Vector Interaction in Beta Decay*

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Further experimental consequences of a beta-decay interaction generated by a conserved vector current, as proposed by Feynman and Gell-Mann, are studied. Utilizing the strong analogy between the conserved beta-decay current, and the conserved charge current, higher multipole corrections to the usual results for allowed transitions are computed, and are related to the corresponding electromagnetic transitions, as was first done by Gell-Mann. The effects calculated include corrections to spectra shapes, beta-gamma directional correlations, and beta-alpha directional correlations. A multipole expansion of the V-A interaction is carried out, including all those terms normally classed as allowed and second forbidden. Applications to several interesting transitions are discussed.

I. INTRODUCTION

THE concept of a universal V-A coupling has had many successes in the description of weak interactions.[1] In particular, if the vector coupling constant determined[2] from the decay of O^{14} [$(1.41\pm0.01)\times10^{-49}$ erg/cm^3] is used to predict the lifetime of the muon, one obtains a lifetime of $(2.26\pm0.04)\times10^{-6}$ sec, as compared with the experimental value of $(2.22\pm0.22)\times10^{-6}$ sec.[3] This agreement is surprising in view of the fact that beta decay involves nucleons, which participate in strong interactions, while the muon decay involves particles with no strong interactions; one would therefore expect large renormalization effects in beta decay and essentially no such effects in muon decay. To explain this agreement, Feynman and Gell-Mann[1] have noted that the equality would hold rigorously, except for electromagnetic effects, if the vector beta-decay coupling were generated by the interaction of a conserved current with itself. Part of this current,

$$\bar{\psi}_n\tau_+\gamma_\mu\psi_n+\bar{\psi}_\nu\gamma_\mu\psi_e+\bar{\psi}_\nu\gamma_\mu\psi_\mu,$$

would generate the usual Fermi interaction; but in order that the current be conserved under the action of the strong coupling between nucleons and pions, we must add to the expression above the term

$$i[\phi^*T_+\partial_\mu\phi-(\partial_\mu\phi)^*T_+\phi].$$

In analogy with the conservation of total charge in electrodynamics, the conservation of this current implies the identity of the vector coupling constants in beta decay and muon decay.

In a recent paper[4] Gell-Mann has shown that the concept of a conserved vector current has specific experimental consequences for beta decay itself. We can paraphrase his discussion as follows: designating the initial and final nuclear states by $|i\rangle$ and $|f\rangle$, the conventional vector beta-decay interaction leads to the matrix element $\langle f|\bar{\psi}_n\gamma_\mu\tau_+\psi_n|i\rangle$ while the Feynman–Gell-Mann scheme would lead to the matrix element

$$\langle f|\bar{\psi}_n\tau_+\gamma_\mu\psi_n+i[\phi^*T_+\partial_\mu\phi-(\partial_\mu\phi)^*T_+\phi]|i\rangle.$$

This latter matrix element is the same as that occurring in the isotopic vector part of the electromagnetic interaction of the nucleus,

$$\langle f|\bar{\psi}_n\tau_3\gamma_\mu\psi_n+i[\phi^*T_3\partial_\mu\phi-(\partial_\mu\phi)^*T_3\phi]|i\rangle,$$

except for the change in the isotopic spin dependence $\tau_+\to\tau_3$, $T_+\to T_3$. If charge independence of the nuclear states is assumed, the substitution $\tau_+\to\tau_3$, $T_+\to T_3$ alters the matrix element only by a factor. Thus, up to a factor, the vector beta transitions and the electromagnetic transitions from one isotopic multiplet to another, involve exactly the same nuclear matrix elements; in particular, the multipole expansions of the transition matrix elements for the emission of a photon, and for vector beta decay, generate the *same* multipole operators. Comparison of the observed decays of the different members of an isotopic multiplet leads to a test of the theory. Note that only the vector part of the beta coupling has been altered; the axial vector part is treated as before.[5]

As an example, Gell-Mann discussed the decay of the $T=1$, $J=1^+$ multiplet in the $A=12$ system, into the $T=0$, $J=0^+$ ground state. The electromagnetic transition is pure $M1$, isotopic vector, and according to the new theory the observed rate of the electromagnetic transition enables one to predict the vector contribution to the beta decays of B^{12} and N^{12}. In the conventional vector coupling, this transition would involve the

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[1] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958); E. C. G. Sudarshan and R. E. Marshak, Phys. Rev. **109**, 1860 (1958); J. J. Sakurai, Nuovo cimento **5**, 649 (1958).

[2] Bromley, Almquist, Gove, Litherland, Paul, and Ferguson, Phys. Rev. **105**, 957 (1957).

[3] W. E. Bell and E. P. Hincks, Phys. Rev. **84**, 1243 (1951).

[4] M. Gell-Mann, Phys. Rev. **111**, 362 (1958). We are also indebted to Professor Gell-Mann for helpful private communications during the course of this work.

