{3}{a^2}+3\sum_{r=0}^{\infty}\frac{(-1)^r a^{-2r-4}}{2r+1}\prod_{n=0}^{r}\{1+(n\gamma^{-1})^2\}\right], \tag{35}$$

and

$$\sigma_{zz}=\frac{\sigma}{1+i\omega\tau}\left[\frac{3\pi}{4a}\coth(\pi\gamma)+\frac{3\pi}{4a^3}\left(1+\frac{1}{4\gamma^2}\right)\coth(\pi\gamma)+3\sum_{r=0}^{\infty}\frac{(-1)^r a^{-2r-2}}{(2r+1)(2r-1)}\prod_{n=0}^{r}\{1+(n\gamma^{-1})^2\}\right]. \tag{36}$$

In the calculations we have performed until this point, we have kept the small term $\omega\tau$. From now on, this term will be neglected, except in the discussion of Sec. IV, whenever it appears in the combination $1+i\omega\tau$. Thus $a\approx ql$ and $\gamma\approx(\omega_c\tau)^{-1}$. Then, we see that, for all practical purposes, the coefficients $\sigma_{xx}, \cdots, \sigma_{zz}$ are approximately real. It is convenient to give, for reference in the following sections, the limiting values of the components of the conductivity tensor for very large magnetic fields ($\omega_c\tau\gg 1$) and for zero magnetic field ($\gamma=\infty$). For $\omega_c\tau\gg 1$, we have

$$\sigma_{xx}=\frac{\sigma}{(\omega_c\tau)^2}(1+\tfrac{2}{5}a^2), \tag{37}$$

$$\sigma_{xy}=-\sigma_{yx}=-\frac{\sigma}{\omega_c\tau}\left[1-\frac{3}{10}\frac{a^2}{(\omega_c\tau)^2}\right], \tag{38}$$

$$\sigma_{yy}=\frac{\sigma}{(\omega_c\tau)^2}\left[1-\frac{3}{20}\frac{a^2}{(\omega_c\tau)^2}\right], \tag{39}$$

$$\sigma_{zz}=\sigma-\frac{1}{5}\frac{\sigma a^2}{(\omega_c\tau)^2}. \tag{40}$$

When $H_0=0$ we find

$$\sigma_{xx}=\sigma_{zz}=\frac{3\sigma}{2a^2}\left[\frac{a^2+1}{a}\tan^{-1}a-1\right], \tag{41}$$

$$\sigma_{xy}=-\sigma_{yx}=0, \tag{42}$$

$$\sigma_{yy}=(3\sigma/a^3)(a-\tan^{-1}a). \tag{43}$$

Equations (41) and (43) are valid for all values of a.

III. ATTENUATION OF TRANSVERSE WAVES IN A TRANSVERSE MAGNETIC FIELD

In this section we shall apply the results obtained in Sec. II to the particular case of the attenuation of

transverse waves in a transverse magnetic field, i.e., the case when $\mathbf{q}$ is perpendicular to both $\mathbf{u}_0$ and $\mathbf{H}_0$. For a transverse wave, $N_1=0$. Two cases will be considered. The first will be that in which $\mathbf{u}_0$ is parallel to $\mathbf{H}_0$, and the second that in which $\mathbf{u}_0$ is at right angles with $\mathbf{H}_0$.

When $\mathbf{u}_0$ is in the direction of $\mathbf{H}_0$, the only nonvanishing component of the current density is

$$J_z=\sigma_{zz}\mathcal{E}_z=\sigma\zeta[E_z+(mu/e\tau)], \tag{44}$$

with

$$\sigma_{zz}=\zeta\sigma. \tag{45}$$

In this geometry, $E_x=E_y=0$ and $H_x\neq 0$ while the other components of $\mathbf{H}$ vanish. From Eqs. (44), (45), (4), (5), and (6) we obtain

$$E_z=\frac{m}{e\tau}\frac{(1-\zeta)iq_0^2}{q^2+i\zeta q_0^2}u, \tag{46}$$

$$J_z=Neu\zeta\frac{q^2+iq_0^2}{q^2+i\zeta q_0^2}. \tag{47}$$

q_0 is defined by

$$q_0^2=4\pi\omega\sigma/c^2. \tag{48}$$

The attenuation α_{11} for this situation is obtained from (1), (21), (46), and (47). The result is

$$\alpha_{11}=\frac{Nm}{\rho\tau v_s}\frac{\zeta(1-\zeta)q_0^4}{q^4+\zeta^2q_0^4}. \tag{49}$$

