Scattering of Protons from Helium and Level Parameters in Li^5†

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The scattering of protons from helium has been investigated experimentally from 2.0 to 5.5 Mev with the Rice Institute Van de Graaff accelerator and a differentially pumped, large-volume scattering chamber. Excitation curves reveal no new states in Li^5 up to an excitation energy of 6.5 Mev. A phase-shift analysis has been made of the angular distributions at laboratory bombarding energies of 3.03, 3.51, 4.02, 4.50, and 5.00 Mev. The phase shifts derived from this experiment and from experiments at other laboratories, in the energy range 1 to 18 Mev, have been interpreted in terms of the dispersion theory, and level parameters have been extracted. The nuclear radius which best fits the P-wave phase shifts is 2.6×10^{-13} cm. For the ground state, $(E_{\mathrm{res}})_{\mathrm{lab}}=2.6$ Mev, $J=\frac{3}{2}^-$, $\gamma_p{}^2=12\times10^{-13}$ Mev-cm, and $\theta_p{}^2=0.40$. For the first excited state, $(E_{\mathrm{res}})_{\mathrm{lab}}=10.8$ Mev, $J=\frac{1}{2}^-$, $\gamma_p{}^2=30\times10^{-13}$ Mev-cm, and $\theta_p{}^2=1.0$. The S-wave phase shift is moderately well fit by a hard-sphere interaction with a radius of 2.0×10^{-13} cm. At the present time, the D-wave phase shifts are so inaccurately known, that very little interpretation is possible.

INTRODUCTION

THE mirror nuclei He^5 and Li^5 have been of considerable theoretical importance. They are the lightest, and consequently simplest, examples of mirror nuclei that consist of a closed shell plus one nucleon. One of the most profitable ways of studying these nuclei is the scattering of neutrons and protons from He^4. The nucleus Li^5 has been investigated by the scattering of protons from He^4 at a variety of energies. The experimental literature on Li^5 has been well reviewed by Brockman,[1] and the theoretical literature has been reviewed by Gammel and Thaler.[2] The purpose of the present paper is, first of all, to present new data for bombarding energies from 2.0 to 5.5 Mev, and secondly, to present the results of a dispersion theory analysis of the P-wave phase shifts which are now available in the energy range from 1 to 18 Mev.

One of the earliest experiments on the scattering of protons from helium was done by the Minnesota group.[3] This experiment covered the energy range from 0.95 to 3.58 Mev, and a phase-shift analysis of the data was made by Critchfield and Dodder.[4] Isolated angular distributions have been observed at 5.78 Mev,[5] at 7.5 Mev,[6] at 9.73 Mev,[7] at 9.76 Mev,[8] and very recently from 11.4 to 18 Mev.[1] In Putnam's paper[6] the results of their 7.5-Mev experiment, and an earlier experiment at 9.48 Mev, are phase-shift analyzed. Potential-well analyses of the phase shifts have been given by Sack, Biedenharn, and Breit,[9] and more recently by Gammel and Thaler.[2] Adair[10] and Dodder and Gammel[11] have given dispersion-theory analyses of both the neutron and proton scattering experiments on He^4. Their analyses of Li^5, however, use only the data of the Minnesota group,[3,4] the data of Kreger *et al.*,[5] and an early analysis of the 9.48-Mev data.[11] Consequently a variety of reduced widths and radii may be found which fit the data below 3.5 Mev.

EXPERIMENTAL DATA

The large-volume scattering chamber, used for the proton-helium experiments, has been described in detail by Russell *et al.*[12]

The estimated root-mean-square uncertainties of the cross section measurements due to geometry, detection efficiency, current integration, and energy scale for the present experiment is 2.4%. To this must be added the uncertainties due to counting statistics. These were at worst 2% at angles near 90°, and they were less than 1% at the more forward and backward angles. No evidence of groups of protons scattered from heavier impurities was found at any time during the experi-

