Relative Parity of Charged and Neutral *K*-Particles

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Neither from the experimental nor from a theoretical point of view is it obvious that the relative parity $p(K)$ of charged and neutral K-particles is even. This paper is devoted to a study of the case of odd $p(K)$. It is shown (Sec. II) that in the approximation $\delta\equiv(M_\Sigma-M_\Lambda)/M_\Lambda=0$ the π-nucleon phenomena are charge independent provided certain coupling-constant relations are satisfied. The strong Λ-neutron and Λ-proton forces are no longer equal, but if π-interaction between Λ and nucleon predominates this equality is still nearly true. Σ- and K-mass differences may occur which are not mediated by the electromagnetic field. The effects of finite δ on π-nucleon charge independence deviations are explored in Sec. III. Charge symmetry is also violated. These deviations appear not to be disturbing. It is shown that the possibility exists of a negative contribution of a new kind to the proton-neutron mass difference. π-baryon and K-baryon couplings alone would lead to contradictions with experiment. These can be overcome by introducing a $KK\pi$-coupling; for odd $p(K)$ this is compatible with parity conservation (Sec. IV). A qualitative explanation of K-exchange scattering is given which is not unsatisfactory. The consequences for elastic K-scattering are discussed. For ΣK-production in π-nucleon collisions the possibility exists of a violation of the triangle inequalities. There appears to be a good promise for understanding the forward (backward) peaking of charged (neutral) hyperons in associated production. It is pointed out that the K-pair effects due to $KK\pi$-interaction lead to further violations of charge independence. The main effect anticipated is for π-nucleon S-scattering. The influence of a lack of charge symmetry on the nucleon-charge distribution is commented on. Section V deals with the symmetries of the strong interactions classified in terms of the four-dimensional real orthogonal group. Full four-invariance is broken by parity. It is conjectured that each symmetry class is characterized by one universal coupling constant. In Sec. VI the main points of direct experimental interest are summarized.

I. INTRODUCTION

THEORETICAL attempts toward an understanding of the strong baryon-meson[1] interaction phenomena currently are mainly guided by the following two ideas:

(1) The extension of the charge independence property of the strong π-nucleon coupling to all strong couplings. This seems a plausible procedure for two reasons: First, it is indicated by the typical multiplet structure of the mass spectrum of the particles concerned. Secondly, the couplings involving the new unstable particles could not appreciably violate charge independence without likewise affecting the charge-independence of the π-nucleon subsystem itself.

Here it should be remembered that the point of departure, the charge independence of π-nucleon phenomena, needs further experimental exploration. Both in the nuclear force problem and in such phenomena as π-scattering and production[2] the verification of or consistency with charge independence is known so far only up to moderately high frequencies. It is not known whether, apart from the effects due to electromagnetic and weak interactions, charge independence is a strict property at all energies.[3]

(2) The realization that the isotopic spin quantum numbers alone are inadequate to account for properties such as the stability of the new particles. At least one more quantum number is needed. The strangeness number S has so far satisfactorily accounted for the phenomena in question.

The requirements of charge independence, that is I-invariance, and of S-conservation impose important restrictions on the general form of the couplings. Even so, there remains considerable arbitrariness. For example, if one considers all couplings that are bilinear in the baryons and linear in the mesons (which in itself is a restriction on interactions that is made for the sake of simplicity of the argument, and not by any means a necessary requirement, for all we know), one gets involved with a set of eight coupling constants whose relative magnitude in general remains free.

Several proposals have been made to impose conditions on the theory which further restrict this freedom.[4] To explore these questions in a systematic way it seems indicated the make, to begin with, further restrictions of a minimal type.

An attempt in this direction was made recently[5] which starts from the observation that to our knowledge the smallest dimensionless parameter characteristic of the baryon meson system in its strong manifestations is the relative Σ, Λ mass difference

$$\delta=(M_\Sigma-M_\Lambda)/M_\Lambda. \tag{1}$$

As $\delta\ll1$, an approximation suggests itself in which one neglects in first instance the effect of finite δ. It turned out that in this situation a set of assumptions could directly be confronted with experiment. As we shall have much occasion to return to these assumptions they will be repeated here.

(a) Σ and Λ have the same spin and the (Σ,Λ) parity is even.

(b) Charge independence in the conventional sense holds true.

(c) The presently known baryon spectrum is complete.

[1] π- and K-particles shall collectively be denoted as mesons.

[2] See, e.g., R. Hildebrand, Phys. Rev. **89**, 1090 (1953); K. Bandtel *et al.*, Phys. Rev. **106**, 802 (1957).

[3] See also J. Sakurai, Phys. Rev. **107**, 1119 (1957).

[4] M. Gell-Mann, Phys. Rev. **106**, 1296 (1957); J. Schwinger, Phys. Rev. **104**, 1164 (1957); Ann. Phys. **2**, 407 (1957).

[5] A. Pais, Phys. Rev. **110**, 574 (1958), quoted here as A.

(d) The corresponding strong Σ- and Λ-couplings are equal in strength. That is to say, the $[\Sigma\Sigma\pi]$ and $[\Sigma\Lambda\pi]$ coupling constants are equal; likewise for the $[\Lambda NK]$, $[\Sigma NK]$ couplings, and also for the $[\Lambda\Xi K]$, $[\Sigma\Xi K]$ couplings. In the notation of A which will be followed throughout in this paper, these equalities were written as[6]

$$G_2=G_3;\quad F_1=F_2;\quad F_3=F_4. \tag{2}$$

It was shown that the conditions (a)–(d) are incompatible with experiment. This means, for example, that under the conditions (a)–(c) it is impossible to have only one universal π-coupling constant and one universal K-coupling constant. Subsequently, a more exhaustive result was obtained[7] based on the conditions (a)–(c) only. It was found that it is impossible to obtain any result stronger than is implied by I-invariance and S-conservation by the mere use of any postulated relations between the baryon-meson coupling constants other than Eq. (2).

Thus the situation is now the following. If the statements (a)–(c) (which are all in principle open to experimental verification) are correct then the whole strong interaction problem, and in particular the question of the relative magnitude of the strong coupling constants can from now on only be approached by more detailed dynamical methods. This is a none too pleasing prospect if we remember how limited our present theoretical knowledge is about the π-nucleon subsystem with its single strong coupling constant. However this may be, it would seem useful in the present state of affairs to ask to what extent the conditions (a)–(c) themselves are to be considered as definite statements of fact.

There is now good evidence[8] for spin $\frac{1}{2}$ for Λ and Σ and we shall take this spin value for granted in what follows. The (Σ,Λ)-parity has not yet been determined.[9] In the present paper we shall continue to assume that this parity is even. This is a necessary condition for the subsequent considerations to be of possible physical interest.[10]

As to the completeness of the baryon spectrum, it seems idle to speculate about this point. Suffice it to say that the completeness is not a necessary condition for the validity of the main ideas developed below. (We shall assume the existence of the as yet undiscovered Ξ^0.)

Next we come to condition (b) the discussion of which is the principal purpose of this paper. Let us first state the main contents of I-invariance in the way it is commonly understood at present.

(α) The isotopic spin assignments are[11]: $\frac{1}{2}$ for nucleon and Ξ, 0 for Λ, 1 for Σ, 1 for π, $\frac{1}{2}$ for K.

(β) Particles belonging to the same multiplet have equivalent space time properties like spin and parity, to the extent that the latter quantity is physically definable.

We now look at these statements more closely and, to begin with, do so in the approximation $\delta=0$. (Of course we also neglect at this stage the mass differences within multiplets.) If the relations (2) hold true, one has an equivalent description of all baryons as I-doublets, the pairings being[12]

$$N_1=\begin{pmatrix} p \\ n \end{pmatrix},\quad N_2=\begin{pmatrix} \Sigma^+ \\ Y^0 \end{pmatrix},$$

$$N_3=\begin{pmatrix} Z^0 \\ \Sigma^- \end{pmatrix},\quad N_4=\begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix}, \tag{3}$$

where

$$\begin{aligned} Y^0&=2^{-\frac{1}{2}}(\Lambda^0-\Sigma^0),\\ Z^0&=2^{-\frac{1}{2}}(\Lambda^0+\Sigma^0). \end{aligned} \tag{4}$$

This case will from now on be called the doublet approximation. As was further noted in A, the K-particles are now described in terms of two I-singlets, one for $K^0(\bar{K}^0)$, and one for $K^\pm$. The pions continue to be described as an I-triplet. (The formal aspects of this equivalent description will be discussed more fully in Sec. V.) The existence of the doublet approximation is independent of the Ξ spin, of the Ξ-nucleon parity and of the K-parity, the latter being defined relative to the Λ-nucleon system. In what follows we assume Ξ-spin $\frac{1}{2}$, K-spin zero. In Sec. II the following is shown.

Let the effect of finite δ in first instance be neglected. Then if and only if all the conditions (2) are satisfied, all π-nucleon phenomena are charge independent in the usual way, not only if the parity of charged relative to neutral K-particles is even but also if it is odd.

Here the case of odd parity constitutes a new possibility. The fact that this can at all be considered is, of course, connected with the two-singlet description of the K-multiplet. For ease of writing we shall from now on use the symbol $P(K^+)$ to denote the parity of K^+ relative to Λ-nucleon, while $p(K)$ shall mean the parity of K^+ relative to K^0.

At this stage we may state the purpose of this paper more precisely: it is not to propose that $p(K)$ is necessarily odd, but rather to point out that one should not take for granted that it is even. It should be noted that

[6] As was noted in A the same conclusions can be reached by putting $G_2=-G_3$; $F_1=-F_2$; $F_3=-F_4$. This trivial alternative which follows from the possibility of redefining the phase of the Λ-wave function modulo 180° will not be explicitly mentioned in the following.

[7] A. Pais, Phys. Rev. **110**, 1480 (1958).

[8] See F. Eisler *et al.*, Nuovo cimento **7**, 222 (1958); also T. D. Lee and C. N. Yang, Phys. Rev. **109**, 1755 (1958).

[9] See A. Pais and S. B. Treiman, Phys. Rev. **109**, 1759 (1958); also G. Feinberg, Phys. Rev. **109**, 1019 (1958); G. Feldman and T. Fulton (to be published).

[10] For odd (Σ,Λ) parity it is impossible to define the doublet approximation of Sec. II.

[11] See, however, J. Tiomno, Nuovo cimento **6**, 69 (1957).

[12] See A, Eqs. (12)–(15) and various earlier papers like d'Espagnat, Prentki, and Salam, Nuclear Phys. **3**, 446 (1957); M. Gell-Mann reference 4; J. Tiomno, reference 11.

if $p(K)$ would turn out to be odd, there would be a strong presumption as to the existence of a single universal π-baryon and a single universal K-baryon coupling constant.

But now immediately a number of questions arise:

(1) What do we know about $P(K^+)$ and $p(K)$?
(2) What are the further consequences of odd $p(K)$?
(3) What are the effects of finite δ?
(4) Why should the charged and neutral K-particles be nearly mass degenerate if they have opposite parity?

The answer to the first question is that we know very little. Perhaps the best proposal to determine $P(K^+)$ is Dalitz's suggestion[13] on the K^- absorption in He^4. Statements about the dependence of threshold behavior on $P(K^+)$ are impossible to establish without additional assumptions.[14] As to $p(K)$ the most interesting reaction is, of course, the exchange scattering

$$K^+ + n \rightarrow K^0 + p, \tag{5}$$

which will be discussed in some detail in Secs. II and IV. If one believes perturbation theory, photoproduction of K particles perhaps indicates that $P(K^+)$ is even, but even in this approximation the situation is obscured by the possibility of anomalous magnetic moment contributions.[15] We come back to these questions in Sec. IV. For the moment let us record, however, that there is no conclusive experimental indication that $p(K)$ has to be even. The rest of this paper is devoted to a discussion of the case of odd $p(K)$.

Further consequences of odd $p(K)$ are discussed in Sec. II. It is shown that in this case the Λ-neutron force is not rigorously equal to the Λ-proton force. Similar conclusions are reached concerning Σ-nucleon forces. However, the deviations from the customary relations are relatively small as long as π-exchange is the main mechanism for hyperon-nucleon interaction. For the following processes amplitude relations are established in the doublet approximation: hyperon production in π-nucleon collisions and in γ-nucleon collisions; K-nucleon scattering; K^--absorption. It is most essential to note that some of these relations disagree with experiment; this point is discussed further in Sec. IV. It is seen that mass differences may occur in the Σ-multiplet as well as in the K-multiplet which are not mediated by the electromagnetic field.

