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Magnetostatic Modes in Ferrimagnetic Spheres*

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Ferrimagnetic resonance experiments (9000 Mc/sec, 300°K) have been performed using single-crystal spheres of yttrium iron garnet. Rf magnetic fields of several different configurations were used to excite specific magnetostatic modes. A mounting technique was employed such as to eliminate dielectric inhomogeneities near the sample. Such inhomogeneities arise from the usual quartz or sapphire mounting rods. They have been observed to be responsible for the excitation of more complicated modes than would be expected from the unperturbed cavity fields. It has been found that supposedly spherical specimens produced by the usual tumbling procedures deviate slightly from true sphericity. In these experiments highly polished truly spherical samples were used. With these conditions there is no ambiguity about the identity of the mode representing the spacially uniform precession of the entire spin system. The spacings of the various modes do not vary with the crystal direction along which H_{dc} is applied. These results contrast with those of White and Solt on manganese ferrite in several respects, viz. the clear identity of the uniform precession and the constant mode spacing for the various crystal directions.

WHITE, Solt, and Mercereau,[1] working with manganese ferrite spheres, reported a number of absorption peaks in ferrimagnetic resonance experiments. This multiplicity of absorption maxima was excited principally by placing the specimen at points in the microwave cavity other than the position of uniform rf magnetic field. Dillon[2] described the observation of these magnetostatic modes in thin disks of manganese ferrite. The use of transverse rf magnetic fields of several different symmetries was found to excite sets of modes. In the case of the disks these fell in well-defined series and the identification was relatively straightforward. Walker's[3] solution of the boundary-value problem appeared to give us an understanding of the magnetostatic modes for situations in which the propagation of radiation through the specimen did not have to be considered. This was essentially a restriction to small samples. The theory described modes which should vary in relative spacing with frequency, magnetization, and sample shape. The anisotropy would be expected to move the mode patterns up or down without changing the relative spacings in those cases in which the steady field was along one of the principal crystallographic directions.

After the initial reports and the apparent agreement between theory and experiment, White and Solt[4,5] made observations apparently in disagreement with the theory. They found that in the case of spheres, at least, the mode spacings seemed to vary with the crystal direction. A theoretical[6] explanation of this effect was proposed. Further they found that there was considerable ambiguity about the identity of the mode which represented the spacially uniform precession of the entire spin system. There seemed to be several modes near the uniform precession. These were found to vary in relative intensity and spacing not only with crystal direction, but also with frequency and temperature. It is believed that the experiments reported in this paper clear up the question of the variation of spacing with crystal direction. They also have relevance to the ambiguity of the uniform mode.

Microwave absorption was measured by placing the sample at the desired point in a transmission cavity which was mounted between the pole pieces of a magnet. A stabilized oscillator was used, and direct detection was employed. The cavities were mounted on a rocking mechanism which could rotate about a vertical or a horizontal axis perpendicular to the field. The angular settings about either of these axes could be made to one minute. The cavities were rectangular, one operated in the TE_{012} mode and the other in the TE_{022} mode. Both cavities could be opened in such a way that no current flowed cross the seam.

The samples were located at special positions in the cavities in order to subject the sample to different rf magnetic field configurations. Note that only the components of the rf field in the plane perpendicular to the steady field are effective in exciting precessional modes of the spin system. In Fig. 1 is shown a drawing of the TE_{022} cavity with an indication of the magnetic field. The three special positions are designated O, I, and X. If the sphere is placed at the point I, and a steady field is applied along the axis y [case (e)], the exciting field is essentially uniform over the sample. In practice the absorption in this case is so great for the spheres used that we cannot apply the steady field along the axis without completely destroying the Q of the cavity for hundreds of oersteds on each side of the resonance. It is sufficient to have a small component of the steady field along the axis. If the sample is placed

* The results given here were described in a paper delivered to the American Physical Society April 2, 1958 [J. F. Dillon, Jr., Bull. Am. Phys. Soc. Ser. II, **3**, 194 (1958)].

[1] White, Solt, and Mercereau, Bull. Am. Phys. Soc. Ser. II, **1**, 12 (1956); R. L. White and I. H. Solt, Phys. Rev. **104**, 56 (1956).

[2] J. F. Dillon, Jr., Bull. Am. Phys. Soc. Ser. II, **1**, 125 (1956).