In deriving (49) we have assumed that ζ is real. As we have remarked before this is approximately true if $\omega\tau\ll 1$. In this expression the symbol v_s stands for the shear velocity of sound. For an ultrasonic frequency $\nu=\omega/2\pi=26$ Mc/sec, with $\sigma=2\times10^{20}$ esu and $v_s=3\times10^5$ cm/sec, $(q_0/q)^2=(2\sigma/\nu)(v_s/c)^2$ turns out to be of the order of 10^3. As ζ is of the order of unity, the approximate relation

$$\alpha_{11}=\frac{Nm}{\rho\tau v_s}\left(\frac{1}{\zeta}-1\right) \tag{50}$$

holds. The dependence of α_{11} on the magnetic field comes in through the dependence of ζ on $\omega_c\tau$. In Fig. 1 we have given a graph of

$$\frac{\alpha_{11}(\omega_c\tau)}{\alpha(0)}=\left(\frac{1}{\zeta(\omega_c\tau)}-1\right)\Big/\left(\frac{1}{\zeta(0)}-1\right) \tag{51}$$

as a function of $\omega_c\tau$ and for several values of ql.

Let us now turn to the case where $\mathbf{u}_0$ is perpendicular to $\mathbf{H}_0$, i.e., when $\mathbf{u}_0$ is along the x axis. In this case the only nonvanishing components of the fields are E_x and H_z. One might think at first sight that there will be a Hall field in the y direction. Such a field cannot be sustained within the metal. In fact, a field of this nature would have the periodicity of (2); therefore, it would change direction after a distance of half a wavelength.

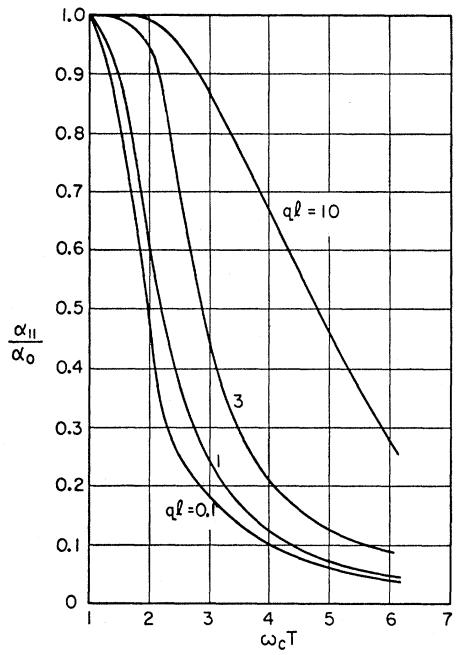

FIG. 1. Ratio (α_{11}/α_0), defined in Eq. (51), as a function of $\omega_c\tau$ for the values of $ql=0.1$, 1, 3, and 10.

This requires space charges which cannot be in equilibrium inside a conductor. Charges of this sort decay exponentially in a time much shorter than the period of the acoustic wave. Then, we have

$$J_x=\sigma_{xx}(E_x+mu/e\tau), \tag{52}$$

$$J_y=\sigma_{yx}(E_x+mu/e\tau). \tag{53}$$

The attenuation turns out to be given by

$$\frac{\alpha_1(\omega_c\tau)}{\alpha(0)}=\left(\frac{1}{\xi(\omega_c\tau)}-1\right)\Big/\left(\frac{1}{\zeta(0)}-1\right), \tag{54}$$

after the same approximations used before to derive (50) have been made and where we have defined $\sigma_{xx}=\sigma\xi(\omega_c\tau)$. The behavior of $\alpha_1(\omega_c\tau)$ is similar to that of $\alpha_{11}(\omega_c\tau)$.

