

elastic penetrabilities  $T_n(U, E')$ . This represents, in the high-energy limit, the number of levels of both parities, and all spins, lying below an excitation energy equal to the incident neutron energy. It will be observed that this quantity varies with energy in a manner very similar to  $T_f$ . This similarity of behavior with energy of  $T_f$  and  $a(J)$  must also persist at higher energies since, as has been remarked by Huizenga,<sup>19</sup> the fission cross section is normally constant over a neutron energy range of 2–5 Mev. This constancy indicates a

<sup>19</sup> J. R. Huizenga, Phys. Rev. **109**, 484 (1958).

constant division of the available cross section between fission and inelastic scattering, and hence a similar behavior of the two corresponding level densities with energy.

#### ACKNOWLEDGMENTS

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### Some Measurements of Atmospheric Neutrons\*†

JOHN D. GABBE‡

Physics Department, New York University, University Heights, New York

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A series of airplane flights carrying neutron counters to an altitude of 700 g cm<sup>-2</sup> was made at 52°20' north geomagnetic latitude in 1955. Enriched and normal BF<sub>3</sub> counters were covered with Cd, Sn, Pyrex, and lime-glass shields. The ratios of the counting rates of the variously shielded counters are compared with the ratios calculated theoretically on a thick-shield basis using the neutron energy distribution function derived by Freese and Meyer. The neutron-production data taken by Davis and by Staker with Pyrex-glass-envelope BF<sub>3</sub> counters are corrected for the absorption of neutrons by the boron in the Pyrex. The corrected neutron-production rates are recalculated to be 2.1±0.4 cm<sup>-2</sup> sec<sup>-1</sup> and 0.9±0.2 cm<sup>-2</sup> sec<sup>-1</sup> at 54°36' and 30°24' north geomagnetic latitude, respectively, using recent values for the various neutron cross sections. These corrected rates agree, to within 5%, with the recent measurements reported by Soberman. The above energy distribution function was found, within the limited accuracy of the experiment, to describe the energy distribution of atmospheric neutrons.

#### INTRODUCTION

IT is generally accepted, at present, that almost all of the neutrons in the atmosphere are secondary particles produced by the events generated when primary cosmic rays impinge on the atmosphere.<sup>1,2</sup> Davis,<sup>3</sup> Staker,<sup>4</sup> and many other investigators<sup>5</sup> have concerned themselves with the processes in which these neutrons are produced and absorbed. A recent calculation of the energy spectrum of a portion of the neutrons in the atmosphere was made by Freese and Meyer.<sup>1</sup> The neutrons treated by Freese and Meyer<sup>1</sup> will be referred to as *lambda* neutrons. Lambda neutrons

comprise atmospheric neutrons having energies less than  $E_0$ , where  $E_0$  lies between 10 kev and 0.1 Mev.

The purposes of this experiment were (1) to obtain further experimental information on the energy spectrum of lambda neutrons in that portion of the atmosphere in which the neutron density varies exponentially with pressure altitude; and (2) to correct the measurements of Davis<sup>3</sup> and of Staker<sup>4</sup> for the effect of the B<sup>10</sup> in the Pyrex-glass envelopes of the boron-trifluoride neutron counters they used, and by using more recent information on neutron cross sections bring their calculations up to date.

The region of the atmosphere in which the neutron density varies exponentially with pressure altitude (namely, the region between an altitude of about 200 g cm<sup>-2</sup> and an altitude  $H$ , where  $H$  is not more than about 750 g cm<sup>-2</sup> and is at least 100 g cm<sup>-2</sup> above the ground<sup>6-8</sup>) will be referred to as the equilibrium region of the atmosphere. More than 90% of the neutrons in the atmosphere originate in evaporation stars.<sup>1,2,6,9</sup> Theoretically one would expect no variation in the

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‡ Present Address: Bell Telephone Laboratories, Incorporated, New York, New York.

