Slow Neutron Resonances in U^{235}†

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The total and fission cross sections of U^{235} have been measured in the energy range 0.1 to 10 ev on the BNL crystal spectrometer. The capture cross section is derived by combining these data with published scattering cross section data. Analysis of the low-energy resonance structure depends strongly on the interpretation of the "background" cross section which dominates the low-energy region. This "background" cannot be accounted for in terms of observed resonances; however, if it is attributed to the presence of a bound level, approximate parameters for such a level can be obtained. It is seen that the strength Γ_n^0 of the hypothetical bound level is anomalously large.

The capture component of the resonances is found to be symmetrical, i.e., of Breit-Wigner shape. However, the fission component of the same resonances is asymmetrical, which indicates that a multilevel formula is necessary for fitting the resonances in the fission cross section. A good but not perfect fit to the data between 0 and 2 ev has been achieved using a multilevel formula which applies to the case of a single fission channel and many capture channels. The remaining small discrepancies in fitting the data indicate that the assumption of a single fission channel is too restrictive, and that one or more additional channels contribute weakly to slow neutron fission in U^{235}.

I. INTRODUCTION

THE total and fission cross sections of U^{235} for low-energy neutrons have been extensively studied with many different types of neutron spectrometers. Results obtained prior to August, 1955, were summarized in several papers[1–6] at the International Conference on the Peaceful Uses of Atomic Energy (Geneva, Switzerland, August, 1955), and in various review articles.[7,8] More recent results were published by Simpson *et al.*[9] and by Pilcher *et al.*[10] These many independent measurements show encouraging qualitative agreement; but unfortunately, appreciable quantitative discrepancies are present and there has been no consistent quantitative interpretation of the resonant structure in terms of resonance parameters. The parameters quoted in the various summaries[6–10] do not faithfully reproduce the observed cross sections when inserted in the usual cross-section formulas.

There are several reasons for the lack of a satisfactory analysis. The resonances in U^{235} are unusually closely spaced and relatively weak, especially below 10 ev where the strength function Γ_n^0/D is anomalously small. This coupled with the usual complications introduced by Doppler broadening and instrument resolution make analysis difficult. In addition, the problem is inherently more complex than in cases of typical nonfissile heavy isotopes, because, as we shall show, the fission component of the resonances cannot be satisfactorily fitted by a simple Breit-Wigner single-level formula.[6,11] Furthermore, the thermal region is dominated by a "background" cross section which is large compared to the thermal contributions of the observable resonances. At present, it is customary to explain this "background" in terms of one or more bound levels ("negative-energy" resonances). As will be seen in a later section, no unique analysis of the "background" is possible with available data, and, in fact the interpretation in terms of a bound level has objections which raise doubt as to its validity.

The purpose of this paper is to present the total and fission cross section data obtained with a high-resolution crystal spectrometer. We attempt to analyze these data in terms of a multilevel dispersion formula derived by Reich and Moore[12] which is valid for the case of a single fission channel. It is shown that a reasonable but not perfect fit can be obtained. In an accompanying paper, Vogt[13] derives a multichannel, multilevel dispersion

† Work performed under the auspices of the U. S. Atomic Energy Commission.

1 Auclair, Galula, Hubert, Jacrot, Joly, Netter, and Vendryes, *Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1955* (United Nations, New York, 1956), Vol. 4, p. 235.

2 J. E. Lynn and N. J. Pattenden, *Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1955* (United Nations, New York, 1956), Vol. 4, p. 210.

3 Nikitin, Galanina, Ignatiew, Okorokow, and Suchorutchkin, *Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1955* (United Nations, New York, 1956), Vol. 4, p. 224.

4 Adamchuk, Gerasimov, Yefimov, Zenkevich, Mostovoi, Pevzner, Chernyshov, and Tsitovich, *Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1955* (United Nations, New York, 1956), Vol. 4, p. 216.

5 J. A. Harvey, *Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1955* (United Nations, New York, 1956), Vol. 4, p. 147.