[5] It has been shown independently by M. Goldberger and S. Treiman [Phys. Rev. **110**, 1478 (1958)] and by R. Norton and K. Watson (private communication) that the assumption of a conserved axial vector current leads to contradictions. The former authors show that there would be too large an effective pseudoscalar coupling in the beta decay of the neutron, while the latter authors show that the π-μ decay would be forbidden completely.

second-forbidden matrix element $\int\tau_{\pm}\boldsymbol{\alpha}\times\mathbf{r}$, which is the ordinary magnetic moment operator for a Dirac particle. Here the new theory results in the replacement of the *ordinary* magnetic moment operator by the *total* magnetic moment operator, Dirac plus pionic.

The axial vector part of the interaction contributes the allowed matrix element $\int\tau_{\pm}\boldsymbol{\sigma}$, which in fact dominates the transition matrix element, as well as further second-forbidden terms. The observable consequences of the theory are therefore restricted to the small corrections to allowed transitions introduced by the second-forbidden matrix elements. These corrections are, of course, present in the Fermi theory as well as the Feynman–Gell-Mann theory; in the latter the corrections due to the vector interaction are directly related to the electromagnetic multipole moments, and can be determined separately. For B^{12} and N^{12} the corrections to the spectrum have been shown by Gell-Mann to be $\sim$10%, and of opposite sign for the two transitions.

In this paper, we shall extend the calculations of Gell-Mann in several respects. Firstly, rather than keeping only first-order terms in the momentum transfer, we will include all the contributions normally classed as second forbidden. There seems to be no justification in general for considering matrix elements like $\int\tau_{+}r_ir_j$ as small compared to matrix elements like $\int\tau_{+}\alpha_ir_j$. Elsewhere in the literature these have been called "moment" and "velocity" terms, respectively; the first is of order $(pR)^2$ and the second of order $(v/c)pR$. The two matrix elements $\int r_ir_j\tau_{+}$ and $\int\alpha_ir_j\tau_{+}$ are in fact both replaced by the electric quadrupole operator in the Gell-Mann theory, and contribute essentially equally. It is important to realize that the inclusion of more matrix elements in the analysis makes it much more difficult to obtain a clear-cut proof or disproof of the idea of a conserved current, since there are more free parameters in the theory. We shall discuss in Sec. III the possibility of reducing the number of unknowns sufficiently to test the theory.

Secondly, we shall extend the calculations to include angular correlations[6] as well as beta spectra. Since the allowed transitions give rise to no directional correlations, the observation of a correlation constitutes a direct measurement of the effect of the corrections due to higher multipole moments.

Thirdly, we shall include some Coulomb effects in the calculations. There is, of course, no rigorous way of including electromagnetic effects, since the electromagnetic interaction destroys the validity of the conserved beta-decay current, and of the charge independence of nuclear states. However, there is probably some meaning to attempting an approximate Coulomb correction by using Coulomb wave functions for the electrons, rather than plane waves. For example, this correction serves to bring into agreement the ft values of decays from different states of the same isotopic multiplet even when the multiplet is tilted by the Coulomb energy differences; the equality of the ft values of the decays of P^{28} and Al^{28} provides a case in point.

It seems appropriate to make a few general comments on the problem of designing a conclusive test of the concept of a conserved current. Since we want to make use of the charge independence of nuclear states, the applications must be limited to the light nuclei, where with few exceptions beta transitions are allowed. Transitions *along* the multiplet do not seem to lead to an unambiguous test of the theory; one notes first that the corrections due to higher multipole moments will be very small due to the low energy transfer and the superallowed nature of the allowed matrix elements. Second, there seems to be no way, at least insofar as the nucleus can be represented as a collection of independent nucleons, of distinguishing experimentally between the old and the new forms of the allowed vector matrix element. The only difference between them is that in the Fermi theory the operator which appears is τ_{+}, while in the Feynman–Gell-Mann theory the operator is $I_{+}=\tau_{+}+T_{+}$. Since for a single nucleon these two operators have matrix elements which are proportional to one another, any difference between them can be absorbed into the (arbitrary) coupling constant. Thus by appropriate adjustment of the coupling constant, the Fermi theory and the Feynman–Gell-Mann theory can be made to predict the same over-all rate for allowed transitions.

The only remaining transitions are those in which the isotopic spin changes[7]; the only contribution of the vector interaction to such transitions is in second-forbidden matrix elements, and we are forced to look for small corrections due to such terms. These corrections are only expected to be discernible when the momentum transfer is very high, or the main Gamow-Teller matrix element very small, or both. In the discussion in Sec. III we have therefore concentrated attention on those transitions which have the most favorable combination of high momentum transfers, and small matrix elements.

We also note that the most striking difference be-

[6] The importance of studying angular correlations in allowed transitions as a check on the new beta-decay theory has also been stressed by M. Gell-Mann and B. Stech (private communication); See F. Boehm *et al.*, Phys. Rev. Letters **1**, 77 (1958).