Section III is devoted to an orienting discussion of the effects due to finite δ. This leads to deviations from charge independence in the π-nucleon system. Estimates indicate that these deviations need not be disturbing, at least at not too high energies. The decisive factors for this are the very short-range character of K-effects in π-nucleon physics combined with the circumstance that the K-couplings are not necessarily as strong as the π-couplings. It is shown that for suitable $P(K^+)$ a negative contribution to the proton-neutron mass difference must be anticipated.

In Sec. IV the problem is taken up again of the inacceptable amplitude relations which were established in Sec. II on the basis of a strong interaction dynamics which involves only π-baryon and K-baryon couplings. It is noted that for odd $p(K)$ a $KK\pi$ coupling of the type

$$\bar{K}^+ K^0 \pi^+ + \bar{K}^0 K^+ \pi^- \tag{6}$$

may be envisaged as compatible with parity conservation in strong interactions, a principle to which we adhere in this paper. One shows that (6) breaks those symmetries (separate S_1, S_2-conservation, see A) which lead to the difficulties just mentioned. Moreover, it is observed that the couplings considered prior to the introduction of the interaction (6) would lead to a γ-stable Σ^0, whereas this new interaction allows for $\Sigma^0 \rightarrow \Lambda + \gamma$. It is attempted to estimate the corresponding coupling constant f (with dimensions of a charge) from K^+-exchange scattering. This yields $f \sim e$ if it is assumed that the π-nucleon vertex is not appreciably damped. It is further noted that the interaction (6) leads to consequences for elastic K-scattering and for associated production in π-nucleon collisions which are in qualitative agreement with experiment. It is observed that, according to the views presented here, there exists the possibility of violating the triangle relations for associated ΣK production. Similar violations may occur in other instances. A further study of the associated production reactions, which goes in part along the lines originally suggested by Goldhaber,[16] shows that $P(K^+)$ more probably than not is even. The interaction (6) leads here to interesting consequences for the angular distribution. It should be strongly emphasized, however, that while a number of results obtained in Sec. IV make the case of odd $p(K)$ seem to be an attractive possibility, all conclusions are quite tenuous because of the general lack of reliable methods of computation where strong interactions are involved.

It is, of course, essential to verify whether or not the somewhat unfamiliar looking interaction (6) leads to any untoward consequences for the charge independence of π-nucleon phenomena. It is noted that here we meet on the one hand finite δ-effects, discussed before in Sec. III from a more phenomenological point of view. On the other hand, new effects appear which are not δ-proportional and which involve virtual K-pairs. Due to the circumstance that the interaction (6) leads to a

[13] R. Dalitz, *Proceedings of the Sixth Annual Rochester Conference on High-Energy Physics* (Interscience Publishers, Inc., New York, 1956), Sec. V. See also a recent paper by R. Dalitz and B. Downs, Phys. Rev. **111**, 967 (1958), Sec. 3 and Appendix D. We assume the validity of charge-conjugation invariance in strong interactions.

[14] See for example G. Costa and B. T. Feld, Phys. Rev. **109**, 606 (1958).

[15] A. Silverman *et al.*, Phys. Rev. **108**, 501 (1957); M. Kawaguchi and M. J. Moravcsik, Phys. Rev. **107**, 563 (1957); A. Fujii and R. E. Marshak, Phys. Rev. **107**, 571 (1957).

[16] M. Goldhaber, Phys. Rev. **101**, 433 (1956). See also S. Barshay, Phys. Rev. **104**, 851 (1956); J. Schwinger, Phys. Rev. **104**, 1164 (1956); J. Sakurai, Nuovo cimento **5**, 1340 (1957).

probability of π-creation which in the nonrelativistic limit is of order $e(2\omega)^{-\frac{1}{2}}$, it would seem that all these effects are of the general order anticipated from the standard electromagnetic violations of charge independence. In low-energy π-nucleon physics the consequences of a conceivable interaction of the type (6) would probably make themselves most clearly felt in π-nucleon S-wave scattering.

Section V deals with the more formal aspects of the problem of symmetries for the case of odd $p(K)$. For the present, a description in terms of the symmetry classes of the four-dimensional rotation group appears to be a most simple possibility. The π-baryon couplings have the full symmetry. The K-baryon interactions are of such a nature that parity breaks this full symmetry. The doublet nature of the (K^+,K^0) states, and of their charge conjugates, reappears in a certain sense. A relatively small mass difference between K^+ and K^0 would not be too unnatural if the K-baryon coupling strength is not too large. Actually the value of this mass difference is not too well known experimentally.[17]

In turn, the K-baryon couplings are of a higher symmetry than the electromagnetic and the $KK\pi$-couplings which have closely related, although not identical symmetries. The following conjecture is made (which is, however, far more speculative then the foregoing considerations). All particle interactions fall in a hierarchy of distinct symmetry classes and each symmetry class is characterized by a single universal coupling constant. It should be added that one may perhaps even go further and assume that there exists a simple connection between f and e. If true one would be left with the following fundamental constants: G, F, e, and "the" weak interaction strength. Considering G to be fixed from π-nucleon physics, there would then remain one constant to be determined, namely F, "the" K-baryon constant. The present scheme would become quite tight in this way.

While it may be hard to devise a practicable truly crucial experiment to determine whether $p(K)$ could be odd, it is clear that such a general demand is made on inner consistency that it will be far from hopeless to see whether the possibility that $p(K)$ is odd will stand or fall.

The concluding remarks of Sec. VI are devoted to an enumeration of those effects whose experimental study seems of particular interest in the present context.

II. THE DOUBLET APPROXIMATION

We start from the expressions given in A, Eqs. (12)–(13) for the trilinear π-interactions $[\pi]$ and K-interactions $[K]$, in the approximation $\delta=0$:

$$[\pi]=i[G_1\bar{N}_1\boldsymbol{\tau}\gamma_5N_1+G(\bar{N}_2\boldsymbol{\tau}\gamma_5N_2+\bar{N}_3\boldsymbol{\tau}\gamma_5N_3)+G_4\bar{N}_4\boldsymbol{\tau}\gamma_5N_4]\cdot\boldsymbol{\pi}, \quad (7)$$

where γ_5 symbolizes[18] the pseudoscalar nature of the π-couplings. For the case of even $p(K)$, $[K]$ can be written as

$$[K]=F_I\sqrt{2}[\bar{N}_1\eta N_2K^0+\bar{N}_1\eta N_3K^+]+F_{II}\sqrt{2}[\bar{N}_4\eta' N_2\bar{K}^+-\bar{N}_4\eta' N_3\bar{K}^0]+\text{H.c.}, \quad (8)$$

where

$$\eta=1 \quad\text{or}\quad i\gamma_5;\quad \eta'=1 \quad\text{or}\quad i\gamma_5. \quad (9)$$

$\eta=1$ $(i\gamma_5)$ corresponds to even (odd) $P(K_+)$. And $\eta=\eta'$ if the Ξ-nucleon parity is even. If the latter is odd, one has $\eta=1$, $\eta'=i\gamma_5$ or $\eta=i\gamma_5$, $\eta'=1$.

The usual charge independence of the π-nucleon system finds its formal expression in the invariance of the theory for I-rotations, with the understanding that N_1 and $\boldsymbol{\pi}$ transform like a spinor and a vector, respectively. Imposing this transformation law on Eqs. (7) and (8) we see that I-invariance is guaranteed provided also N_2, N_3, and N_4 transform like spinors while K^0, K^+, and their Hermitian conjugates stay invariant.

For odd $p(K)$ we can write $[K]$ as follows:

$$[K]=F_I\sqrt{2}[\bar{N}_1i\eta\gamma_5N_2K^0+\bar{N}_1\eta N_3K^+]+F_{II}\sqrt{2}[\bar{N}_4\eta' N_2\bar{K}^+-\bar{N}_4i\eta'\gamma_5N_3\bar{K}^0]+\text{H.c.} \quad (10)$$

The rest of this paper is devoted to the study of the combined interactions (7) and (10). Evidently we have again I-invariance of the total interaction by

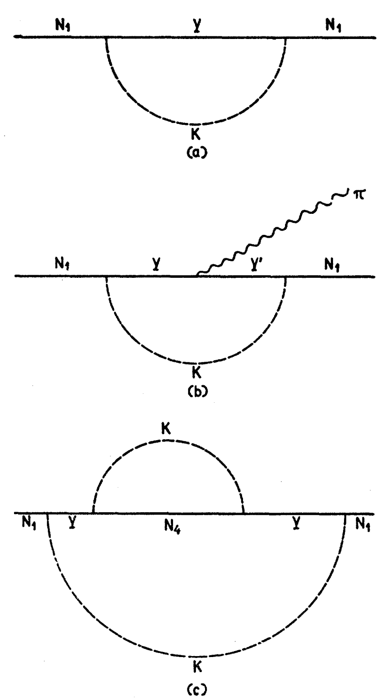

FIG. 1. Nucleon self-energy and π-nucleon vertex corrections due to K-baryon couplings. Y, Y' stands for Λ or Σ-states. For other notations see Eq. (3).

[17] See for example L. W. Alvarez's survey table, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics* (Interscience Publishers, New York, 1957). *Note added in proof*: see, however, W. Barkas and A. Rosenfeld, University of California Radiation Laboratory Report UCRL-8030 (unpublished), for more recent data.

[18] See also A, reference 6. As in A we put $G_2=G_3=G$, $F_1=F_2=F_I$, $F_3=F_4=F_{II}$. H.c. means Hermitian conjugate.

specifying all transformation properties in the same way as we did in connection with the set of interactions (7), (8). This proves one-half of the statement made in Sec. I, namely that for the charge independence of π-nucleon effects to hold true it is sufficient, for odd $p(K)$, that the relations (2) are satisfied.

We next show that these relations are also necessary, again for odd $p(K)$. This is done by a similar method as used previously,[7] namely by asking that some physical requirement be satisfied in a given order in the power series expansion. In the present case one proceeds most easily as follows.

First consider the requirement: proton mass is equal to neutron mass, which is part of the charge independence conditions. Consider, for $\delta=0$, the mass contributions to the second order in the K-couplings. The simple type of graph is given in Fig. 1(a). We shall now not require from the outset that the relations (2) be valid. Thus we now use a more general set of coupling which take the place of A Eqs. (6)–(9):

$$[\Lambda,N_1,K]=F_1[\bar{p}\eta\Lambda K^+ + \bar{n}i\eta\gamma_5\Lambda K^0]+\text{H.c.}, \tag{11}$$

$$[\Sigma,N_1,K]=F_2[\bar{p}\eta\Sigma^0K^+ - \bar{n}i\eta\gamma_5\Sigma^0K^0 + 2^{\frac{1}{2}}\bar{n}\eta\Sigma^-K^+ + 2^{\frac{1}{2}}\bar{p}i\eta\gamma_5\Sigma^+K^0]+\text{H.c.}, \tag{12}$$

$$[N_4,\Lambda,K]=F_3[\bar{\Xi}^-\eta'\Lambda\bar{K}^+ - \bar{\Xi}^0i\eta'\gamma_5\Lambda\bar{K}^0]+\text{H.c.}, \tag{13}$$

$$[N_4,\Sigma,K]=F_4[-\bar{\Xi}^-\eta'\Sigma^0\bar{K}^+ - \bar{\Xi}^0i\eta'\gamma_5\Sigma^0\bar{K}^0 + 2^{\frac{1}{2}}\bar{\Xi}^0\eta'\Sigma^+\bar{K}^+ - 2^{\frac{1}{2}}\bar{\Xi}^-i\eta'\gamma_5\Sigma^-\bar{K}^0]+\text{H.c.} \tag{14}$$

The condition of mass equality then is

$$(F_1^2-F_2^2)(a_1-a_2)=0, \tag{15}$$

where the terms proportional to $a_1(a_2)$ are those involving a virtual $K^+(K^0)$. The general form of Eq. (15) also holds true for even $p(K)$, but there we have $a_1=a_2$ and (15) is an identity giving us no information about coupling constants, as it should be.[19] For odd $p(K)$, $a_1\neq a_2$ and we have

$$F_1^2=F_2^2. \tag{16}$$

Consider next the second order K-corrections to the π-nucleon vertex; the graph is drawn in Fig. 1(b). Charge independence requires that the correction to the $(\bar{p}p\pi^0)$-vertex is equal and opposite to that[20] for $(\bar{n}n\pi^0)$. This condition yields

$$F_2(F_1G_2-F_2G_3)(b_1-b_2)=0, \tag{17}$$

where the $b_1(b_2)$-terms correspond to $K_+(K_0)$ contributions, respectively. Again we have a triviality[19] for even $p(K)$. For odd $p(K)$, where $b_1\neq b_2$, Eqs. (16)–(17) yield[6] $F_1=F_2$, $G_2=G_3$. Thus we have now shown two of the three relations (2) to be necessary.

