Hall Effect, Magnetoresistance, and Size Effects in Copper*

TED G. BERLINCOURT
Atomics International, Division of North American Aviation, Incorporated, Canoga Park, California
(Received June 23, 1958)

Measurements of the Hall coefficient of several annealed polycrystalline Cu strips of resistivity ratio $\rho(273°K)/\rho(4.2°K)\sim450$ and thicknesses ranging from 0.05 to 1.6 mm have revealed the existence of a marked size effect at low temperatures. The effect is orders of magnitude greater than that to be expected on the basis of free-electron theory. It is suggested that earlier low-temperature data on the Hall effect of thin high-purity samples are subject to uncertainties arising from such effects. Size effects were also evident in the transverse magnetoresistance. At high fields, a tendency toward saturation in the transverse magnetoresistance was observed for thick samples. The temperature dependence of the Hall coefficient has also been studied.

I. INTRODUCTION

When the electron mean free path in a metal becomes comparable with sample dimensions, the electron transport properties become functions of the sample dimensions. The theory and experimental situation for such size effects have been reviewed by Sondheimer,[1] who developed a free-electron theory for the transverse magnetoresistance and the Hall effect in thin samples.[2] The important parameters in this theory are the sample thickness t, the electron mean free path l, the radius $r_0=m\bar{v}c/eH$ (where $\bar{v}$ is the Fermi velocity) of a free electron orbit in a magnetic field H, and the surface scattering coefficient p, which has the values 0 for diffuse scattering and 1 for specular reflection. An oscillatory dependence of transverse magnetoresistance upon magnetic field strength[3] was predicted for thin samples, and such behavior was subsequently observed by Babiskin and Siebenmann[4] in a thin wire of Na, strikingly verifying the applicability of Sondheimer's free electron theory to Na.

The experiments to be described below were undertaken for the purpose of studying the temperature dependence of the Hall coefficient in Cu. However, it soon became apparent that size effects considerably greater than those predicted by the free-electron theory were important in high-purity Cu at liquid helium temperatures.[5] Disagreement with free electron theory is not unexpected in view of Pippard's recent anomalous skin effect measurements[6] which provide evidence that the Fermi surface touches the Brillouin zone boundary in Cu. Nevertheless, the large magnitude of the disagreement is surprising and suggests that recent Hall effect studies by Chambers[7] and Borovik[8–10] also involved size effects.

The measurements on thick samples have bearing on electron transport theory for bulk samples. The semiclassical theory developed by Kohler[11,12] and extended by Lifshitz, Azbel', and Kaganov[13] and Chambers[7] is based on the assumptions that (1) collisions can be described by a field-independent relaxation time $\tau(\mathbf{k})$, (2) changes in purity and temperature alter all $\tau(\mathbf{k})$ by the same factor, and (3) the size of the filled region in $\mathbf{k}$ space does not change appreciably over the ranges of temperature and purity considered. If these conditions are fulfilled, all components ρ_{ij} and σ_{ij} of the resistivity and conductivity tensors should obey Kohler's rule; i.e., for a given metal each component should be a function only of a reduced field $H/\rho(0)$, where $\rho(0)$ is the resistivity in zero field. The theory further states that for a group I metal in which the Fermi surface does not touch the Brillouin zone boundary, the Hall coefficient and the transverse magnetoresistance should saturate in a high magnetic field. The saturation value for the Hall coefficient R should be the free-electron value $1/(nec)$, where n is the number of conduction electrons per cm^3. For Cu, Chambers[7] has estimated an upper limit for the saturation value for the resistivity of 3 to 4 times $\rho(0)$. On the other hand, if the Fermi surface touches the Brillouin zone boundary, the transverse magnetoresistivity should saturate at a much higher value.

The semiclassical theory is subject to the criticisms that (1) the detailed quantum mechanical nature of the motion of the electron has been ignored, and (2) at high fields it is unlikely that a field-independent relaxation time exists. A quantum mechanical treatment of

* This research was supported by the U. S. Air Force Office of Scientific Research and the U. S. Atomic Energy Commission.

[1] E. H. Sondheimer, in *Advances in Physics*, edited by N. F. Mott (Taylor and Francis, Ltd., London, 1952), Vol. 1, p. 1.

[2] E. H. Sondheimer, Phys. Rev. **80**, 401 (1950).

[3] These oscillations are periodic in H and are not to be confused with the oscillations of the de Haas-van Alphen type, which arise from orbital quantization and are periodic in H^{-1}.

