for the cross section for associated production by protons on copper is, however, in agreement with values obtained by Baumel *et al.*,[13] Lindenbaum and Yuan.[14]

Other experiments have been performed to measure the cross section for production of heavy unstable particles by *p-p* collisions and the results are conflicting. The diffusion chamber experiment by Cool *et al.*[15] indicated extremely small or possibly zero cross sections for the production of Σ^+, Λ^0, or K^0 particles. Baumel *et al.*[13] report a cross section of between 0.1 and 0.3 mb for K^+-meson production obtained from differential cross sections measured at 0°. Lindenbaum and Yuan[14] have also observed K^+ production at 60°.

[13] Baumel, Harris, Orear, and Taylor 108, 1322 (1957).
[14] S. J. Lindenbaum and L. C. L. Yuan, Phys. Rev. 105, 1931, (1957).
[15] Cool, Morris, Rau, Thorndike, and Whittemore, Phys. Rev. 108, 1048 (1957).

ACKNOWLEDGMENTS

We are indebted to Dr. R. M. Sternheimer and Dr. R. K. Adair for helpful discussions. We also wish to thank Dr. J. Cumming for his help with the beam-monitoring foils.

PHYSICAL REVIEW VOLUME 112, NUMBER 2 OCTOBER 15, 1958

Electron-Deuteron Scattering by the Impulse Approximation*

A. GOLDBERG†
Stanford University, Stanford, California
(Received June 19, 1958)

The cross section for inelastic scattering of high-energy electrons by deuterons is calculated using the impulse approximation. The results agree with those of Jankus. The cross sections are given for several neutron charge and moment distributions. The peak cross sections are simply related to the free-nucleon cross sections with this approximation.

I

RECENT high-energy electron-deuteron scattering experiments by Hofstadter and Yearian[1] have yielded information about the neutron's electromagnetic structure. In these experiments, the electrons are scattered inelastically, disintegrating the deuteron, and the energy spectrum of the outgoing electrons is measured. The results show an inelastic peak of about 26- to 45-Mev width, centered slightly below the energy of elastic scattering from a free nucleon. The objective of this paper is to relate the peak cross sections or the area under these curves to the sum of the free proton and neutron cross sections. Since the proton cross section has been measured independently, the neutron cross section can then be determined.

Jankus[2] has calculated these cross sections by considering the Møller potential acting on the nuclear charge and current. At small momentum transfers, where the nucleons can be treated nonrelativistically, his results can be fitted to the experimental data. However, for large values of q^2, Jankus' curves show the peak at too low an energy. Blankenbecler[3] has suggested that the corrections to Jankus' results can be estimated by replacing the three-momentum transfer by the corresponding four-vector. With this modification, Hofstadter and Yearian have been able to fit the peak cross sections to their data within experimental limits.

Although Jankus' model breaks up the nuclear charge and current into separate proton and neutron terms, the electrons interact with the deuteron as a whole. Hence he obtains interference effects between the proton and the neutron. At large bombarding energies, with the electron wavelength much shorter than the separation between nucleons, the interference is expected to be small. To test the importance of these interference effects, we have used the impulse approximation to calculate the inelastic scattering cross section as a function of outgoing electron energy.

In this approximation, the nucleons are free particles, with a distribution of momenta due to the deuteron binding. The electron interacts separately with the neutron or the proton as a moving free particle. As a result, the interacting nucleon is given a large outgoing momentum, while the other is unaffected. The scattering from the two nucleons is then incoherent, and one may add the cross section for each process. This must then be averaged over the initial nucleon momenta to give the total differential cross section.