[19] For even $p(K)$ one obtains, of course, the same result for $\delta\neq 0$.

[20] The discussion of the second order K-corrections to the other π-nucleon vertices yields no new information.

To find the third relation, consider the fourth order mass contributions involving the cascade states N_4. Mass equality yields

$$W_1+W_2=0,$$
$$W_1=(F_1F_3+F_2F_4)(F_1F_3-F_2F_4)(c_1-c_2), \tag{18}$$
$$W_2=(F_1F_3-F_2F_4)^2(c_3-c_4).$$

One will verify that the terms proportional to $c_1\cdots, c_4$ correspond to different pairings of the virtual charged and neutral K-particles. For even $p(K)$ all c's are equal; for odd $p(K)$ they are all distinct. Finally, by studying the corresponding contribution to the $(\bar{p}p\pi^0)$ and $(\bar{n}n\pi^0)$ vertices, one shows that W_1 and W_2 have to be zero separately. This yields the third relation (2) and completes the proof of necessity which has been spelled out at some length to show how the formalism gets tightened, as it were, in the case of odd $p(K)$. This concludes the discussion of π-nucleon charge independence[21] and we now turn, still in the doublet approximation, to the further consequences of odd $p(K)$.

(1) *Λ-nucleon forces.*—The Λn and Λp forces are still equal in as far as they are brought about by pions only.[22] The same is no longer true, however, for the contributions involving K-mesons.

The main argument for assuming (as one currently does) strict equality of the strong Λn and Λp interactions[13] comes from the approximate equality of the binding energies of ${}_\Lambda H^4$ and ${}_\Lambda He^4$. The present values[23] for these quantities are 1.8±0.2 Mev, 2.0±0.3 Mev, respectively. It is therefore not clear whether a difference in binding energy of rough relative magnitude of as much as ten percent is excluded by the data.

In this context it is quite instructive to consider the results of Lichtenberg and Ross[24] about the relative importance of π- and K-forces for the Λ-nucleon interaction. Their analysis, of course, refers to even $p(K)$. On the basis of a static baryon model they conclude that neither for even nor for odd $P(K^+)$ can K-mesons be principally responsible for hyperon-nucleon forces if the main contributions come from K- and $K\pi$-exchange. On the other hand, they find, for universal π-coupling (even $\Sigma\Lambda$-parity, $G_1=G=G_4$), that agree-

[21] The use of relations like (15)–(18) implies perhaps more faith in field theory than is entirely warranted. For example the statement: $a_1\neq a_2$ for odd $p(K)$ refers to quantities whose definition is ambiguous. All one can really say is that any reasonable way of comparison yields the inequalities concerning a, b, c which were used in the argument. In addition it may be noted that the study of more complicated effects like K-contributions to nucleon-nucleon forces leads to the same conclusions.

The logical possibility that charged and neutral K's would have different spin will not be considered in this paper.

[22] In particular single π-emission by a Λ remains forbidden as a strong reaction. See further Dalitz and Downs, reference 13, footnote 19.

[23] Levi-Setti, Slater, and Telegdi, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics* (Interscience Publishers, Inc., New York, 1957), Sec. VIII. See also reference 13.

[24] D. Lichtenberg and M. Ross, Phys. Rev. **107**, 1714 (1957); **109**, 2163 (1958). See also N. Dallaporta and F. Ferrari, Nuovo cimento **5**, 111 (1957).

ment with experiment is possible provided K-exchange effects are small.

For odd $p(K)$, it is, of course, out of the question that K-exchange could be the main mechanism for the Λ-nucleon interaction. On the other hand, a picture of preponderant π-pair interaction between Λ and nucleons with relatively small noncharge symmetric corrections due to K-exchange is not necessarily excluded in the present state of knowledge.

If there exists at all a difference in binding energy between ${}_\Lambda H^4$ and ${}_\Lambda He^4$ which could not be ascribed to electromagnetic effects, one must ask how nuclear configuration affects the manifestation of a possible difference in the two-body Λn- and Λp-interactions. Let ΔV be the nuclear potential energy difference due to different K-coupling in the Λn- *versus* the Λp-system, that is to say

$$\Delta V = V({}_\Lambda H^4) - V({}_\Lambda He^4) \tag{19}$$

(barring electromagnetic effects). It should be emphasized that ΔV depends first, on the angular momentum J of the hypernuclei, and secondly, on $P(K_+)$. Consider for example the contributions to ΔV due to single K-exchange. By Wentzel's method[25] one finds:

$$\begin{aligned} P(K^+)\text{ odd}: \quad \Delta V &= -2(\Lambda n)_0, \quad (J=0), \\ &= -\tfrac{2}{3}(\Lambda p)_0, \quad (J=1); \\ P(K^+)\text{ even}: \quad \Delta V &= 2(\Lambda p)_0, \quad (J=0), \\ &= \tfrac{2}{3}(\Lambda n)_0, \quad (J=1). \end{aligned} \tag{20}$$

Here $(\Lambda n)_0$ stands for the two-body Λn-interaction due to single K-exchange in the singlet state; likewise for $(\Lambda p)_0$. For either $P(K^+)$, the quantities $(\Lambda n)_0$ and $(\Lambda p)_0$ are negative.[25] Thus, independently of the value of J, ${}_\Lambda H^4$ will be more (less) bound than ${}_\Lambda He^4$ if $P(K^+)$ is even (odd).

The results of Eq. (20) are quoted mainly to show what kind of arguments one meets in a study of a possible ΔV-effect. The actual conclusion on the sign of ΔV is dubious for many reasons. First it is questionable whether such a subtle effect can be discussed under the neglect of tensor forces and of core polarization.[13,25] Secondly it may be anticipated that the effects of $K\pi$-exchange, for example, may be comparable to those of K-exchange.[24] Here we shall not go into further detail, and rest content with the above illustration of some of the qualitative aspects of the problem. (See also Sec. IV for further conceivable causes of differences between the Λn- and the Λp-force.)

(2) *Σ-nucleon forces.*[26]—Again, the pure π-interactions remain the same as usual, whereas effects involving K-exchange would be modified. Perhaps the most interesting question is here the relation between (Σ^-,n) and (Σ^+,p) interaction. These two should be the same if charge symmetry is valid, which is not true if $p(K)$ is odd. But if the π-interactions are preponderant, one will still have a nearly charge symmetric situation.

There has been some indication recently that the (Σ^+,p) system has a bound state.[27] If this is so and if charge symmetry holds true, then (Σ^-,n) is certainly bound, as both the absence of Coulomb repulsion and the larger Σ^--mass favor the latter case. From the present point of view, these Σ-nucleon systems are perhaps even more interesting than the Λ-hyperfragments, as here we may more easily obtain information about the question of charge symmetry of the hyperon-nucleon two-body system, on which we touch for odd $p(K)$. The recent discussion of a possible method[27] to obtain (Σ^-,n) fragments, if they exist, remains unaffected as the value of the binding energy was treated as a free parameter.

(3) *Hyperon production in π-nucleon collisions.*—In the doublet approximation this problem can be discussed qualitatively along similar lines as was done in A. In particular it will be evident that the role of the auxiliary quantum numbers S_1, S_2 (see especially A, Table I) remains the same whether $p(K)$ is even or odd. Consider the reactions

$$\pi^- + p \to \Lambda + K^0, \quad (A_\Lambda), \tag{21}$$

$$\pi^- + p \to \Sigma^0 + K^0, \quad (A_0), \tag{22}$$

$$\pi^- + p \to \Sigma^- + K^+, \quad (A_-), \tag{23}$$

$$\pi^+ + p \to \Sigma^+ + K^+, \quad (A_+). \tag{24}$$

The symbol in parentheses denotes the production amplitude. Thus we have, as in A, Eq. (20):

$$A_\Lambda \approx -A_0. \tag{25}$$

From now on the symbol ≈ will be used to mean equality for $\delta = 0$.

For even $p(K)$ it was possible to relate A_- to A_0 by $A_- \approx -2^{\frac{1}{2}} A_0$, see A Eq. (24). However, the reasoning which led to this relation[28] is valid only for even $p(K)$. In fact we can state that for odd $p(K)$, the triangle relations for Σ-production in π-nucleon collisions are not necessarily satisfied for $\delta \neq 0$.

Thus the relation A Eq. (24) which is in disagreement with experiment[29] is here invalidated. However, the theoretical prediction about Σ^+-production causes trouble at this stage. Indeed, the relation A Eq. (26):

$$A_+ \approx 0 \tag{26}$$

still holds true, as it follows from (S_1,S_2) conservation only. We shall come back to this point in Sec. IV, where a possibility is indicated to avoid having $|A_+|^2$ proportional to δ^2, and where the question of angular distribution is taken up in somewhat more detail.

[25] G. Wentzel, Phys. Rev. **101**, 835 (1956).
[26] See also F. Ferrari and L. Fonda, Nuovo cimento **6**, 1027 (1957).
[27] M. Baldo-Ceolin *et al.*, Nuovo cimento **6**, 144 (1957). For theoretical aspects see A. Pais and S. Treiman, Phys. Rev. **107**, 1396 (1957); G. Snow, Phys. Rev. **110**, 1192 (1958).
[28] See A, Sec. II, rule B.
[29] For references, see A, footnote 9.

We conclude the present remarks on production by noting the existence of the following mathematical relation. Let $A^{(\pm)}$ denote a production amplitude for the cases that $P(K^+)$ is $(\pm)$. Then one easily shows that

$$A_-^{(\pm)} \approx -2^{\frac{1}{2}} A_0^{(\mp)}. \tag{27}$$

Evidently this relation is without physical content. It is nevertheless a useful one as it saves labor in the discussion of how the production cross sections depend on $P(K^+)$, see Sec. IV.

(4) *Photoproduction of K-particles.*—For the case of even $p(K)$ it has been pointed out previously[7] that the production amplitudes for $\gamma+p \rightarrow \Lambda + K^+$, Σ^0+K^+ are related by

$$A_\Lambda^\gamma \approx -A_0^\gamma. \tag{28}$$

It is readily seen that Eq. (28) also holds true for odd $p(K)$.

(5) *K-nucleon forces and K-nucleon scattering.*—Clearly the strong (K^+p)- and (K^+n)-interactions are equal and the same is true for the (K^0p)- and (K^0n)-interaction. But the (K^+, nucleon) and (K^0-nucleon) interactions are now distinct and no longer related to each other by the conventional I-rules. If $p(K)$ would turn out to be odd, one must then be cautious in the use of K^--data for the analysis of absorption and regeneration experiments involving long-lived neutral K-particles.[30]

For the present our main concern will be the scattering of K^+ on p and n. In the doublet approximation we have, in obvious notation

$$A_{\text{el}}(K^+, p) \approx A_{\text{el}}(K^+, n), \tag{29}$$

$$A_{\text{exch}}(K^+, n) \approx 0, \tag{30}$$

where Eq. (30) follows, as in A, by the use of (S_1,S_2)-conservation. These relations are true for either choice of $P(K^+)$. They disagree with experiment. At this point we state this without further comment; the problem of K^+-scattering is taken up in more detail in Sec. IV.

(6) *K^--absorption.*—Consider the processes

$$K^-+p \rightarrow \Lambda+\pi^0, \quad (B_\Lambda), \tag{31}$$

$$\Sigma^0+\pi^0, \quad (B_0), \tag{32}$$

$$\Sigma^-+\pi^+, \quad (B_-), \tag{33}$$

$$\Sigma^++\pi^-, \quad (B_+). \tag{34}$$

By means of the methods given in A one can establish relations between the B's for $\delta=0$. Thus for example one has $B_\Lambda \approx B_0$. Such a relation does not seem very trustworthy, as the neglect of δ in comparing Λ- and Σ_0-production is not very meaningful. This is also true for the relation between B_- and B_0 which can be established by using the present isotopic spin assignments:

$$B_- \approx 2B_0. \tag{35}$$

Note further that

$$B_+ \approx 0. \tag{36}$$

This latter result will again be modified by the considerations of Sec. IV.