[4] J. Babiskin and P. G. Siebenmann, Phys. Rev. **107**, 1249 (1957).

[5] A brief preliminary report of the present work was presented at the *Fifth International Conference on Low-Temperature Physics and Chemistry, Madison, Wisconsin 1957* (to be published).

[6] A. B. Pippard, Trans. Roy. Soc. (London) **A250**, 325 (1957).

[7] R. G. Chambers, Proc. Roy. Soc. (London) **A238**, 344 (1956).

[8] E. S. Borovik, Doklady Akad. Nauk S.S.S.R. **70**, 601 (1950).

[9] E. S. Borovik, J. Exptl. Theoret. Phys. U.S.S.R. **23**, 83 (1952). (Translation: Naval Research Laboratory Report NRL–462.)

[10] E. S. Borovik, J. Exptl. Theoret. Phys. U.S.S.R. **27**, 355 (1954).

[11] M. Kohler, Ann. Physik **32**, 211 (1938).

[12] M. Kohler, Ann. Physik **5**, 99 (1949).

[13] Lifshitz, Azbel', and Kaganov, J. Exptl. Theoret. Phys. U.S.S.R. **31**, 63 (1956) [translation: Soviet Phys. JETP **4**. 41 (1957)].

TABLE I. Sample characteristics.

Source		t (mm)	s	sl_{273} (mm)	l_g (mm)
JM[a]	u[c]	0.145	37	0.0015	...
		0.145	316	0.013	0.020
AS and R[b]		0.054	393	0.016	0.021
		0.099	439	0.018	0.033
		0.165	443	0.018	0.023
		0.391	472	0.019	0.051
		0.798	455	0.018	0.054
		1.616	487	0.020	0.051

[a] JM denotes Johnson-Matthey.
[b] AS and R denotes American Smelting and Refining.
[c] u denotes unannealed sample.

the problem has been carried out by Argyres[14] for the case of a single isotropic energy surface and scattering by phonons. More applicable to the present work is the quantum mechanical theory of Lifschitz,[15] which considers the case of equally spaced Landau levels, Fermi energy much greater than the Landau level spacing, elastic impurity scattering, and a closed Fermi surface. The result for this model is a conductivity tensor which is the same as the semiclassical one except for an oscillating part and a small correction which varies smoothly with H. In light of these theories the data on thick samples appear to support the belief that the Fermi surface touches the Brillouin zone boundary in Cu.

II. EXPERIMENTAL PROCEDURE

The polycrystalline Cu samples were cold-rolled to the desired thicknesses, cut to size (4.5 mm wide and 37 mm long), and etched. With one exception, the samples were annealed at 490°C for one hour under high vacuum. The six American Smelting and Refining (AS and R) samples were obtained from the same small section of material, and great care was taken to give them identical treatments except for the amount of reduction in thickness. The characteristics of the samples are listed in Table I, where s is the ratio of the resistivity at 273°K to that at 4.2°K, and $l_{273}=4.06\times10^{-5}$ mm is the electron mean free path in Cu at the ice point as determined by Chambers[16] from anomalous skin effect measurements. The product sl_{273} is then an approximation to $l_{4.2}$, the mean free path at 4.2°K. (It is not accurate because s is itself size dependent for l comparable with t.) The parameter l_g is related to grain size and is defined as the average distance an electron moving in a straight line would travel before encountering a grain boundary. Values for l_g were derived from the number of grain boundaries intersected by straight lines of known length drawn on photomicrographs of the samples.

[14] P. N. Argyres, Phys. Rev. **109**, 1115 (1958).
[15] I. M. Lifshitz, J. Exptl. Theoret. Phys. U.S.S.R. **32**, 1509 (1957) [translation: Soviet Phys. JETP **5**, 1227 (1957)].
[16] R. G. Chambers, Proc. Roy. Soc. (London) **A215**, 481 (1952).

The usual Hall geometry was employed as illustrated in Fig. 1. All potential leads were attached at the extreme edges of the sample with a minimum of nonsuperconducting solder.