II

If we consider scattering by a free proton of initial momentum $\mathbf{p}_p$, the transition rate for the electron from the state of momentum $\mathbf{k}$ to momentum $\mathbf{k}'$ (energy k

* This research was supported in part by the U. S. Air Force through the Air Force Office of Scientific Research.
† Holder of a National Science Foundation Fellowship for the years 1955–56, 1956–57, and 1957–58.
[1] M. R. Yearian and R. Hofstadter, Phys. Rev. 110, 552 (1958).
[2] V. Jankus, Phys. Rev. 102, 1586 (1956).
[3] R. Blankenbecler, Phys. Rev. 111, 1684 (1958).

and k') is

$$Wd^3k'=\frac{\langle|K|^2\rangle_{\text{Av}}}{(2\pi)^2}\delta(E_p+k-E_p'-k')\frac{m}{k'}\frac{M}{E_p'}d^3k'.$$

E_p and E_p' are the initial and final proton energies,

$$E_p=(M^2+\mathbf{p}_p^2)^{\frac{1}{2}},\quad E_p'=(M^2+(\mathbf{p}_p+\mathbf{q})^2)^{\frac{1}{2}}.$$

$\langle|K|^2\rangle_{\text{Av}}$ is the square of the transition matrix element, which has been summed and averaged over initial and final spin states, and also summed over final proton momenta, to give

$$\langle|K|^2\rangle_{\text{Av}}=\frac{(2\pi)^2}{m^2M^2}e^4\frac{1}{q^4}\Big\{F_{1p}^2[M^2q^2+2(k\cdot p_p)^2+2(k'\cdot p_p)^2]$$
$$-\frac{q^2}{2M^2}(\kappa_pF_{2p})^2\left[(k\cdot p_p)^2+(k'\cdot p_p)^2-\frac{M^2q^2}{2}+q\cdot p_p\frac{q^2}{2}\right]$$
$$+q^4F_{1p}\kappa_pF_{2p}\Big\}.$$

$k\cdot p_p$ and $k'\cdot p_p$ are the invariant four-products,

$$k\cdot p_p=kE_p-\mathbf{k}\cdot\mathbf{p}_p,\quad k'\cdot p_p=k'E_p-\mathbf{k}'\cdot\mathbf{p}_p.$$

F_{1p} and F_{2p} are the proton electric and magnetic form factors, and are functions only of the invariant q^2.

This must now be averaged over the initial proton momentum states. Since the neutron and proton masses are equal, the relative and center-of-mass momenta of the deuteron are

$$\mathbf{p}=\tfrac{1}{2}(\mathbf{p}_1-\mathbf{p}_n),\quad \mathbf{P}=\mathbf{p}_p+\mathbf{p}_n.$$

Taking the deuteron at rest, $P=0$, $p_p=-p_n=p$. Hence the probability of finding the proton with momentum p_p is $|\phi(\mathbf{p})|^2$ where $\phi(\mathbf{p})$ is the deuteron wave function in momentum space,

$$\phi(\mathbf{p})=\frac{1}{(2\pi)^{\frac{3}{2}}}\int d^3r\, e^{-i\mathbf{p}\cdot\mathbf{r}}\psi(\mathbf{r}),$$

where $\mathbf{r}$ is the relative coordinate. $\phi(\mathbf{p})$ is normalized to unit volume, while in the matrix element the plane wave states are normalized to volume $(2\pi)^3E/M$. The initial density of states is then $|\phi(\mathbf{p})|^2(M/E)d^3p$.

Assuming a spherically symmetric wave function, and doing the angular integrals over d^3p, we find that the cross section for electrons scattering into solid angle $d\Omega$ and energy interval dk' is

$$\frac{d^2\sigma}{d\Omega dk'}=e^4\frac{k'}{k}\frac{(I_1+I_2)}{q^4},$$

$$I_1=\frac{2\pi}{|\mathbf{q}|}\frac{q^2}{4}\Big\{4M^2F_{1p}^2+q^2\left(1+\frac{q^2}{2M^2}\right)\times(K_pF_{2p})^2+4q^2F_{1p}\kappa_pF_{2p}\Big\}\mathcal{I}_3,\quad (1)$$

$$I_2=\frac{2\pi}{|\mathbf{q}|}\Big\{F_{1p}^2-\frac{q^2}{4M^2}(\kappa_pF_{2p})^2\Big\}\{T_1\mathcal{I}_1+T_2\mathcal{I}_2+T_3\mathcal{I}_3\}.$$