<sup>1</sup> E. Freese and P. Meyer, "Neutronen in der Atmosphäre," in *Kosmische Strahlung*, edited by W. Heisenberg (Springer-Verlag, Berlin, 1953), second edition.

<sup>2</sup> J. A. Simpson, Jr., Phys. Rev. **83**, 1175 (1951).

<sup>3</sup> W. O. Davis, Phys. Rev. **80**, 150 (1950).

<sup>4</sup> W. P. Staker, Phys. Rev. **80**, 52 (1950).

<sup>5</sup> Bibliographies may be found in references 1, 8, and 16.

<sup>6</sup> Bethe, Korff, and Placzek, Phys. Rev. **57**, 573 (1940).

<sup>7</sup> L. C. L. Yuan, Phys. Rev. **81**, 175 (1951).

<sup>8</sup> R. K. Soberman, Phys. Rev. **102**, 1399 (1956).

<sup>9</sup> Simpson, Fonger, and Treiman, Phys. Rev. **90**, 934 (1953).

energy spectrum of lambda neutrons with latitude, longitude, or time in the equilibrium region of the atmosphere.<sup>1,6,10</sup> This expectation is born out experimentally by the constancy of the cadmium ratio in the above region.<sup>2,7,11-14</sup>

The data for this experiment were obtained by recording the counting rates of boron-trifluoride neutron counters which were covered with shields of various materials.

### THEORY

The energy distribution function derived by Freese and Meyer<sup>1</sup> was used in the calculations in this paper. This function is given by

$$n(E)dE = BE^{-\frac{1}{2}}dE \exp[-\alpha E^{-\frac{1}{2}} + \beta E^{-1} - \gamma E^{-\frac{3}{2}} + \dots], \quad (1)$$

where  $n(E)dE$  is the number of lambda neutrons with energies between  $E$  and  $E+dE$ , the energy-independent factor  $B$  is connected with the strength of the source of the lambda neutrons,  $\alpha = 0.388$  (ev)<sup>1/2</sup>,  $\beta = 0.00343$  ev, and  $\gamma = 0.0000482$  (ev)<sup>3/2</sup>.<sup>15</sup> The approximations and physical assumptions used in the derivation of Eq. (1) together with the limitations governing its application are fully discussed in the article by Freese and Meyer.<sup>1</sup>

It may be shown<sup>16</sup> that the ratio of the expected values of the counting rates of two identical, thin, boron-trifluoride, neutron detectors covered with dissimilar shields is given by

$$\frac{M_1}{M_2} = \frac{\int_0^\infty n'(E)dE \exp[-U_1 y_1 S_1(E)]}{\int_0^\infty n'(E)dE \exp[-U_2 y_2 S_2(E)]}, \quad (2)$$

where  $M$  is the expected value of the counting rate,  $U$  is the density of the neutron-absorbing centers in the shield,  $y$  is the apparent thickness of the shield (see section entitled Theoretical Calculations),  $S(E)$  is the neutron-absorption cross section of the neutron-absorbing centers in the shield, the subscripts indicate different shield materials, and

$$Bn'(E)dE \equiv n(E)dE. \quad (3)$$

Equation (2) is used for the calculations that follow, because it allows several constants which are not amenable to measurement or calculation to be eliminated by cancellation. A detailed examination of the

counters and shields<sup>17</sup> used in this experiment and the conditions under which the measurements were carried out leads to the conclusion that, assuming no error in  $n'(E)$ , Eq. (2) is correct to within about 3%.