A special sample holder permitted thermal contraction of the samples to take place without constraint and protected the fragile samples from forces arising from the interaction of the measuring current I and the magnetic field. The Hall voltage V_H was taken as $\frac{1}{2}[V_{12}(H)-V_{12}(-H)]$, where $V_{12}(H)$ is the voltage between probes 1 and 2 for the magnetic field in the positive direction and $V_{12}(-H)$ applies to the reverse field direction. Field reversal was actually accomplished by a 180-degree rotation of the sample about its long axis. Under these conditions the Hall coefficient is given by $R=V_Ht/IH$. The resistive voltage V_{34} was measured for forward and reverse current directions so that the effects of thermal voltages could be eliminated. Measuring currents from 0.3 to 3 amperes were employed, and V_{12} and V_{34} were measured with a Rubicon microvolt potentiometer and a photoelectric galvanometer. In general, settings could be made to $\pm10^{-9}$ volt, and over-all accuracy is estimated at $\pm1\%$ for the smallest voltages measured (i.e., for the thickest samples and lowest magnetic fields) and considerably better in general.

Magnetic fields up to 30 kilogauss were supplied by a large air-cooled electromagnet with 6-inch diameter pole tips and a gap of 1.4 inches. The generator supplying power to the magnet was electronically regulated, and measuring circuit loops in the high-field region were minimized so that induced voltages arising from field fluctuations were negligible. The magnet was calibrated below 11 kilogauss with a nuclear magnetic resonance fluxmeter. Calibration at higher fields was accomplished

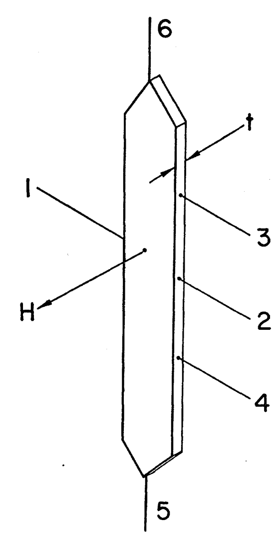

FIG. 1. Geometry for the Hall effect and magnetoresistance measurements. 1 and 2 are Hall probes located opposite each other as accurately as possible; 3 and 4 are resistance probes; 5 and 6 are current leads.

with a Grassot-type fluxmeter, which was standardized against the nuclear resonance fluxmeter at lower fields.

Measurements of the temperature dependence of the Hall coefficient between the boiling point of helium and the freezing point of oxygen and between the boiling point of oxygen and room temperature were accomplished during warmup. This warmup was quite slow because of the large thermal mass and good thermal insulation of the Dewar, and the data proved to be accurately reproducible. Carbon resistance thermometry was used between 4.2°K and 20°K, and a calibrated copper-constantan thermocouple with a liquid helium temperature reference junction was utilized for higher temperatures.

III. EXPERIMENTAL RESULTS AND COMPARISON WITH THEORY

A. Hall Coefficient Size Effect

Initial measurements were carried out on the unannealed Johnson-Matthey sample, $Cu_{0.145}$. (Hereafter, samples will be designated by subscripts denoting their thicknesses in mm.) As indicated in Fig. 2, the Hall coefficient was independent of H at room temperature and 4.2°K. Annealing increased the resistivity ratio s of $Cu_{0.145}$ from 37 and 316 and had negligible effect on the room temperature Hall coefficient. However, at 4.2°K the annealed sample exhibited a magnetic-field-dependent Hall coefficient, Fig. 2. (Data taken at 1.5°K were identical within experimental error.) Similar field dependences were reported earlier by Borovik[10] for a Cu sample, with $t=0.02$ mm and $s=137$, and by Chambers[7] for two Cu samples with $t=0.1$ mm and values for s of 243 and 825.

Chambers[7] noted a deviation from Kohler's rule in his measurements, and it is evident from Fig. 2 that the results on $Cu_{0.145}$ are also inconsistent with Kohler's rule. In the present instance, the discrepancy might be explained in terms of the argument used by van Bueren[17] to account for the failure of Kohler's rule to apply to magnetoresistance data on Cu before and after cold work. He pointed out that the dislocations in a

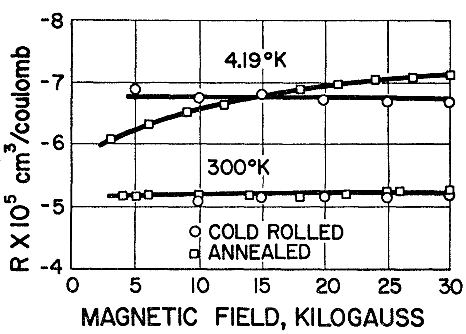

FIG. 2. Hall coefficient *versus* magnetic field for $Cu_{0.145}$ before and after annealing.

[17] H. G. van Bueren, thesis, University of Leiden, 1956 (unpublished).