6 V. L. Sailor, *Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1955* (United Nations, New York, 1956), Vol. 4, p. 199. This paper summarized the unpublished works done in the United States by Foote, Friesen, Gaerttner, Harvey, Havens, Hughes, Landon, Leonard, Levin, Melkonian, Perez-Mendez, Pilcher, Rainwater, Sailor, Seppi, and Yeater.

7 J. A. Harvey and J. E. Sanders, *Progress in Nuclear Energy, Series I* (Pergamon Press, London, 1956), Vol. 1, pp. 1–54.

8 P. A. Egelstaff and D. J. Hughes, *Progress in Nuclear Energy, Series I* (Pergamon Press, London, 1956), Vol. 1, pp. 55–89.

9 Simpson, Fluharty, and Simpson, Phys. Rev. **103**, 971 (1956).

10 Pilcher, Harvey, and Hughes, Phys. Rev. **103**, 1342 (1956).

11 F. J. Shore and V. L. Sailor, Bull. Am. Phys. Soc. Ser. II, 2, 70, 219 (1957).

12 C. W. Reich and M. S. Moore, Phys. Rev. **111**, 929 (1958).

13 E. Vogt, following paper [Phys. Rev. **112**, 203 (1958)].

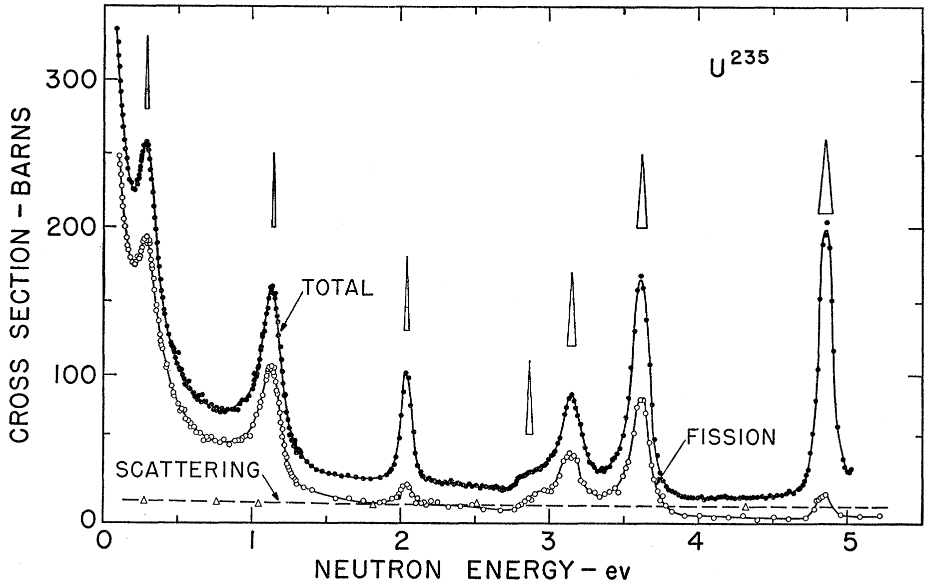

FIG. 1. Observed total, fission, and scattering cross sections of U^{235} from 0.1 to 5 ev. The fission cross section was normalized to 582 barns at 0.0253 ev. The scattering data are those of Foote (see reference 25).

formula and shows that such a formula can also give a satisfactory fit to these same data.

II. CROSS-SECTION MEASUREMENTS

A high-resolution crystal spectrometer[14] was used to measure the total and fission cross section of U^{235} for neutrons in the energy range from about 0.1 to 10 ev. Both sets of data were obtained under identical conditions of resolution (0.17 μsec/m for energies greater than 0.3 ev, and 0.26 μsec/m below 0.3 ev). Although the instrument resolution operates on the total and fission cross sections in a different manner,[15] the distortions produced in the two cases are nearly the same provided that the resolution effect is small.