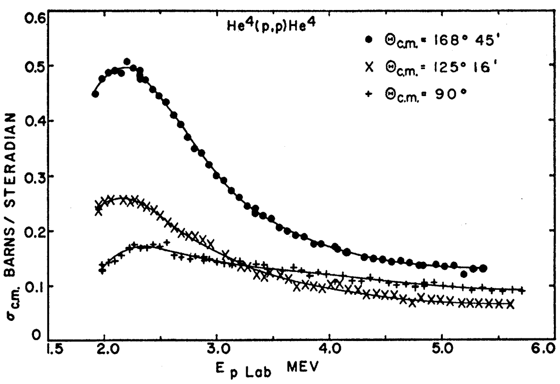

FIG. 1. $He^4(p,p)He^4$ excitation curves at three center-of-mass angles from 2.0 to 5.5 Mev bombarding energy. The cross sections are expressed in the center-of-mass system. Arbitrary smooth curves are drawn through the data points.

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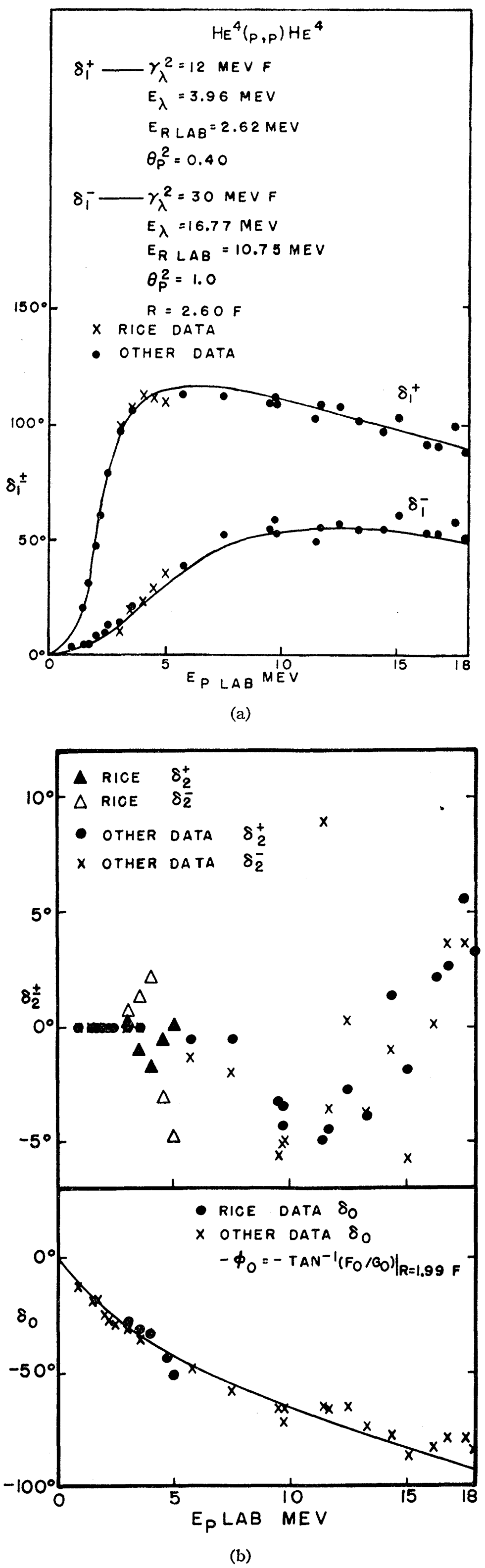


FIG. 2. $He^4(p,p)He^4$ phase shifts in degrees plotted *versus* bombarding energy. The Rice data are shown, and references to the other data shown are given in the text. The reanalysis of the 11.4- to 18-Mev data is also discussed in the text. The dispersion theory fits to the *P*-wave phase shifts are shown for the parameters given, and the hard-sphere fit to the *S*-wave phase shift is shown. The radius *R* is given in fermis [1 fermi (f) $\equiv 10^{-13}$ cm].