The quantities T_1, T_2, and T_3 are given by

$$T_1=\frac{2\mathbf{q}^2-q^2}{\mathbf{q}^2}(k^2+k'^2)-\frac{4q_0}{\mathbf{q}^2}[k\mathbf{k}\cdot\mathbf{q}+k'\mathbf{k}'\cdot\mathbf{q}]+\frac{2q_0^2+q^2}{\mathbf{q}^4}[(\mathbf{k}\cdot\mathbf{q})^2+(\mathbf{k}'\cdot\mathbf{q})^2],$$

$$T_2=-q_0\frac{q^2}{\mathbf{q}^2}(k^2+k'^2)-\frac{2q^2}{\mathbf{q}^2}[k\mathbf{k}\cdot\mathbf{q}+k'\mathbf{k}'\cdot\mathbf{q}]+\frac{3q_0q^2}{\mathbf{q}^4}[(\mathbf{k}\cdot\mathbf{q})^2+(\mathbf{k}'\cdot\mathbf{q})^2],$$

$$T_3=-\frac{(M^2\mathbf{q}^2+\frac{1}{4}q^4)}{\mathbf{q}^2}(k^2+k'^2)+\frac{(M^2\mathbf{q}^2+\frac{3}{4}q^4)}{\mathbf{q}^4}\times[(\mathbf{k}\cdot\mathbf{q})^2+(\mathbf{k}'\cdot\mathbf{q})^2].$$

$\mathcal{I}_1$, $\mathcal{I}_2$, and $\mathcal{I}_3$ are integrals over the deuteron wave function,

$$\mathcal{I}_1=\int_{-\frac{1}{2}q_0+\frac{1}{2}[\mathbf{q}^2(1-4M^2/q^2)]^{\frac{1}{2}}}^{\infty}E^2|\phi(p)|^2dE,$$

$$\mathcal{I}_2=\int_{-\frac{1}{2}q_0+\frac{1}{2}[\mathbf{q}^2(1-4M^2/q^2)]^{\frac{1}{2}}}^{\infty}E|\phi(p)|^2dE,$$

$$\mathcal{I}_3=\int_{-\frac{1}{2}q_0+\frac{1}{2}[\mathbf{q}^2(1-4M^2/q^2)]^{\frac{1}{2}}}^{\infty}|\phi(p)|^2dE.$$

Equation (1) gives the cross section for scattering by the proton only. Since the neutron differs only in its form factors, it gives an identical term and the total cross section is the sum of these two. Hence in I_1 and I_2, F_{1p}^2, $F_{1p}K_pF_{2p}$, and $(K_pF_{2p})^2$ are replaced by

$$F_1^2=F_{1p}^2+F_{1n}^2,$$
$$F_1KF_2=F_{1p}K_pF_{2p}+F_{1n}K_nF_{2n},$$
$$(KF_2)^2=(K_pF_{2p})^2+(K_nF_{2n})^2.$$

Equation (1) then gives the total cross section for scattering by the deuteron.

It can be shown that the lower limit for the integrals $\mathcal{I}_1$, $\mathcal{I}_2$, and $\mathcal{I}_3$ is greater than or equal to M. Hence the peak cross section occurs when these integrals extend over their maximum range, from M to ∞. In this case, one has

$$k'=k/[1+2(k/M)\sin^2(\tfrac{1}{2}\theta)],$$

that is, the peak energy should occur at the energy of elastic scattering by a free nucleon. Experimentally, the peak is about 2 Mev below this, probably due to the deuteron binding energy. To account for this, we must note that the energy available to the nucleon after the collision is decreased by the deuteron binding energy ϵ. The energy of the peak is then approximately

$$k'=\frac{k}{1+(2k/M)\sin^2\frac{1}{2}\theta}-\frac{k}{M}\frac{\epsilon}{1+(2k/M)\sin^2\frac{1}{2}\theta}.$$