The cross sections of many nonfissile isotopes have been studied on this spectrometer, and the properties of the instrument, e.g., resolution,[14] second-order contamination,[16] and precision of energy scale have been thoroughly investigated.

A. Total Cross Section

The total cross sections were obtained by conventional transmission measurements on metallic foils of several thicknesses, each highly enriched in U^{235}. Each energy region was covered by several independent sets of data with several sample thicknesses. In general, each individual transmission measurement had a statistical accuracy of 3% or better. Only those transmissions which satisfied the criteria $0.85>T>0.05$ were used for computing the total cross sections, so as to avoid excessive uncertainties due to magnification of the statistical error in the case of $T>0.85$ and second-order contamination corrections for $T<0.05$. Occasional "wild" points lying outside the statistical limits appear in the data. These were due to the misbehavior of electronics, errors in the processing of data, and other common maladies of experimentation. Care was taken to avoid the arbitrary elimination of "bad" data. Whenever inconsistencies appeared between different sets of data additional measurements were made to eliminate the doubtful results.

The results are summarized in Figs. 1 and 2. It is impractical to present the entire data in detail, or even to plot all available data on a reasonable size graph. All analysis was carried out on large-scale plots of small regions of the curve, or on tabulations of the data. Such tabulations have been made available to persons having special interest in the detailed data.[17]

B. Fission Cross Section

1. *Experimental Procedure*

The fission cross section was measured relative to B^{10} by simultaneous observation of the count rates in a twenty-plate ionization chamber[18] containing U^{235}, and in a "thin" BF_3 proportional counter enriched in B^{10}. The fission chamber, which was shielded by a two inch paraffin and boron carbide mixture, was mounted on the end of the spectrometer arm. The BF_3 counter was

[14] Sailor, Foote, Landon, and Wood, Rev. Sci. Instr. **27**, 26 (1956); and L. B. Borst and V. L. Sailor, Rev. Sci. Instr. **24**, 141 (1953).

[15] For total cross section measurements the spectrometer resolution function $R(E-E')$ is folded into the transmission, $\int \exp[-N\sigma_{nT}(E')]R(E-E')dE'$; however, for fission measurements it is folded directly into the cross section $\int \sigma_{n,f}(E') \times R(E-E')dE'$.

[16] R. Haas and F. J. Shore, Bull. Am. Phys. Soc. Ser. II, **3**, 19 (1958).

[17] For example, see reference 13.

[18] We are indebted to Professor W. W. Havens, Jr., Columbia University for lending us this fission chamber. It is described by Melkonian, Perez-Mendez, Melkonian, Havens, and Rainwater, Nuclear Sci. and Eng. **3**, 435 (1958).

interposed between the fission chamber and the arm collimator so that the monoenergetic neutron beam traversed the BF_3 counter before entering the fission chamber. The proportional counter had a well-defined counting volume obtained through the use of guard tubes.[19] It was filled with one-third atmosphere of BF_3 (enriched to 95% in B^{10}), and absorbed approximately 1.5% of the neutrons at 1 ev. In addition to these two counters a small BF_3 counter, which responded to neutrons scattered from the first collimator, served as a monitor of pile power. All three counting circuits employed conventional pulse amplifiers, scalers, and print-out circuits. Counts were taken with the spectrometer crystal adjusted for the Bragg condition and background with the crystal rotated "off Bragg" by one degree so that the net count rate could be obtained. The ratio, R, of net count rate in the fission chamber to that in the thin BF_3 counter was closely proportional to the square root of the energy times the fission cross section. An effort was made to accumulate at least 3000 fission counts for each point, although for most of the data below 1.5 ev 10 000 counts were obtained. In the high-energy region where the count rate in the thin BF_3 counter was small, care was taken that the statistical uncertainty in R did not exceed 3% except in a few isolated spots between resonances where the fission cross section fell below 10 barns.