It has been observed[31] that the experimental determination of the relative rates of production of hyperons by K^--absorption in deuterium and also in He^4 gives information about the validity of charge independence as applied to the new particles. It is easily shown that for $\delta=0$ the same probability ratios are found as in the usual theory.[31] As was stressed above, the doublet approximation is least reliable in these relatively low-energy processes. Thus the study of the relative reaction rates in question becomes also interesting from the point of view of finding out whether or not possible deviations from these ratios could reasonably be ascribed to electromagnetic effects only.

7. *Mass splits between Σ-states and between K-states.*—In the present formulation of the case of odd $p(K)$, the various Σ-states get split by the K-couplings. Thus mass differences may arise which are not mediated by the electromagnetic field. This possibility is perhaps not unwelcome as the Σ^+, Σ^--mass difference is rather large.[17] Also there arises a nonelectromagnetic K^+-K^0 split. The question of these mass differences will be taken up in a little more detail in a subsequent paper.

This concludes the qualitative survey of the doublet approximation. From the remarks made in connection with Eqs. (26), (29), (30), (36) it is evident that the possibility of odd $p(K)$ with its concomitant requirement of validity of Eq. (2) leads to contradictions. In fact, these are some of the same contradictions which led us to question the validity of Eq. (2) for even $p(K)$. But as already mentioned in the introduction, the case of odd $p(K)$ leads to new possibilities for overcoming the mentioned difficulties. This will be discussed in Sec. IV. First, however, we shall go back to the question of π-nucleon charge independence and we shall ask to what extent the conclusions reached at this point must be modified if effects proportional to powers of δ are taken into consideration. Before doing so, it should be emphasized that the developments of Sec. IV will demonstrate that the conclusions of the next section can at best be of a very preliminary character.

III. EFFECTS OF FINITE δ

It will be clear from the foregoing that the finiteness of δ leads to nonelectromagnetic deviations in π-nucleon systems. Estimates of the importance of these effects cannot be made without considerably more commitment to dynamical details than was necessary in the previous considerations. Now, as is well known, it is

[30] See, for example, W. Panofsky *et al.*, Phys. Rev. **109**, 1353 (1958).

[31] T. D. Lee, Phys. Rev. **99**, 337 (1955).

as easy to write down strong interactions as it is hard to evaluate their quantitative implications. This is especially true in the present instance where, due to the massiveness of the K-particles, the effects in question depend sensitively on high virtual frequencies so that baryon recoil and pair effects are of the essence. We are in fact moving into distances comparable to or smaller than the nuclear core radius $[\sim(2m_\pi)^{-1}]$ which is a characteristic length in the present half-phenomenological theories of π-nucleon interaction.[32]

However, the aim of the present section is only a limited one. For the moment we shall be mainly interested in the comparison of effects that violate charge independence due to finite δ on the one hand and due to electromagnetism on the other. We intend to show that the δ-effects do not obviously lead to inadmissibly large violations as compared to the order anticipated from electromagnetic effects only. The circumstance that δ is only twice as large as the relative $\pi^+-\pi^0$ mass difference may perhaps indicate that this is not unreasonable. However this may be, the author does not at all hold it to be excluded that the above qualitative statement may turn out to be wrong. This in itself might then provide an argument against odd $p(K)$.

After having sounded warning, let us now come to some details. For the purpose of illustration we consider the interactions (7) where now we take a concrete pseudoscalar point coupling and where the definitions (3), (4) are reinserted whenever it is necessary to distinguish between Λ and Σ; and the K-interactions Eqs. (11)–(14), likewise with $\delta\neq0$, but with Eqs. (2) supposed to be satisfied. The K-couplings are taken as point scalar or pseudoscalar, as the case may be.

It seems reasonable to consider only effects of the first order in δ. In this approximation there is one simplifying feature which has to do with the dependence of the corrections on $P(K^+)$: observe that, insofar as the interactions (7), (11)–(14) are concerned, the substitutions

$$\eta\rightarrow i\eta\gamma_5,\quad \eta'\rightarrow i\eta'\gamma_5 \qquad (37)$$

that is the transition from even to odd $P(K^+)$, are equivalent to the substitutions[33]

$$\begin{aligned} &N_1\rightarrow\tau_1N_1, && N_4\rightarrow-\tau_1N_4,\\ &N_2\leftrightarrow\tau_1N_3, && K_0\leftrightarrow K_+,\\ &\pi_\pm\rightarrow\pi_\mp, && \pi_0\rightarrow-\pi_0. \end{aligned} \qquad (38)$$

Equation (38) implies in particular that

$$\Lambda\rightarrow\Lambda. \qquad (39)$$

Hence, taking also into account the effect of Eq. (38) on electromagnetic phenomena we have the following conclusion:

To all orders in all strong coupling constants (but ignoring electromagnetic effects), any charge-dependent correction to π-nucleon phenomena for the case: $P(K^+)$ even is equal to the corresponding correction for the isotopically rotated situation given by Eq. (38) for the case of odd $P(K^+)$.

This rather general theorem sheds an interesting light on the proton-neutron mass difference problem which we discuss first.

(1) *Proton-neutron mass difference* $M_{pn}=M_p-M_n$.—It has been suggested[34] that electromagnetic effects alone could conceivably lead to a theoretical value for M_{pn} of the right sign and order of magnitude. Here a perturbation argument was used which is, of course, open to doubt.[35]

Consider now the effect on M_{pn} due to finite δ. It is clear from the theorem just stated that for one choice of $P(K^+)$ the contribution to M_{pn} will be negative (if it does not happen to be zero). Thus if $p(K)$ would turn out to be odd, a future dynamical theory might possible relate the sign of M_{pn} to $P(K^+)$.

As a further illustration, consider the effect on M_{pn} of single K-exchange [see Fig. 1(a)] for even $P(K^+)$, i.e., $\eta=1$. The self-energy difference operator is

$$2iF^2\bar{p}(1)\{M_\Lambda\Delta_\Lambda(12)-M_\Sigma\Delta_\Sigma(12)\}\Delta_K(21)p(2), \quad (40)$$

where Δ is the appropriate Green's function for the particle by which it is labeled. The contribution to M_{pn} is divergent. Using a Feynman cutoff (with mass λ) on the K-propagator, one gets

$$\frac{M_{pn}}{M}=C(\lambda)\frac{\delta}{2\pi}\frac{F_I^2}{4\pi}, \qquad (41)$$

where M is the nucleon mass. Characteristic values of the constant $C(\lambda)$ are: $C(M)=-0.5$; $C(2M)=1.5$. This shows that Eq. (41) could give a contribution to M_{pn} of the experimental order of magnitude, for either choice of $P(K^+)$, with $F_I^2/4\pi$ of general order unity and with a cutoff between M and $2M$. This proves next to nothing. But perhaps it indicates that one need not necessarily expect quite unreasonable contributions to M_{pn} due to this particular way in which charge independence may be violated. A full treatment of this problem would, of course, have to take into account the balance of electromagnetic and of δ-effects. Moreover, it will become clear from the developments in the next section that the present description of the influence of the couplings on M_{pn} may well be a very provisional one.

[32] See, for example, G. Chew, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics* (Interscience Publishers, Inc., New York, 1957), Sec. I.

[33] For the purpose of this section we neglect the mass differences within the Σ-multiplet. Actually, it is not difficult to take these differences along, but no further insight is gained thereby.

[34] R. Feynman and G. Speisman, Phys. Rev. **94**, 500 (1954).

[35] See G. Wick, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics* (Interscience Publishers, New York, 1957), Sec. I, and especially R. Sorensen, Ph.D. thesis, Carnegie Institute of Technology, 1958 (unpublished).

(2) *Modifications of the π-nucleon vertex.*—This we take as a second and last example. More specifically, we consider the δ-effect due to the type of graph of Fig. 1(b). The quantities to be considered are

$$\left.\begin{array}{l}(\bar{p}p\pi^0)-(\bar{p}n\pi^+)/\sqrt{2}\\(\bar{n}n\pi^0)-(\bar{p}n\pi^+)/\sqrt{2}\\(\bar{p}p\pi^0)-(\bar{n}n\pi^0)\end{array}\right\}=2iF_I{}^2G\bar{N}_1(1)\int d2\,d3\,\pi(3)$$

$$\times\left\{\begin{array}{l}\Gamma(1,2,3)N_1(2)\\\Gamma'(1,2,3)N_1(2)\\\{\Gamma(1,2,3)-\Gamma'(1,2,3)\}N_1.\end{array}\right. \quad (42)$$

Again it is sufficient to take $\eta=1$. One finds

$$\Gamma=\{M_\Sigma\Delta_\Sigma(13)i\gamma_5[M_\Sigma\Delta_\Sigma-M_\Lambda\Delta_\Lambda]_{32}+\gamma\cdot\partial_1\Delta_\Sigma(13)i\gamma_5\gamma\cdot\partial_3(\Delta_\Sigma-\Delta_\Lambda)_{32}\}\Delta_K(12), \quad (43)$$

$$-\Gamma'=\{M_\Sigma(M_\Sigma\Delta_\Sigma-M_\Lambda\Delta_\Lambda)_{13}i\gamma_5\Delta_\Sigma(32)+\gamma\cdot\partial_1(\Delta_\Sigma-\Delta_\Lambda)_{13}i\gamma_5\gamma\cdot\partial_3\Delta_\Sigma(32)\}\Delta_K(12), \quad (44)$$

which shows that we have here a finite effect. Moreover one verifies that $\langle\Gamma\rangle=-\langle\Gamma'\rangle$, where $\langle\ \rangle$ means a matrix element between free nucleon states. Thus it suffices to consider the first line of Eq. (42). One finds

$$(\bar{p}p\pi^0)-\frac{(\bar{p}n\pi^+)}{\sqrt{2}}=G\bar{N}_1i\gamma_5N_1\left(\frac{\delta}{2\pi}\right)\left(\frac{F_I{}^2}{4\pi}\right)\times\left[0.6+0.2\frac{\Box}{M^2}+\cdots\right]\pi, \quad (45)$$

where the term in [ ] is a finite distance operator acting on the π-field. Note that the coupling constant G is not necessarily equal to the π-nucleon constant G_1, see Eq. (7). It is in the spirit of this work, however, to try and think of G as being of the same order as G_1. Then we have a charge-dependent change in coupling constant of order 0.006 $F_I{}^2/4\pi$. Even if the latter constant is ≈ 5, this amounts to a three percent correction. As before, this result does not signify very much. It indicates perhaps that also here the δ-corrections are of the same order as other effects which are anyway to be anticipated on general grounds. For example, corrections due to the $\pi^\pm-\pi^0$ mass difference are of the same order of magnitude.[36] In this general context, it is well to remember that it has not definitely been settled whether the charge independence deviations of the nuclear forces are entirely of electromagnetic nature.[37]

Apart from such vertex corrections (only incompletely treated here) there are, of course, other contributions which do not satisfy the usual charge independence criteria either. For example, there is the two K-exchange between nucleons. This is an effect that does not extend appreciably further than the nucleon Compton wavelength. The deviation effect is proportional to δ and of relative order (F^2/G^2) as compared to 2π-exchange. Thus it seems reasonable to consider this as a small effect. It should always be borne in mind, however, that what may be small in the low-energy nucleon physics and π-nucleon scattering domain need not be small at higher energies.

IV. $KK\pi$-INTERACTION

In a certain sense the considerations of the previous section are phenomenological: we have not considered what may be the dynamical reason for the existence of a nonzero δ, but rather we have treated the δ-effects by simply putting in the experimental Σ, Λ-masses first and thereafter performing the perturbation estimates of the foregoing section. One may ask what dynamics might underlie the (Σ,Λ)-mass separations.

The logical procedure is then to go back to the doublet approximation. We know that if the states N_2, N_3 are degenerate in the absence of all coupling, they remain so in the presence of the interaction (7). However, the interaction (10) splits N_2 from N_3 but in such a way that (Σ^+, Y^0) remain degenerate; likewise for (Z^0, Σ^-). While this split leaves the (S_1,S_2)-rules intact, it has nevertheless implicitly been neglected in Sec. II, in accordance with the program stated in Sec. I of ignoring all (Σ,Σ) and (Σ,Λ) mass differences. Taking this split into account now, we are still far from the desired (approximate triplet+singlet) spectrum. One may ask if electromagnetic effects could bring about the necessary modifications, in such a way that the higher (lower) neutral mass state would by definition be called $\Sigma^0(\Lambda)$. Even if true, the situation would still be inacceptable. From the definition of the electric current[38] it follows that electromagnetism does not mix Y^0 and Z^0 so that the reaction

$$\Sigma^0\to\Lambda+\gamma \quad (46)$$

would be forbidden. This adds to the difficulties, stated at the end of Sec. II, which follow from the dynamics of G- and F-interactions only.