[7] It is interesting to note another distinction between the old Fermi theory and the new theory which might be studied experimentally. In a *J-J* transition the usual Fermi matrix element would involve τ_{+}, the contribution to the isotopic spin from the nucleons alone, while in the new theory the Fermi matrix element involves I_{+}, the total isotopic operator. Hence in the new theory *J-J* (no) transitions off an isobar are forbidden by the $\Delta I=0$ selection rule. In the old Fermi theory one would have to evaluate the matrix elements of τ_{+}, which differs from I_{+}, and allowed Fermi transitions off an isobar are possible in principle. If the nucleus is adequately described by an independent-particle model then the same selection rules hold for τ_{+} and I_{+}, which is to say, with this assumption the old theory and the new one would both imply the impossibility of allowed Fermi transitions off of the isobar. A case in point is the mass-24 system in which the $4^{+}\rightarrow4^{+}$ $I=1$ to $I=0$ transition of Na^{24} could have a Fermi matrix element according to the old theory (apart from that due to electrodynamic effects).

tween the old and new forms of the vector interaction should appear in the $M1$ matrix element, rather than the $E2$ matrix element, since the anomalous contribution to the $M1$ operator is quite large, and that to the $E2$ operator is probably small.[8]

II. GENERAL THEORY

As we have already noted, the matrix element appearing in the vector beta-decay interaction differs from that appearing in the electromagnetic interaction only in its isotopic spin dependence, and so the multipole expansion of these matrix elements contains the same multipole operators. This can be explicitly verified by constructing the expansions of both matrix elements to zeroth order in the meson field. Such a derivation serves the purpose of relating the multipole operators to their conventional definitions, and of relating the multipole expansions of the Feynman–Gell-Mann interaction to that of the Fermi interaction. We shall only give the results of this procedure.

Starting with the actual beta-decay Hamiltonian written in the notation of reference 4,

$$-(G/\sqrt{2})(\mathcal{J}_{\mu^+}{}^{V}+\lambda\mathcal{J}_{\mu^+}{}^{A})i[\bar{e}\gamma_\mu(1+\gamma_5)\nu] +\text{Herm. conj.}, \quad (1)$$

with

$$\mathcal{J}_{\mu^+}{}^{V}=\bar{\psi}(\tau_+\gamma_\mu)\psi+\text{Herm. conj.} +i[\phi^*T_+\partial_\mu\phi-(\partial_\mu\phi)^*T_+\phi], \quad (2)$$

$$\mathcal{J}_{\mu^+}{}^{A}=\bar{\psi}(\tau_+i\gamma_\mu\gamma_5)\psi+\text{Herm. conj.}$$

and $\lambda=-C_A/C_V$ (in the V-A theory λ is a positive number), it can be shown that the effective Hamiltonian has the form

$$H_{\text{eff}}=(G/\sqrt{2})u_e{}^\dagger(p)\mathcal{L}(1+\gamma_5)u_\nu(q), \quad (3)$$

$$\begin{aligned}\mathcal{L}=\Big[&E0+i(\rho_1\boldsymbol{\sigma}W_0-\mathbf{k})\cdot\mathbf{E1}\\ &-i\rho_1\boldsymbol{\sigma}\times\mathbf{k}\cdot\mathbf{M1}+i(\tfrac{1}{2}W_0\rho_1\sigma_ik_j-\tfrac{1}{2}k_ik_j)(E2)_{ij}\\ &+(\tfrac{1}{3}W_0\rho_1\boldsymbol{\sigma}\cdot\mathbf{k}-\tfrac{1}{6}k^2)E0'\\ &+\lambda\Big(\boldsymbol{\sigma}\cdot\int\boldsymbol{\sigma}+i\rho_1\mathbf{k}\cdot\int\rho_1\mathbf{r}-\tfrac{1}{2}\sigma_ik_jk_l\int\sigma_ir_jr_l\\ &-i\sigma_ik_j\int\sigma_ir_j-\rho_1\int\rho_1\Big)\Big]. \end{aligned} \quad (4)$$

Here $\mathbf{k}=\mathbf{p}+\mathbf{q}$, where $\mathbf{p}$ and $\mathbf{q}$ are the momenta of electron and neutrino; the isotopic spin dependence has been suppressed. We have included all those terms normally classed as allowed, first and second forbidden. In zeroth order, the multipole operators are given in the nonrelativistic limit as follows:

$$E0=\int 1, \qquad E0'=\int r^2,$$

$$\mathbf{M1}=\frac{1}{2M}\int\mathbf{r}\times\mathbf{p}+\boldsymbol{\sigma}, \quad \mathbf{E2}=\int(\mathbf{rr}-\tfrac{1}{3}r^2\mathbf{I}), \quad (5)$$

where $\mathbf{I}$ is the unit dyadic. Of course, in this approximation the Feynman–Gell-Mann theory is identical with the Fermi theory. Including higher order meson effects changes the explicit form of the multipole operators, but does not change the form of the effective Hamiltonian.