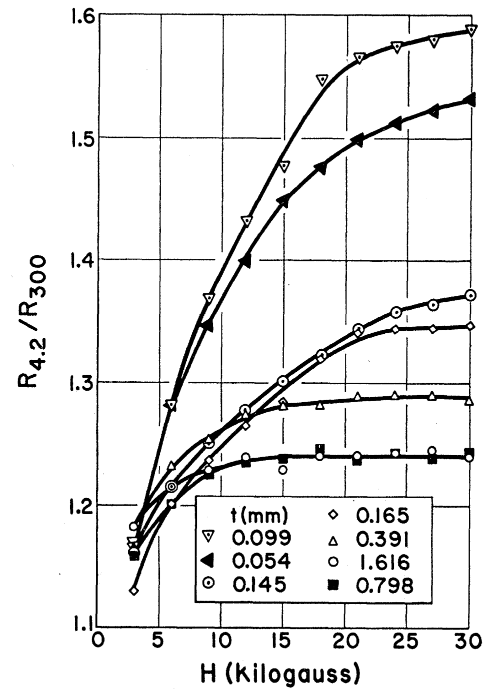

FIG. 3. Ratio of Hall coefficient at 4.2°K to value at 300°K *versus* magnetic field for Cu samples of various thicknesses.

cold-worked sample constitute anisotropic scattering sites. Hence, the assumption of the theory that changes in purity alter all $\tau(\mathbf{k})$ by the same factor is not valid for samples characterized by different dislocation densities. Because van Bueren found that annealing Cu above 300°C brought the magnetoresistance data back into accord with Kohler's rule, it is unlikely that such effects are important in the annealed samples used in the present investigation.

The above argument does not rule out the possibility that size effects of the type considered by Sondheimer[2] could be partly responsible for the failure of the $Cu_{0.145}$ data to obey Kohler's rule. According to the theory such size effects should become appreciable only if $l \sim t$; whereas for $Cu_{0.145}$, $l_{4.2} \sim 0.1t$. Nevertheless, the existence of a size effect is clearly indicated in Fig. 3, where $R_{4.2}/R_{300}$ is plotted against H for all the annealed samples studied. The results for the two thickest samples ($Cu_{0.798}$ and $Cu_{1.616}$) are nearly superposed and are probably characteristic of the bulk material. The observed size effects become appreciable only for the thinner samples. With a single exception, $R_{4.2}/R_{300}$ increases with decreasing t at the highest fields. The fact that the curve for $Cu_{0.054}$ lies below the $Cu_{0.099}$ curve could mean that the trend is reversed for thinner samples, but more likely a difference in mean free path or grain size is responsible, as indicated by the s and l_g values for these two samples. Unfortunately, a good quantitative description of the size effects is limited by difficulties inherent in producing samples of uniform

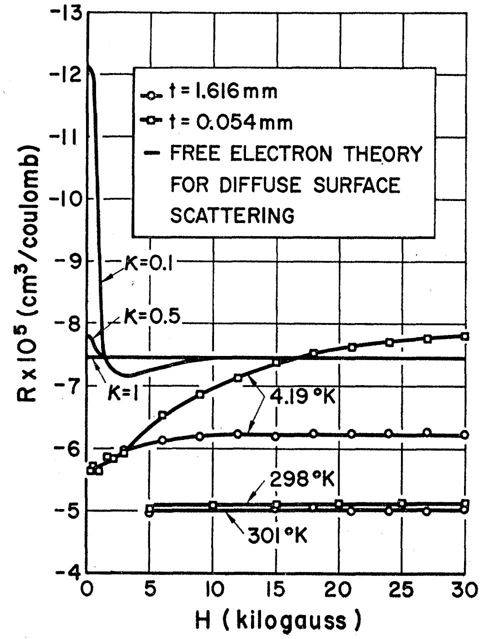

FIG. 4. Hall coefficient *versus* magnetic field for $Cu_{0.054}$ and $Cu_{1.616}$ compared with predictions of free-electron theory for various ratios κ of thickness to mean free path.

quality. Although the s values are fairly uniform throughout the AS and R samples, l_g increases by a factor of 2.5 in going from the thinnest to the thickest sample. While it thus appears that (in zero field at least) the major contribution to scattering comes from scattering sites other than grain boundaries, the possibility is not ruled out that the influence of grain boundaries on the size effect in a magnetic field could still be appreciable.