Consider two identical counter shells, one filled with a gas consisting of 18.8% B<sup>10</sup>F<sub>3</sub>, and 81.2% B<sup>11</sup>F<sub>3</sub> (regular filling), and the other filled to the same pressure with a gas consisting of 96% B<sup>10</sup>F<sub>3</sub> and 4% B<sup>11</sup>F<sub>3</sub> (enriched filling). Let  $w$  be the number of neutron counts per minute that would be registered by a counter containing pure B<sup>10</sup>F<sub>3</sub>, and  $b$  be the number of background counts per minute. The total number of counts per minute,  $W$ , indicated by the counter will be given by

$$W_E = 0.96w + b, \quad (4)$$

or

$$W_R = 0.188w + b, \quad (5)$$

where  $E$  and  $R$  appearing as subscripts indicate the enriched and regular fillings, respectively.<sup>3</sup> When  $W_E$  and  $W_R$  are recorded concurrently, one may solve Eqs. (4) and (5) simultaneously for  $w$  and  $b$ . This method of eliminating the background is applicable only if the thin-detector approximation can be made.

### PLAN OF THE EXPERIMENT

The experiment being reported is a thick-shield experiment.<sup>17</sup> To accomplish the objectives set forth, eight brass-walled neutron counters were flown simultaneously in the equilibrium region of the atmosphere. The flights were made at the highest altitude, 700 g cm<sup>-2</sup>, at which it proved practical to operate the Piper Pacer aircraft in which the equipment was installed. The counters were divided into three groups.

Group I consisted of three counters, two enriched and one regular. The regular and one of the enriched counters were covered with lime-glass shields. The other enriched counter was covered with a shield made from Pyrex glass. The lime glass, which, except for its boron content, has a chemical composition very similar to that of Pyrex, was included to reproduce all the effects of the Pyrex shield (e.g., star production) other than the absorption of neutrons by the B<sup>10</sup>. The desirability of an arrangement of shields of the kind just described has been indicated by Bagge and Finke<sup>10</sup> and by Yuan.<sup>13</sup> The counting rates of the two lime-

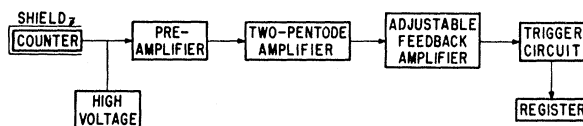


Fig. 1. The block diagram of one of the eight counting trains.

<sup>17</sup> If  $S_1(E)$  and  $S_2(E)$  are proportional to  $E^{-1}$  and the thin-shield approximation  $\exp[-UyS(E)] \approx 1 - UyS(E)$  may be made, it may be shown that Eq. (2) is independent of  $n'(E)$ . With the exception of the no-shield to lime-glass ratio, one or the other of these two conditions was violated for each ratio of counting rates measured.

<sup>10</sup> E. Bagge and K. Finke, Ann. Physik 6, 321 (1949).

<sup>11</sup> Agnew, Bright, and Froman, Phys. Rev. 72, 203 (1947).

<sup>12</sup> Simpson, Baldwin, and Uretz, Phys. Rev. 76, 165 (1949).

<sup>13</sup> L. C. L. Yuan, Phys. Rev. 76, 1267 (1949).

<sup>14</sup> Korff, George, and Kerr, Phys. Rev. 73, 1133 (1948).

<sup>15</sup> E. Fermi, Nuclear Physics (University of Chicago Press, Chicago, 1950), revised edition, compiled by Orear, Rosenfeld, and Schluter, gives a very similar distribution on page 221.

<sup>16</sup> J. D. Gabbe, doctoral thesis, New York University, New York, New York, 1956 (unpublished).