TABLE I. $He^4(p,p)He^4$ angular distribution at 3.03-Mev bombarding energy. Differential cross sections in the center-of-mass system are given, and the phase-shift fits are given. The phase shifts for the *S* and *P* waves column are $\delta_0-27.5°$, $\delta_1^+=99.3°$, and $\delta_1^-=10.2°$. The rms deviation for this set of phase shifts is 1.8%. The phase shifts for the *S*, *P*, and *D* waves column are $\delta_0=-27.3°$, $\delta_1^+=99.3°$, $\delta_1^-=10.3°$, $\delta_2^+=0.1°$, and $\delta_2^-=0.7°$. The rms deviation for this set of phase shifts is 1.7%.

$\theta_{c.m.}$ (degrees)	$\sigma_{experiment}$ (barns/steradian)	σ_{phases} *S* and *P* waves (barns/steradian)	Error %	σ_{phases} *S*, *P*, and *D* waves (barns/steradian)	Error %
30	0.742	0.746	0.5	0.738	−0.5
40	0.548	0.560	2.2	0.557	1.6
50	0.438	0.440	0.5	0.439	0.2
60	0.344	0.342	−0.6	0.341	−0.9
70	0.263	0.259	−1.5	0.259	−1.5
80	0.198	0.194	−2.0	0.195	−1.5
90	0.146	0.149	2.1	0.150	2.7
100	0.126	0.124	−1.6	0.125	−0.8
110	0.123	0.120	−2.4	0.121	−1.6
120	0.136	0.134	−1.5	0.134	−1.5
130	0.156	0.160	2.6	0.160	2.6
140	0.189	0.193	2.1	0.192	1.6
150	0.221	0.227	2.7	0.226	2.3
160	0.259	0.256	−1.2	0.255	−1.5
168	0.277	0.273	−1.4	0.271	−2.2

ments. The energy scale was derived from the 2.660-Mev resonance from protons on O^{16}. The energies are accurate to ±10 kev.

Excitation curves were taken in the bombarding energy range from 2.0 to 5.5 Mev. Figure 1 shows the results of three typical excitation curves at center-of-mass angles of 168° 45′, 125° 16′, and 90°. The smooth curves were drawn arbitrarily through the data points. There is no evidence of any narrow structure in any of the excitation curves taken. For this reason it was decided to base the phase-shift analysis on angular distributions taken at intervals of 500 kev from 3.0 to 5.0 Mev. Tables I through V give the angular distributions at 3.03, 3.51, 4.02, 4.50, 5.00 Mev. The fifteen center-of-mass angles chosen for each angular distribution were the even multiples of 10° from 30° through 160°, and in addition 168°.

THE PHASE SHIFT ANALYSIS

The formula for the differential cross section in the center-of-mass is[4]

$$\sigma(\theta)=|f_c(\theta)|^2+|f_i(\theta)|^2, \qquad (1)$$

where

$$kf_c(\theta)=(-\eta/2)\csc^2(\theta/2)\exp[i\eta\ln\csc^2(\theta/2)]$$
$$+\sum_{l=0}^{\infty} e^{i\alpha_l}P_l(\cos\theta)[(l+1)\exp(i\delta_l^+)\sin\delta_l^+ + l\exp(i\delta_l^-)\sin\delta_l^-], \qquad (2)$$

and

$$kf_i(\theta)=\sum_{l=1}^{\infty} e^{i\alpha_l}P_l^0(\cos\theta)\sin\theta \times[\exp(i\delta_l^+)\sin\delta_l^+-\exp(i\delta_l^-)\sin\delta_l^-]. \quad (3)$$

In these formulas, $\eta=(Z_1Z_2e^2)/\hbar v$, $\alpha_0=0$, and

$$\alpha_l=2\sum_{s=1}^{\infty}\tan^{-1}(\eta/s).$$

Z_1 and Z_2 are the charges of the incident and target nuclei, respectively; $k=\mu v/\hbar$; v is the relative velocity of the incident and target nuclei at infinity, and μ is the reduced mass of the system.