It was necessary to accumulate the data over a period of months due to the low counting rates obtained. The low intensity resulted from the small amount of U^{235} in the beam (approximately 20 mg/cm^2 in 40 layers), the small fission cross section over most of the region studied, and the decrease with energy of the neutron flux. The procedure was adopted of returning to a standard point at intervals of about 2 days to insure that the sensitivity of the system had not changed. Occasional corrections of the order of a few percent were applied to compensate for drifts in sensitivity.

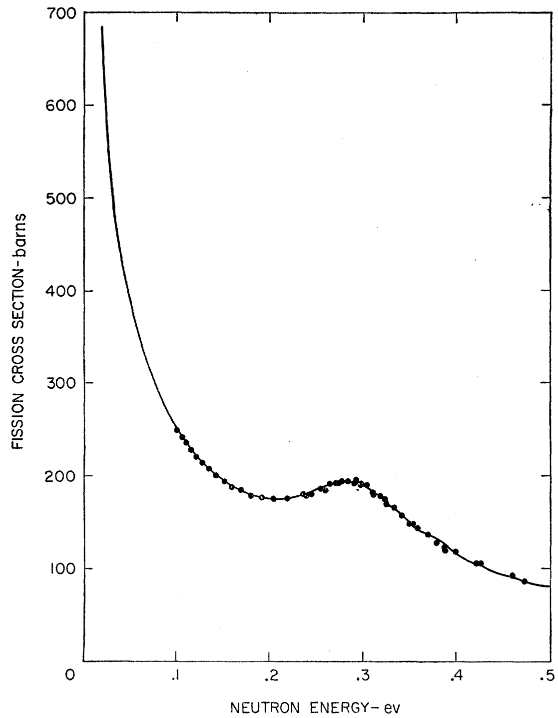

FIG. 3. Normalization curve. The solid curve represents the Hanford U^{235} fission cross section data (see reference 20) normalized to 582 barns at 0.0253 ev. The solid circles are the BNL crystal spectrometer data normalized to the curve in the region 0.1 to 0.5 ev. Wiggles in the curve near 0.4 ev reflect small fluctuations in the Hanford data.

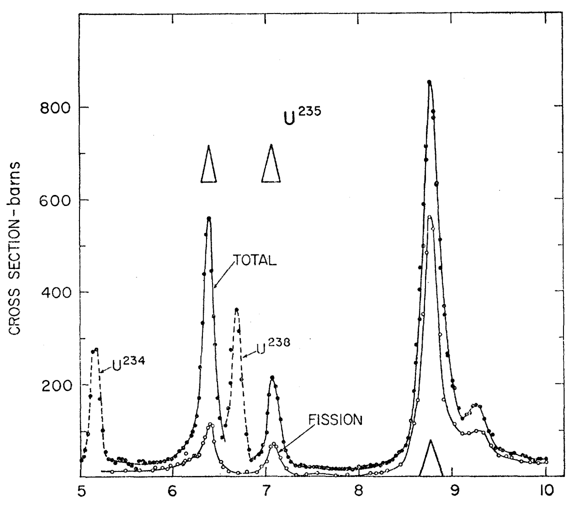

FIG. 2. Observed total and fission cross sections of U^{235} from 5 to 10 ev. The fission cross section was normalized to 582 barns at 0.0253 ev. Gaps exist in the total cross section at 5.3 and 6.7 ev due to contaminations of U^{234} and U^{238} in the samples.

Corrections were also applied to the data to account for the absorption of neutrons in the beam by nuclear matter other than B^{10} and U^{235} and for the effect of second order contamination of the neutron beam. At higher energies the second order correction became substantial at isolated spots for those points which fell at one-fourth the energy of strong resonances. For example, the strong resonances at 8.8 and 19.3 ev caused large corrections at 2.2 and 4.8 ev, respectively.

For the region below 2 ev the instrument resolution had negligible effect on the shape of the observed cross section. The Doppler correction was negligible at all points below 2 ev except for a correction of 4% at the peak of the 1.14-ev resonance and 0.5% at the 0.29-ev peak.