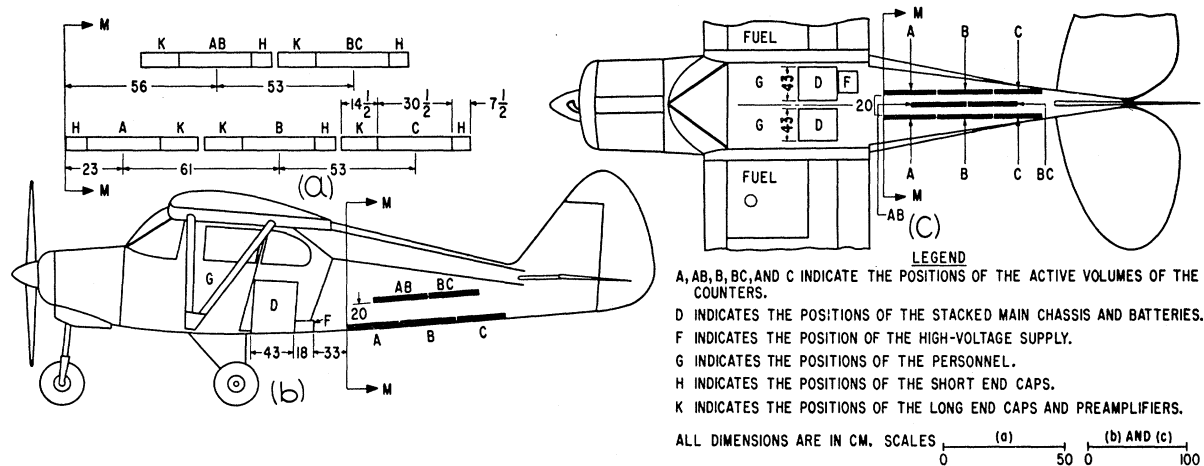


Fig. 2. The position of the equipment in the aircraft. (a) Detail of the counters; (b) side view of the aircraft; (c) top view of the aircraft.

glass-covered counters were used to calculate the number of neutron counts per minute and the number of background counts per minute from Eqs. (4) and (5). The background counting rate was assumed to be the same in the Pyrex-covered counter. Thus the neutron counting rate of the Pyrex-covered counter, and the ratio of the neutron counting rate of the lime-glass-shielded enriched counter to the neutron counting rate of the Pyrex-shielded enriched counter were calculated.

An analogous calculation was made for the counters in group II, which consisted of an enriched counter covered with a cadmium shield and an enriched and a regular counter both shielded with tin.

Group III consisted of two unshielded counters, one enriched and one regular. For reasons of symmetry these two counters will be referred to as being covered with "no-shield." The counters in group III were flown so that the data from the various flights could be compared. The counting rates of the shielded counters could not be used as a basis for normalizing the data, because both the shields and the relative positions of the counters in groups I and II were interchanged between flights.

The ratios of the counting rates obtained in these flights are, after correction for the effect of the airplane and sundry other systematic errors, compared with the theoretically expected ratios calculated from Eq. (2).

#### EQUIPMENT AND INSTALLATION

A block diagram of one of the eight counting trains appears in Fig. 1. Each counter, *C*, will be designated by two superscripts and two subscripts. The superscripts indicate whether the counter was enriched (*E*) or regular (*R*); and of what material the shield enclosing the counter was made, cadmium (*C*), tin (*S*), Pyrex (*P*), lime glass (*G*), or no-shield (*N*). The first subscript identifies the counter with its built-on preamplifier.

The second identifies the remainder of the electronic counting train and the register to which the counter was attached.

The matched counters and most of the electronic circuits have been described previously.<sup>8,18</sup> The counter plateaus, which were 300 v long, were centered at about 2400 v and had slopes of 3% per 100 v. The adjustable-feedback amplifier was used to match the circuits. The Pyrex shield was of No. 7740, Pyrex brand, glass tubing; 46 cm long; 6.4 cm in outside diameter; and nominally 0.32 cm thick. The boron content was 4.2%.<sup>19</sup> The dimensions of the lime-glass shields were identical with those of the Pyrex shield except that the wall thickness was only 0.16 cm. The lime glass proved to contain 0.82% boron.<sup>20</sup> The greater mass of the Pyrex shield is not regarded as an important source of additional background counts. The cylindrical cadmium shield was 46 cm long, 5.4 cm in diameter, and 0.061 cm thick. One end of the cylinder was closed by a disk 0.10 cm thick. The otherwise identical tin shields were 0.064 cm thick.

The one-tube preamplifiers were attached directly to the counters. The remainder of the electronic circuits and all of the batteries were grouped away from the counters. The installation in the airplane is shown in Fig. 2.