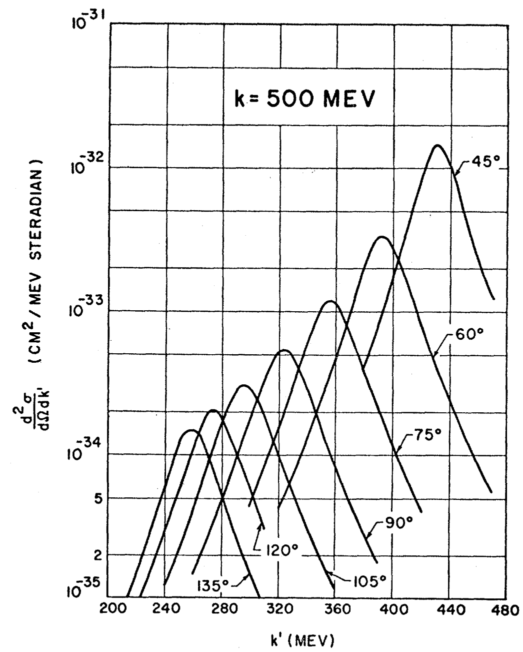

FIG. 1. Each curve is the cross section for 500-Mev electrons at the angle shown *vs* outgoing energy.

This is roughly equivalent to considering the mass of the nucleon to be decreased by some average potential.[4] For the purposes of computation, we have taken the nucleons to be in an attractive well of average depth 20 Mev. The major effect of this is to shift the curve down to the right peak energy. The peak heights are raised by approximately 5%.

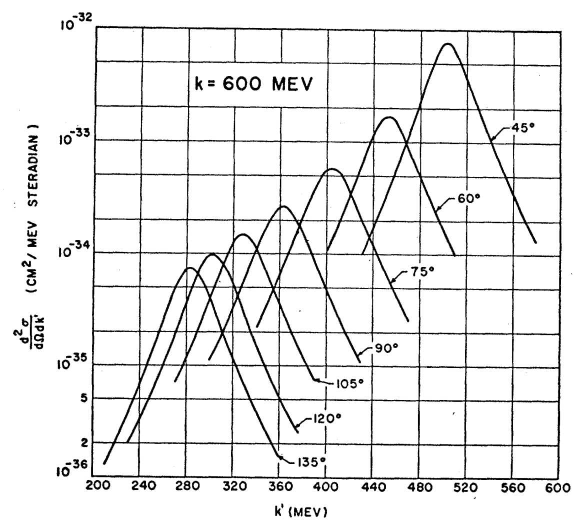

FIG. 2. Each curve is the cross section for 600-Mev electrons at the angle shown *vs* outgoing energy.

[4] R. Blankenbecler; the theoretical consequences of this are discussed in a paper to be published shortly.

III

The proton form factors are known from electron-proton scattering[5] to be[6]

$$F_{1p}=F_{2p}=\alpha^4/(\alpha^2-q^2)^2, \quad \alpha=4.32 \text{ fermi}^{-1} \quad (1 \text{ fermi}=10^{-13} \text{ cm}).$$

This corresponds to a charge distribution of mean square radius 0.80 fermi. The neutron electric form factor must satisfy the requirement that the total charge and mean square radius be zero.[7] Elastic electron-deuteron scattering data[8] are consistent with zero neutron charge distribution, i.e., $F_{1n}=0$. Hofstadter and Yearian also fit their data to Jankus' model with $F_{1n}=0$, finding the magnetic form factor equal to that of the proton. With these choices, we have computed

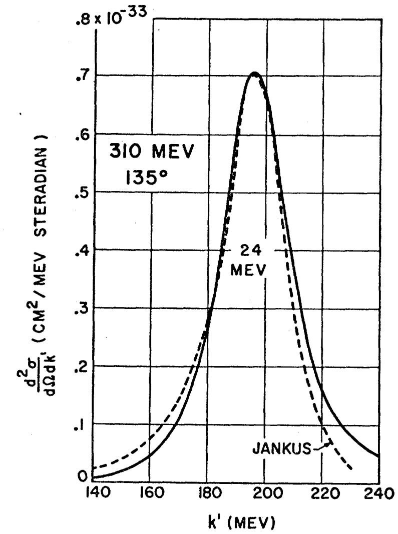

FIG. 3. Cross section at 310-Mev incident energy and scattering angle of 135° *vs* outgoing energies. (The dashed-line curve is that based on Jankus' model.)