[3] L. R. Walker, Bull. Am. Phys. Soc. Ser. II, **1**, 125 (1956); L. R. Walker, Phys. Rev. **105**, 390 (1957).

[4] I. H. Solt, Jr., and R. L. White, Bull. Am. Phys. Soc. Ser. II, **2**, 22 (1957).

[5] Solt, White, and Mercereau, J. Appl. Phys. **29**, 324 (1958).

[6] R. L. White and I. H. Solt, Jr., Bull. Am. Phys. Soc. Ser. II, **2**, 22 (1957).

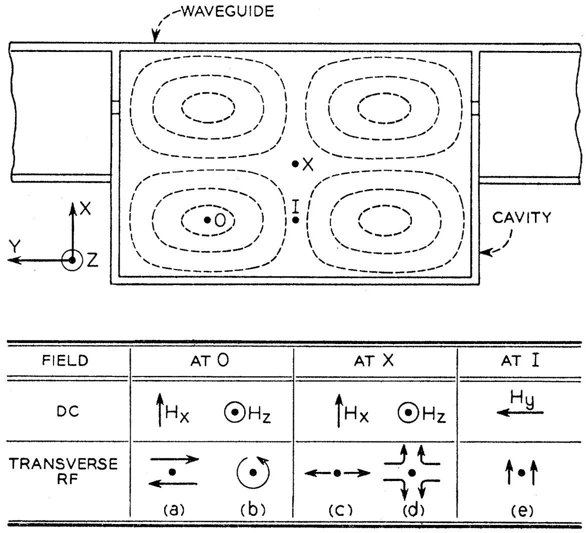

Fig. 1. Drawing of TE_{022} cavity showing the transverse rf fields available at various points in the cavity.

at O with the steady field along the x or y axes, it experiences a transverse rf field which is of the type (a). This contrasts with the transverse field seen by the sample at O when the dc field lies along z, case (b). If, on the other hand, the sample is placed at X, there are two different configurations of the exciting field available depending on whether the steady field lies along y or z. Obviously the fields at points other than these special positions, or for H_{dc} along a general direction, may be considered as a linear combination of the fields at these and other similar special positions. We can use these various special field configurations to excite a few appropriate modes. Having associated particular modes with field configurations, we can hope to use the symmetry of the exciting field in the identification of the magnetostatic modes.

SAMPLE PREPARATION

In general the hardness of a crystal is an anisotropic property. Spheres made by the tumbling procedure[7] have apparently been used by all those who have done ferrimagnetic resonance experiments up to this time. A careful examination of samples made by this procedure shows that they depart very slightly from sphericity. This departure is a consequence of the anisotropy of hardness. In the case of the ferrimagnetic garnets the derivation from a perfect sphere is very slight, and must be consistent with the cubic symmetry of the crystal structure. These two factors presumably account for the failure to recognize the deviation from sphericity before. The actual amount of the deviation would certainly vary with the abrasive used. Thus an abrasive with a hardness only slightly greater than that of the sample would surely produce much more strongly perturbed spheres than an abrasive very much harder than the sample. A 1.10-mm "sphere" was ground against a diamond abrasive, and polished against emery. When examined in an optical comparator variations in the radius of about one part in seventy-five were observed. Aside from this observation, no attempt has been made to correlate the deviation from sphericity with some of the anisotropic effects which have been observed. The point being made is that the most carefully prepared samples do not show these anisotropic effects.

Bond[8] described another method of making spheres which grinds only those parts of the crystal projecting above some mean radius. This overrides the effect of the hardness anisotropy and produces considerably better approximations to spheres than the tumbling process. In this alternate procedure the sample is held between the ends of two tubes, one of which is rotating about its axis. The other is moved around in angle, always maintaining enough pressure on the sample to keep it in place. An abrasive slurry kept on the sample does the actual grinding. Our procedure has consisted in grinding with emery (American Optical Company $303\frac{1}{2}$), then polishing with Linde A Alumina. With care it is possible to produce an almost scratch-free surface. The spheres made by this method have ranged down to about 0.05-cm diameter. The tubes used were machined from a fiber-impregnated plastic.