At this stage it may be instructive to return for a moment to the results obtained in A for even $p(K)$, under the assumptions (a)–(d) recapitulated in Sec. I. The essential reasons why these conditions could not simultaneously be fulfilled was the existence of two invariance principles, called rules (A) and (B) in A, Sec. II, which turned out to be too strong. It was noted in Sec. II of the present paper that rule (B) is invalidated if $p(K)$ is odd, but that rule (A), the separate conservation of S_1 and S_2, still holds true. Experiment tells us that this is still too stringent a condition, as it leads to certain unwanted δ-proportional probability amplitudes (see end of Sec. II). The question then is,

[36] A. Sugie, Progr. Theoret. Phys. Japan 11, 333 (1954). See, however, R. Sorensen, reference 35.
[37] E. Salpeter, Phys. Rev. 91, 994 (1953).
[38] The current operator as given in A does not depend on whether $p(K)$ is even or odd.

whether and, if so, how we can break the invariance which leads to separate S_1, S_2-conservation and yet retain the relations (2).

This is indeed possible. For odd $p(K)$, one can namely envisage a new type of coupling:

$$[K\pi]=f(2m_K)(\bar{K}^+K^0\pi^+ + \bar{K}^0K^+\pi^-). \quad (47)$$

Here the factor $(2m_K)$ has dimensions such as to make the coupling constant f dimensionless [in units $(\hbar c)^{\frac{1}{2}}$]. The magnitude of this factor is chosen such that in the nonrelativistic limit f is a measure for the probability of creating a π-meson. It is not against the postulate of strict parity conservation of all strong interactions to have a coupling of this kind with a strength f which is large compared to weak-interaction strengths. The interaction (47) has the following basic properties.

(1) It violates the separate S_1, S_2-conservation, as is easily seen with reference to A, Table I. Thus there is now, at least in principle, a way open to avoid the difficulties mentioned in Sec. II.

(2) By the same token it makes the reaction (46) to an allowed one.

The question remains, of course, whether the combined f- and F-interactions could lead to reasonable separations within the Σ, Λ-states. No definite statement can be made, but some further comments on this question will be given in a subsequent paper.

To summarize: if $p(K)$ is odd, we are naturally led to the relations (2). This opens the possibility of universal π-baryon and of universal K-baryon coupling. The simplest conceivable (but not necessarily unique) scheme then seems to be the existence of three distinct types of interactions, with three corresponding coupling constants.

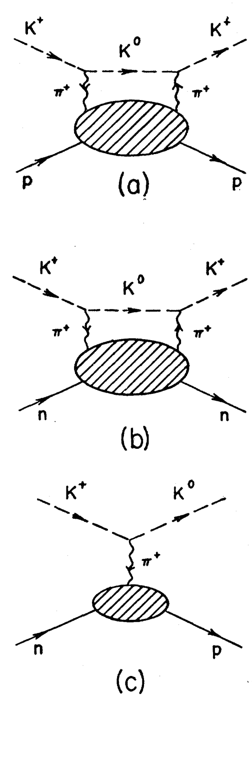

FIG. 2. $KK\pi$-contributions of lowest order in f to K-nucleon scattering: (a) (K,p)-, (b) elastic (K^+,n)-, (c) (K,n)-exchange scattering. In (a) and (b) the shaded box denotes the effective π-nucleon scattering mechanism. In (c) it denotes the effective π-nucleon vertex.

Our primary concern must again be the bearing of a coupling of the type (47) on π-nucleon charge independence. However, as it is necessary for this purpose to have some insight into the magnitude of f, we leave this question till the end of this section and we shall rather start to confront the dynamics with the experiments on the new particles.

First consider the reaction (5). We saw in Sec. II [see Eq. (30)] that for $f=0$ the cross section is proportional to δ^2. Thus it seems reasonable to ignore in first instance the F-contributions. To lowest order in f the scattering amplitude is then a folding of the $KK\pi$-coupling into the π-nucleon vertex part, as sketched in Fig. 2(c). Note that the virtual π must give a retardation factor $\{1+\alpha(1-\cos\theta)\}^{-2}$, where $\alpha=2p^2m_\pi^{-2}$. We do not know how to treat the strong vertex rigorously. But at not too high energies it seems safe to say that the scattering is due to an S-wave π-emission in the $KK\pi$-vertex and a P-wave π-absorption by the nucleon. This leads us to write the cross section as

$$d\sigma=8\pi\left(\frac{G_1^2}{4\pi}\right)\left(\frac{f^2}{4\pi}\right)\left(\frac{m_K}{m_\pi}\right)^2\left(\frac{p}{E+\Omega}\right)^2 \times\frac{\psi(\theta)}{\{1+\alpha(1-\cos\theta)\}^2}\frac{d\cos\theta}{m_\pi^2}, \quad (48)$$

$$\psi(\theta)=1+\lambda\cos\theta,$$

where λ is a parameter which lies between -1 and $+1$. In perturbation theory (where also the equivalence theory holds) one obtains Eq. (40) with $\lambda=-1$. This is by no means a justification for normalizing the leading term in ψ to unity as the π-nucleon vertex may be damped. This possible damping effect clearly makes for uncertainty in estimating the magnitude of f. Apart from this, it would not seem unreasonable that Eq. (48) represents a fair description of the angular and energy dependence over a limited energy domain where λ does not vary too much with energy. The momentum dependence of the total cross section is given by the factor:

$$\left(\frac{p}{E+\Omega}\right)^2\varphi(\alpha),$$

$$\varphi(\alpha)=\frac{1}{\alpha^2}\left[2\alpha\left(\frac{\alpha(1+\lambda)+\lambda}{1+2\alpha}\right)-\lambda\ln(1+2\alpha)\right], \quad (49)$$

which yields the following characteristics: σ starts with a swift rise due to the p^2-factor [$\varphi(0)=2$] and then flattens out, mainly due to the α-dependence; $\alpha\sim 1$ at 25 Mev. The rise is more gradual and the flattening more pronounced the larger λ is. For $\lambda\gtrsim 0$ the cross section varies little with energy from 80–200 Mev, for smaller λ it slowly decreases. On this picture it seems possible to harmonize the experimental results of Hoang *et al.*[39] who find very little exchange in the region below 65 Mev with those of Lannutti *et al.*[40] and others who find a fairly constant cross section $\sim 4.0\pm 0.8$ mb in the region 60–180 Mev. It does not appear to be necessary, as has been suggested,[40] to assume S-wave scattering from 0–200 Mev.

Note further that according to Eq. (48) $d\sigma$ should show a marked forward peaking. At not too low energies this is true for all λ and is the more pronounced the larger λ is. This effect is of special importance for K^+-exchange scattering in heavy nuclei where the Pauli principle will tend to suppress forward scattering. It would be extremely interesting to study the K^+-exchange angular distribution in a deuterium bubble chamber. And it might be enlightening to analyze the data in terms of Eqs. (48) and (49), in order to see whether energy and angular dependence can be simply correlated in this way for suitably chosen λ.

With $G_1^2/4\pi\approx 15$ and $\sigma\approx 4$ mb at 100 Mev, we find that $f^2/4\pi\sim 0.007$; 0.011; 0.03 for $\lambda=1, 0, -1$, respectively, corresponding to a range for f from about e to $2e$. We conclude that

$$f\sim e, \tag{50}$$

if it is justified to use the undamped value for G_1 (or for the corresponding small pseudovector constant). This is a suggestive order of magnitude (see Sec. V), but we must stress again that the theoretical foundation for this estimate is poor. Note that in K-phenomena the development goes, in powers of $f^2/4\pi.\ (m_K/m_\pi)^2 \sim 0.2$. Thus we may perhaps make some progress in expanding if the estimate (50) is not too far off.

Whereas the f-interaction is rather long-ranged for K^+-exchange scattering, corresponding to a rise of α to unity at ~ 25 Mev, this interaction is effectively shorter-ranged where elastic scattering is concerned. In the latter case the coupling (47) acts only via the exchange of π-pairs between K and nucleon and thus has a range $\sim(2m_\pi)^{-1}$. Experimental indications are[40] that the range for elastic scattering could be of this order of magnitude. The F-interaction also contributes in this case, in fact equally for scattering on protons and on neutrons, see Eq. (29). Experimentally[40] the p and n cross sections vary little with energy in the region 20–200 Mev, while the former (14.5 ± 2.2 mb) is larger than the latter (5.8 ± 3.1 mb). As we have seen, this p, n difference was another complication if F-couplings only were invoked. To see qualitatively how the interaction (47) affects the situation we shall for a moment consider the extreme case where F-effects are completely ignored.

It is at once clear that the f-interactions give a different contribution for proton *versus* neutron scattering; see for example Figs. 2(a), 2(b). These figures illustrate that, as long as the f-coupling is taken to lowest relevant order (not so with the G-interaction), we deal with a folding of π^+-nucleon scattering into an effective $K^2\pi^2$ interaction. Suppose now, as is not quite unreasonable, that in the 100–200 Mev region the (virtual) π^+-nucleon scattering would go mainly via the $I=\frac{3}{2}$ state. In the pure case one would then have a 9:1 ratio for (K^+, p) *versus* (K^+, n) scattering. At any rate it seems plausible that an f-coupling leads to a larger (K^+p) than a (K^+n) cross section.

The c.m. angular distribution is fairly isotropic[40] in the region 20–200 Mev. Now at low energies the effective $K^2\pi^2$ interaction, which itself has a range $\sim m_K^{-1}$, is very nearly a point interaction and will thus give an isotropic distribution. This is precisely a feature of the interesting work of Barshay[41] who first introduced effective $K^2\pi^2$-couplings. For larger energies the non-point character of our $K^2\pi^2$-interaction will become more effective, tending to give a preference to forward scattering.

Now we must ask for the influence of the F-interaction. To get some idea about this we go to the opposite extreme, ignore the f-couplings, and, for all it is worth, consider the K-baryon coupling in Born approximation. Here we can profit from the work of Ceolin and Taffara.[42] Their investigations are also instructive from the present point of view as one can immediately read off[43] the effects of finite δ. In terms of the relations (2) these authors find: first, a fairly isotropic c.m. angular distribution at 80-Mev lab energy,[44] for either even or odd $P(K^+)$; second a decreasing cross section as function of energy,[45] for either K^+-parity. Taking a mean K^+-nucleon cross section of 10 mb at 100 Mev, one can read off: $F_I^2/4\pi=0.9$ (1.7) for scalar (pseudoscalar) K^+. The main purpose of quoting this last result is to show that the orders of the F_I-coupling and the effective f-coupling quoted previously may well be comparable.[46]

For this very reason a more detailed discussion of the elastic K-scattering, taking into account both F- and f-effects appears to be rather complicated. In particular it is not clear whether K-scattering is the easiest means to obtain information about F_I, and in fact about $P(K^+)$. In this latter connection it should be observed that if $p(K)$ would turn out to be odd, the

[39] Hoang, Kaplon, and Cester, Phys. Rev. **107**, 1698 (1957).

[40] J. E. Lannutti *et al.* Phys. Rev. **109**, 2121 (1958) where extensive references to further literature can also be found.

[41] S. Barshay, Phys. Rev. **109**, 2160 (1950); **110**, 743 (1958). In this work the $K^2\pi^2$-coupling is charge independent and so gives equal contributions for p- and n-scattering. The split between these quantities is supposed to come about via K-baryon interactions with $F_1\neq F_2$. In this respect the present view is rather the opposite.

[42] C. Ceolin and L. Teffara, Nuovo cimento **5**, 435 (1957). Under the present conditions of coupling, the connections with their notation are as follows: $g_\Lambda^2=g_\Sigma^2=F_I^2/4\pi$, $s=1$.

[43] Reference 42, Figs. 7 and 8.

[44] Reference 42, Figs. 4 and 5.

[45] Reference 42, Figs. 9 and 10.