For $\omega_c\tau\gg 1$ and a, we obtain the approximate expressions

$$\frac{\alpha_{11}}{\alpha(0)}=\frac{a^2}{5(\omega_c\tau)^2}\left(\frac{\zeta(0)}{1-\zeta(0)}\right), \tag{55}$$

$$\frac{\alpha_1}{\alpha(0)}=\frac{(\omega_c\tau)^2}{1+\frac{2}{5}a^2}\left(\frac{\zeta(0)}{1-\zeta(0)}\right). \tag{56}$$

When the external magnetic field vanishes, the attenuation is

$$\alpha(0)=\frac{Nm}{\rho\tau v_s}\left(\frac{1}{\zeta(0)}-1\right), \tag{57}$$

where

$$\zeta(0)=\frac{3}{2a^2}\left[\frac{a^2+1}{a}\tan^{-1}a-1\right].$$

This result agrees with that obtained by Pippard.[9] For $a\ll 1$,

$$\alpha(0)=\frac{Nmv_0^2\omega^2\tau}{5\rho v_s^3}\left(1-\frac{8}{35}a^2+\frac{22}{175}a^4-\cdots\right). \quad (58)$$

For[11] $a\gtrsim 1$,

$$\alpha(0)\approx 4Nmv_0\omega/(3\pi\rho v_s^2). \quad (59)$$

IV. ATTENUATION OF LONGITUDINAL WAVES IN A TRANSVERSE MAGNETIC FIELD

In the case of a longitudinal wave propagating in the y direction and in the presence of a magnetic field in the z direction, there will be a Hall field directed along the x direction. Here $\mathbf{u}_0$ and $\mathbf{q}$ are parallel to each other and $\mathbf{H}_0$ is perpendicular to both $\mathbf{u}_0$ and q. The argument given in Sec. III holds only for drift currents that originate in the y direction. Because the fields have a space-time dependence of the form $\exp(i\omega t-iqy)$; at each instant they are constant on any plane which is transverse to the direction of propagation of the wave. Therefore, a Hall field can be sustained by surface charges on the boundaries of the sample and no internal charge densities are required. We determine the Hall field E_x by setting $J_x=0$ and remembering that $\mathbf{u}$ is directed along the y axis. From Eq. (28) and setting $J_x=0$, it follows that

$$E_x=-\frac{\sigma_{xy}}{\sigma_{xx}}\left[\mathcal{E}_y+\frac{ia}{3}\frac{mv_0}{e\tau}\frac{N_1}{N}\right]. \quad (60)$$

Substituting (60) into (26), we find

$$J_y=\sigma\eta\left[E_y+\frac{mu}{e\tau}\left(1+\frac{ia}{3}\frac{v_0}{v_l}\right)\right], \quad (61)$$

where v_l is the longitudinal velocity of sound and

$$\eta=\frac{\sigma_{yy}}{\sigma}+\frac{\sigma_{xy}^2}{\sigma\sigma_{xx}}. \quad (62)$$

The attenuation α_l can now easily be obtained. From (61), (24), and (3) we get

$$J_y=\sigma\eta\left[E_y+\frac{mu}{e\tau}\left(1+\frac{i(ql)^2}{3\omega\tau(1+i\omega\tau)}\right)\right]. \quad (63)$$

[11] It may be argued that the attenuation given in (59) is independent of the Coulomb interaction between ions and electrons as the expression does not contain the electronic charge as a parameter. However, we must remember that (59) is an approximation in which the large ability of the electrons to screen unbalanced charges has been considered. If the charge of the electron were much smaller than what it actually is, the better approximation (49) should be used. It is seen that, if the electron charge tends to zero, so does the attenuation.

But, $J_y=Neu$ as can be seen from (7) and (8). Of course, this is only valid when the displacement current is neglected. At the acoustic frequencies that are of interest to us the displacement current is extremely small as compared with Neu. Proceeding as before, it can be shown that

$$\alpha_l=\frac{Nm}{\rho v_l\tau}\left[\frac{1}{\eta}-1-\frac{|a|^2}{3}\right]. \quad (64)$$

For $\omega_c\tau\gg 1$,

$$\alpha_l=\frac{Nm}{\rho v_l\tau}\left[\frac{a^2}{15}-\frac{(1+\frac{2}{5}a^2)(1-\frac{1}{5}a^2)}{(\omega_c\tau)^2}\right]. \quad (65)$$