#### DATA

Flights *i* and *ii* provided no usable data as the apparatus was inadequately shielded against interference from the ignition system of the aircraft. The registers were read at 10-minute intervals during the three successful flights. The average counting rate for each counter is recorded in Table I. The aircraft flew

<sup>18</sup> Pavalow, Davis, and Staker, *Rev. Sci. Instr.* **21**, 529 (1950).

<sup>19</sup> Wickers, Finn, and Clabaugh, *J. Research Natl. Bur. Standards* **26**, 537 (1941).

<sup>20</sup> *Report of Tests*, July 27, 1955, City Testing and Research Laboratories, Inc., New York, New York.

TABLE I. Collected flight data and standard deviations.

Date Time Duration at 700 g cm <sup>-2</sup> Air temperature Weather Altitude Geomagnetic latitude Counting train	Flight <i>iii</i> June 25, 1955 1255 hours EST 200 minutes 4° C Clear 3050±60 m 52°20'±15' N			Flight <i>iv</i> July 12, 1955 1130 hours EST 200 minutes 7° C Clear 3050±90 m 52°20'±15' N			Flight <i>v</i> July 22, 1955 0814 hours EST 80 minutes 9° C Clear 3000±60 m 52°20'±15' N		
	Counts/min	a	b	Counts/min	a	b	Counts/min	a	b
C <sub>11</sub> <sup>B</sup>	3.15±0.13	P	B	4.68±0.16	G	A	3.11±0.20	P	B
C <sub>22</sub> <sup>B</sup>	4.90±0.16	N	AB	4.92±0.16	N	AB	5.05±0.25	N	AB
C <sub>33</sub> <sup>B</sup>	5.34±0.17	G	A	3.18±0.13	P	B	4.00±0.23	G	C
C <sub>44</sub> <sup>B</sup>	5.98±0.17	S	A	4.78±0.16	S	C	2.01±0.16	C	B
C <sub>56</sub> <sup>R</sup>	1.43±0.08	N	BC	1.26±0.08	N	BC	1.29±0.13	N	BC
C <sub>65</sub> <sup>B</sup>	Failed	C	B	Failed	C	B			
C <sub>77</sub> <sup>R</sup>	1.89±0.10	G	C	1.66±0.09	G	C	1.71±0.15	S	C
C <sub>88</sub> <sup>R</sup>	1.64±0.09	S	C	1.86±0.10	S	A	1.90±0.15	G	A
C <sub>95</sub> <sup>B</sup>							5.91±0.28	S	A

<sup>a</sup> Shield material; see text (nominally the second superscript of C).

<sup>b</sup> Position of the counter in the aircraft; see Fig. 2.

TABLE II. Final corrected neutron (*w*) and background (*b*) counting rates and crude estimates of the standard deviations for each shield material.

Shield	Flight <i>iii</i>		Flight <i>iv</i>		Flight <i>v</i>	
	<i>w</i> counts/min	<i>b</i> counts/min	<i>w</i> counts/min	<i>b</i> counts/min	<i>w</i> counts/min	<i>b</i> counts/min
N	3.94±0.31	0.68±0.12	3.93±0.31	0.45±0.12	4.07±0.44	0.46±0.18
S	3.90±0.44	0.87±0.14	3.96±0.27	0.82±0.15	3.70±0.56	0.97±0.22
C					1.06±0.28	
G	3.25±0.44	1.28±0.15	2.66±0.44	1.15±0.15	2.91±0.40	1.17±0.24
P	1.92±0.23		2.08±0.23		1.99±0.35	

on the line connecting Dover, New Jersey, and Montauk Point, Long Island, New York. Flight *v* was cut short because the aircraft developed engine trouble. All the flights were made in clear weather.<sup>21</sup> Occasional stray cumulus cloud formations (at altitudes of about 800 to 770 g cm<sup>-2</sup>) were given a very wide berth.<sup>11</sup>

#### CALCULATIONS FROM THE FLIGHT DATA

Corrections for the following effects were considered: (1) the interception of neutrons that would ordinarily be counted by one counter by any other counter or shield; (2) the differences in background counting rates caused by differences in the masses of shields in the same counter group; (3) the changes in  $n'(E)$  brought about by inserting the counters and shields into the atmosphere; (4) the moderation of neutrons by the gasoline in the airplane under the assumption that the number of neutrons so moderated would decrease as the gasoline was consumed; (5) the recovery time of the counting trains (0.3 sec); and (6) the moderation of neutrons by the frontal mass of the airplane (engine, crew, batteries, etc.).