2. *Normalization*

To convert the *relative* cross sections into *absolute* cross sections it was necessary to normalize some point on the curve to a known absolute value. Unfortunately,

[19] A. L. Cockcroft and S. C. Curran, Rev. Sci. Instr. 22, 37 (1951); we are indebted to Mr. C. Z. Nawrocki of Brookhaven National Laboratory for fabricating this counter.

it was impossible to normalize directly to the absolute value of the thermal fission cross section, $\sigma_{n,f}(0.0253$ ev), because properties peculiar to this spectrometer make it impractical to work at energies below ~0.08 ev. However, the very excellent data of Leonard *et al.*[20] extending from ~0.02 to ~1.0 ev made the normalization possible. The Leonard data were normalized to the so-called "world value,"[21] $\sigma_{n,f}(0.0253\text{ ev})=582\pm10$ barns, and our data were in turn normalized to Leonard's curve in the region of overlap from 0.1 to 0.4 ev (see Fig. 3). The fission cross sections obtained after the normalization are shown in Figs. 1 and 2.

There is some question as to the reliability of the "world value" because recent measurements of $\sigma_{n,f}(0.0253$ ev) have yielded widely divergent results. Bollinger *et al.*[22] obtained the value 606±6 barns using a new and very elegant method for making a direct absolute fission cross section measurement. This result cannot be ignored, although the method should have further experimental tests before it can be accepted with full confidence. Leonard *et al.*[23] obtain 552±6 barns in a more conventional measurement, and Tunnicliffe *et al.*[24] have reported the value 575±6 barns. It is obvious that this important cross section needs further experimental attention.

C. Scattering Cross Section

The scattering cross section of U^{235} has been measured at a few energies by Foote.[25] At low energies, the scattering is almost entirely potential scattering because the resonances are so weak that resonant scattering is entirely negligible. A possible exception is the "negative energy" resonance discussed in Sec. IIIA. Foote's data reproduced in Fig. 1 show a slight increase in $\sigma_{n,n}$ at lowest energies. In calculations which follow the appropriate value of $\sigma_{n,n}$ was obtained at each energy by fairing a smooth curve through Foote's measured values.

D. Derivation of the Capture Cross Section

The capture cross section, $\sigma_{n,\gamma}(E)$, can be derived from the other cross section data by taking the difference, $\sigma_{n,\gamma}=\sigma_{nT}-\sigma_{n,f}-\sigma_{n,n}$. We have done this by plotting the σ_{nT}, and $\sigma_{n,n}$ data on large scale graphs, then fairing smooth curves through the points and taking the difference between the smoothed curves to get the absorption cross section, σ_{nX}. The individual points for $\sigma_{n,f}$ were then subtracted to yield $\sigma_{n,\gamma}$. The capture cross sections derived by this procedure are shown in Figs. 4, 6, 7, and 8.

III. INTERPRETATION OF THE DATA

A. Evidence for a Bound Level

One of the most difficult problems in the interpretation of the U^{235} cross sections arises from the shape of the curves in the region below 2 ev. This is the region of most precise data because of relatively good counting statistics and small corrections. The general trend of the data shows a rapid rise in cross section as the energy is decreased with small peaks superimposed at 0.29 and 1.14 ev. It should be noted that it is this large "background" which is the dominating feature of the low-energy cross section.

In this energy region the cross section is composed of contributions from all neighboring levels in the compound nucleus which have the proper spin and parity to be formed from the target nucleus by *s*-wave neutrons. Of course, the primary contributions come from those levels which lie close to the binding energy. These contributions depart from a $1/v$ behavior depending on the location of the level. Levels which lie further away from the binding energy contribute only a very small $1/v$ component. In most isotopes one or two levels contribute the major portion of the thermal cross section. Such a level might lie above the binding energy in which case it would be observable as a resonance in the cross section; or it might lie below the binding energy (a "negative energy" resonance) in which case it would make its presence felt by the shape of the cross section curve, by the existence of resonant scattering, and various other properties such as the magnitudes of the coherent and incoherent scattering cross sections.