For the particular applications we wish to discuss, several of the terms will not contribute. For allowed transitions, the parity does not change, which rules out the matrix elements $E1$, $\int\rho_1$, and $\int\sigma_ir_j$. For transitions in which the total isotopic spin changes, the matrix element $E0$ vanishes. Finally, the matrix element $E0'$ does not contribute to either the beta spectrum or the angular correlation functions, but only to such effects as the circular polarization of gammas, and so it is dropped. Hence, we consider only the simpler Hamiltonian

$$\begin{aligned}H_{\text{eff}}=(G/\sqrt{2})u_e{}^\dagger(p)\Big[&i(\tfrac{1}{2}W_0\rho_1\sigma_ik_j-\tfrac{1}{2}k_ik_j)(E_2)_{ij}\\ &-i\rho_1\boldsymbol{\sigma}\times\mathbf{k}\cdot\mathbf{M1}+\lambda\Big(\boldsymbol{\sigma}\cdot\int\boldsymbol{\sigma}+i\rho_1\mathbf{k}\cdot\int\rho_1\mathbf{r}\\ &-\tfrac{1}{2}\sigma_ik_jk_l\int\sigma_ir_jr_l\Big)\Big](1+\gamma_5)u_\nu(q). \end{aligned} \quad (6)$$

The rather tedious derivation of the beta spectra and angular correlation functions resulting from this Hamiltonian is discussed in the appendix. The results are as follows:

The beta spectrum has the allowed shape, multiplied by the factor

$$\textstyle\sum_L b_0(L,L). \quad (7)$$

The parameters $b_k(L,L')$ are tabulated for $k=0$, 2 in Table I of the appendix. One may easily check that this result reduces to that of Gell-Mann[4] if one drops the Coulomb corrections, and terms quadratic in the momentum transfer.

The beta-gamma directional correlation function is given by[9] (A12):

$$W_\gamma(\theta)\propto\sum_{kLL'}\delta_\lambda\delta_{\lambda'}b_k(L,L')F_k(LL',j_1j_2) \times F_k(\lambda\lambda',j_3j_2)P_k(\cos\theta). \quad (8)$$

Here j_1, j_2, j_3 are the spins of the initial, intermediate and final nuclear states; the quantities F_k are defined

[8] See, for example, R. J. Blin-Stoyle, Revs. Modern Phys. **25**, 75 (1956).

[9] We have given the correlation functions only up to constant factor; hence the $\propto$ symbol.

in the appendix, and are tabulated in the literature.[10] δ_λ is the reduced matrix element for the electromagnetic multipole operator of order λ, and P_k is the Legendre polynomial.

Finally, the beta-alpha directional correlation function for a $2^+\rightarrow 0^+$ transition is given by (A15):

$$W_\alpha(\theta)\propto\sum_{kLL'}(2k+1)^{\frac{1}{2}}F_k(LL',j_12)\times b_k(L,L')\begin{pmatrix}2 & 2 & k\\ 0 & 0 & 0\end{pmatrix}P_k(\cos\theta).\quad(9)$$

III. APPLICATION AND DISCUSSION

We have already emphasized that the most evident practical test[7] of the new theory lies in the small corrections to allowed transitions, and that these can only be measured in transitions with a large momentum transfer and/or anomalously large ft values. We will now discuss separately the various special cases of such transitions, concentrating especially on the $A=4n$ nuclei. Since the relevant gamma lifetimes are not generally available, we are forced to estimate them, making use of the empirical systematics of gamma lifetimes in light nuclei. We will assume, in each case, that the gamma widths have the *average* value given by Wilkinson: that is, $\Gamma(M1)=0.15\Gamma_W$, where Γ_W is the Weisskopf unit, $\Gamma_W=5.5\omega^3\times10^{-9}$ in units in which $\hbar=m=c=1$. The $E2$ width is much more difficult to estimate, and so no attempt will be made to do so.

$A=8$

The highest energy component of the decays of Li^8, B^8 are interpreted as transitions from a $T=1$, $J=2^+$ state to the first excited state $T=0$, $J=2^+$ of Be^8, which then breaks up into two alpha particles. The transition is of considerable interest due both to its high energy, and its rather large ft value[11] ($\log ft=5.6$). Assuming the gamma transition to have a normal lifetime leads one to expect large contributions from the second-forbidden matrix elements. For example, the ratio $(\|M1\|)/(\lambda\|\sigma\|)$, which is just the parameter a introduced by Gell-Mann aside from a sign, is estimated to be

$$a=\left[\frac{3}{4}\frac{137\Gamma(M1)}{\omega^3}\times\frac{(ft)}{(ft)_{O^{14}}}\right]^{\frac{1}{2}}\cong3\times10^{-3},$$

so that aE becomes as large as $\sim7.5\%$. Of course, in general, there are three unknown matrix elements in the axial vector contribution to the beta spectrum, and two additional ones which appear in the correlation functions. In this, and in future applications, we must carefully consider the number of unknown constants and the number of available experimental data; we shall see that often it is the case that there are more adjustable parameters than there are experimental data, so that nothing can be concluded about the validity of the new theory.