Because Sondheimer's free-electron theory predicts the largest size effects for low magnetic field strengths, additional data were obtained at low fields on the thinnest sample. The results are plotted in Fig. 4 along with the results for the thickest sample and the theoretical predictions of the free-electron model for $p=0$. (For $p>0$ the theoretical size effects are still smaller.) κ is the ratio t/l which has the value 3 for $Cu_{0.145}$ if $l_{4.2}$ is taken as the value of sl_{273} from Table I. Clearly, both quantitative and qualitative discrepancies exist between the free electron theory and the experimental results for Cu. If the estimates sl_{273} of the mean free path were too small by a factor of 10 to 40, size effects of the observed order of magnitude (but not of the same functional dependence on H) could be accounted for, but a larger variation than was observed would exist in values of s in going from thick to thin samples.

It is also possible that an internal size effect of the type suggested by MacDonald[18,19] and Steele[20] in regard to other work could give rise to the observed size effects. Striations or laminae could be introduced in the rolling process and would probably be more closely spaced in the thinner samples. However, this argument also suffers from the objection that a larger variation in s with thickness would be expected under these conditions.

Probably the most likely sources of the difficulty lie in the assumption of a magnetic-field-independent relaxation time and the neglect of anisotropies in the Fermi surface and scattering in the formulation of the free electron theory. Lehman[21] has carried out calculations, based on more general anisotropic Fermi surfaces, which indicate that a variety of size-effect field dependences may be expected for different types of constant

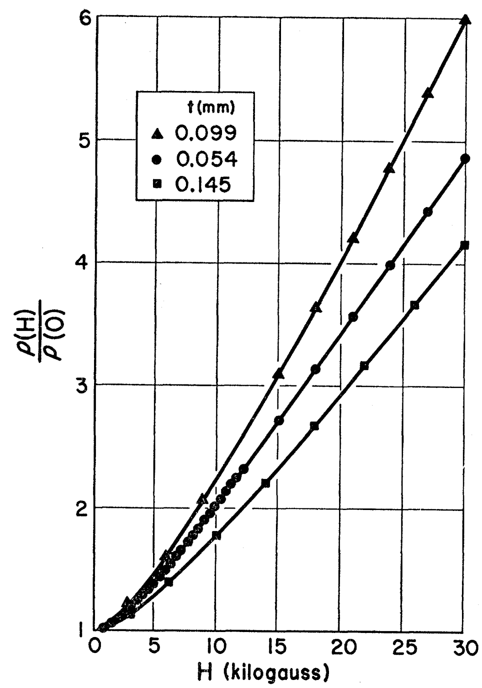

FIG. 5. Magnetoresistance *versus* magnetic field at 4.2°K for the three thinnest Cu samples. All three curves exhibit positive curvature over entire range of measurement.

energy surfaces. However, the requirement $l\sim t$ is retained. Whether or not a still more general treatment of the problem would relax this condition is not known.

B. Magnetoresistance Size Effect

The magnetoresistance data appear in Figs. 5, 6, and 7, where $\rho(H)/\rho(0)$ is the ratio of the resistivity in a field H to that in zero field. The curves for the three thinnest samples exhibit positive curvature over the entire range of measurement; the curve for $Cu_{0.165}$ is linear above about 5 kilogauss; and the curves for the three thickest samples exhibit negative curvature or a tendency toward saturation at high fields, which is

[18] D. K. C. MacDonald, Phil. Mag. 42, 756 (1951).
[19] D. K. C. MacDonald, Phil. Mag. 43, 124 (1952).
[20] M. C. Steele, Phys. Rev. 97, 1720 (1955).
[21] G. W. Lehman (private communication).

probably characteristic of the bulk material. Such a tendency toward saturation has not previously been reported for Cu, possibly because earlier measurements might have failed to fulfill the necessary observational conditions: (1) the use of a thick sample, and (2) the attainment of large values of l/r_0. However, it must be admitted that the effect is quite small and should be verified by the extension of the measurements to still larger values of l/r_0. Chambers noted an even more marked tendency toward saturation in his Ag results, but the possibility exists that size effects were present.

