Magnetoresistance in n-Type Germanium at Low Temperatures*

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Magnetoresistance was studied for n-type single-crystal germanium of 4×10^{15} effective donors/cm^3. The effective anisotropy parameter K was found to decrease from $\sim$20 at 300°K to $\sim$6 at 20°K. Values close to 20 were again obtained at 7°K and 4.2°K. By introduction of compensating acceptors with heat treatment, it was shown directly that the decrease of K was due to anisotropic scattering by ionized impurities, and the anisotropy was investigated by using different degrees of compensation. Below 7°K, the scattering is determined by neutral impurities, and the high value of K indicates that the scattering is isotropic.

INTRODUCTION

THE nature of the multivalleyed conduction band in n-type germanium, with four energy minima in the $\langle 111\rangle$ directions in momentum space, has been well established from cyclotron resonance[1,2] and other measurements. The anisotropy of magnetoresistance gave the earliest evidence for an anisotropic band structure.[3,4] The quantitative interpretation of magnetoresistance depends, however, not only upon the band structure but also upon the scattering mechanism. An effective anisotropy parameter K can be defined in terms of magnetoresistance and Hall coefficients. If the scattering of electrons can be represented by a relaxation time τ which is isotropic in momentum space, the value of K is simply $K=m_{\parallel}/m_{\perp}\equiv K_m$, i.e., the ratio of effective masses along and perpendicular to the major axes of the energy ellipsoid. If τ has the same symmetry as the energy ellipsoids, and if the quantity $\tau_{\parallel}/\tau_{\perp}\equiv K_\tau$ is independent of energy, the effective K can be represented as $K\equiv K_m/K_\tau$.

Measurements by other workers[5–7] in the carrier exhaustion range from 300°K to 50°K have shown that very pure samples give values of K about equal to the cyclotron resonance value $K_m=20$ (obtained at 4.2°K). Since scattering at high temperatures in pure samples is dominated by lattice vibrations, these measurements indicate that lattice scattering is isotropic. For samples in which ionized impurity scattering is important, K is found to decrease with decreasing temperature and increasing impurity concentration.[8] Thus these measurements indicated qualitatively the anisotropic character of ionized impurity scattering, since K_m is not expected to exhibit appreciable temperature dependence.

The present measurements have been extended down to 4.2°K. Using samples of 4×10^{15} effective donors/cm^3, the value of K was found to decrease from 20 at 300°K to 6 at 20°K. However, at 7.0°K, when most of the effective impurity atoms become electrically neutral, the value of K was again 20. The value was greatly reduced when compensating acceptors were introduced into the sample. The experiments provide a clear proof that scattering by ionized impurities is anisotropic, with a reduced value of K. The results also lead to the conclusion that the scattering at low temperatures was controlled by neutral impurities with essentially isotropic relaxation time. The introduction of increasing amounts of compensating acceptors into the same sample provided a means for studying the anisotropy of scattering by ionized impurities. This method has an advantage over higher-temperature measurement for various impurity concentrations, in that screening of ionized centers by free electrons is unimportant.

THEORY USED FOR INTERPRETATION

The conduction band for n-type germanium may be described by 4 energy ellipsoids of revolution oriented with major axes in the $\langle 111\rangle$ directions of momentum space. The electron energy for each ellipsoid is described by the relation

$$\epsilon=\pm\left(\frac{p_x^2}{2m_x}+\frac{p_y^2}{2m_y}+\frac{p_z^2}{2m_z}\right);\quad m_x=m_y=m_\perp\neq m_{\parallel}=m_z. \quad (1)$$

Herring and Vogt[9] have derived expressions for the magnetoresistance coefficients in the limit of small magnetic field, under the condition that a relaxation time for carrier scattering exists and has the symmetry of the energy ellipsoids, so that it may be described as a diagonal tensor with $\tau_x=\tau_y=\tau_\perp$, $\tau_z=\tau_{\parallel}$. Under this assumption, the magnetoresistance and Hall mobility for the case of $\langle 111\rangle$ ellipsoids are related in the expression

$$W\equiv\lim_{H\to 0}\frac{M_{100}/H^2}{M_{100}{}^{010}/H^2+\mu_H{}^2}$$
$$=\frac{2[K_m{}^2\langle\tau_\perp{}^3\rangle-2K_m\langle\tau_\perp{}^2\tau_{\parallel}\rangle+\langle\tau_\perp\tau_{\parallel}{}^2\rangle]}{[2K_m{}^2\langle\tau_\perp{}^3\rangle+5K_m\langle\tau_\perp{}^2\tau_{\parallel}\rangle+2\langle\tau_\perp\tau_{\parallel}{}^2\rangle]}, \quad (2)$$