For example, in the Li^8 beta spectrum, the axial vector contribution contains three real parameters, the reduced nuclear matrix elements, which we will treat as completely unknown. But examination of the parameters s_0 in Table I shows that the general form of the correction factor to the spectral shape is a quadratic function of the electron energy, so that there are only two experimental constants, the coefficients of the linear and quadratic terms. Thus the deviations from the allowed shape, insofar as they can be fitted by a quadratic function can always be fitted by the contributions from the axial vector terms alone, and cannot possibly lead to any conclusion concerning the vector interaction. The same is true of the B^8 decay, and of the beta-alpha correlation functions of Li^8 and B^8, taken individually.[12]

If, however, we compare the decays of Li^8 and B^8 and consider both sets of data simultaneously, the situation is quite different, due to the almost complete charge independence of the system. The charge independence implies the identity of the nuclear matrix elements. Hence the number of parameters remains the same, while the number of experimental data is doubled. This observation simplifies the analysis sufficiently to consider testing the nature of the vector interaction; for example, in the spectra of Li^8 and B^8 there are now four experimental parameters, and if one assumes a pure $M1$ electromagnetic transition, there are only four theoretical parameters.

However, in this case there is a further simplification due to the fact that the energy differences along the multiplet are small compared to the energy differences between multiplets. Here one can compare the spectra (or correlation functions) at each energy. There are at each energy effectively only two parameters—the total contribution of A-A terms and of V-A terms. As Gell-Mann has pointed out, the A-A contribution is the same for Li^8 and B^8, and the V-A contribution is of opposite sign. Therefore the vector contribution can

[10] The notation for the various spherical functions introduced here and hereafter is that of A. R. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton University Press, Princeton, New Jersey, 1957). The functions F_k have been tabulated for even k by L. C. Biedenharn and M. E. Rose, Revs. Modern Phys. **25**, 729 (1953), and by M. Ferentz and N. Rosenzweig, Argonne National Laboratory Report ANL-5324 (unpublished). Alder, Stech, and Winther [Phys. Rev. **107**, 728 (1957)] have given a tabulation including the odd k values needed for effects in which parity nonconservation is detectable. Our derivation of the correlation functions follows a rather similar derivation made by the latter authors.

[11] The ft values given here and below are quoted from F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. **27**, 77 (1955), and R. M. Endt and C. M. Braams, Revs. Modern Phys. **29**, 683 (1957), as well as nuclear data cards.

[12] M. Morita and H. Yamada [Progr. Theoret. Phys. (Japan) **13**, 114 (1955)] have given a calculation of the β-α correlations in Li^8 on the basis of the old Fermi theory.

TABLE I. The parameter $b_k(L,L')$ is the product of the reduced matrix element of the tensor operator in the third column of the table, times the fourth column of the table, times the reduced matrix element $(\|\sigma\|)$. An over-all factor has been extracted so that $s_0=1$ for the main Gamow-Teller term. For $L\neq L'$, we have tabulated $b_k(L,L')+b_k(L',L)$. We have denoted the electron energy, momentum, by W, p, and the neutrino energy q.

L	K	Γ	L'	K'	Γ'	Tensor operator	s_k
1	0	1	1	0	1	$C_A\sigma^M$	$s_0=1$
1	0	1	1	0	1	$C_A\sigma^M r^2$	$s_0=(2/9)\{\frac{1}{3}(p^2q/W)-\frac{3}{2}q^2-\frac{3}{2}p^2+\xi[q-3(6W-p^2/W)]\}$ $s_2=(8/27)p^2q/W$
1	0	1	1	2	1	$T_1^M=C_A\sum C_{\mu QM}{}^{121}\sigma^\mu C_2{}^Q r^2$	$s_0=(4/9)(\sqrt{}10)q[\xi+\frac{1}{3}p^2/W]$ $s_2=\frac{1}{3}(\sqrt{}10)(p^2/W)[\xi+\frac{2}{5}W+(2/9)q]$
1	0	1	2	2	1	$T_2^M\equiv C_A\sum C_{\mu QM}{}^{122}\sigma^\mu C_2{}^Q r^2$	$s_2=(2/\sqrt{}15)(p^2/W)[\xi-\frac{2}{3}q+(2/15)W$
1	0	1	3	2	1	$T_3^M\equiv C_A\sum C_{\mu QM}{}^{123}\sigma^\mu C_2{}^Q r^2$	$s_2=-\frac{1}{3}(14/15)^{\frac{1}{2}}p^2$
1	0	1	2	2	0	$C_V C_2{}^M r^2$	$s_2=-(4/3\sqrt{}10)(p^2/W)[\xi+\frac{2}{3}q+\frac{4}{5}W]$
1	0	1	1	1	0	$C_A i\rho_1 C_1{}^M r$	$s_0=2[\xi+\frac{1}{3}(p^2/W)+\frac{1}{3}q]$ $s_2=-\frac{4}{3}(p^2/W)$
1	0	1	2	1	1	$C_V i\rho_1 \sum_{\mu Q} C_{\mu QM}{}^{112}\sigma^\mu C_1{}^Q r$	$s_2=(4/3\sqrt{}5)(p^2/W)$
1	0	1	1	1	1	$C_V i\rho_1 \sum_{\mu Q} C_{\mu QM}{}^{111}\sigma^\mu C_1{}^Q r$	$s_0=-2\sqrt{2}[\xi+\frac{1}{3}(p^2/W)-\frac{1}{3}q]$ $s_2=-(4/3\sqrt{2})(p^2/W)$

be deduced directly by comparison of the two spectra point by point.