[46] K^+-nucleon scattering has also been studied by D. Amati and B. Vitale, Nuovo cimento **5**, 1533 (1957), for $P(K^+)$ even. A scattering integral equation with cutoff is solved. With $F_1=F_2$ (in our notation) they find $F_I^2/4\pi\approx 0.3$. This is again of the general order of magnitude quoted above for scalar K^+.

question of the K-nucleon dispersion relations[47] would have to be re-examined. If one can at all prove these relations (for forward scattering), their interpretation would have to be modified. The constant f would explicitly appear due to the presence of equal-time commutators. We conclude this cursory survey of the K^+-scattering problems with the remark that both the repulsive nature of the K^+-proton potential and the relative rate of exchange to elastic scattering may throw more light on the relative importance of K-baryon to $KK\pi$ couplings.

One may ask how the interaction (47) affects the K^- and also the K^0, $\bar{K}^0$ (or K_1^0, K_2^0) scattering. The situation is summarized as follows: if we treat the f-coupling to the approximation indicated above, then the isotopic spin structure of V_f, the f-contribution to the elastic scattering matrix element, is given by[48]

$$V_f = 2\bar{K}K\bar{N}_1N_1 + \bar{K}\tau_3K\bar{N}_1\tau_3N_1. \tag{51}$$

Hence if the f-interaction were preponderant, the K^+, p and K^-, p elastic scattering would be equal, and the elastic K_2^0 scattering on protons (neutrons) would equal the K^+ scattering on neutrons (protons), etc. The K^-p elastic cross section is actually larger[49] than the one for K^+p. It is to be anticipated, however, that for the former an appreciable role will be played by the virtual annihilation effects which will also tend to produce an isotropic distribution. The K_2^0 elastic scattering is, of course, difficult to measure directly. In the case of odd $p(K)$ this would be a particularly interesting effect.

According to Eq. (47) the f-proportional part of the amplitude for charge exchange scattering of K^- on protons and of $\bar{K}^0$ on neutrons should be identical with the one for K^+-exchange, in lowest order. Note, however, that the δ-proportional part of the amplitude, due to K-baryon coupling, is of a different nature for the K^- as compared to the K^+ case: the K^--exchange can go via virtual pure Λ and Σ^0 states. It is easily seen that, at low energies, the neglect of the (Σ^0, Λ) mass difference is here quite bad, and in particular worse than for K^+-exchange at comparable energies. From the present point of view the $K^- \leftrightarrow \bar{K}^0$ exchange may therefore be more difficult to interpret. Experimentally no K^--exchange events in hydrogen have definitely been identified.[50] The estimates of Alles *et al.*[49] lead to a rather large exchange cross section, however.

Also for K^--absorption phenomena the neglect of δ is unwarranted. We shall therefore not discuss the

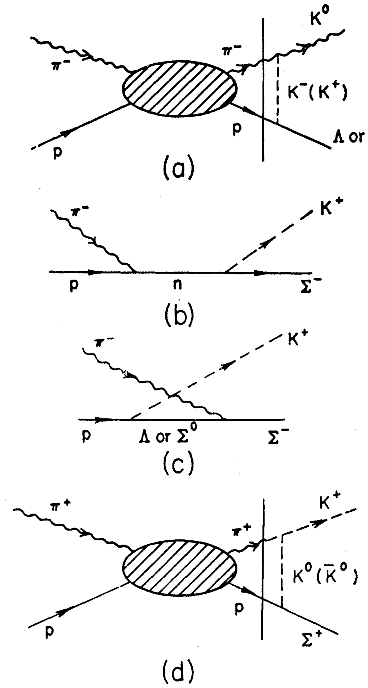

FIG. 3. Mechanisms of associated production: (a) of K^0 and Λ or Σ^0; (b), (c) of Σ^-, K^+; (d) of Σ^+, K^+. The shaded boxes have the same meaning as in Fig. 2.

absorption in this preliminary survey, but only note that the unwanted relation (36) is invalidated due to the presence of the interaction (47).

Concerning the determination of the constant F_I, it would seem that, also from the present point of view, Gell-Mann's original suggestion[4] to use photoproduction of K-mesons for this purpose is still the best one. In this effect the interaction (47) only enters in higher order, and thus is perhaps not too important. If this is indeed true, the near equality of Λ- and of Σ^0-production in (γp)-interactions should continue to be a good approximation, see Eq. (28).

Next we turn to the discussion of associated production in π-nucleon collisions. There is some indication[51] that charge independence in the usual sense does not apply to these processes. This would not be too surprising from the present point of view. We shall show now that it is possible, for odd $p(K)$, to reproduce the trend of the experiments on Λ, Σ production provided $P(K^+)$ is even.

We start with the discussion of the reactions (21) and (22); for definiteness consider the production at 1.1 Bev, and take first the extreme case where F-couplings are neglected. At this energy the (π^-, p) elastic scattering is $\sim$20 mb and the production cross sections are about a percent of this amount. Thus it seems reasonable to picture the Λ, Σ^0-production as schematically drawn in Fig. 3(a): after a virtual (π^-, p) scattering, the π emits a K^0 and a K^-, the latter being absorbed by the proton (or $p \to K^+ + \Lambda$ or

[47] See P. Matthews and A. Salam, Phys. Rev. **110**, 565, 569 (1958); C. Goebel, Phys. Rev. **110**, 572 (1958); M. Polivanov, Doklady Akad. Nauk. U.S.S.R. **116**, 943 (1957) [translation: Soviet Phys. 2, 472 (1958)].

[48] Here K denotes an I-spinor with components K^+, K^0. For a further discussion of the switch to such a spinor representation at this stage, see Sec. V. Following the notation of Eq. (70) below, we should actually write the second term in Eq. (51) as $\bar{K}\rho_3K \times \bar{N}_1\tau_3N_1$.

[49] W. Alles *et al.*, Nuovo cimento **6**, 571 (1957).

[50] L. W. Alvarez *et al.*, Nuovo cimento **5**, 1026 (1957).

[51] J. L. Brown *et al.*, Phys. Rev. **107**, 906 (1957).

Σ^0, the K^+ being absorbed by the scattered π^-). We know that, apart from important refinements, the π^--scattering shows a very marked forward peaking in the c.m. system due to shadow scattering.[52] Then according to Fig. 3(a), the K^0-production is essentially a final state interaction of a forward scattered π^- and a proton. This last interaction depends in an important way on $P(K^+)$. In fact, for odd $p(K)$, the differential cross section for the reaction (24) as brought about by the mechanism to the right of the vertical line in Fig. 3(a) has an angular dependence[53]

$$\frac{1\pm(MM_Y/EE_Y)+v_pv_Y\cos\theta}{(1+v_\pi v_K\cos\theta)^2}, \tag{52}$$

where v denotes the velocity of the particle in question; E, E_Y are the nucleon and hyperon energy, respectively. The plus (minus) sign in Eq. (52) corresponds to $P(K)$ odd (even). θ is the angle between π and hyperon.

Suppose for a moment that the π^- after its virtual scattering would have its real energy and momentum, that is it would go preponderantly forward. Call $R(\theta)d\cos\theta$ the relative rate of production at angle θ. Then one has for Λ-production at 1.1 Bev:

$$\begin{aligned}R(0):R(\tfrac{1}{2}\pi):R(\pi)&=0.4:1:6,\quad P(K^+)\text{ even}\\&=0.7:1:0.9,\quad P(K^+)\text{ odd.}\end{aligned} \tag{53}$$

Thus the observed backward peaking of the Λ fits in naturally for even $P(K^+)$, that is pseudoscalar K^0. Actually it is not unreasonable that for this particular parity the virtual π^-, p scattering would not differ much from the real one: in this case the $(\bar{p}\Lambda K^+)$-vertex is a scalar one which leads in the main to a transition between positive energy baryon states. For the case of odd $P(K^+)$, where Eq. (53) does not yield the direct backward peaking, the vertex in question has pseudoscalar character and leads to a negative $\rightarrow$ positive energy baryon transition. For the sake of illustration let us assume that the proton after virtual scattering is in a state of opposite energy and momentum compared to the real scattering and preparatory to its transition to a Λ-state. Then one would have, always for odd $P(K^+)$:

$$R(0):R(\tfrac{1}{2}\pi):R(\pi)=6:1:0.4, \tag{54}$$

that is, a strong forward peaking. Now the assumption which leads to Eq. (54) is obviously an extreme one and it refers to states very far off the energy shell. Yet it would not seem unreasonable that for odd $P(K^+)$ the answer lies somewhere between the results of Eq. (53), second line and Eq. (54). I would therefore conclude that $P(K^+)$ cannot be odd always if $p(K)$ is odd.

Assuming then that $P(K^+)$ is even, it should further be noted that one will have to fold a π-distribution off the energy shell, but not far off, into the production mechanism. It would seem reasonable to suppose that this does not spoil the backward trend of the produced Λ's. Note that the observed amount of (π^-, p) back scattering[52] will tend to reduce the backward-forward ratio of Λ-production.

If it is supposed that $P(K^+)$ is even, one is also committed as to the Σ^0-production. In the same approximation one finds in this case $R(0):R(\pi/2):R(\pi)=0.5:1:3.7$ that is a smaller backward/forward ratio at the same energy as compared for Λ-production. It will not be attempted here to give a more complete treatment of Λ, Σ^0-production, where one has to consider the combined effects of $KK\pi$- as well as K-baryon interaction. However, we shall see below that pure F-interaction seems to yield an inadmissible angular distribution for Λ-production. As to the energy dependence of the cross sections, the decrease of (π^-, p) scattering with increasing energy beyond 1.1 Bev would tend to lead to a similar decrease in Λ, Σ^0-production.[54]

To see the effects of F-interactions, the Σ^--production is particularly interesting as here the mechanism of Fig. 3(a) does not contribute. It turns out to be instructive to consider the Σ^--production in Born approximation see Figs. 3(b), (c). From a comparison of these two graphs it is at once clear that the ratio $G_1/G\equiv\rho$ of the π-coupling constants plays an important role. It is in the spirit of the present attempt to consider only the cases $\rho=\pm1$. Table I summarizes the rates of production at angles 0, $\pi/2$, π for $\rho=\pm1$ and for $P(K^+)$ even or odd. Note that the experimentally observed forward peak in the Σ-distribution[55] could, in this presumably clumsy approximation, equally well be understood for $P(K^+)$ even, $\rho=1$ as for $P(K^+)$ odd, $\rho=-1$.* Considering ourselves committed to even $P(K^+)$, it is interesting to see that this goes with

TABLE I. Relative rates of Σ^--production at 1.1 Bev as function of $P(K^+)$ and of ρ.

	$P(K^+)$ even		$P(K^+)$ odd	
	$\rho=1$	$\rho=-1$	$\rho=1$	$\rho=-1$
$R(0)$	2.8	1.05	1.1	2.6
$R(\pi/2)$	1	1	1	1
$R(\pi)$	0.2	1	1	0.5

[52] L. M. Eisberg *et al.*, Phys. Rev. **97**, 797 (1955); W. D. Walker and J. Crussard, Phys. Rev. **98**, 1416 (1956); W. D. Walker *et al.*, Phys. Rev. **104**, 526 (1956); M. Chretien *et al.*, Phys. Rev. **108**, 383 (1957); A. R. Erwin and J. K. Kapp, Phys. Rev. **109**, 1364 (1958).

[53] This and subsequent formulas refer to the c.m. system. Terms of order m_π/m_K or smaller have been neglected.

[54] See reference 55 and also C. Besson *et al.*, Nuovo cimento **6**, 1168 (1957).

[55] See reference 51 and W. B. Fowler *et al.*, Phys. Rev. **98**, 121 (1954); R. Budde *et al.*, Phys. Rev. **103**, 1827 (1956); J. L. Brown *et al.*, Phys. Rev. **108**, 1036 (1957); L. B. Leipuner and R. K. Adair, Phys. Rev. **109**, 1358 (1958); F. S. Crawford *et al.*, Bull. Am. Phys. Soc. Ser. II, **3**, 25 (1958).

* *Note added in proof.*—The latter case corresponds rather closely to the results of C. Warner, Phys. Rev. Letters **1**, 246 (1958).

$\rho=1$ which tends to fit in with the idea of a universal π-coupling. Using Eq. (50) the Born approximation yields $F_I^2/4\pi\sim0.1$ if we take[55] $\sigma_-\sim0.17$ mb. This is the same general order of magnitude as was found[42,46] from K-scattering. Even so, the author does not believe that such numbers should necessarily be trusted.