Equation (65) tells us that the attenuation of a longitudinal wave increases asymptotically to a constant value as $(\omega_c\tau)$ increases. When $\mathbf{H}_0=0$,

$$\alpha_l(0)=\frac{Nm}{\rho v_l\tau}\left[\frac{a^2\tan^{-1}a}{3(a-\tan^{-1}a)}-1\right], \quad (66)$$

which again is in agreement with the result given in reference 9. Once more we give the expressions for the limiting cases of $a\ll 1$ and $a\gg 1$. They are

$$\alpha_l(0)=\frac{4Nmv_0^2\omega^2\tau}{15\rho v_l^3}\left[1-\frac{9}{35}a^2+\frac{23}{175}a^4-\cdots\right] \quad (67)$$

for $a\ll 1$, and

$$\alpha_l(0)=\pi Nmv_0\omega/6\rho v_l^2 \quad (68)$$

for $a\gg 1$.

V. ATTENUATION OF ULTRASONIC WAVES IN A LONGITUDINAL MAGNETIC FIELD

The attenuation of acoustical waves in the presence of a magnetic field $\mathbf{H}_0$ pointing in the direction of propagation of the wave has been considered by Kjeldaas.[12] Let us consider a wave propagating in the z direction with the particle velocity $\mathbf{u}$ in an arbitrary direction. The case of a transverse wave is particularly interesting as we shall see later. If we consider the portion of the Fermi surface corresponding to electrons having a component of velocity $v_0\cos\theta$ in the z direction, then if $\mathbf{H}_0$ is such that

$$\omega_c=\omega\pm qv_0\cos\theta, \quad (69)$$

we find an absorption that is very similar to that obtained in cyclotron resonance. In fact, the electrons travelling with velocity $\pm v_0\cos\theta$ along the z axis will experience an electric field having a frequency $\nu'=\nu[1\pm(v_0/v_s)\cos\theta]$. Then if the field $\mathbf{H}_0$ is such that (69) holds, the electrons we are concerned with will observe a field $\mathbf{E}$ which is always in the same direction. Therefore, an absorption of energy results.

To develop the theory quantitatively, we start from

[12] T. Kjeldaas, Jr., Bull. Am. Phys. Soc. Ser. II, 3, 180 (1958).

Eqs. (18) and (20). These equations become

$$\gamma=\frac{1+i\omega\tau-iql\cos\theta}{\omega_c\tau}, \tag{70}$$

$$J_\pm=\tfrac{3}{4}\sigma\mathcal{E}_\pm\int_0^\pi\frac{d\theta\sin^3\theta}{1+i\omega\tau-iql\cos\theta\mp i\omega_c\tau}, \tag{71}$$

where $\mathcal{E}$ is defined as in (28) and

$$J_\pm=J_x\pm iJ_y, \tag{72}$$

with a similar expression for $\mathcal{E}_\pm$. The z component of the current density J_z turns out to be completely independent of the presence of the magnetic field. The attenuation in this case, is simply that given in Eq. (66). It only remains to consider the transverse waves. From (71) we get

$$J_\pm=\frac{3\sigma\mathcal{E}_\pm}{4(1+i\omega\tau\mp i\omega_c\tau)}\int_0^\pi\frac{d\theta\sin^3\theta}{1-ia_\pm\cos\theta}, \tag{73}$$

with $a_\pm$ defined as

$$a_\pm=\frac{ql}{1+i(\omega\mp\omega_c)\tau}. \tag{74}$$

Performing the integration in Eq. (73), we find

$$J_\pm=\frac{\sigma\mathcal{E}_\pm}{1+i\tau(\omega\mp\omega_c)}\frac{3}{2a_\pm^2}\left\{\frac{a_\pm^2+1}{a_\pm}\tan^{-1}a_\pm-1\right\}. \tag{75}$$

The attenuation is now calculated as before.

A more detailed analysis of the situation encountered here is desirable. However, it turns out to be no simple matter to get an expression for the attenuation for the interesting range of values of the parameters ql and $\omega_c\tau$. As we have stated before, electroacoustic resonances should occur for electrons satisfying (69). The important values of the parameters are $ql\gtrsim1$ and $\omega_c\tau\sim1$. An order of magnitude estimate of the effect is given in the next section.