The beta spectra of Li^8 and B^8 may not provide a good test of this type owing to the probable existence of additional broad excited states nearby. It would be very difficult to distinguish the effects of complexity in the beta spectra, from the deviations from the allowed shape, especially if the states are broad and overlapping.

On the other hand, the beta-alpha correlation function seems to provide a very worthwhile test. The essential difference is that one can study the angular dependence at a fixed energy, rather than the energy dependence. A remeasurement of the correlation functions for Li^8 and B^8 near the upper end point seems particularly interesting. Present evidence[11] on the correlation in Li^8 is consistent with no correlation within about 3%. If we use our crude estimate of the $M1$ lifetime, and neglect completely the $E2$ contribution, the expected correlation in B^8 at the upper end point is, assuming zero correlation in Li^8,

$$W(\theta)\cong 1+(10/7)^{\frac{1}{2}}(8/3)aEF_2(11,22)P_2(\theta)$$
$$\cong 1\pm 0.10P_2.$$

The origin of the sign ambiguity lies in the fact that the estimate of the $M1$ matrix element used above gives only its magnitude, but not its sign.

A=12

The only possible experiment here is a measurement of the spectrum; Gell-Mann[4] has already discussed the deviations from allowed shape due to terms of first order in the momentum transfer, and has pointed out that the additional terms in the axial vector part of second order, will cancel out in the ratio of the spectra of B^{12} and N^{12}.

A=14

The anomalously large ft value of C^{14} ($ft=1.1\times 10^9$) might seem to make the mass-14 system ideal for a study of the vector contribution. However, Gell-Mann has noted[13] that since the major difference between the new and old theories for $M1$ transitions is in the spin contribution to the magnetic moment, which is presumably small in l-forbidden transitions, both theories should predict the same deviations in the spectra, assuming, of course, the validity of the independent-particle model. Nonetheless, it would seem to make sense to test the theory by measuring the gamma lifetime in N^{14}, and comparing it with the expected deviations in the O^{14} spectrum. It appears that the experimental deviations in the C^{14} spectrum are certainly less than a few percent,[13] and that therefore one might expect observable deviations in the O^{14} spectrum. In this system, the splitting of the multiplet is by no means negligible, nor does the charge independence of the nuclear states hold accurately. However, owing to the wealth of information about the mass-14 system, one can hope to calculate the nuclear matrix elements and predict the spectral shape.[14]

A=20 and 24

These systems provide examples of transitions which should show beta-gamma correlations due to the forbiddenness corrections. In the mass-20 system one can,

[13] We are indebted to B. C. Carlson for communicating the results of his analysis of the spectral deviations in the mass-14 system from the point of view of the Fermi theory.

[14] Such calculations have been reported, see for example W. M. Visscher and R. A. Ferrell, Phys. Rev. **107**, 781 (1957), and D. T. Goldman and R. A. Ferrell, Bull. Am. Phys. Soc. Ser. II, **2**, 206 (1958). We would like to thank Professor Ferrell and Dr. Goldman for a discussion of their work.

unfortunately, study only the decay of F^{20}, and the analysis is therefore complicated by the appearance of several unknown matrix elements from the axial vector part.[15] The mass-24 system is of interest due to its large ft value ($\log ft=6.1$), and large energy release. Presumably both members of the multiplet, Na^{24} and Al^{24}, can be studied and compared. It has already been established that there is no correlation in Na^{24} to within 0.1%.[16] One would therefore expect a possible correlation in Al^{24}; the situation is however different from that in the mass-8 case, since the splitting of the multiplet is no longer negligible, and the analysis is complicated thereby.

$A=32$

It is of some interest to note that the beta spectrum of P^{32} is the only one in which deviations due to higher multipole corrections seem well established experimentally; deviations linear in the electron energy from the allowed shape have been reported, to the extent of about 3%.[17] One readily sees that this experimental result can be explained by the axial vector matrix elements alone, and therefore no conclusion can be drawn concerning the vector contributions.

In conclusion, it is clear that a detailed test of the Feynman–Gell-Mann scheme will be quite difficult, but several very interesting experiments seem to be suggested.

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APPENDIX

We shall describe here some of the more important steps in the multipole expansion of the transition matrix for the V-A beta interaction.[9] We write the Hamiltonian in the conventional form

$$H=\sqrt{2}C_A\psi_p^\dagger\boldsymbol{\sigma}\psi_n\cdot\psi_e^\dagger\boldsymbol{\sigma}\psi_\nu+\sqrt{2}C_A\psi_p^\dagger i\rho_1\psi_n\psi_e^\dagger i\rho_1\psi_\nu$$
$$+\sqrt{2}C_V\psi_p^\dagger\psi_n\psi_e^\dagger\psi_\nu+\sqrt{2}C_V\psi_p^\dagger i\rho_1\boldsymbol{\sigma}\psi_n\cdot\psi_e^\dagger i\rho_1\boldsymbol{\sigma}\psi_\nu. \quad \text{(A1)}$$