The phase-shift analysis was programmed for an IBM-650 computer using the partial waves through $l=4$. The method of computation is as follows. First, initial guesses of the phase shifts are made, and the angular distribution is calculated. The following expression for the error, $\mathcal{E}$, is then formed:

$$\mathcal{E}=\sum_{i=1}^{N}[\sigma(\theta_i)_{\text{calc}}-\sigma(\theta_i)_{\text{obs}}]^2/[\nu(\theta_i)\sigma(\theta_i)_{\text{obs}}]. \quad (4)$$

In this expression, N is the number of points on the angular distribution, $\sigma(\theta_i)_{\text{calc}}$ is the differential cross section at the angle θ_i, as calculated from (1), $\sigma(\theta_i)_{\text{obs}}$ is the experimentally observed differential cross section at the angle θ_i, and $\nu(\theta_i)$ is a statistical weight factor which measures the relative accuracy of each particular datum point. A sequence of phases to be varied is specified to the computer, and the expression for $\mathcal{E}$ is

TABLE II. $He^4(p,p)He^4$ angular distribution at 3.51-Mev bombarding energy. Differential cross sections in the center-of-mass system are given, and the phase-shift fits are given. The phase shifts for the S and P waves column are $\delta_0=-31.7°$, $\delta_1^+=107.6°$, and $\delta_1^-=19.2°$. The rms deviation for this set of phase shifts is 2.1%. The phase shifts for the S, P, and D waves column are $\delta_0=-30.9°$, $\delta_1^+=107.6°$, $\delta_1^-=18.8°$, $\delta_2^+=-1.1°$, and $\delta_2^-=1.3°$. The rms deviation for this set of phase shifts is 1.7%.

$\theta_{\text{c.m.}}$ (degrees)	$\sigma_{\text{experiment}}$ (barns/steradian)	σ_{phases} S and P waves (barns/steradian)	Error %	σ_{phases} S, P, and D waves (barns/steradian)	Error %
30	0.666	0.668	0.3	0.668	0.3
40	0.510	0.505	−1.0	0.504	−1.2
50	0.400	0.399	−0.3	0.398	−0.5
60	0.312	0.311	−0.3	0.311	−0.3
70	0.230	0.237	3.0	0.236	2.6
80	0.175	0.176	0.6	0.176	0.6
90	0.134	0.133	−0.7	0.132	−1.5
100	0.109	0.107	−1.8	0.105	−3.7
110	0.095	0.098	3.2	0.095	0.0
120	0.098	0.103	5.1	0.101	3.1
130	0.118	0.120	1.7	0.118	0.0
140	0.140	0.142	1.4	0.141	0.7
150	0.165	0.167	1.2	0.167	1.2
160	0.194	0.188	−3.1	0.189	−2.6
168	0.202	0.200	−1.0	0.202	0.0

TABLE III. $He^4(p,p)He^4$ angular distribution at 4.02-Mev bombarding energy. Differential cross sections in the center-of-mass system are given, and the phase-shift fits are given. The phase shifts for the S and P waves column are $\delta_0=-34.5°$, $\delta_1^+=112.2°$, and $\delta_1^-=23.5°$. The rms deviation for this set of phase shifts is 3.4%. The phase shifts for the S, P, and D waves column are $\delta_0=-32.9°$, $\delta_1^+=112.4°$, $\delta_1^-=23.6°$, $\delta_2^+=-1.7°$, and $\delta_2^-=2.1°$. The rms deviation for this set of phase shifts is 1.9%.