* Work supported by an Office of Naval Research Contract.
[1] Dexter, Kip, and Kittel, Phys. Rev. **98**, 368 (1955).
[2] Dexter, Zeiger, and Lax, Phys. Rev. **104**, 637 (1956).
[3] G. L. Pearson and H. Suhl, Phys. Rev. **78**, 646 (1950).
[4] B. Abeles and S. Meiboom, Phys. Rev. **95**, 31 (1954).
[5] R. M. Broudy and J. D. Venables, Phys. Rev. **103**, 1129 (1956).
[6] H. P. Furth and R. W. Waniek, Phys. Rev. **104**, 343 (1956).
[7] C. Goldberg, Phys. Rev. **109**, 331 (1958).
[8] M. Glicksman, Phys. Rev. **108**, 264 (1957).
[9] C. Herring and E. Vogt, Phys. Rev. **101**, 944 (1956).

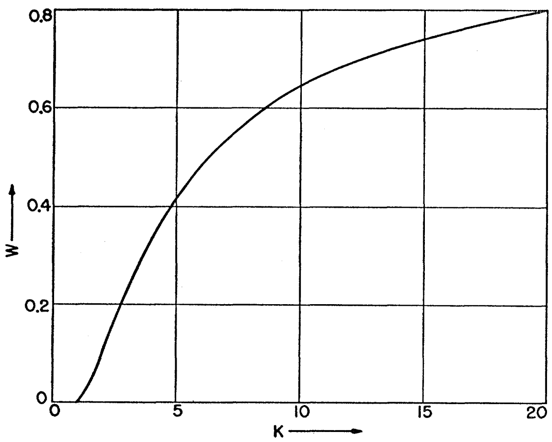

FIG. 1. Value of the function W as a function of K.

where $M_{100}=\Delta\rho/\rho_0)_{I(100)}{}^{H(100)}$, $M_{100}{}^{010}=\Delta\rho/\rho_0)_{I(100)}{}^{H(010)}$, μ_H is the Hall mobility, $K_m=m_{\parallel}/m_{\perp}$, and

$$\langle\tau_\perp{}^r\tau_\parallel{}^s\rangle\equiv\int_0^\infty \tau_\perp{}^r\tau_\parallel{}^s\epsilon^{\frac{3}{2}}\exp(-\epsilon/kT)d\epsilon.$$

In the special case where

$$K_\tau\equiv\tau_\parallel/\tau_\perp \tag{3}$$

is independent of energy, W reduces to the simple form

$$W=\frac{2(K-1)^2}{(2K+1)(K+2)}, \tag{4}$$

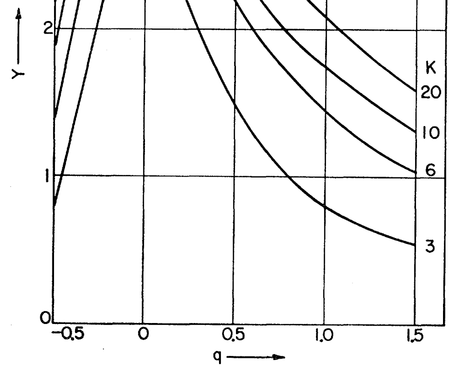

FIG. 2. Value of the function Y as a function of q for various values of K, assuming relaxation time $\tau\propto\epsilon^q$.

where

$$K=K_m/K_\tau. \tag{5}$$

More generally, K may be regarded as a parameter defined operationally in terms of M_{100}, $M_{100}{}^{010}$, and μ_H by means of (4) and (2), irrespective of the validity of (5). W as a function of K is shown in Fig. 1.