In U^{235} the known resonances can account for only a small fraction of the thermal absorption cross section (about 75 out of 685 barns), so it appears reasonable to ascribe the remaining large "background" to one or more bound levels. We can attempt to learn more about this hypothetical bound level (or levels) by fitting the "background" curve. Of course, the process of curve fitting is complicated by the presence of the resonances, particularly the ones at 0.29 and 1.14 ev, whose effects must be removed by a reiteration process.

Before proceeding with the analysis we shall anticipate the results of later sections and assume that the bound level (or levels) is of the opposite spin state from the 0.29- and 1.14-ev resonances, and hence cannot interfere with them. It should be noted that Vogt in the accompanying paper,[13] arrives at the opposite assignment; i.e., the bound level, the 0.29- and the

[20] Leonard, Seppi, and Friesen, Hanford Atomic Products Operation, General Electric Company, 1954 (unpublished).

[21] *Neutron Cross Sections*, compiled by D. J. Hughes and R. Schwartz, Brookhaven National Laboratory Report BNL-325, Suppl. No. 1 (Superintendent of Documents, U. S. Government Printing Office, Washington, D. C., 1957), p. vii.

[22] Bollinger, Saplakoglu, Coceva, Coté, and Thomas, Bull. Am. Phys. Soc. Ser. II, 2, 196 (1957). The value has recently been revised to 606±6 barns [L. M. Bollinger (private communication)].

[23] B. R. Leonard (private communication).

[24] Tunnicliffe, Bigham, Campion, and Hanna, International Conference on the Neutron Interaction with the Nucleus, Columbia University, September 9–13, 1957 (unpublished). *Note added in proof.*—The value has recently been changed to 570±6 barns. Bigham, Hanna, Tunnicliffe, Campion, Lounsbury, and MacKenzie, Second International Conference on the Peaceful Uses of Atomic Energy, Geneva Switzerland, September 1–13, 1958, paper P/204, unpublished.

[25] H. L. Foote, Jr., Phys. Rev. **109**, 1641 (1958).

1.14-ev resonances are all in the same spin state and thus can exhibit interference. These two conflicting interpretations represent the most important unsolved puzzle in the U^{235} slow neutron cross section. Until the correct interpretation has been determined by experiment, the results of analysis of the lower energy resonances must be considered as provisional.

Information about the location and strength of the bound level can be obtained from the absorption cross section curve, $\sigma_{nX}(E)$. The shape of this curve has been tested against the following analytical forms to see which gives the best fit to the background:

$$\sigma_{nX}=KE^{-Y}, \tag{1}$$

$$\sigma_{nX}=KE^{-\frac{1}{2}}(E-E_0)^{-2}, \tag{2}$$

$$\sigma_{nX}=K_1E^{-\frac{1}{2}}+K_2E^{-\frac{1}{2}}(E-E_0)^{-2}, \tag{3}$$

$$\sigma_{nX}=K_1E^{-\frac{1}{2}}+K_2E^{-\frac{1}{2}}[(E-E_0)^2+\Gamma^2]^{-1}, \tag{4}$$

where $\sigma_{nX}=\sigma_{n,\gamma}+\sigma_{n,f}$ is the absorption cross section.

The method of least squares was used to solve for the constants, K, Y, E_0, etc., in each case. If the "background" were due to one or more noninterfering bound levels, the above four forms would cover most eventualities, e.g., cases for which $|E_0|\approx 0$, $|E_0|\approx\Gamma$, and $|E_0|\gg\Gamma$. The analysis is extremely tedious, and, in addition, the corrections for the 0.29- and 1.14-ev levels introduce considerable uncertainty; hence, the conclusions obtained are not beyond question. Within these limitations we found that Eq. (2) fit the "background" the best, and, in fact, the solutions using Eqs. (3) and (4) gave $K_1\approx 0$ and $E_0^2\gg\Gamma^2$.