MOUNTING TECHNIQUE

Ferrimagnetic resonance experiments are usually performed with the sample affixed to the end of a quartz, polystyrene, or sapphire rod. This is convenient, since if the [110] axis lies along the axis of the rod, and the axis of the rod is perpendicular to the magnetic field, one may observe the resonance with the field along each of the principal crystal directions merely by

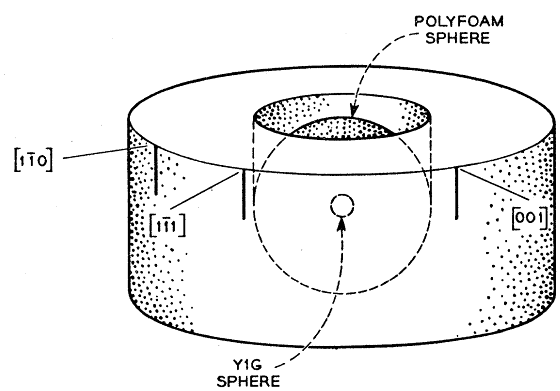

Fig. 2. In order to eliminate dielectric inhomogeneities at the sample, the yttrium iron garnet sphere was introduced into the cavity in the center of this Polyfoam mounting block. First the sample was pressed into the center of a Polyfoam sphere. By x-ray goniometer techniques, that sphere was then pressed into the outer Polyfoam ring in such a way that (110) was parallel to the end faces of the ring.

[7] W. L. Bond, Rev. Sci. Instr. **22**, 344 (1951).

[8] W. L. Bond, Rev. Sci. Instr. **25**, 401 (1954).

rotating the rod. In these experiments it was found that the distortion of the rf field produced by the dielectric rod supporting the specimen resulted in the excitation of many extra modes. This point will be illustrated later. The simplest rf field configurations would be obtained with the sphere suspended in space within the cavity. This effect was achieved by mounting the sphere as shown in Fig. 2. The sphere was pressed into the center of a Polyfoam sphere. The Polyfoam sphere was placed in a Polyfoam ring which was placed in the cavity at the desired position. The garnet sphere imbedded within the Polyfoam sphere was crystallographically oriented with an x-ray goniometer. The Polyfoam sphere was placed within the ring so that the [110] axis of the garnet sphere was parallel to the axis of the ring. Finally, the principal directions in the (110) plane were marked about the edge of the ring. This mounting technique allowed us to introduce a sphere with a known orientation into a cavity without significant dielectric inhomogeneity. At least for some rf field configurations, H_{dc} may be made parallel to the principal crystal axes. It precludes a measurement of the temperature of the sphere.

All the absorption traces shown in this paper were taken with a 1.10-mm diameter polished sphere mounted as just described.

MAGNETOSTATIC MODES

Figure 3 shows the absorption encountered in rf exciting fields of the various symmetries available in our rectangular cavities. These figures are traces of the actual experimental recordings. In order to facilitate the comparison of these curves, the horizontal axes have been shifted so the positions of the uniform mode are in register. Note that the field axes are not quite linear. The uniform resonance occurs at various fields in the various traces because of orientation differences, and frequency differences. In cases (b) and (e) the uniform rf magnetic field component at a point could be eliminated by very careful positioning of the specimen. In cases (b) and (e) it could also be eliminated by making the steady field parallel to the uniform rf field. This characteristic could be used to identify it. For the purpose of this figure, the uniform precession was desirable as a reference position, so it was adjusted to some reasonable depth of absorption.

In Fig. 3(a) the absorption of the uniform precession can be reduced at wlll without affecting the height of the two small maxima above it in field. Thus they apparently arise from some residual field inhomogeneity. This may well be a propagation effect.

In Fig. 3(b) there are two principal modes excited, one at 53 oe below the uniform precession, the other about 130 oe below. But in 3(c) and (d) we see that the lower of these two peaks can be associated with the hyperbola-like rf field configuration shown in Fig. 1(d). The other is excited by the circular rf field shown in