From Table I it is immediately possible to read off the contribution of a pure F-interaction to $d\sigma_0$ by means[56] of Eq. (27): one has simply to interchange the headings $P(K^+)$ even, odd. Thus, again for $P(K^+)$ even, $\rho=1$ one gets a nearly isotropic contribution to $d\sigma_0$ and likewise to $d\sigma_\Lambda$ see Eq. (25). It is hoped that a more quantitative theory will make it possible to maintain a general picture where contributions of the type of Fig. 3(a) are at least comparable with those of Figs. 3(b), (c) in Λ, Σ^0-production.

A difference in parity properties of charged and neutral K-particles will in general also induce distinctions between polarization patterns of the associated neutral and charged hyperons produced in π-nucleon collisions. Thus the absence of up-down asymmetry in Σ^--decay, as compared to the large Λ-effect[57] may be due to a difference in production mechanisms.[58] This would open the possibility of further universalizing the weak decay interactions.

As the Σ^0-production mechanism is much akin to that for Λ, it is to be anticipated that also Σ^0 should be strongly polarized, if the present picture is correct. Of course, the degree of polarization of the secondary Λ is cut down by a factor three[59] but still one may hope for a sizable effect.

We conclude the discussion of production processes with the Σ^+-reaction (24). Here we can invoke the same type of mechanism as for Λ-production, see Fig. 3(d). No angular distributions for (π^+, p)-scattering in the Bev range have been published so far. However, it seems reasonable to expect also here a forward peak from shadow scattering. Arguing as before, we then would have again to consider, to start with, an angular distribution of the type (52). However, it is important to note that for Σ^+-production the plus (minus) sign in Eq. (52) now refers to $P(K^+)$ even (odd). This is of course due to the oddness of $p(K)$. It is the switch in sign which prevents the Σ^+-distribution from being strongly peaked backward, as for the Λ. We see at this point that the original suggestion, based on the parity doublet picture, that $KK\pi$-interactions might account for the angular distributions in ΣK-production[16] can possibly only be maintained in conjunction with odd $p(K)$.

However, for even $P(K^+)$ one now finds from Eq. (52) $R(0):R(\pi/2):R(\pi)=0.75:1:1.2$ which is rather flat, but which is still peaked backward and thus contradicts the experimentally found forward peaking[51] of the Σ^+. But now we must remember that for even $P(K^+)$, hence odd $P(K^0)$, the $(\bar{p}\Sigma^+K^0)$-vertex has pseudoscalar character. For orienting purposes we may then argue again in the way which led up to Eq. (54). This yields:

$$R(0):R(\pi/2):R(\pi)=3.7:1:0.5. \qquad (55)$$

One cannot say more than that this result looks very promising. It is based on an assumption about the nature of π^+-proton scattering matrix elements far off the energy shell and it remains entirely to be seen whether this assumption is justified. For this same reason it would be premature to speculate about polarization in Σ^+-production. One may hope, nevertheless, that the combined assumptions of odd $p(K)$ and the existence of the coupling (47) could satisfactorily account for the associated production phenomena. It should further be remembered that for Σ^+-production the F-effect is proportional to δ. On the other hand, this effect [due to a crossed graph of the kind drawn in Fig. 3(d)] gives a retardation factor $(1-v_\pi v_\Sigma\cos\theta)^{-4}$. The high power in the exponent is due to the circumstance that the δ-effect comes about as a differentiation of a hyperon propagator with respect to the mass. This factor is extremely strongly peaked forward. It may therefore be that the interference between f- and F-terms is important at small angles.

The f-interaction leads to further differences between the Λn- and Λp-forces, see for example Fig. 4. This does not change the qualitative reasoning of Sec. II as long as the π-induced forces remain preponderant. (The fourth order effect drawn is or order $fF_I^2G_1$.)

Now we must come back once again to the charge independence in π-nucleon phenomena. To some extent we are repeating ourselves. As noted at the beginning of this section, the interaction (49) either accounts for or else at least contributes to the (Σ^0, Λ)-mass split. Thus we may now have a possible dynamical description of the δ-effects which were treated more phenomenologically in Sec. III. However, it is important to note that the interaction (47) also gives rise to virtual effects in π-nucleon systems of another kind then those of the general type discussed in Sec. III.

For the nuclear forces we have two distinct types of effects; one graph representing each class is drawn in

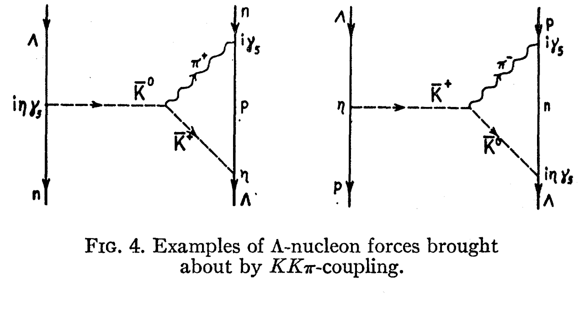

FIG. 4. Examples of Λ-nucleon forces brought about by $KK\pi$-coupling.

[56] Equation (27) is valid to all orders in F and G. This theorem is invalidated by the presence of the f-interactions. But that is clearly immaterial for the present discussion of the Born term.

[57] F. S. Crawford *et al.*, Phys. Rev. **108**, 1102 (1957); F. Eisler *et al.*, Phys. Rev. **108**, 1353 (1957).

[58] From this point of view it would be interesting to study the up-down asymmetry of Σ^- produced in $\pi^-+d\rightarrow\Sigma^-+K^0+p$.

[59] A. Pais and S. Treiman, Phys. Rev. **109**, 1759 (1958).

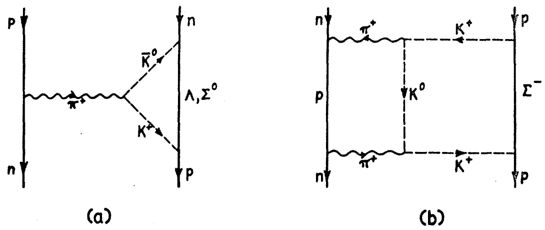

FIG. 5. Examples of noncharge independent effects of the f-interaction for the case of nuclear forces.

Fig. 5. In Fig. 5(a) we see a vertex correction proportional to $\delta f F_I^2$, with an extension $\sim 1/2m_K$ which is the order of the nucleon Compton wavelength. It seems safe to consider this as a small correction. Then there is a class represented by Fig. 5(b) where corrections appear that are obviously not proportional to δ. This sixth order effect is $\sim f^2 F_I^2 G_1^2$; it is charge dependent as can be seen by studying the class exhaustively. I believe that these non-δ-type effects are actually the most interesting corrections, and that they are not of an obviously disturbing magnitude, due to the smallness of f and to the possibility that also $F_I^2/G_1^2<1$. No detailed estimates have been made so far.

The situation is similar for π-nucleon scattering. In Fig. 6 we have δ-proportional vertex corrections whereas the K-pair effects shown in Fig. 7 are not δ-proportional. We note the following features of the latter effect. (a) It violates charge independence. For in this order there is no contribution to charge exchange scattering, while at the same time the effect is different for (π^+, p) *versus* (π^-, p) elastic, as $p(K)$ is odd. (b) The effect depends on $P(K^+)$. (c) We have a typical pair effect which will thus show up predominantly in S-scattering. Now according to present views, S-scattering is mainly due to virtual nucleon pair effects. In this respect the K-pairs have an edge as they are lighter. On the other hand, we have seen that for low-energy π's the probability of creation is just given by $f(2\omega)^{-\frac{1}{2}}$ where, according to Eq. (50), f is of the order of the electric charge. This means that the effects in question are of electromagnetic order of magnitude. Even so, a refined study of π-nucleon S-wave scattering might be

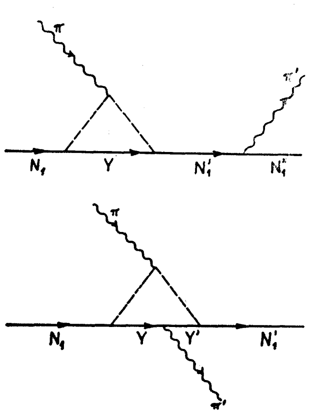

FIG. 6. $f\delta$-effects in π-nucleon scattering. N_1, N_1', and N_1'' denote nucleon states. Y and Y' are Λ or Σ-states.

the most natural procedure for localizing the possible existence of $KK\pi$-effects in low-energy π-nucleon physics. That this will not be easy is seen from the uncertainties that presently attach to the analysis of electromagnetic effects in S-wave scattering.[60]

In conclusion we note that the lack of charge symmetry between proton and neutron, where K-baryon couplings are involved, may perhaps shed new light on Sandri's suggestion[61] that K-particle interactions may be of help in explaining the puzzling properties of the charge and magnetic-moment distributions of the nucleon. We note that the evenness of $P(K^+)$ favors a trend to cancellation between the π- and the K-cloud of the neutron and leads to small contributions to the magnetic moments. On the other hand, there is a contribution to the proton moment due to the pseudoscalar $p\rightarrow\Sigma^+ + K^0$ coupling. It has been pointed out[62] that perturbation estimates for these effects are quite unreliable. One would perhaps anticipate that the K-effects are not sufficient to explain the effects in question, especially if F_I is smaller than G_1. However, the effects are not easy to estimate reliably and may deserve further study.

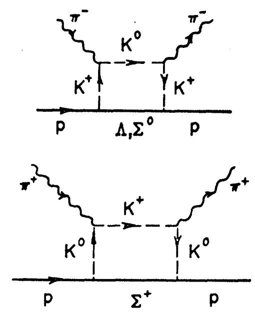

FIG. 7. K-pair effects in π-nucleon scattering.

V. SYMMETRIES OF THE STRONG INTERACTIONS

The purpose of this section is to provide a formal basis for classifying the symmetries employed thus far. While no new physical results are obtained, a clear understanding of the symmetries involved must eventually lead to further progress. The results of this section, if at all correct, should be considered as preliminary in nature.

The four-dimensional real orthogonal group excluding reflections provides an adequate framework for a classification of the couplings introduced so far. This is not the first time that the use of the 4-group is suggested. As we go along, we shall see how previous attempts can be characterized as compared to the present one.

The generators M_{ij} of the 4-group satisfy

$$[M_{ij},M_{kl}]=-i(M_{il}\delta_{jk}+M_{jk}\delta_{il}-M_{ik}\delta_{jl}-M_{jl}\delta_{ik}). \quad (56)$$

[60] See for example H. Noyes, Phys. Rev. **101**, 320 (1956).
[61] G. Sandri, Phys. Rev. **101**, 1616 (1956).
[62] Federbush, Goldberger, and Treiman (to be published).

The fact that we use the real group makes all M_{ij} Hermitean. The linear combinations

$$\begin{aligned} I_1&=\tfrac{1}{2}(M_{23}+M_{14}), \quad K_1=\tfrac{1}{2}(M_{23}-M_{14}),\\ I_2&=\tfrac{1}{2}(M_{31}+M_{24}), \quad K_2=\tfrac{1}{2}(M_{31}-M_{24}),\\ I_3&=\tfrac{1}{2}(M_{12}+M_{34}), \quad K_3=\tfrac{1}{2}(M_{12}-M_{34}),\end{aligned} \tag{57}$$

satisfy

$$\begin{aligned}&[I_1,I_2]=iI_3, \text{ cycl.}; \quad [K_1,K_2]=iK_3, \text{ cycl.};\\ &[I_j,K_l]=0 \quad \text{for} \quad j, l=1, 2, 3.\end{aligned} \tag{58}$$

$\mathbf{I}^2$, I_3, $\mathbf{K}^2$, K_3 are a complete set of commuting operators. The labeling of the representations of the 4-group excluding reflections goes by the pair of numbers (i,k) characterizing the lengths of $\mathbf{I}^2$, $\mathbf{K}^2$; and the dimension of the representation is $(2i+1)(2k+1)$.

The vector $\mathbf{L}$ defined by

$$L_1=M_{23}, \quad L_2=M_{31}, \quad L_3=M_{12} \tag{59}$$

characterizes the (123) subspace. We have

$$\mathbf{L}=\mathbf{I}+\mathbf{K}. \tag{60}$$

Define further

$$\mathbf{M}=\mathbf{I}-\mathbf{K}. \tag{61}$$

Note that $\mathbf{L}^2$, $\mathbf{M}^2$, and L_3 commute.