If (3) is valid, we get the expression

$$Y\equiv\lim_{H\to0}\frac{M_{100}}{M_{100}{}^{010}}=\frac{\nu\left[\frac{2}{3}\frac{(2K+1)(K-1)^2}{K(K+2)^2}\right]}{\left\{\nu\left[\frac{(2K+1)^2}{3K(K+2)}\right]-1\right\}}, \tag{6}$$

where

$$\nu\equiv\langle\tau_\perp\rangle\langle\tau_\perp{}^3\rangle/\langle\tau_\perp{}^2\rangle^2.$$

Through ν, Y depends on the variation of τ with energy. Therefore the relation (6) can be used to study the energy dependence of τ. Figure 2 shows Y for various values of K as a function of q, under the assumption that $\tau\propto\epsilon^q$. Y is seen to be strongly dependent both upon K and q. For a given K, two values of q satisfy the measured Y, so that other information is necessary to choose between a positive or negative q. The temperature dependence of Hall mobility could be expected to provide this information: i.e., increasing mobility with increasing temperature implies a positive power dependence of τ upon energy.

The validity of (3) also implies that the high-field theory of Abeles and Meiboom[4] is applicable, with K substituted for K_m. By using this theory, information can be obtained from the field dependence of magnetoresistance. K uniquely determines the saturation value of longitudinal magnetoresistance in the high-field limit, independent of the nature of the variation of relaxation time with energy.

The assumption of a relaxation time tensor having the symmetry of the energy ellipsoids leads to the "symmetry relation" $\gamma=-\beta$, where γ and β are phenomenological magnetoconductance coefficients introduced by Seitz.[10] This relation is equivalent to the expression

$$(M_{110}{}^{\bar{1}10}/M_{110}{}^{001})-1=M_{110}/M_{110}{}^{001}. \tag{7}$$

The violation of this relation would indicate that the relaxation time assumption is not valid. Careful experiments thus far[11] have shown that the symmetry relation is quite well satisfied, justifying the assumption. However, Herring and Vogt have shown that calculations based upon the above assumption of relaxation time may give good results even when the scattering is actually not describable by a relaxation time.

[10] F. Seitz, Phys. Rev. **79**, 373 (1950).
[11] C. Goldberg and W. Howard, Phys. Rev. **110**, 1035 (1958).

EXPERIMENTAL RESULTS

The samples were single crystal n-type germanium, oriented by x-ray. Measurements of K were made with samples having length, width, and thickness in the $\langle 100 \rangle$ directions. The symmetry relation was checked using samples in which the current direction was [110], the other two directions being $[\bar{1}10]$ and [001]. The measurements were made in a low-temperature cell which makes use of helium exchange gas and a heater to provide temperatures above bath temperature. At 7.0°K, the temperature could be maintained constant to within ±0.004°K. Such careful temperature control was necessary because of the extremely large dependence of resistivity upon temperature in this range. Determination of W from magnetoresistance and Hall mobility was obtained by computing W at each field, and extrapolating to zero magnetic field, as illustrated in Fig. 3.

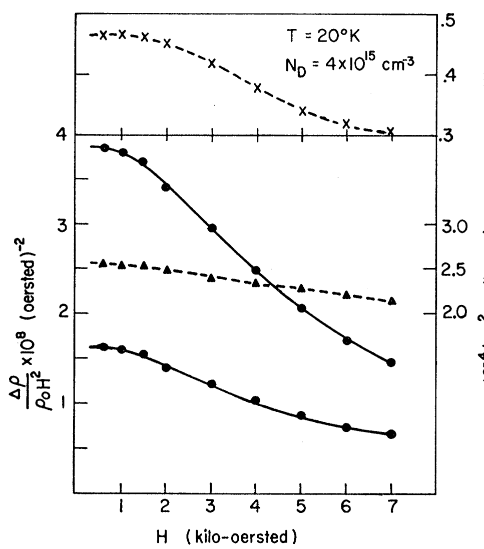

FIG. 3. The magnetic field dependence of magnetoresistance, Hall mobility, and function W for an uncompensated sample.

The temperature dependence of the effective anisotropy parameter K determined for a sample of $N_D = 4\times10^{15}/\text{cm}^3$, where N_D is the original concentration of the effective donors is shown in Fig. 4. The decrease of K with decreasing temperature from the room-temperature value of ~20 is due to the increase of ionized impurity scattering. The rise at low temperature shows that the scattering mechanism has changed between 20°K and 7°K. The value of $K\sim20$ at 7°K is in agreement with cyclotron resonance measurements for K_m, and thus indicates that the scattering is isotropic. It is estimated from the Hall coefficient that about 20% of the effective donors are ionized at 20°K, while the number is negligible at 7°K.