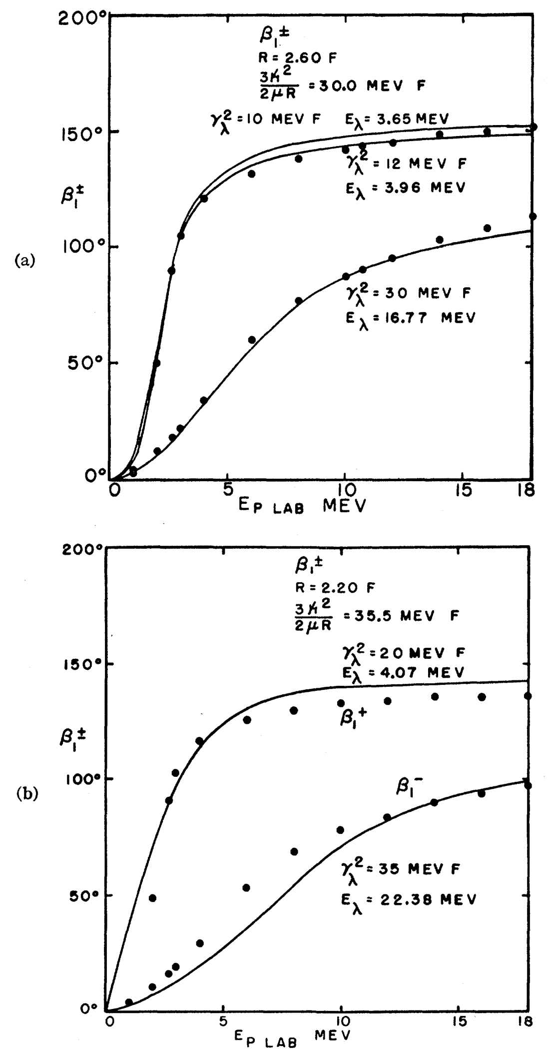
$\theta_{\text{c.m.}}$ (degrees)	$\sigma_{\text{experiment}}$ (barns/steradian)	σ_{phases} S and P waves (barns/steradian)	Error %	σ_{phases} S, P, and D waves (barns/steradian)	Error %
30	0.588	0.588	0.0	0.587	−0.2
40	0.445	0.447	0.4	0.447	0.4
50	0.356	0.355	−0.3	0.355	−0.3
60	0.282	0.278	−1.4	0.279	−1.1
70	0.210	0.213	1.4	0.213	1.4
80	0.158	0.160	1.3	0.158	0.0
90	0.114	0.120	5.3	0.117	2.6
100	0.094	0.095	1.1	0.090	−4.3
110	0.080	0.083	3.8	0.078	−2.5
120	0.079	0.084	6.3	0.079	0.0
130	0.088	0.094	6.8	0.091	3.4
140	0.107	0.111	3.7	0.109	1.9
150	0.131	0.128	−2.3	0.129	−1.5
160	0.147	0.144	−2.0	0.147	0.0
168	0.160	0.154	−3.8	0.158	−1.2

TABLE IV. $He^4(p,p)He^4$ angular distribution at 4.50-Mev bombarding energy. Differential cross sections in the center-of-mass system are given, and the phase-shift fit is given. The phase shifts are $\delta_0=-43.1°$, $\delta_1^+=111.6°$, $\delta_1^-=29.1°$, $\delta_2^+=-0.5°$, and $\delta_2^-=-3.1°$. The rms deviation for this set of phase shifts is 1.3%.

$\theta_{\text{c.m.}}$ (degrees)	$\sigma_{\text{experiment}}$ (barns/steradian)	σ_{phases} (barns/steradian)	Error %
30	0.578	0.577	−0.2
40	0.446	0.446	0.0
50	0.352	0.353	0.3
60	0.276	0.275	−0.4
70	0.204	0.207	1.5
80	0.155	0.153	−1.3
90	0.111	0.112	0.9
100	0.089	0.086	−3.4
110	0.075	0.074	−1.3
120	0.073	0.074	1.4
130	0.083	0.083	0.0
140	0.097	0.097	0.0
150	0.113	0.113	0.0
160	0.127	0.128	0.8
168	0.139	0.136	−2.2

TABLE V. $He^4(p,p)He^4$ angular distribution at 5.00-Mev bombarding energy. Differential cross sections in the center-of-mass system are given, and the phase-shift fit is given. The phase shifts are $\delta_0=-51.8°$, $\delta_1^+=109.9°$, $\delta_1^-=35.5°$, $\delta_2^+=0.1°$, and $\delta_2^-=-4.7°$. The rms deviation for this set of phases is 2.4%.