Consider the π-baryon interactions (7). We make contact with the present language by assigning representations as follows.

$$\left.\begin{array}{ll}\text{Nucleon}: (\tfrac{1}{2},0); & N_2: (\tfrac{1}{2}\,\tfrac{1}{2})\\ \Xi: \quad (\tfrac{1}{2},0); & N_3: (\tfrac{1}{2}\,\tfrac{1}{2})\end{array}\right\}\text{with } K_3=\left\{\begin{array}{c}+\tfrac{1}{2}\\ -\tfrac{1}{2}\end{array}\right\}\text{only}, \tag{62}$$
$$\pi: \quad (1,0).$$

Thus (N_2,N_3) jointly are assigned the full representation $(\frac{1}{2},\frac{1}{2})$. Clearly a 4-scalar is characterized by (0,0). The invariant couplings are now constructed as follows:

$$\begin{aligned}&\pi \text{ to nucleon}: && (\tfrac{1}{2},0)\times(\tfrac{1}{2},0)\times(1,0),\\ &\pi \text{ to } \Sigma, \Lambda: && (\tfrac{1}{2},\tfrac{1}{2})\times(\tfrac{1}{2},\tfrac{1}{2})\times(1,0),\\ &\pi \text{ to } \Xi: && (\tfrac{1}{2},0)\times(\tfrac{1}{2},0)\times(1,0),\end{aligned} \tag{63}$$

where that part of the product is projected out which leads to (0,0). According to Eq. (58) one may use the usual vector addition on I, K separately. Observe:

(1) The usual π-nucleon coupling separately may trivially be embedded in a four-frame.

(2) The coupling π to nucleon, Ξ is formally on a somewhat different footing than π to Σ, Λ. For $G_1=G_4$, but not necessarily equal to G, one can unite π to (N_1,N_4) in the same way as π to (N_2,N_3) provided one "neglects" the N_1-N_4 mass difference. In the present paper we shall not consider any commitments on this score.

(3) For $G_1=G_4=G$ and neglecting all baryon mass differences, one can unite all baryons to a single eight-dimensional representation of a group which is the direct product of three two-dimensional unitary unimodular groups. This is what is called "global symmetry."[4]

(4) In the $\mathbf{L}^2$, $\mathbf{M}^2$, L_3 language one has, with the help of Eqs. (3), (4), (60), and (63), the following l-values:

$$\text{nucleon}: \tfrac{1}{2}; \Lambda: 0; \Sigma: 1; \Xi: \tfrac{1}{2}; \pi: 1. \tag{64}$$

One is at this stage equally entitled to call L the isotopic spin as one may give I that same name.[11] In each language one has an extension of charge independence from π-nucleon to π-baryon physics. This is for example the reason that the charge independence tests from K^--absorption are unaltered in the present paper as long as δ is neglected, see Sec. II. However, the introduction of the K-couplings makes it necessary to let I be "the" isotopic spin. We shall call K the K-spin.

The introduction of the K-baryon couplings leaves the assignments (62) unaltered but, due to the oddness of $p(K)$, the equivalence between $(\frac{1}{2}\,\frac{1}{2})$, K-spin up and $(\frac{1}{2}\,\frac{1}{2})$, K spin down is destroyed. In other words, odd $p(K)$ breaks the K-quantum number:

$$\begin{aligned}&\pi\text{-baryon couplings}: \mathbf{I}^2, I_3, \mathbf{K}^2, K_3 \text{ are good},\\ &K\text{-baryon couplings}: \mathbf{I}^2, I_3, K_3 \text{ are good};\\ &\qquad\qquad \mathbf{K}^2 \text{ is "out."}\end{aligned} \tag{65}$$

To see this, we assign $(0,\frac{1}{2})$ to K and likewise to $\bar{K}$. Hence K and its charge conjugate are K-spinors, just as the nucleon and its charge conjugate are I-spinors. For even $p(K)$ the K-baryon couplings could be written as $(\frac{1}{2},0)\times(\frac{1}{2}\,\frac{1}{2})\times(0,\frac{1}{2})$. For odd $p(K)$, we may still use the $(0,\frac{1}{2})$ representation for K, but the loss of k as a good quantum number precisely means that the above characterization of the K-baryon couplings is no longer meaningful.

In all previous attempts to use the 4-group it was always the main stumbling block that there was no rational way to get rid of k as a good quantum number. Thus the initial attempt in this direction,[63] where k was assumed to be good, had to be discarded as too high degeneracies were involved. Subsequent attempts lifted the k-degeneracy in a more or less phenomenological way.[64] The present attempt may be characterized by the statement: K-invariance is broken by parity.

The electromagnetic interactions are characterized by a charge density operator

$$Q=L_3+\tfrac{1}{2}S+\tfrac{1}{2}N. \tag{66}$$

According to Eq. (59) L_3 generates rotations in the 12-plane, "around" the 34-plane. Just as the electromagnetic interactions led to a preferred axis in the 3-group language, so they are now characterized by a

[63] A. Pais, Proc. Natl. Acad. Sci. U. S. 40, 484 (1954).

[64] A. Pais, *Proceedings of the Fifth Annual Rochester Conference on High-Energy Physics, 1955* (Interscience Publishers, Inc., New York, 1955); A. Salam and J. C. Polkinghorne, Nuovo cimento 2, 685 (1955). See further A, reference 16.

preferred plane in the 4-group description. As always, restrictions have to be imposed rather *ad hoc* as to the absence of "off-diagonal Pauli terms."[65] We note:

(1) The definition of Q fits in with the "old" isotopic spin picture where L_3 is the 3-component of I-spin; see Eq. (64) and the comments subsequent thereto. According to Eq. (60) we have a further degeneracy due to the separate conservation of I_3 and K_3. Consequences of this degeneracy are the relation (28) and the γ-stability of the Σ^0.

(2) The relation[66]

$$\mu(\Sigma^+)+\mu(\Sigma^-)=2\mu(\Sigma^0), \tag{67}$$

holds true where virtual π-currents, but not where virtual K-currents are involved; μ denotes the magnetic moment.

(3) In A the definition $Q=I_3+S_1+\frac{1}{2}N$ was used. This is identical with Eq. (66), as we have $S_1=K_3+\frac{1}{2}S$ (see A, Table I).

(4) It is at this stage that we first explicitly meet S and N, the baryon number. It would seem as if S becomes somewhat of a counterpart to N: we may think of N-conservation as due to a gauge invariance in I-space, of S-conservation as related to a similar invariance in K-space. As things stand, these are at best suggestive terms, however. In previous attempts,[63,64] it was conjectured that S stood in a simple relation to K_3.

(5) Continuing with the classification started in (65), we next have:

$$\text{Electromagnetic interactions: } I_3,\ K_3 \text{ are good; } \quad \mathbf{I}^2,\ \mathbf{K}^2 \text{ are "out."} \tag{68}$$

Finally there are the $KK\pi$-interactions. To see their symmetry character, introduce a Pauli spin vector $\boldsymbol{\varrho}$ which plays the same role in K-space as does the usual $\boldsymbol{\tau}$ in I-space. Note that $\bar{K}\boldsymbol{\varrho}K$ is characterized by the representation (0,1) of the 4-group. We have

$$\bar{K}^+K^0\pi^+ + \bar{K}^0K^+\pi^- = 2^{\frac{1}{2}}\bar{K}(\rho_1\pi_1+\rho_2\pi_2)K, \tag{69}$$

and this expression is invariant for rotations in the (12)-plane, leaving the (34)-plane unchanged. That is to say, the $KK\pi$-interaction (47) belongs to the same symmetry class of the 4-group as does the electromagnetic interaction, with the one important distinction, however, that the separate conservation of I_3 and K_3 no longer holds (and the Λ- and Σ^0-mass in principle get separated):

$$KK\pi\text{-interactions: } I_3+K_3 \text{ is good; } \quad I_3,\ K_3 \text{ separately and } \mathbf{I}^2,\ \mathbf{K}^2 \text{ are "out."} \tag{70}$$

[65] A. Pais, Phys. Rev. **86**, 663 (1952), Eqs. (5) and (6); M. Gell-Mann, *Proceedings of the Sixth Annual Rochester Conference on High-Energy Physics, 1956* (Interscience Publishers, Inc., New York, 1956).

[66] R. Marshak *et al.*, Phys. Rev. **106**, 599 (1957); H. Katsumori, Progr. Theoret. Phys. Japan **18**, 375 (1957).

Of course, all interactions written down so far are in accordance with N- and S-conservation.

We conclude this section by stating two conjectures which are suggested by the present considerations, but not a necessary consequence thereof: we have recognized four symmetry classes of the 4-group, one for π-baryon, one for K-baryon, one for electromagnetic, and one for $KK\pi$-interactions. It is tempting to assume (a) that each class is characterized by a single universal coupling constant; (b) that the close relation between the electromagnetic and the $KK\pi$ symmetries on the one hand and the relation (50) on the other are no coincidence, and that in fact there exists a simple connection between f and e. These are no logical steps in the argument but there seems to be no obvious reason why one could not envisage such a situation. It is appealing in its economy of constants. The conjectures (a) and (b) are to some extent independent of each other.

It may be objected that if one accepts the conjectures, there is no strong coupling which would provide for the large split between the masses of the nucleon (M_0), the mass center of Σ and Λ (M_1), and the Ξ mass (M_2). This objection may be serious. However, it should not be forgotten that the view that the differences between M_0, M_1, and M_2 are entirely due to strong-field self energies is not compelling. It may be that we are in a mixed situation of mass displacements due to strong-field couplings and a different mechanism of mass shifts. The approximate relation

$$M_1-M_0=M_2-M_1 \tag{71}$$

may perhaps be a clue to the mechanism of a novel type of quantization. However this may be, the implications of a situation where the dynamics is satisfactory but couplings cannot account for the large baryon mass splits, would be of the utmost importance.

In the author's opinion, the mysterious μ-meson may be considered as a further possible clue to an as yet unknown mechanism of mass displacement.

In conclusion we note the following. (1) A further dynamical quantity remains to be specified, namely the Ξ-nucleon parity. (2) the ΔI-rules for weak interactions will have to be overhauled. In the present language there is no longer question of half-integer ΔI in π-decays of K-particles and hyperons. Instead one has integer ΔI and the rule $\Delta I=\pm 1$, 0 seems to be the simplest one. Note that the $KK\pi$-interaction sheds new light on the relative rate of 2π-decay of K^+ and K^0. These points will be discussed elsewhere.

VI. CONCLUDING REMARKS

The case of odd $p(K)$ appears to have several attractive features. However, as already stated in the introduction, the main purpose of this work has been and remains to point out that $p(K)$ could be even, not that it must be even. The considerations of Sec. IV lack the quantitative reliability necessary for making a stronger statement.

One may ask, however, what further experimental information could possibly lead to circumstantial evidence for odd $p(K)$. Some points in this connection have already been raised in the foregoing. We wish to summarize them here and add a few further remarks.

(1) As noted in Sec. II, evidence about a possible new type of deviation from charge independence, connected with odd $p(K)$, may come from the study of hyperon-nucleon systems. The comparison of (Σ^-, n) and (Σ^+, p) seems especially suited for this purpose.

(2) It has been proposed[31] that the absorption of K^- on deuterium and He^4 may provide tests for the validity of charge independence. As noted in Sec. II, the present picture gives identical results only for $\delta=0$. Thus deviations which are not due to electromagnetic effects may possibly occur also here. The influence of the $KK\pi$-interaction will be discussed elsewhere.

(3) It is of special interest from the present point of view to measure the low energy excitation function and the angular distribution of K-exchange scattering under circumstances where no suppression due to the exclusion principle can take place, for example in K^+-d scattering; see Sec. IV.

(4) It is anticipated that not too close to threshold the ΛK^+ and $\Sigma^0 K^+$ production amplitudes in γ-proton collisions are substantially the same. (This is conditional upon the relative unimportance of $KK\pi$-interactions in photoproduction.) See Sec. IV.

(5) It is anticipated that the Σ^0's produced in $\pi^- p$ collisions may have polarization characteristics closely similar to those of the Λ's produced in the same way; see Sec. IV.

(6) Experiment indicates[40] that the K^+-nucleon forces are repulsive. For odd $p(K)$ this does not imply the same for K^0-nucleon forces. Thus it is not inconceivable that a K^0 could be bound in nuclear matter.[67] Such K-fragments, if they at all exist, are known to be rare, to say the least. Nevertheless it will be clear that the existence of K-fragments, however relatively rare, would be of great interest in the present context.

(7) A $KK\pi$-interaction of the type considered here may lead to interesting effects concerning the relative frequencies of production of K-pairs of various charge combinations. We hope to come back later to such higher energy effects.

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[67] See also A. Pais and R. Serber, Phys. Rev. **99**, 1551 (1955).