The first four effects were found to be unimportant (less than about 2% of the counting rates in all cases). The last effect is very large. It is possible to make a

correction for the last effect by considering the counting rates of counters whose positions in the aircraft were exchanged between the various flights, and making several simplifying assumptions regarding the geometry of the moderating material and the energy of the scattered neutrons. Account was also taken of the effect of the shields on these moderated neutrons. The effect of this correction on the ratios of the counting rates is fairly insensitive to the exact nature of the assumptions made. The data of Table I were corrected for the recovery time of the counting trains in the usual manner.<sup>22</sup> Then the correction for the effect of the airplane was applied, and the data were processed through Eqs. (4) and (5) to give the results in Table II. The application of the last correction tends to smooth out the statistical variations among the flights and gives a false uniformity to the results.

#### THEORETICAL CALCULATIONS

Equation (2) was evaluated numerically for the shields used in this experiment. The ratios in Table III were obtained for the indicated shield materials. The last result in the column titled "Theoretical ratio" assumes a step-function cadmium cutoff with the step at 0.4 ev.<sup>1</sup> The value of 2.43 for the no-shield to cadmium ratio is reached using the actual neutron-capture cross section of cadmium. The difference

<sup>21</sup> The author is indebted to the members of Project Scud, Department of Meteorology, College of Engineering, New York University, for special weather forecasts of unimpeachable accuracy.

<sup>22</sup> S. A. Korff, *Electron and Nuclear Counters* (D. Van Nostrand Company, Inc., New York, 1955), second edition, p. 254 et seq.

between the value of 2.28 and that of 2.43, which corresponds to a step function with a cutoff at 0.55 ev, is not considered significant. The values of  $S(E)$  inserted into Eq. (2) were taken from the literature.<sup>23</sup> It is possible to evaluate  $U$  for the various cases from the densities of the shields [(2.23±0.03) g cm<sup>-3</sup> for Pyrex and (2.51±0.03) g cm<sup>-3</sup> for the lime glass used] and information previously given. The calculation of  $y$  is straightforward but the details depend on the exact placement of the shields. In the present case it was found that  $y$  was equal to  $\frac{4}{3}$  the wall thickness of the shields. The standard deviations in the first column of Table III were crudely estimated on the basis of the following considerations: (1) the 3% error in evaluating Eq. (2) was assumed random and treated as a percent standard deviation; and (2) the standard deviations of the experimentally determined values used in the calculation of the ratios were taken into account.

No average energy or average velocity can be calculated from the energy distribution given in Eq. (1), because  $n(E)$  does not go to zero fast enough as  $E$  becomes infinite. This is not unexpected as Eq. (1) is not valid above 0.1 Mev. If the distribution is considered to be cut off at  $E_0$  and the average energy is then calculated, one finds that the average is sensitive to the exact value chosen for  $E_0$ . However, it is possible to calculate the average neutron cross section for B<sup>10</sup>,  $\bar{S}'$ , in the usual manner,

$$\bar{S}' = \left[ \int S'(E)n'(E)dE \right] / \left[ \int n'(E)dE \right] \\ = 1900 \text{ barns.} \quad (6)$$

The neutron energy corresponding to 1900 barns is about 0.12 ev.

TABLE III. Ratios of counting rates for variously shielded counters in the equilibrium region of the atmosphere.