The detailed low-field magnetoresistance data appearing in Fig. 5 for the thinnest sample, $Cu_{0.054}$, were obtained on the possibility that an oscillatory size

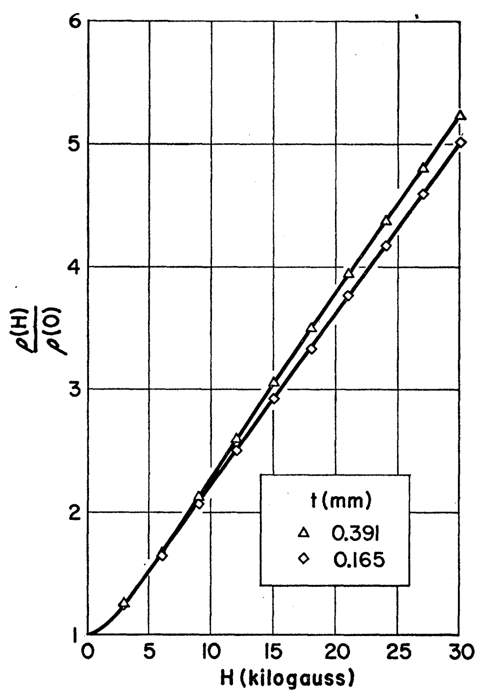

FIG. 6. Magnetoresistance *versus* magnetic field at 4.2°K. The curve for $Cu_{0.165}$ is linear above about 5 kilogauss while the curve for $Cu_{0.391}$ exhibits a negative curvature at high fields.

effect (of the type predicted by Sondheimer's free-electron theory and observed in Na by Babiskin and Siebenmann) might be observed. The lack of evidence for such an effect is in keeping with expectation for a polycrystalline sample with an anisotropic Fermi surface and for a sample in which $t/l\sim 3$.

The failure of the thin-sample magnetoresistance data to obey Kohler's rule is illustrated in Fig. 8 where $\rho(H)/\rho(0)$ is plotted against sH for the highest fields. This provides additional evidence for the existence of the size effect, for, with the exception of the curve for $Cu_{0.054}$, the thinner the sample, the higher its curve falls on the graph. Again, as in the case of the Hall measurements (and for the same reasons), the size-effect data are not as quantitative as would be desirable.

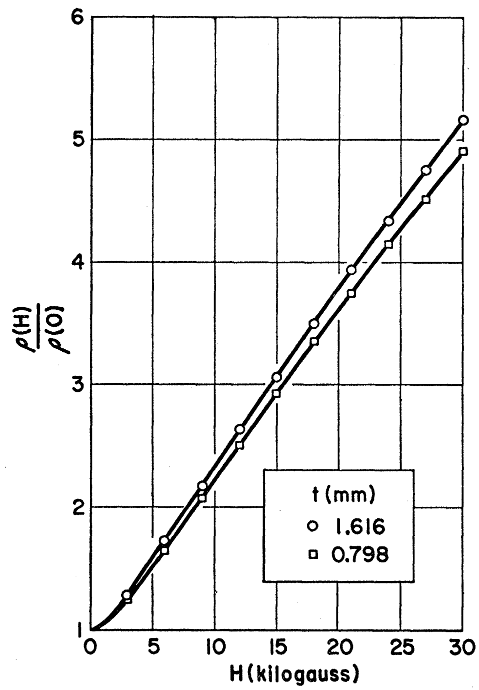

FIG. 7. Magnetoresistance *versus* magnetic field strength at 4.2°K for the two thickest Cu samples. Both curves have negative curvature at high fields.

C. Temperature Dependence of R

It is meaningful to characterize the temperature dependence of the Hall coefficient by a single curve only in the case of $Cu_{0.145}$ before it was annealed, for only in this instance was the Hall coefficient independent of magnetic field. The data are presented in Fig. 9. Such a temperature dependence cannot be explained by existing theory, and it is probable that a satisfactory explanation will require the introduction into the theory of anisotropies in both the scattering and the Fermi surface. In this connection it is of interest that Na exhibits a Hall coefficient which is very nearly temperature and magnetic field independent[22] and closely approximates the free electron value.[23] The fact that the Hall coefficient of Cu is only slightly temperature dependent in the residual and linear resistivity regions while being quite strongly temperature dependent in the small-angle electron-phonon scattering region supports the view[24] that scattering can be comparable in importance with charge carrier concentration in determining R. Bearing on this point is the fact that at 4.2°K the magnitude of R for the unannealed sample (in which size effects should be absent)

[22] D. K. C. MacDonald, Phil. Mag. 2, 97 (1957).

[23] It should be pointed out, however, that a close examination of MacDonald's Na data[22] at 4.2°K reveals some nonlinearity in the plot of Hall voltage *versus* H. If genuine, this field dependence could arise from a size effect of the type reported in the present paper for Cu.

[24] T. G. Berlincourt, Bull. Am. Phys. Soc. Ser. II, 2, 136 (1957).