VI. DISCUSSION

The present theory of the attenuation of acoustic waves is unsatisfactory from the experimental point of view. In fact, the behavior both at very large fields ($\omega_c\tau\gg1$) and in the region where maxima and minima of the attenuation as a function of $|\mathbf{H}_0|$ occur, is not obtained on this model. The attenuation for large fields in a transverse wave such that $\mathbf{u}_0$ is parallel to $\mathbf{H}_0$ ($\mathbf{q}$ being perpendicular to $\mathbf{u}_0$) decreases inversely as the square of $|\mathbf{H}_0|$ while the experimental result shows a decrease as $|\mathbf{H}_0|^{-1}$. This result probably arises from the limitations in the validity of the Boltzmann transport theory for large magnetic fields, and is also encountered in other galvanomagnetic phenomena such as magnetoresistance.[13] The failure of the theory to account for the oscillations in α before decreasing with $|\mathbf{H}_0|$ is a more serious one from our standpoint. We have discussed the reason why our treatment fails to show the oscillations in Sec. I. Similar difficulties arise when $\mathbf{u}_0$ is perpendicular to both $\mathbf{q}$ and $\mathbf{H}_0$, with $\mathbf{u}_0$ again normal to $\mathbf{q}$. Here, for large fields, the attenuation increases indefinitely as $|\mathbf{H}_0|^2$. The difference in the large magnetic field behavior for the two possible directions of polarization of the transverse wave can easily be understood on the electron gas viscosity picture. In fact, in the case of $\mathbf{H}_0$ parallel to $\mathbf{u}_0$ we expect the attenuation to decrease with increasing $|\mathbf{H}_0|$ because the presence of such a field decreases the viscosity of the electron gas. The electron gas viscosity decreases in this case because $|\mathbf{H}_0|$ makes the electrons less effective in the transfer of momentum in the direction of propagation of the wave. When $\mathbf{u}_0$ is perpendicular to $\mathbf{H}_0$, for large $|\mathbf{H}_0|$ the electrons which are accelerated in the direction of $\mathbf{u}_0$ are immediately bent by the magnetic field so that their direction of motion is reversed. This produces large shear stresses in the electron gas and therefore a large attenuation results. This process creates a drift of the electronic cloud in the direction of propagation of the wave. This drift has the periodicity of $\mathbf{u}$. If the drift were stopped by a Hall field, for example, then the attenuation would decrease with $\omega_c\tau$ for large fields. The experimental result in polycrystalline copper[3] is that the attenuation decreases with $|\mathbf{H}_0|$. We believe this effect to be probably caused by Hall fields that can be maintained inside the metal if the size of the grains is smaller than the ultrasonic wavelength.

The most interesting case is, perhaps, the one considered in Sec. V. Here we have an absorption that can, with justice, be called cyclotron resonance absorption. Let us assume that we have a circularly polarized transverse ultrasonic wave. The effect of the presence of such a wave will be to set up a screw of radial electric fields in the metal. The pitch of the screw is the wavelength λ of the acoustic wave. The system of fields as a whole will be travelling along the direction of propagation of the wave with the velocity of sound. The electrons, in the presence of the external magnetic field, move in helical paths the pitch of which depends on the region of the Fermi surface in which they are. If the electrons turn in the same sense as the sense of polarization of the wave, and they have a component of velocity $v_0\cos\theta_0$ along the direction of propagation of the wave such that in one cyclotron period of time the electrons travel a distance λ, i.e., if

$$(2\pi/\omega_c)v_0\cos\theta_0=\lambda, \tag{76}$$

then the electrons will experience a field which accelerates them away from the axis of the helix in much the same way as in cyclotron resonance absorption. We observe that we can separate the contributions to the attenuation coming from carriers with charges of

[13] R. G. Chambers, Proc. Roy. Soc. (London) **A238**, 344 (1957).

different sign by using circularly polarized acoustic waves with the planes of polarization rotating in different directions. To give a detailed account of the ultrasonic attenuation in this case, we need to express (75) in terms of simple functions of the parameters ql and $\omega_c\tau$. This is not easy and we prefer to give a rather crude calculation of the effect.