the inelastic cross sections at electron energies of 500 and 600 Mev from 45° to 135° scattering angle. We have used the same deuteron wave function as Jankus, the Hulthén wave function

$$\psi(r)=\left[\frac{\gamma_1\gamma_2(\gamma_1+\gamma_2)}{2\pi(\gamma_1-\gamma_2)^2}\right]^{\frac{1}{2}}\left(\frac{e^{-\gamma_1 r}-e^{-\gamma_2 r}}{r}\right),$$

$$\phi(p)=\left[\frac{\gamma_1\gamma_2(\gamma_1+\gamma_2)}{\pi^2(\gamma_1-\gamma_2)^2}\right]^{\frac{1}{2}}\left\{\frac{1}{p^2+\gamma_1^2}-\frac{1}{p^2+\gamma_2^2}\right\},$$

$$\gamma_1=2.316\times10^{12} \text{ cm}^{-1}=46.03 \text{ Mev}, \quad \gamma_2=6.21\gamma_1.$$

The results are shown in Figs. 1 and 2. Each curve gives the outgoing electron energy spectrum for given

[5] E. E. Chambers and R. Hofstadter, Phys. Rev. **103**, 1454 (1956).
[6] In this metric q^2 is negative.
[7] L. Foldy, Phys. Rev. **87**, 693 (1952).
[8] J. McIntyre and S. Dhar, Phys. Rev. **106**, 1074 (1957).

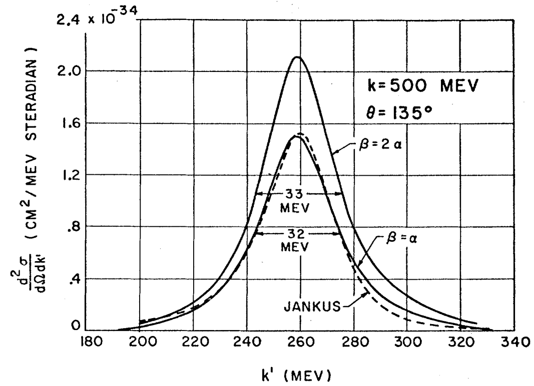

FIG. 4. Cross section at 500-Mev incident energy and scattering angle of 135° *vs* outgoing energies. (The dashed-line curve is that based on Jankus' model.) The $\beta=2\alpha$ curve shows the effect of Schiff's form factor (see below) over the whole spectrum.

incident energy and scattering angle. In Figs. 3, 4 and 5, several curves are compared with those of Jankus.[9] The agreement is typical of all the results. At large angles greater than 75°, the peak heights agree within 2%. At smaller angles, where the impulse approximation is expected to break down, they disagree by about 5%. The half-widths seem to agree throughout. The largest discrepancies are on the wings of the curves. At small energy transfer (the high sides of the curves), Jankus' cross sections go to zero more rapidly, while at large energy transfer, they extend to a longer tail. The choice of zero neutronelectric form factor is, however, not required by the experiments. To give a charge distribution to the neutron, Schiff[10] has suggested the form factor

$$F_{1n}=-\frac{\alpha^4}{(\alpha^2-q^2)^2}+\frac{\beta^6/\alpha^2}{(\beta^2-q^2)^2}+1-\frac{\beta^2}{\alpha^2}.$$

This gives both zero charge and zero mean square radius. With $\beta=\alpha$, $F_{1n}=0$ while with $\beta=2\alpha$, F_{1n} corresponds to a charge density at large distances similar to that of the proton (although opposite in sign). To test the effect of this, we have also computed the in-

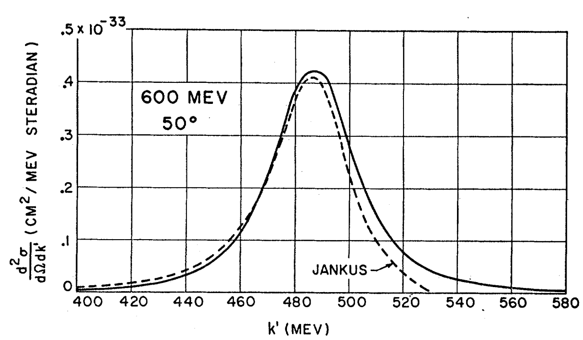

FIG. 5. Cross section at 600-Mev incident energy and scattering angle of 50° *vs* outgoing energies. (The dashed-line curve is that based on Jankus' model.)