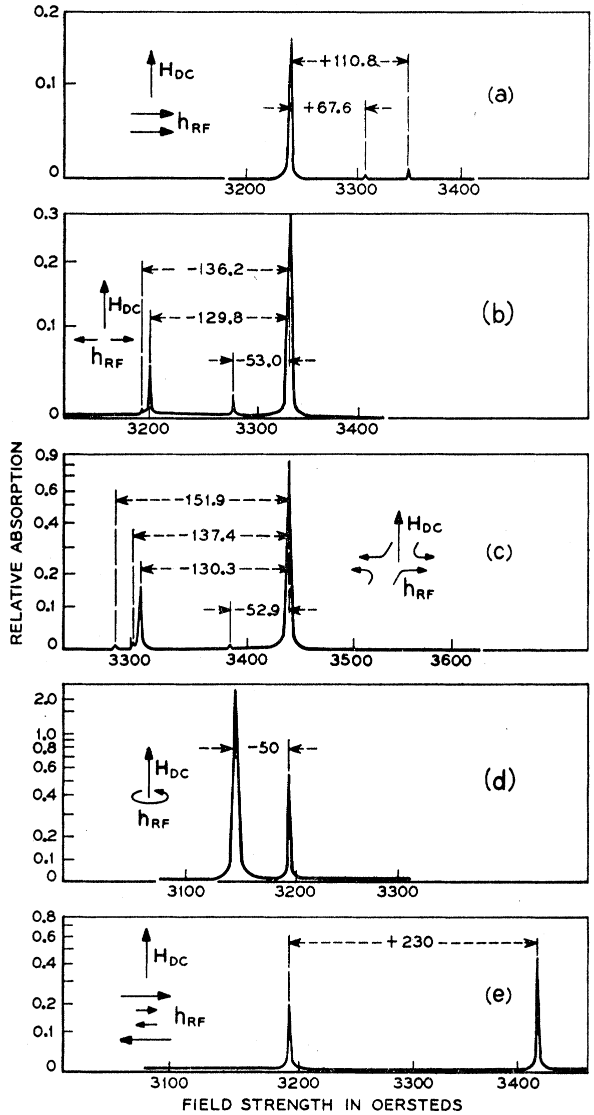

FIG. 3. Traces showing the different absorption patterns obtained with rf magnetic fields of the various symmetries available in a TE_{022} cavity. The horizontal axes have been shifted so the positions of the uniform precession on all five traces are in register. Note the nonlinear vertical axes, and the fact that the horizontal axis is not quite linear.

Fig. 1(b). Walker has given an expression for the magnetic potential Ψ for some of the magnetostatic modes. For the mode (2,0,1), Ψ contains x and y, the coordinates in the azimuthal plane, only in the form x^2+y^2. The correspondence between the symmetry of the rf field and the magnetic potential of the mode suggests strongly that the mode at -53 is (2,0,1).

Similarly it is believed the mode at -130 oe is (4,2,0). In this case the magnetic potential Ψ contains the term x^2-y^2, and thus has the same symmetry as the hyperbolic rf field which appears to excite this mode. Referring now to Fig. 3(e), we see in addition to the uniform mode another at about $+230$ oe. This is

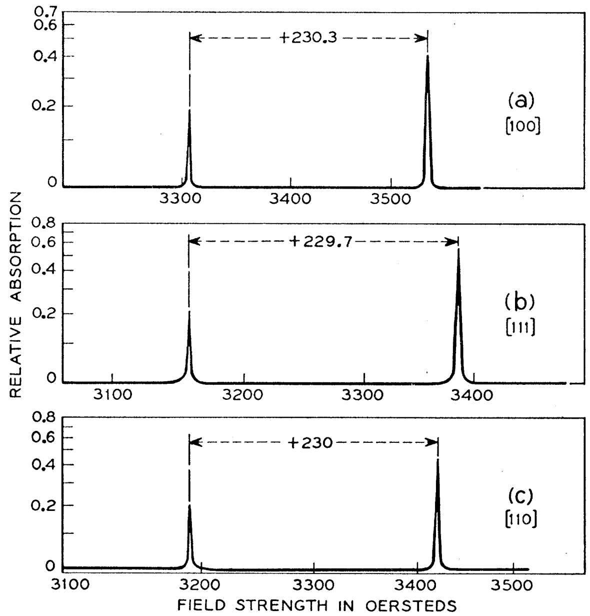

FIG. 4. With the sphere in an rf field having the symmetry of Fig. 1(a) plus a small uniform component, traces were taken with the steady field along each of the principal crystal directions. The field intervals between the modes (1,1,0) and (2,1,0) are seen to vary by only a fraction of a percent. Most of this variation was associated with slight changes in the temperature of the specimen. Frequency 9000 Mc/sec.

excited in a field which is odd along z. The mode (2,1,0) is the simplest to have this symmetry. Walker's theory predicts that it will lie $(2/15)4\pi M$ above the uniform precession at all frequencies. In this case a value of $4\pi M=1690$ gauss at 300°K was derived from Gilleo's[9] data. This would lead us to predict a spacing of 225 oe. This is considerably better agreement than the knowledge of the temperature justifies. Apparently, one could use the spacing between (1,1,0) and (2,1,0) as a microwave measure of the magnetization. Thus in observations of line width or anisotropy over a wide temperature range, the magnetization could be measured conveniently at each temperature point.