From Eq. (71) we see that the electrons which give rise to the cyclotron resonance are those having a polar angle θ_0 in the Fermi surface such that

$$\cos\theta_0=(\omega\pm\omega_c)/qv_0\approx\mp\omega_c\tau/ql. \tag{77}$$

We see that in order to obtain a resonance we must have $(\omega_c\tau/ql)\leqq 1$. For fields such that $1<ql\ll\omega_c\tau\ll(q_0/q)^2$ the attenuation becomes independent of the field, but when $\omega_c\tau\gg(q_0/q)^2$ the attenuation decreases as $(\omega_c\tau)^{-1}$. When $\omega_c\tau\leqq ql$ there will always be a portion of the Fermi surface around the polar angle θ_0 which will give an extra power absorption. The number of electrons that contribute to the power absorption depends on the relaxation time. If the relaxation time is very large, few of the electrons remain in phase with the electric field. The criterion to decide how many electrons contribute to the cyclotron absorption is the following. All those electrons with polar angle θ on the Fermi surface such that they do not get out of phase with $\mathbf{E}$ by more than $\lambda/2$ in their mean free path will contribute to the power absorption. Therefore, the range in θ for the electrons giving cyclotron resonance is defined by

$$|v_0\tau\cos\theta-v_0\tau\cos\theta_0|<\lambda/2. \tag{78}$$

If $ql\gg 1$ this range $\Delta\theta$ of θ satisfying (78) is approximately given by

$$\Delta\theta=2\pi/(ql\sin\theta_0). \tag{79}$$

The number of electrons contributing to the effect is proportional to the portion of Fermi surface in between the angles $\theta_0\pm\frac{1}{2}\Delta\theta$, i.e., proportional to $\frac{1}{2}\sin\theta_0\Delta\theta$. Thus, the change in attenuation from the value in the absence of cyclotron resonance is roughly

$$\Delta\alpha=\frac{Nm}{\tau\rho v_s}\frac{1}{\zeta}\left(\frac{1}{\zeta}-1\right)\frac{\pi}{ql}, \tag{80}$$

where ζ is defined in (45). We see that $\Delta\alpha$ is inversely proportional to both ω and l, as we expect. In fact, the larger l is, the fewer is the number of electrons that contribute to the absorption. Also, if ω is very large the screw of fields $\mathbf{E}$ we have considered can, in a relatively short time, get out of phase with the resonant electrons.

According to the argument presented above, if we change the magnetic field slightly the attenuation α will remain practically unchanged. However, if we are in the region where $\omega_c\approx qv_0$ an increase in the magnetic field will obliterate the increase in the attenuation. An experiment of ultrasonic attenuation in a longitudinal magnetic field may, thus, give information concerning the extent of the Fermi surface in the different directions. For a successful application of this technique single crystals should be used. However, we do not believe that the distribution of effective masses for different sections of the Fermi surface can be resolved by an experiment based on this geometry.

VII. ACKNOWLEDGMENTS

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MATHEMATICAL APPENDIX

We shall briefly indicate here how Eqs. (33)–(36) were obtained from (29)–(32). Let us consider σ_{zz}, which is the simplest example. We are interested in expanding the function

$$F(a,\gamma)=\tfrac{3}{4}\sum_{r=0}^{\infty}\frac{(-1)^r a^{2r}}{(r+\frac{1}{2})(r+\frac{3}{2})}\prod_{n=0}^{r}\frac{1}{1+(n\gamma^{-1})^2} \tag{A1}$$

in a power series of a^{-1}. The product in the right-hand side of (A1) can be expressed in terms of Γ functions.[14] Then we have

$$F(a,\gamma)=\frac{3\pi\gamma}{4\sinh(\pi\gamma)}\sum_{r=0}^{\infty}\frac{(-1)^r t^{2r}}{(r+\frac{1}{2})(r+\frac{3}{2})}\times\frac{1}{\Gamma(i\gamma+r+1)\Gamma(-i\gamma+r+1)}, \tag{A2}$$

with $t=a\gamma$. Consider now the function of the complex variable z defined by[15]

$$P(z)=\frac{\pi t^{2z}}{(z+\frac{1}{2})(z+\frac{3}{2})}\frac{1}{\Gamma(i\gamma+z+1)\Gamma(-i\gamma+z+1)\sin\pi z}. \tag{A3}$$