The following values, which we shall consider to be provisional, were obtained for the constants in Eq. (2): $K=208.5$ barn $(\text{ev})^{\frac{5}{2}}$ and $E_0=-1.45$ ev, i.e., the shape of the "background" is consistent with a single strong level lying 1.45 ev below the binding energy. The constant K yields a reduced neutron width $\Gamma_n{}^0=3.04\times10^{-3}$ ev. More information about the characteristics of this hypothetical level can be obtained from the ratio of capture to fission at thermal energies,[21] $\alpha(0.0253\text{ ev})=\sigma_{n,\gamma}/\sigma_{n,f}=0.192\pm0.008$. Correcting this ratio for the effects of the higher energy resonances we obtain $\alpha(-1.45\text{ ev})\approx0.1475$. If we assume the radiation width of this level to be "average" $\Gamma_\gamma\approx33\times10^{-3}$ ev we obtain $\Gamma=256\times10^{-3}$ and $\Gamma_f=223\times10^{-3}$ ev. These results are summarized in Table I. Similar results were obtained by Harvey[26] in an independent analysis based on independent data. However, Vogt[13] obtains slightly different parameters from his analysis: $E_0=-0.95$ ev, and $\Gamma_n{}^0=1.49\times10^{-3}$ ev (equivalent to $K=64.0$).

The scattering cross section data support the conclusion that a relatively strong bound level is present somewhere close to the binding energy. Measurements by Foote[25] show that the scattering cross section increases with decreasing energy, rising from the value at 5 ev which approximates the predicted potential scattering. This departure from potential scattering as the energy is decreased can be ascribed to resonance scattering by a very strong bound level. Unfortunately the scattering data are not sufficiently precise or detailed to permit an exact determination of the strength or position of the level, and thus make no strong preference for either set of parameters.

At very low energies ($E<0.05$ ev) a marked curvature occurs in the cross section (not shown in our data) which Harvey[26] has analyzed, and ascribed to a weak resonance at $E_0=-0.02$ ev. This weak resonance has only minor effect at thermal energies and is negligible above $\sim$0.1 ev; however it cannot be ignored completely. In Table I we have listed slightly modified parameters for this resonance which fit our data better. Vogt[13] was able to fit the thermal data ($E=0.0253$ ev) without invoking this resonance. It is not clear whether his analysis would also account for the curvature at the lowest energies mentioned above.

The "negative energy" resonance interpretation of the U^{235} "background" has several features which taken together tend to make it questionable. First it should be noted that "backgrounds" of similar magnitude (but not similar shape) are also present in U^{233}, Pu^{239}, and Pu^{241}. Furthermore, the U^{235} bound level is by no means average in its properties, e.g., both $\Gamma_n{}^0$ and Γ_f are anomalously large. In fact $\Gamma_n{}^0$ is five times larger than for any other resonance observed in the range 0 to 50 ev,[6,9,10] and it is about twenty times larger than the average. That Γ_f is unusually large can best be appreciated in terms of $\alpha=\sigma_\gamma/\sigma_f$. The value of α associated with the bound level lies slightly below a smooth extrapolation of the data of Diven *et al.*[27] obtained in the region 200 to 1000 kev. Yet the average α for the intermediate energies is much larger: in the range 0–50 ev[6,9] $\alpha\approx0.6$, and in the low kev region[28] $\alpha\approx0.45$. However, in spite of the oddities mentioned here, the hypothesis of a strong negative energy resonance is the only plausible explanation for the "background" at the present time. In what follows we hold to this interpretation.

B. Failure of the Single-Level Formula

Figure 4 shows $E^{\frac{1}{2}}\sigma_{n,\gamma}$ *vs* E from 0.1 to 1.5 ev derived as discussed in Sec. IID from the total, fission, and scattering cross sections. Also shown is the "background" cross section due to resonances other than those at 0.29 and 1.14 ev, including the "negative energy" resonance. The open circles are the result of subtracting the background from the derived capture cross section. The smooth curve is the best single-level Breit-Wigner curve which can be drawn through the

[26] J. A. Harvey, see reference 7, p. 46.