$\theta_{\text{c.m.}}$ (degrees)	$\sigma_{\text{experiment}}$ (barns/steradian)	σ_{phases} (barns/steradian)	Error %
30	0.567	0.561	−1.1
40	0.440	0.446	1.4
50	0.357	0.356	−0.3
60	0.276	0.275	−0.4
70	0.202	0.205	1.5
80	0.153	0.148	−3.3
90	0.107	0.106	−0.9
100	0.083	0.079	−4.8
110	0.066	0.067	1.5
120	0.066	0.066	0.0
130	0.074	0.075	1.3
140	0.087	0.089	2.3
150	0.109	0.104	−4.6
160	0.115	0.118	2.6
168	0.128	0.125	−2.3



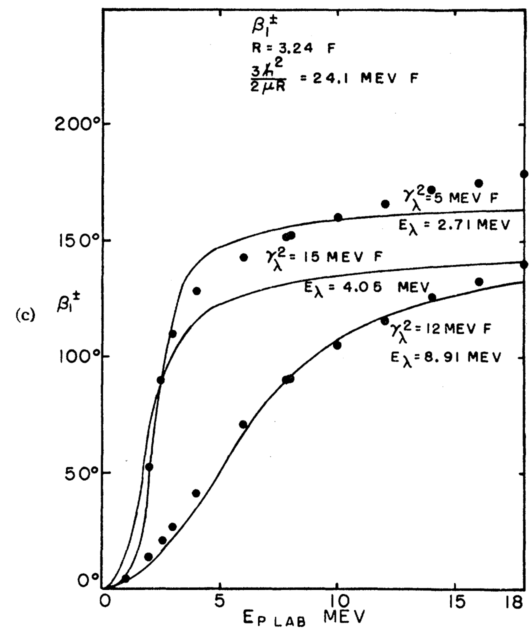


FIG. 3. The behavior of $\beta_1^\pm$ for three values of R, the nuclear radius, is shown for various values of the reduced width, γ_p^2 plotted *versus* bombarding energy. The circles were taken from a smooth curve drawn through the phase shifts of Fig. 2. The smooth curves are the fits for the parameters shown. (R in fermis; γ^2 in Mev f.)

minimized by taking each phase shift of this sequence and changing it by a given amount, Δ, first in the positive direction, and then in the negative direction. When a minimum is found in $\mathcal{E}$ with respect to variation of that particular phase shift by integral multiples of Δ, then the next phase shift from the sequence is selected, and the process is repeated until $\mathcal{E}$ is a minimum for as many phases as desired through $l=4$. For the present experiments, $\mathcal{E}$ was minimized with respect to δ_0, $\delta_1^\pm$, and $\delta_2^\pm$, for an increment, Δ, equal to 0.1°.

The angular distributions from the present experiment were fitted only with that solution of the phase-shift formula, (1), which implies an inverted doublet for the ground and first excited states of Li^5; this fact has been established by spin polarization measurements.[13–16] The lowest three angular distributions were first fitted using only S and P wave phase shifts; the fits were not noticeably improved by the addition of D-wave phase shifts. The two upper angular distributions were only fitted using S, P, and D wave phase shifts. The results of the phase shift analyses are shown in Tables I through V with their angular distributions, and in Fig. 2 as "Rice data."

Figure 2 shows the results of all of the $He^4(p,p)He^4$ experiments including the present one. The references to the other data were given in the first section of this paper. The data from 11.4 to 18 Mev[1] were reanalyzed using the actual data, instead of using smoothed data as was done by Brockman.

DISCUSSION OF THE PHASE SHIFTS FROM 1 TO 18 MEV

S Wave

The S-wave phase shift is described reasonably well at low energies by a hard-sphere phase shift with an

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interaction radius of 2.0×10^{-13} cm. At the high-energy end, as can be seen from Fig. 2, there is some deviation in the positive direction from the computed hard sphere value. This is most likely the result of a very high energy, broad $2S$ state.