This function is analytic except at the poles at $z=0$, ±1, ±2, $\cdots$ and $z=-\frac{1}{2}$, $-\frac{3}{2}$. Take now the integral $\int_{-\infty}^{(0+)}P(z)dz$, where the contour of integration is a curve that runs from $-\infty$ to 0 below the real axis, turns around the origin, and goes back to $-\infty$ above the real axis. The integral of $P(z)$ along a circle of infinite radius

[14] E. T. Copson, *An Introduction to the Theory of Functions of a Complex Variable* (Oxford University Press, London, 1935), pp. 205–232.

[15] The consideration of a function of this type was suggested to the author by G. Dresselhaus.

and centered at the origin, which is complete except for a small angle around $|\arg z|=\pi$ gives zero (see reference 14, p. 219). This shows that

$$-\frac{1}{2\pi i}\int_{-\infty}^{(0+)}P(z)dz=\frac{4\sinh(\pi\gamma)}{3\pi\gamma}F(a,\gamma)-\frac{4}{3}\frac{\sinh(\pi\gamma)}{\pi\gamma}. \quad \text{(A4)}$$

The integral on the left can be performed directly. We get

$$F(a,\gamma)=\frac{3\pi}{4a}\coth(\pi\gamma)+\frac{3\pi}{4a^3}\left(1+\frac{1}{4\gamma^2}\right)\coth(\pi\gamma)$$
$$+3\sum_{r=0}^{\infty}\frac{(-1)^r a^{-2r-2}}{(2r+1)(2r-1)}\prod_{n=0}^{r}[1+(n\gamma^{-1})^2]. \quad \text{(A5)}$$

This is the result we needed to prove (36).

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Superconductivity and Ferromagnetism in Isomorphous Compounds

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Isomorphous germanides of some of the rare earth metals are observed to become either ferromagnetic or superconducting. It is concluded that there is a close relationship between the two phenomena.

SINCE the discovery of ferromagnetism in $ZrZn_2$,[1] it has become increasingly more likely that the relation between superconductivity and ferromagnetism may be a much closer one than had until now been anticipated. We have recently found a number of isomorphous compounds which are either superconducting of ferromagnetic and which illustrate this point of view further. These compounds are being formed between elements of the third column of the periodic system and either germanium or silicon. Superconductivity or ferromagnetism was discovered in the germanides listed in Table I. In Table II measurements of the corresponding silicides are reported.

From Table II it becomes evident that if any of the silicides aside from $PrSi_2$ should ever become superconducting or ferromagnetic, this could only happen at a much lower temperature than the corresponding germanides. From this, one might ask whether an electronic configuration favorable to superconductivity is also favorable to ferromagnetism?

We would like to thank Professor F. H. Spedding for the scandium metal.

TABLE I. Transition temperatures of rare earth germanides.

		Transition temperature	Crystal structure
$ScGe_2$	superconducting	1.30°–1.31°K	?
YGe_2	superconducting	3.8°K	tetragonal $ThSi_2$
$LaGe_2$	superconducting	1.49°K	orthorhombically distorted $ThSi_2$
		Curie point	
$CeGe_2$	ferromagnetic	~4.5°K	orthorhombically distorted $ThSi_2$
$PrGe_2$	ferromagnetic	19°K	tetragonal $ThSi_2$
$NdGe_2$	ferromagnetic	3.6°K	tetragonal $ThSi_2$

TABLE II. Transition temperatures of rare earth silicides.

		Crystal structure
$ScSi_2$	normal above 1°K	?
YSi_2	normal above 1°K	orthorhombically distorted $ThSi_2$
$LaSi_2$	normal above 1°K	tetragonal $ThSi_2$
$CeSi_2$	normal above 1°K	tetragonal $ThSi_2$
$PrSi_2$	Ferromagnetic curie point at 10.5°K	tetragonal $ThSi_2$
$NdSi_2$	normal above 1°K	tetragonal $ThSi_2$

[1] B. T. Matthias and R. M. Bozorth, Phys. Rev. **109**, 604 (1958).