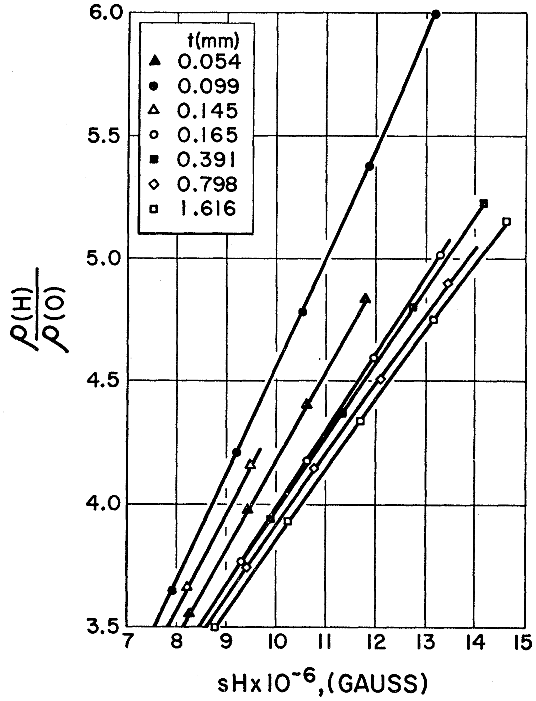

FIG. 8. High-field portion of Kohler plot for Cu samples of various thicknesses, illustrating inapplicability of Kohler's rule. The data were obtained at 4.2°K.

is 8% *greater* than the magnitude of the saturation value of R for the thickest annealed sample (in which size effects should also be absent). The difference is probably largely the consequence of different dislocation densities. If such a large change in R can result from anisotropy in the scattering introduced by dislocations, then it is not surprising that R changes appreciably as a function of temperature, for the anisotropy of the scattering in the impurity-scattering temperature region is appreciably different from that in the lattice-scattering temperature region. Coles[25] has presented arguments, based on Hall coefficient data on alloys, which emphasize the importance of scattering on the Hall coefficient, and Fukuroi and Ikeda[26] have noted a correlation between the Debye temperature and the temperatures at which the Hall coefficients of Cu, Ag, and Au become temperature-dependent.

D. Fermi Surface in Cu

The thick-sample results are of interest in relation to the question of the configuration of the Fermi surface, but in view of the admitted limitations of existing theory it would be presumptuous to attempt to draw definite conclusions. It is fair to state, however, that *if* the assumptions of the semiclassical theory are valid, then the data are consistent with a Fermi surface which touches the Brillouin zone boundary. The observed saturation value for R is -6.25×10^{-5} cm³/coulomb which differs significantly from the free-electron value -7.45×10^{-5} cm³/coulomb. Furthermore, if a saturation value for the magnetoresistivity exists, it is considerably greater than 3 to 4 times the zero-field resistivity. Olson and Rodriguez[27] have obtained magneto resistance data on Cu single crystals at such low fields that objections to the applicability of the semiclassical theory are somewhat moderated. They conclude that the Fermi surface probably touches the zone boundary in the [111] directions, in agreement with Pippard's anomalous skin effect findings.[6]

IV. DISCUSSION

For reasons already noted, it is not surprising that the free electron theory appears to be inapplicable to the size effect in the Hall coefficient of Cu. However, the appearance of size effects in samples where the thickness is considerably greater than the generally accepted value for the mean free path is counter to foreseeable extensions of the theory. The literature reveals some additional data with possible bearing on this problem. Borovik[8–10] has carried out careful low-temperature measurements on the Hall coefficient and magnetoresistance of high-purity samples (single crystals in many instances) of Pb, Cd, Mg, Be, and Al with thicknesses ranging from 0.048 mm to 0.6 mm and s values ranging from 90 to 1.8×10^4. Because many of the observed R *versus* H curves are similar in character to the curve for $Cu_{0.054}$ in Fig. 4, it is possible, though by no means established, that size effects were important.

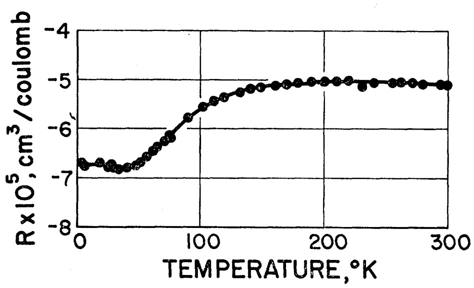

FIG. 9. Hall coefficient *versus* temperature for $Cu_{0.145}$ before annealing.