[9] R. Herman (to be published).
[10] L. I. Schiff, Revs. Modern Phys. 30, 462 (1958).

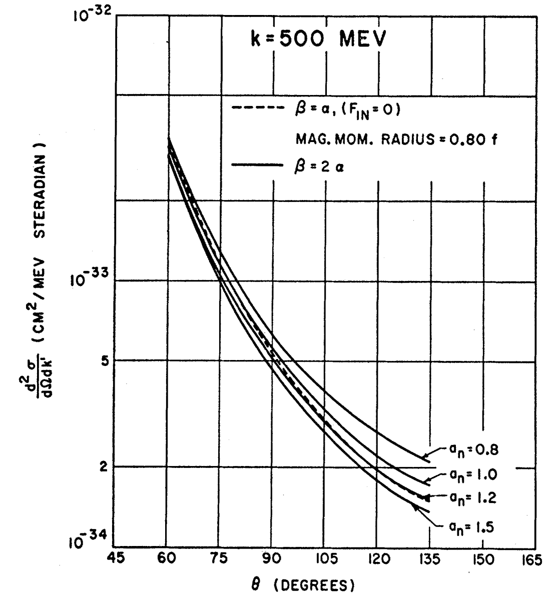

FIG. 6. Peak cross section *vs* scattering angles at 500-Mev incident energy. (The dashed-line curve is for $F_{1n}=0$, $F_{2n}=F_{1p}$. The solid-line curves are for $\beta=2\alpha$, with various neutron moment radii a_n.)

elastic cross sections with this F_{1n}, using $\beta=2\alpha$. The magnetic form factor F_{1n} is of the same form as above. However, several moment radii, $a_n=0.80$, 1.00, 1.20, and 1.50 fermi, were used. The peak cross sections are plotted *versus* scattering angle in Figs. 6 and 7. At

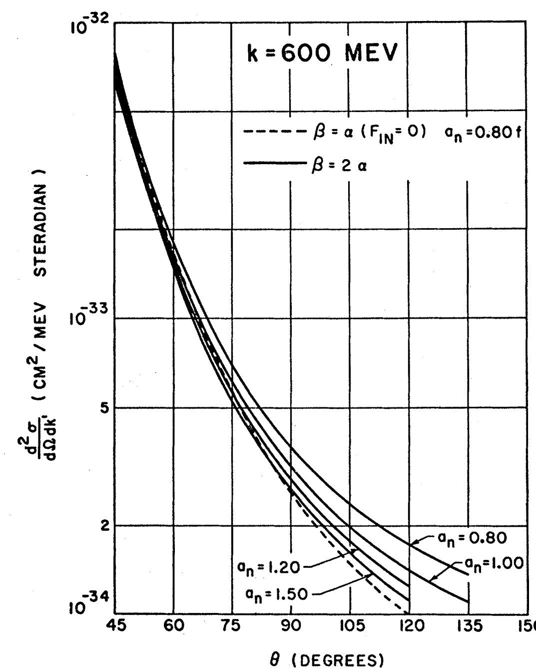

FIG. 7. Peak cross sections *vs* scattering angles at 600-Mev incident energy. (The dashed-line curve is for $F_{1n}=0$, $F_{2n}=F_{1p}$. The solid-line curves are for $\beta=2\alpha$, with various neutron moment radii a_n.)

small angles there is little effect (~5% variations). At large angles, F_{1n} increases the cross sections considerably, and as is seen in the 600-Mev curve, a_n would have to be unreasonably large to bring the curves down.

Since the impulse approximation and Jankus' cross sections agree over the major parts of the curves, the interference effects, as expected, are small.