Shields	Theoretical <sup>a</sup> ratio	Experimental <sup>b</sup> ratio
Lime glass to Pyrex	1.55±0.08	1.5 ±0.2
No-shield to Pyrex	1.68±0.09	2.0 ±0.2
No-shield to lime glass	1.08±0.05	1.35±0.15
No-shield to Davis' counters <sup>c-e</sup>	1.45±0.12	
Cadmium ratio (actual)	2.43±0.12	2.2 <sup>g</sup>
Cadmium ratio (nominal) <sup>f</sup>	2.28	

<sup>a</sup> Except as noted, these ratios apply only to the shields used in this experiment.

<sup>b</sup> Except as noted, the experimental ratios are the result of the present experiment.

<sup>c</sup> See reference 3.

<sup>d</sup> See reference 4.

<sup>e</sup> See reference 24.

<sup>f</sup> After Freese and Meyer (reference 1).

<sup>g</sup> This number has been obtained by several investigators [references 2 and 11, and H. J. Kouts and L. C. L. Yuan, Phys. Rev. **86**, 128 (1952)]. The sharp cutoff of the cadmium neutron-absorption cross section at about 0.4 ev makes the cadmium ratio insensitive to the counter and shield geometries used in cadmium-ratio measurements.

<sup>23</sup> D. J. Hughes and J. A. Harvey, *Neutron Cross Sections*, United States Atomic Energy Commission (McGraw-Hill Book Company, Inc., New York, 1955), second edition of AECU-2040.

TABLE IV. Average neutron production rates.

Author	North geomagnetic latitude	Average neutron production rate per cm <sup>2</sup> per sec		
		Original	Corrected	Recent <sup>a</sup>
Davis <sup>b</sup>	54.7°	1.0	2.1	2.0
Staker <sup>c</sup>	54.7°	0.98	2.1	2.0
Staker <sup>c</sup>	30.4°	0.41	0.9	0.9
SPK <sup>d</sup>	69°	2.0	4.2 <sup>e</sup>	2.1

<sup>a</sup> See reference 8.

<sup>b</sup> See reference 3.

<sup>c</sup> See reference 4.

<sup>d</sup> See reference 24.

<sup>e</sup> These anomalous results are probably due to solar activity at the time the measurements of Staker, Pavalow, and Korff (reference 24) were made [see reference 2 and S. A. Korff (private communication)].

The data of Davis<sup>3</sup> and of Staker<sup>4,24</sup> were multiplied by a factor of 1.4 to correct for the neutrons absorbed by the Pyrex envelopes of their counters. Davis<sup>3</sup> and Staker<sup>4</sup> used a factor of only 1.14 which Davis calculated on the basis of an older energy distribution than that used here. A factor

$$\text{no-shield/Pyrex (Davis)} = 1.45$$

was calculated from Eq. (2) using the length  $y''$  calculated from the wall thickness (2 mm) of the counters of Davis<sup>3</sup> and Staker,<sup>4</sup> which fortunately were still available to us for measurement. The factor 1.45 was decreased to 1.4 to compensate for the reduced effect of the Pyrex above 200 g cm<sup>-2</sup>, where the average neutron energy is higher. Corrected neutron-production rates were calculated using the same method<sup>6</sup> used by Davis<sup>3</sup> and by Staker,<sup>4</sup> but using the values of the neutron-absorption cross sections and the mean absorption coefficient used by Soberman.<sup>8,25</sup> The results are collected in Table IV. The author feels that a standard deviation of 20% should be attributed to the corrected production rates. The neutron-production rates collected by Soberman<sup>8</sup> are included in Table IV for comparison.

## DISCUSSION

The experimental ratios calculated from Table II appear in Table III. The experimental and theoretical values for the lime-glass to Pyrex ratio agree very well. The agreement between the experimental and theoretical results for the no-shield to Pyrex and no-shield to lime-glass ratios is not as good. The design of the experiment permitted the effect of variations among the counters and counting trains to be averaged out of the experimental determination of the lime-glass to Pyrex ratio. Lamentably, the opportunity to carry out the complete experimental design was not available and these variations cannot be averaged out of the two no-shield ratios. The latter must therefore be considered to have a greater variance than the lime-glass to Pyrex

<sup>24</sup> Staker, Pavalow, and Korff, Phys. Rev. **81**, 889 (1951).