Since we intend to compute only scalars, the replacement $C(1+\gamma_5)\rightarrow C\sqrt{2}$ has been made. The Hamiltonian can be written in spherical, rather than Cartesian, form by use of the identity

$$\delta(\mathbf{r}_1-\mathbf{r}_2)=\frac{1}{r_1^2}\delta(|\mathbf{r}_1|-|\mathbf{r}_2|)\sum_{KQ}Y_K{}^Q(\mathbf{r}_1)Y_K{}^{*Q}(\mathbf{r}_2). \quad \text{(A2)}$$

For example, the first term in the Hamiltonian becomes

$$\langle H\rangle=\frac{\sqrt{2}C_A}{4\pi}\sum_{KLM}(-)^M(2K+1)$$
$$\times\int d\mathbf{r}_1\,\psi_p^\dagger T_L{}^M(K,\Gamma)\psi_n(\mathbf{r}_1)$$
$$\times\int d\Omega_2\psi_e^\dagger\tau_L{}^{-M}\psi_\nu(\mathbf{r}_2)\,|_{|\mathbf{r}_2|=|\mathbf{r}_1|}, \quad \text{(A3)}$$

where

$$T_L{}^M\equiv\sum_{\mu Q}C_{\mu QM}{}^{\Gamma KL}\sigma_\Gamma{}^\mu C_K{}^Q,$$
$$\tau_L{}^M\equiv\sum_{\mu Q}C_{Q\mu M}{}^{K\Gamma L}C_K{}^Q\sigma_\Gamma{}^\mu. \quad \text{(A4)}$$

Here Γ is 1 for those terms containing a $\boldsymbol{\sigma}$, and zero otherwise.

To compute the matrix element, use is made of the expansion of the appropriate electron and neutrino wave functions into spherical waves. The expansion for the electron wave function for momentum $\mathbf{p}$, spin σ, with incoming spherical wave is

$$\Psi_{p,\sigma}{}^{(-)}=\frac{1}{p}\left(\frac{2\pi W}{m}\right)^{\frac{1}{2}}\sum_{\kappa m}[2l(\kappa)+1]^{\frac{1}{2}}C_{0\sigma\sigma}{}^{l(\kappa)\frac{1}{2}j(\kappa)}$$
$$\times e^{-i\Delta_\kappa}e^{+i(\pi/2)l(\kappa)}\psi_\kappa{}^m\mathfrak{D}_{m\sigma}{}^{j(\kappa)}(-\varphi,-\theta,0), \quad \text{(A5)}$$

Here

$$\psi_\kappa{}^m=\begin{pmatrix}-if_\kappa\chi_{-\kappa}{}^m\\ g_\kappa\chi_\kappa{}^m\end{pmatrix};$$

use is made of the special representation of the Dirac matrices in which

$$\rho_1=\begin{pmatrix}0&-1\\-1&0\end{pmatrix},\quad \rho_3=\begin{pmatrix}-1&0\\0&1\end{pmatrix}.$$

The radial functions f_κ, g_κ are given by Rose[18]; the Coulomb phase shifts are

$$\Delta_\kappa=\tfrac{1}{2}\arg\left[\frac{-\kappa+i\alpha Z/p}{\gamma+i\alpha ZW/p}\right]-\arg\Gamma(\gamma+i\alpha ZW/p)$$
$$+\tfrac{1}{2}\pi[l(\kappa)+1-\gamma],$$

with $\gamma=(\kappa^2-\alpha^2Z^2)^{\frac{1}{2}}$.

If we call the initial and final nuclear spins j_1 and j_2, the density matrix is

$$\rho_{m_2m_2'}\propto\sum_{m_1,\sigma,\sigma_\nu}\int d\Omega_q\langle m_2|H|m_1\rangle\langle m_2'|H|m_1\rangle^*. \quad \text{(A6)}$$

Substituting the wave functions and carrying out the

[15] Professor Gell-Mann has informed us that an experiment is in progress at the California Institute of Technology to look for β-γ correlations in F^{20}.

[16] R. Steffen (private communication). We are grateful to Professor Steffen for a discussion of the angular correlation measurements in the mass-24 system.

[17] See I. Iben, Phys. Rev. **109**, 2059 (1958) for a discussion of the experimental situation in P^{32} and of the implications of the old theory in connection with the spectral deviations.

[18] M. E. Rose, Phys. Rev. **51**, 484 (1937).

TABLE II. The multipole expansion of the Feynman–Gell-Mann interaction leads to the same parameters $b_k(L,L')$, with the exception of a different identification of the reduced nuclear matrix elements, for the vector part of the interaction. The tensors in the first column are to be replaced by those in the second column to obtain the correct b_k. Note also that the constants C_V, C_A are to be replaced by $G/\sqrt{2}$, $-\lambda G/\sqrt{2}$, as defined by Feynman and Gell-Mann. The multipole operators $M1$, $E2$ are taken to include the effect of the pions.