We may also note that with this approximation, the peak cross sections may be simply related to the free proton and neutron cross sections, σ_p and σ_n. At the peak the integrals $\mathscr{I}_1$, $\mathscr{I}_2$, and $\mathscr{I}_3$ become expectation values over the deuteron,

$$\mathscr{I}_1=(1/4\pi)\langle E/P\rangle,\ \mathscr{I}_2=(1/4\pi)\langle 1/P\rangle,\ \mathscr{I}_3=(1/4\pi)\langle 1/EP\rangle;$$

E is the total energy, including the nucleon rest energy. Therefore replacing E by M should introduce only a small error. For the Hulthén wave function above, this error is about 1% for both $\mathscr{I}_1$ and $\mathscr{I}_3$. Taking $E=M$, the peak cross section becomes

$$\frac{d^2\sigma}{d\Omega dk'}=\tfrac{1}{2}M\langle 1/P\rangle\frac{1}{|\mathbf{q}|}\frac{k}{k'}(\sigma_p+\sigma_n).$$

σ_p and σ_n are given by the Rosenbluth formula,[11]

$$\sigma_p=\frac{e^4\cos^2(\frac{1}{2}\theta)}{4k^2\sin^4(\frac{1}{2}\theta)[1+2(k/M)\sin^2(\frac{1}{2}\theta)]}\times\left\{F_{1p}^2-\frac{q^2}{4M^2}\{2[F_{1p}+(K_pF_{2p})^2]\times\tan^2(\tfrac{1}{2}\theta)+(K_pF_{2p})^2\}\right\}.$$

The only deuteron dependence here is as a constant multiplicative factor $\langle 1/P\rangle$. At the energies and angles plotted above, this agrees with the complete expression to within 3%.

ACKNOWLEDGMENTS

We wish to thank Professor L. I. Schiff for suggesting this calculation and for helpful discussions. Also we are grateful to Professor R. Hofstadter, Dr. R. Blankenbecler, Dr. M. R. Yearian, S. E. Sobottka, and L. N. Hand for several discussions. Lastly, we are indebted to Dr. R. Herman for allowing us to use his computations of Jankus' cross sections.

[11] M. N. Rosenbluth, Phys. Rev. 79, 615 (1950).

PHYSICAL REVIEW VOLUME 112, NUMBER 2 OCTOBER 15, 1958

Negative K-Meson Reactions with Protons: Masses of Charged Σ Hyperons and the Negative K Meson*

WALTER H. BARKAS, JOHN N. DYER, PETER C. GILES, HARRY H. HECKMAN, CONRAD J. MASON, NORRIS A. NICKOLS, AND FRANCES M. SMITH
Radiation Laboratory, University of California, Berkeley, California

(Received June 30, 1958)

New measurements of the masses of the charged Σ hyperons and the negative K meson are reported. The results obtained are $M_{\Sigma^+}=1189.3\pm0.3$ Mev, $M_{\Sigma^-}=1195.8\pm0.5$ Mev, and $M_{K^-}=493.87\pm0.46$ Mev. No evidence for more than one K^--meson mass was found. The terminal behavior of the Σ^- hyperons was also studied.

In continuation of our program of precise mass measurements, we have obtained some improved data on the reactions of negative K mesons with protons in which charged hyperons are formed. Measurements of this sort are best made in calibrated nuclear-track emulsions, and for good statistics one requires a large sample of K-meson capture events. Preliminary results of an effort to improve the K-meson beam from the Bevatron were reported earlier.[1] This beam has been further exploited, and a new type of beam developed by Murray has also been successfully employed.[2] We have taken data from three emulsion stacks, two of Ilford-G.5 emulsion and one of K.5, all carefully calibrated with respect to density, and have adhered to our previously described conventions for range measurements.[3]

About 4700 negative K-meson interactions at rest were examined, and 20 events were found to consist of colinear pion and hyperon tracks with a clean common

* This work was performed under the auspices of the U. S. Atomic Energy Commission.

[1] Barkas, Dudziak, Giles, Heckman, Inman, Mason, Nickols, and Smith, Phys. Rev. 105, 1417 (1957).

[2] Joseph J. Murray, Bull. Am. Phys. Soc. Ser. II, 3, 175 (1958); and Horwitz, Murray, Ross, and Tripp, University of California Radiation Laboratory Report UCRL-8269 (unpublished).

[3] Barkas, Barrett, Cuer, Heckman, Smith, and Ticho, Nuovo cimento 8, 185 (1958).