The identification of modes is largely beyond the scope of this study, and it was not pursued far beyond the few cases just given. Similarly, the question of the displacement of various modes from their theoretical position with frequency and sample size has not been investigated.

MODE SPACING *VS* CRYSTAL DIRECTION

The workers at the Hughes Research Laboratories reported that the mode spacings differ when the steady field is applied along different crystal directions. Walker's treatment of the problem contains no mention of the anisotropy field. As the theory now stands, the anisotropy could only be introduced as an additive field for the case of the steady field along the principal directions. In the first instance we performed experiments on spheres ground by the tumbling procedure, and polished as carefully as possible. They showed this anisotropy of mode spacing. For instance, the spacing between the modes (1,1,0) and (2,1,0) varied by 15 oe as the field was applied along the various crystal directions. Corresponding measurements made on spheres produced by the two-pipe procedure show no variation of mode spacing with crystal direction. In Fig. 4 are shown actual traces of the absorption with the steady field along each of the principal crystallographic directions. In each case the sphere is at the same position in the cavity, namely very close to the point O with the steady field in the xy plane, case (a) of Fig. 1. The actual angle with the y axis is varied slightly to adjust the height of the uniform mode. As noted elsewhere, the temperature of the specimen is a variable which our mounting procedure allows us neither to control nor to measure. During the course of a day's work it was observed that the mode spacings drifted slightly. It was found that we could attribute this drift to the change in the magnetization with temperature.

In those cases in which the measurements with the steady field along different crystal directions were taken in a very short time interval, the mode spacings were the same within the accuracy of our field measurements. Mode spacings are accurate to ± 0.2 oe.

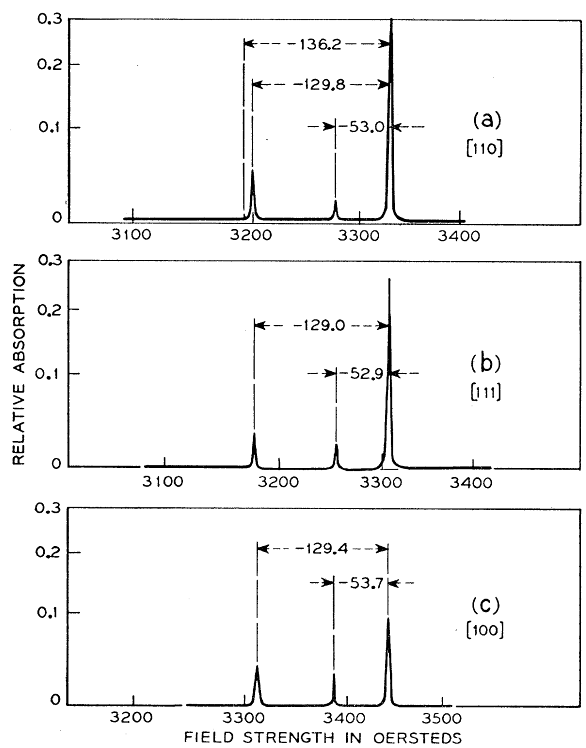

FIG. 5. In this case the rf field was like Fig. 1(c) plus a small uniform component. It is again seen that the spacings of these modes are essentially the same with the steady field along each of the principal crystal directions. Frequency 9395 Mc/sec.

[9] M. A. Gilleo (private communication).