$\int \rho_1(\boldsymbol{\sigma}\times\mathbf{r})_i$	$-2\int(\mathbf{M}1)_i$
$\int(r_ir_j-\frac{1}{3}\delta_{ij}r^2)$	$\int(E2)_{ij}$
$\int i\rho_1(\sigma_ir_j+\sigma_jr_i-\frac{2}{3}(\boldsymbol{\sigma}\cdot\mathbf{r})\delta_{ij})$	$W_0\int(E2)_{ij}$

indicated operations, we obtain

$$\rho_{m_2m_2'}\propto\sum_{LL'k\mu}\sum_{KK'\Gamma\Gamma}(-)^{j_2+m_2'}\left(\frac{2k+1}{2j_2+1}\right)^{\frac{1}{2}}\times F_k(LL',j_1j_2)b_k(LL')\times\mathfrak{D}_{\mu0}{}^{k*}(-\varphi,-\theta,0)\begin{pmatrix}k & j_2 & j_2\\ -\mu & -m_2 & m_2'\end{pmatrix},\quad\text{(A7)}$$

where

$$F_k(LL',j_1j_2)=(-)^{j_1+j_2-1}\times[(2k+1)(2j_2+1)(2L+1)(2L'+1)]^{\frac{1}{2}}\times\begin{pmatrix}L & L' & k\\ 1 & -1 & 0\end{pmatrix}\begin{Bmatrix}L & L' & k\\ j_2 & j_2 & j_1\end{Bmatrix},\quad\text{(A8)}$$

and

$$b_k=\sum_{\kappa,\kappa',\kappa_\nu}(2K+1)(2K'+1)\times\left[\frac{(2l+1)(2l'+1)(2j+1)(2j'+1)}{(2L+1)(2L'+1)}\right]^{\frac{1}{2}}\times(-)^{j+j'+j_\nu-\frac{1}{2}+l+l'+L+L'}\begin{Bmatrix}L' & L & k\\ j & j' & j_\nu\end{Bmatrix}\times\begin{Bmatrix}l & l' & k\\ j' & j & \frac{1}{2}\end{Bmatrix}\begin{pmatrix}l & k & l'\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}L & L' & k\\ 1 & -1 & 0\end{pmatrix}^{-1}\times e^{i(\Delta\kappa-\Delta\kappa')}e^{i(\pi/2)(l'-l)}(j_2\|T_L(K\Gamma)\|j_1)\times(j_2\|T_{L'}(K'\Gamma')\|j_1)^*(\kappa\|\tau_L\|\kappa_\nu)(\kappa'\|\tau_{L'}\|\kappa_\nu).\quad\text{(A9)}$$

The functions F_k have been tabulated elsewhere[9]; the quantities b_k are given in Table I for $k=0$, 2.

The beta spectrum is given by the trace of the density matrix,

$$\mathrm{tr}\rho_{m_2m_2'}\propto\textstyle\sum_L b_0(L,L).\quad\text{(A10)}$$

The beta-gamma directional correlation is

$$W_\gamma(\theta)=\sum_{m_2m_2'}P_{m_2m_2'}{}^\gamma\rho_{m_2m_2'},$$

where

$$P_{m_2m_2'}{}^\gamma=\sum_{J\lambda\lambda'}\left[\frac{2J+1}{2j_2+1}\right]^{\frac{1}{2}}(-)^{j_2+m_2}F_J(\lambda\lambda'j_3j_2)\delta_\lambda\delta_{\lambda'}\times\begin{pmatrix}j_2 & j_2 & J\\ -m_2 & m_2' & M\end{pmatrix}\mathfrak{D}_{M0}{}^{*J}(k),\quad\text{(A11)}$$

so that

$$W_\gamma(\theta)\propto\sum_{LL'k}\sum_{\lambda\lambda'}\delta_\lambda\delta_{\lambda'}b_k(LL')F_k(LL',j_1j_2)\times F_k(\lambda\lambda',j_3j_2)P_k(\cos\theta).\quad\text{(A12)}$$

Finally, the beta-alpha correlation function, for a 2^+ to 0^+ transition is

$$W_\alpha(\theta)=\sum_{m_2m_2'}P_{m_2m_2'}{}^\alpha\rho_{m_2m_2'},\quad\text{(A13)}$$

where

$$P_{m_2m_2'}{}^\alpha=\sum_{\Delta M}\left(\frac{2J+1}{4\pi}\right)^{\frac{1}{2}}(-)^{m_2'}\begin{pmatrix}2 & 2 & J\\ 0 & 0 & 0\end{pmatrix}\times\begin{pmatrix}2 & 2 & J\\ m_2 & -m_2' & M\end{pmatrix}Y_J{}^{*M}(k),\quad\text{(A14)}$$

so that

$$W_\alpha(\theta)\propto\sum_{LL'k}(2k+1)^{\frac{1}{2}}F_k(LL',j_12)\times\begin{pmatrix}2 & 2 & k\\ 0 & 0 & 0\end{pmatrix}b_k(LL')P_k(\cos\theta).\quad\text{(A15)}$$

To obtain the multipole expansion of the interaction defined by Feynman and Gell-Mann, it is just necessary to redefine the multipole moment operators appropropriately; the relevant substitutions are given in Table II.