In order to make sure that the increase of K from 20°K to 7°K is indeed due to the decrease of ionized

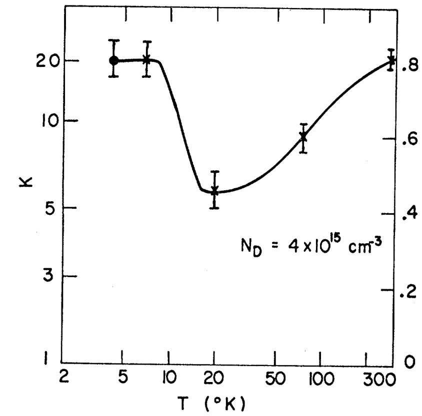

FIG. 4. Temperature dependence of the effective anisotropy parameter K.

impurity scattering, compensating impurities were introduced by heat treatment. The sample was heated in helium atmosphere at 640°C for one hour, and quenched to room temperature. The acceptors introduced were probably copper thermally diffused into the sample, although no copper was deliberately added. As a result of this heat treatment, the concentration of ionized impurities at low temperatures was increased by an amount of $\sim0.15N_D$. The results are shown in Fig. 5. The Hall mobility and factor W at 8.0°K were both reduced to nearly the same values obtained at 20°K before the heat treatment. This result gives

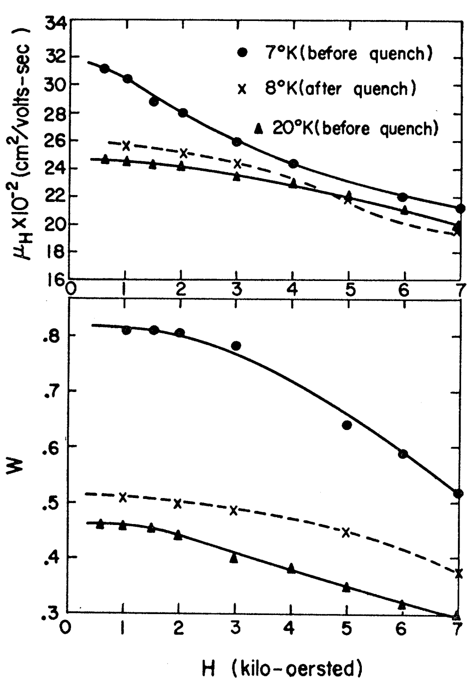

FIG. 5. The magnetic field dependence of the function W and the Hall mobility, showing the influence of additional ionized impurities at low temperature.

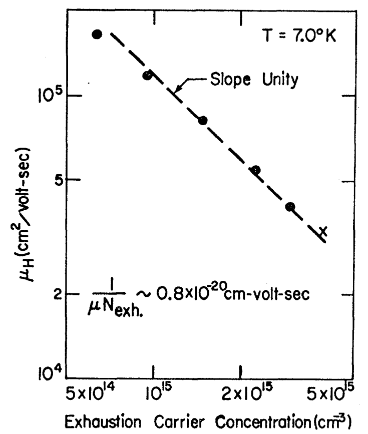

FIG. 6. Hall mobility of electrons at low temperature as a function of exhaustion carrier concentration. Solid points from unpublished data of H. Fritzche.

direct evidence that ionized impurities reduce the value of K. The fact that the samples before the heat treatment had at 7°K a value of $K\sim20$ characteristic of isotropic τ shows that scattering by ionized impurities must have been unimportant. Therefore the concentration of ionized impurities due to any possible compensation must have been much less than $0.15N_D$. This deduction is in agreement with the low value of K obtained at 20°K before the heat treatment. The concentration of ionized impurities was about $0.20N_D$ at 20°K, if there was negligible compensation. Figure 5 shows that the curve of $W(H)$ is somewhat lower than the curve for 8°K after heat treatment, when the concentration of ionized impurities was $0.15N_D$.