Borovik's transverse magnetoresistance determinations[28] on a 1-mm diameter single crystal of Sn with its tetragonal axis perpendicular to the wire axis and $s=2.3\times10^3$ are particularly interesting. When H was parallel to the tetragonal axis of the crystal, Borovik noted that a periodic function, for which he could offer no explanation, was superimposed on the normal

[25] B. R. Coles, Phys. Rev. **101**, 1254 (1956).
[26] T. Fukuroi and T. Ikeda, Science Repts. Tohoku Univ. **A8**, 205 (1956).
[27] R. Olson and S. Rodriguez, Phys. Rev. **108**, 1212 (1957).
[28] E. S. Borovik, J. Exptl. Theoret. Phys. U.S.S.R. **23**, 91 (1952) (translation: Naval Research Laboratory Report NRL-463).

magnetoresistance curve. It can now be argued that he was seeing size-effect oscillations of the type observed by Babiskin and Siebenmann in Na, for the amplitude of the oscillations diminishes with increasing field, and the oscillations appear to be periodic in H as required by Sondheimer's theory. A careful attempt was made to determine the locations in field of the maxima and minima. These locations are given in the theory by

$$H_i = im\bar{v}c/et,$$

where i equals 1, 7, 13, $\cdots$ for maxima and 4, 10, 16, $\cdots$ for minima, and $m\bar{v}$ is the electron momentum at the Fermi level. The experimental values are plotted against i in Fig. 10, and the linearity of the plot supports the belief that these are indeed size-effect oscillations. However, a serious quantitative discrepancy exists in that the slope of this line, in combination with the above equation with $t=1$ mm, yields a momentum $m\bar{v} = 1.37\times10^{-18}$ g cm sec^{-1}, whereas the free-electron value for Sn for four electrons per atom is 1.73×10^{-19} g cm sec^{-1}. The discrepancy is even worse if the more realistic de Haas-van Alphen effect or anomalous skin effect momenta[29] are used in the comparison. Furthermore, the mean free path in Borovik's Sn sample (estimated from the ρl value given by Kunzler and Renton[30]) was probably only about 0.04 mm. This is considerably smaller than the sample diameter so that a condition for observation of the oscillations was violated. An internal size effect could perhaps account for the discrepancy, but such an explanation would require the existence of rather small, well-defined layers of uniform thickness. Measurements on a crystal reduced in size by successive etchings would provide a test of this possibility. If the effect is a genuine size effect, then the discrepancy more than likely arises because the theory applies to a spherical Fermi surface (and to films rather than to wires). A more general theory might provide means for determining the shape of the Fermi surface from magneto-oscillatory size effects.

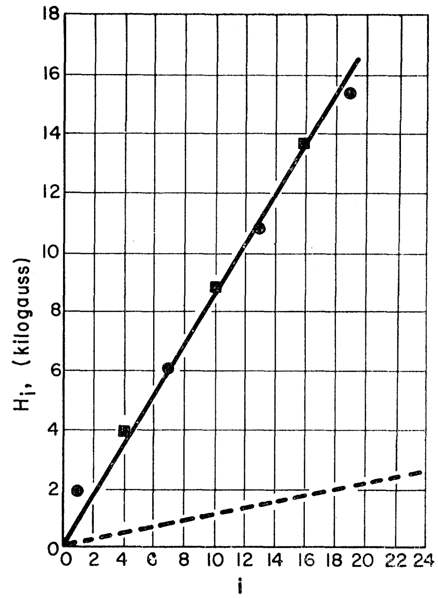

FIG. 10. Values of magnetic field strength at which maxima (circles) and minima (squares) occur in Borovik's magnetoresistance data on a thin Sn single crystal, plotted against integers i. The solid and dashed lines correspond respectively to momenta at the Fermi surface of 1.37×10^{-18} g cm sec^{-1} and 1.73×10^{-19} g cm sec^{-1}. The latter value corresponds to four free electrons per atom in Sn.

V. ACKNOWLEDGMENT

It is a pleasure to acknowledge many stimulating discussions with Dr. G. W. Lehman.

[29] See D. Shoenberg, in *Progress in Low Temperature Physics* (North-Holland Publishing Company, Amsterdam, 1957), Vol. 2, p. 226.

[30] J. E. Kunzler and C. A. Renton, Phys. Rev. **108**, 1397 (1957).