<sup>25</sup> Beiser, Haymes, and Korff (to be published) direct attention to a calculational error in R. K. Soberman, Phys. Rev. **102**, 1399 (1956).

ratio. However, the fact that the experimental no-shield ratios are somewhat higher than the theoretical no-shield ratios suggests that the correction for the effect of the aircraft was insufficient to compensate completely for the softening of the neutron energy spectrum. Enough data were not obtained from the cadmium-shielded counters to warrant a calculation of the cadmium ratio.

The values of  $w$  for the unshielded counters, Table II, agree very well with the value of  $4.2 \pm 1$  counts  $\text{min}^{-1}$  for the 690-millibar point in Fig. 13 of reference 8, which was obtained using counters identical to those used in this experiment. The difference of 3 degrees in the geomagnetic latitudes at which these data were taken is not significant at this altitude and geomagnetic latitude.<sup>8</sup>

The standard deviations given in Table III are crude estimates for which the nature of the experiment does not allow rigorous support. The standard deviation in the experimental results is about 15%. If the many approximations in the derivation of  $n(E)$  itself are also considered, the uncertainty in the theoretical ratios cannot be thought much less. While the agreement between experimental and theoretical results must be viewed in the light of the above, the agreement is helpful nevertheless in two areas.

(1) The theoretically calculated correction for the thick Pyrex envelopes of the counters of Davis<sup>3</sup> and Staker<sup>4</sup> may be applied without degrading the accuracy of their data. This final correction to the work of Davis<sup>3</sup> and Staker<sup>4</sup> brings into agreement all the data on atmospheric neutrons obtained by the Cosmic-Ray Project at New York University between 1948 and 1956.<sup>8</sup> Throughout this period the experiments performed by members of the Project have made use of two counters, one enriched with  $\text{B}^{10}$  and the other either depleted<sup>3,4,24</sup> in  $\text{B}^{10}$  or nonenriched,<sup>8</sup> to eliminate background counts.

(2) In most<sup>14,26,27</sup> previous experiments the cadmium ratio has been used as an index of how well a distribution function represents the actual distribution of the energies of atmospheric neutrons below 200  $\text{g cm}^{-2}$  altitude.<sup>1</sup> However, as pointed out by Freese and

Meyer,<sup>1</sup> the cadmium ratio gives information about  $n(E)$  only for energies above the cadmium cutoff. The cadmium ratio thus gives no information as to the effect of the second- and third-order terms in the exponent of Eq. (1) or about effects perturbing the energy distribution at very low neutron energies. As the cross section of boron varies as  $E^{-1}$  and has no sharp, low-energy cutoff, the ratios calculated here are affected by the details of the distribution below 0.4 ev. However, these ratios are not very sensitive to the exact shape of the distribution function, and are especially insensitive at the very low energies (below 0.025 ev). The agreement between the experimental and theoretical results of this experiment tends to substantiate the calculations of Freese and Meyer.<sup>1</sup> A more refined experiment must be undertaken before any comments may be made about the details of  $n(E)$ .

### CONCLUSIONS

(1) All the data obtained between 1948 and 1956, by members of the Cosmic-Ray Project<sup>3,4,8,24</sup> of New York University, on the production and distribution of atmospheric neutrons, are self-consistent.

(2) Within the sensitivity of this experiment, the function [Eq. (1)] derived by Freese and Meyer<sup>1</sup> adequately describes the energy distribution of lambda neutrons in the equilibrium region of the atmosphere.

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<sup>26</sup> S. A. Korff and B. Hamermesh, *Phys. Rev.* **69**, 155 (1946).

<sup>27</sup> P. Meyer, *Z. Physik* **141**, 28 (1955).