The scattering at low temperature with apparently isotropic τ seems to be due to neutral impurities. The Hall mobility of 7°K in similar samples has been found to be inversely proportional to the number of effective donors as determined from the exhaustion carrier concentration. Figure 6 shows this relationship. This is the behavior to be expected for neutral impurity scattering, since the number of neutral impurities at low temperature is equal to the number of donor atoms, in the absence of compensating acceptors. Early treatment of neutral impurity scattering by Erginsoy[12] gave an energy-independent relaxation time. Determination of ν from Y and K according to (6) shows $\nu\neq1$. Hence the relaxation time must be energy-dependent. Under the assumption of a simple power dependence of τ on energy, the data indicate that the relaxation time should vary roughly as the 0.7 power of the energy. Recent treatment of neutral impurity scattering by Sclar[13] shows that some energy dependence of τ may be expected. However, both theoretical treatments are based upon rather simplified models and isotropic effective mass. The nature of neutral impurity scattering requires further study.

To investigate the anisotropy of scattering by ionized impurities, larger amounts of compensating impurities were introduced by quenching from higher temperatures. Measurements were made at 20°K, a temperature low enough so that lattice scattering would be small, but not so low that impurity-band conduction would influence the results. Figure 7 shows the result for one such measurement, in which compensating centers to the extent of $\sim0.35N_D$ were introduced. The fraction of total impurities, acceptors plus donors, ionized at 20°K is ~0.6. The longitudinal magnetoresistance has

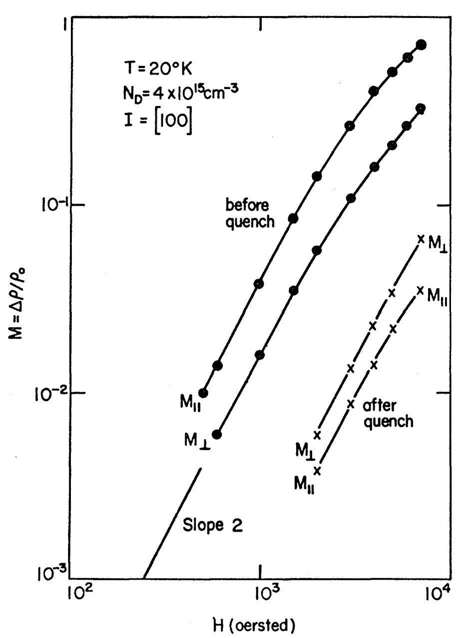

FIG. 7. Magnetic field dependence of magnetoresistance for uncompensated and heavily compensated n-type Ge.

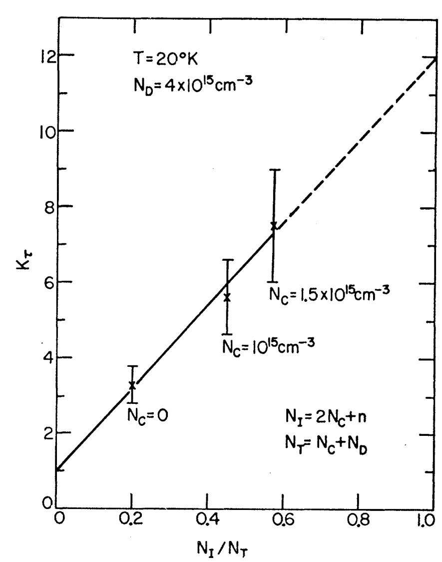

FIG. 8. Scattering anisotropy parameter K_T as a function of fraction of ionized to total impurities for increasing concentration of compensating impurities.

[12] C. Erginsoy, Phys. Rev. **79**, 1013 (1950).
[13] N. Sclar, Phys. Rev. **104**, 1559 (1956).

become smaller than the transverse, in contrast to the behavior before the introduction of acceptors, where only ~0.2 of the total impurities are ionized. Figure 8 shows a plot of the derived K_τ with increasing N_I/N_T. There should be no anisotropy when the scattering is due to neutral impurities, so the curve can be drawn through $K_\tau=1$ at $N_I/N_T=0$. Ham[14] has calculated a $K_\tau=12$ for ionized impurity scattering by generalizing the Brooks-Herring mobility analysis to the case of ellipsoidal energy surfaces with $K_m=19$ and by considering only low-angle scattering. This value is consistent with the extrapolation of the experimental curve to $N_I/N_T=1$. With the introduction of compensating impurities, the Hall mobility decreased from $\mu_H=25\,000$ cm²/volt-second for $N_I/N_T=0.2$ to $\mu_H=5700$ for $N_I/N_T=0.57$. The quantity $1/\mu_H N_I$, however, changed only from 5×10^{-20} to 5.9×10^{-20} cm-volt-second as shown in Fig. 9. This observation indicates that with $N_I/N_T=0.2$, the mobility was already largely limited by ionized impurity scattering. On the other hand, the value of K_τ was only 3.3 at $N_I/N_T=0.2$, considerably lower than is expected for pure ionized impurity scattering. Thus K_τ is much more sensitive to the admixture of other scattering mechanisms than is the mobility. The quantity $1/\mu_H N_I$ is a measure of the scattering cross section of ionized impurities. The curve $1/\mu_H N_T$ is also plotted in Fig. 9. By extrapolating this curve to $N_I/N_T=0$, we get $1/\mu_H N_N \sim 0.45\times10^{-20}$ cm-volt-second for neutral impurities. Erginsoy's expression for neutral impurity scattering gives a value of 0.55×10^{-20} cm-volt-second.