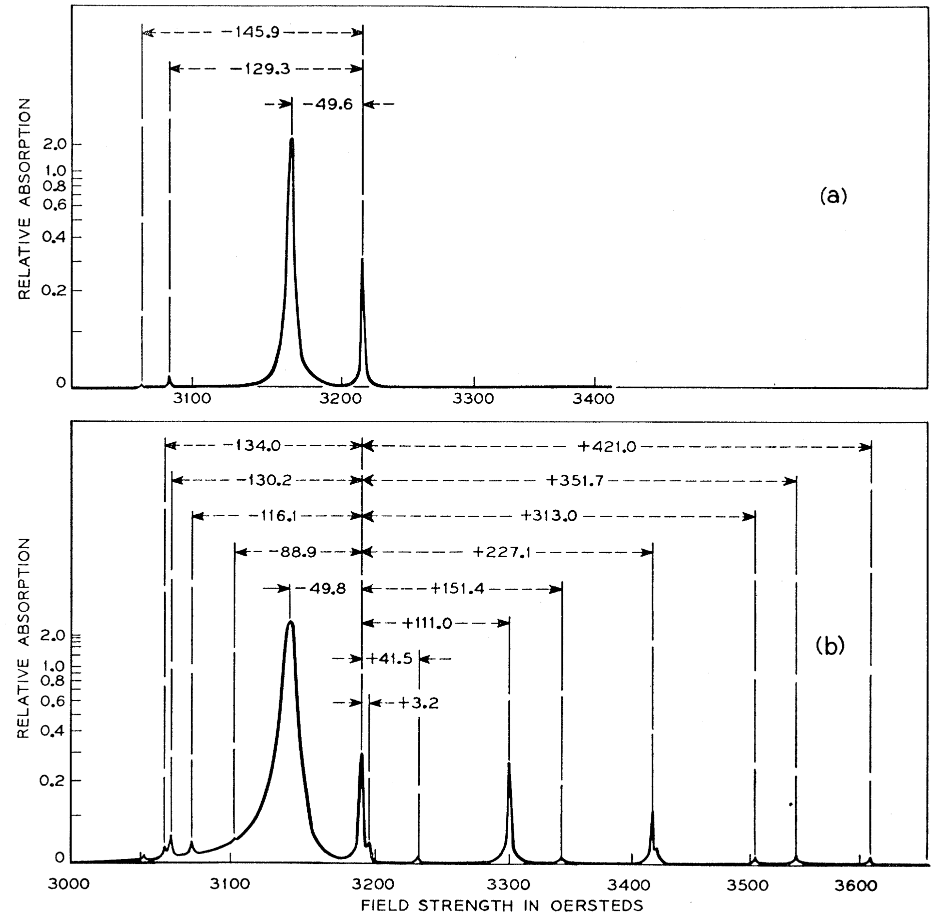

FIG. 6. Both of these traces were taken with the rf field of Fig. 1(b). In (a) the sphere was mounted in its Polyfoam holder. In (b) a sapphire rod 0.050 in. with its axis parallel to the z axis touched the sphere. The presence of a mounting rod was also found to alter the absorption structure seen with the sample at the other special points in the cavity. Frequencies (a) 9037 Mc/sec, (b) 8985 Mc/sec.

In Fig. 5 the rf field was like that shown in Fig. 1(c) plus a small uniform component to excite the (1,1,0) mode. Again we see that the mode spacing is not a function of crystal direction, at least as far as the principal axes are concerned.

EFFECT OF MOUNTING ROD

In Fig. 6 are shown the absorption (a) without and (b) with a dielectric inhomogeneity near the sphere. The sphere was at the position O with the steady field along the z axis. A short length of single-crystal sapphire rod, 0.050 in diameter, such as we generally use for mounting spheres was pushed through the Polyfoam up to the sphere. The trace shown in Fig. 6 (b) was taken. In it one can resolve some sixteen modes spread over 600 oe. The cavity was then opened, the sapphire rod removed, and the trace (a) taken. In this only some four modes are resolvable. Similar curves to these could be shown for the other special positions with the cavity. In cases where the uniform component of the rf field can be eliminated by making the dc field parallel to it, it is found that at least a part of several modes come or go with the uniform field. That is to say, for the case of a sphere mounted on the end of the customary rod other modes than (1,1,0) appear to be associated with the uniform component of the rf magnetic field.

CONCLUSION

The experiments reported in this paper have shown that the positions of the magnetostatic modes are not functions of crystal direction as previously reported. The anisotropy is seen to be a simple material property and not different for the various modes. It appears that the specimens on which ferrimagnetic resonance has been performed up to this time deviated from sphericity slightly. Finally, evidence has been given of the effect of dielectric inhomogeneities in the neighborhood of the sample in a ferrimagnetic resonance experiment.

ACKNOWLEDGMENTS

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