P Wave

The *P*-wave phase shifts will be discussed in terms of the dispersion theory of Wigner and Eisenbud.[17] The pertinent expression from the dispersion theory is

$$\begin{aligned}\delta_l^{\pm} &= -\phi_l + \tan^{-1}(\Gamma_\lambda/2)/(E_\lambda+\Delta_\lambda-E)\\ &= -\phi_l+\beta_l^{\pm}.\end{aligned} \tag{5}$$

In this expression, $-\phi_l$ is the contribution to the phase shift from potential scattering and distant resonances. In the present experiment it will be assumed that

$$\phi_l = \tan^{-1}(F_l/G_l)\,|_{\rho=kR}, \tag{6}$$

and this assumption will be justified by the results. Here R is the assumed nuclear radius, and F_l and G_l are the regular and irregular Coulomb wave functions respectively, as defined by Bloch *et al.*[18] The second term of (5), $\beta_l^{\pm}$, is the resonant portion of the phase shift, and the quantities involved are defined as follows:

$$\Gamma_\lambda/2 = k\gamma_\lambda^2/A_l^2, \tag{7}$$

$$A_l^2 = F_l^2+G_l^2, \tag{8}$$

$$\Delta_\lambda = -[(k\gamma_\lambda^2/\rho)][(\rho/A_l)(dA_l/d\rho)+l]. \tag{9}$$

E_λ is defined as that energy necessary to make the denominator of $\beta_l^{\pm}$ vanish at the value of E where $\beta_l^{\pm}$ is determined to be 90°. The free parameters for fitting the observed phase shifts are the radius, R, and the reduced width γ_λ^2. In the cases where the Coulomb wave functions were available in the tables of Bloch *et al.*,[18] they were used. At high energies, where $\eta<0.1585$, linear interpolation was used between the quantities in the Bloch tables, and the corresponding neutron functions. For low energies (small ρ), the Bessel-function expansion of the irregular Coulomb wave functions was used to determine the shift function.[19] The behavior of $\beta_1^{\pm}$ for radii of (2.2, 2.6, and 3.2) $\times10^{-13}$ cm, and for various assumed values of the reduced width, γ_p^2, is shown in Fig. 3. If the ratio of the reduced width to the Wigner limit,[20] $(3\hbar^2/2\mu R)$, is defined to be θ_p^2, then the parameters determined by reasonable fits to the experimental data for the first two states of Li^5 are given in Table VI. These fits to the phase shifts, $\delta_1^{\pm}$, that result by using the parameters of Table VI are shown in Fig. 2. In the $P_{\frac{1}{2}}$ case there might be a trend of the experimental data away from the dispersion theory fit at high energies, but the trend seems to be within the scatter of the experimental points. The assumption of only hard-sphere potential effects for ϕ_1 seems to be justified.

TABLE VI. Parameters for the first two states of Li^5 as given by the scattering of protons from He^4, using an interaction radius of 2.6×10^{-13} cm.

Excitation energy (Mev)	Spin ($\hbar$)	Parity π	γ_p^2 (Mev-cm)	θ_p^2	$(E_{res})_{lab}$ (Mev)	$(E_\lambda)_{c.m.}$ (Mev)
0	$\frac{3}{2}$	—	12×10^{-13}	0.40	2.6	4.0
8.6	$\frac{1}{2}$	—	30×10^{-13}	1.0	10.8	16.8

D Wave

The values of the *D*-wave phase shifts from the present and other experiments are shown in Fig. 2. The inaccuracies of the existing experiments preclude the possibility of getting any quantitative information from the *D*-wave phase shifts. Qualitatively, it may be said that the *D*-wave phase shifts appear to be approximately unsplit below 10 Mev, and appear to be decreasing with energy. Above 10 Mev, one or both of the *D*-wave phase shifts appear to be increasing with energy. From the form of Eqs. (1), (2), and (3), it may be seen that it is impossible to tell which δ of a given l is about to resonate, below a resonance.

A $\frac{3}{2}^+$ state at an excitation energy of 16.80 Mev is known in Li^5 from the $He^3(d,p)He^4$ reaction as studied by Bonner, Conner, and Lillie.[21] The parameters determined for this state by these authors are $\theta_d^2=0.21$, and $\theta_p^2=0.004$, so that the state is a very good single-particle state of He^3+d. It seems likely that a state with $J=\frac{1}{2}^+$ might lie very near in energy to this $J=\frac{3}{2}^+$ state, and that this state would also be a very good single particle state of He^3+d. No evidence of this second state appears in Brockman's data,[1] as can be seen from Fig. 2. Continuous excitation curves above 6 Mev bombarding energy are still needed to eliminate the possibility of additional narrow states in Li^5.

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