The symmetry relation was checked. At 300°K and 77°K, before quenching, $\gamma/\beta=-1.00\pm0.04$. At 20°K, in a sample quenched to $N_I/N_T=0.45$, $K_\tau\sim5.6$ and $\gamma/\beta=-0.97\pm0.06$. It would thus appear that there is no glaring deviation from the symmetry relation, even with fairly anisotropic scattering.

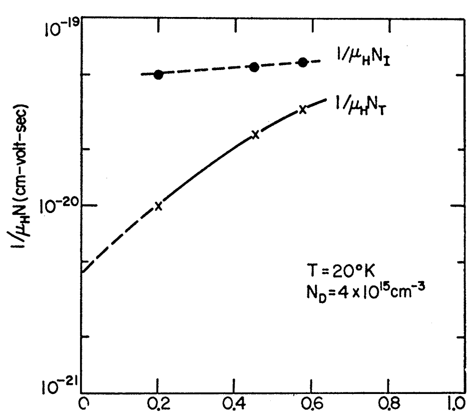

FIG. 9. Dependence of $1/\mu_H N_I$ and $1/\mu_H N_T$ upon the fraction of ionized to total impurities.

[14] F. Ham, Phys. Rev. **100**, 1251(A) (1955).

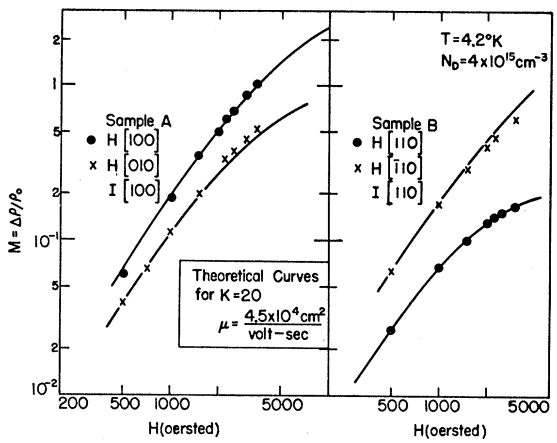

FIG. 10. The magnetic field dependence of magnetoresistance of optically excited electrons, compared with high-field theory of Abeles and Meiboom.

Some measurements were also made at liquid helium temperature. The sample showed impurity-band conduction behavior, with low mobility and magnetoresistance. However, the magnetoresistance of conduction electrons can be determined by exciting them to the conduction band with infrared radiation. Infrared light of lower photon energy than the intrinsic absorption was used, to insure the absence of hole excitation, and to give uniform bulk illumination. The magnetoresistance of the photoconductive electrons was measured using chopped light and ac detection, so as to eliminate effects of conduction by unexcited carriers. Measurements were made (before compensation) upon the samples described above, at fields up to 3500 oersted. The Hall mobility of the excited electrons was not measured, so W would not be determined directly. Figure 10 shows that the data could be fitted with the Abeles-Meiboom high-field theory for a Hall mobility of 45 000 cm²/volt-second and $K=20$, again indicating isotropic scattering. The symmetry relation is also obeyed to within experimental error.

CONCLUSIONS

Ionized impurities decrease the effective anisotropy parameter K. The results obtained with the introduction of different amounts of ionized impurities indicate that K_τ may be as high as 12 for pure ionized impurity scattering. The symmetry relation is found to be quite well obeyed for values of $K_\tau \lesssim 5.6$. In the samples used, the scattering at low temperatures, when nearly all of the carriers are frozen out, is mainly due to neutral impurities. The parameter K reaches the value $K_m=20$, indicating that neutral impurity scattering is isotropic.