[27] Diven, Terrell, and Hemmendinger, Phys. Rev. **109**, 144 (1958).

[28] Kanne, Stewart, and White, *Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1955* (United Nations, New York, 1956), Vol. 4, p. 315.

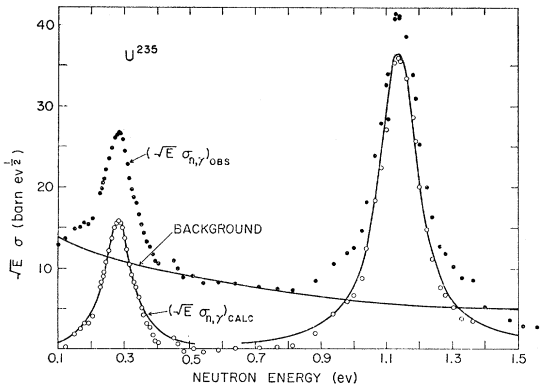

FIG. 4. Capture cross section times $E^{\frac{1}{2}}$ for U^{235} from 0.1 to 1.5 ev. Solid circles give the difference between the absorption and fission cross sections. A strong resonance, presumed to be at -1.45 ev, accounts almost entirely for the background curve. Subtraction of the background curve from the solid circles yields the open circles to which single-level Breit-Wigner curves have been fit.

data. It is seen that the capture cross section is well fitted by this curve. Figure 5 gives the observed fission cross section, normalized to $\sigma_{n,f}(0.0253)=582$ barns, and the background associated with the other resonances including the "negative energy" resonance. The smooth curve drawn through the open circle points is the net fission cross section, and the dashed lines represent the locus of points which bisect the resonances. It is seen that the bisectors are not straight, and are not centered at E_λ, the energy corresponding to the peaks of the capture curves. Figures 6, 7, and 8 represent the capture and fission cross sections for the 2.04-ev, the 3.6-ev, and 4.8-ev resonances, respectively. In each case the capture cross section is fitted by a symmetrical curve, whereas the fission cross section is not. It is important to note that for the last three resonances,

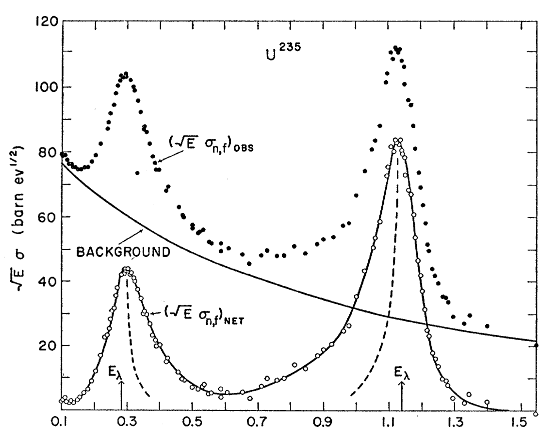

FIG. 5. Fission cross section times $E^{\frac{1}{2}}$ for U^{235} from 0.1 to 1.5 ev. The background curve, due mainly to a presumed -1.45-ev level, when subtracted from the observed fission cross section yields the open circle points. The dashed curves, which are the bisectors of the resonances, are seen to be curved and shifted in energy from E, the energy at the capture peak.

the background due to the "negative energy" resonance is small and varies very slowly; hence, any uncertainty due to it is small. Note, however, for the 2.04- and 4.8-ev resonances the correction for second order contamination becomes important. For three of the five resonances shown, the peak of the fission cross section is shifted toward lower energy; whereas for the other two it is shifted to higher energies. This gives one confidence that the distortion of the fission curves is not caused by some systematic instrumental effect.

The asymmetries noted by Sailor[6] in the total cross section are seen to be associated with the fission component of the cross section. The explanation of the total cross section asymmetry in the 0.1- to 1.5-ev region due to Harvey and Sanders,[7] in which two unresolved

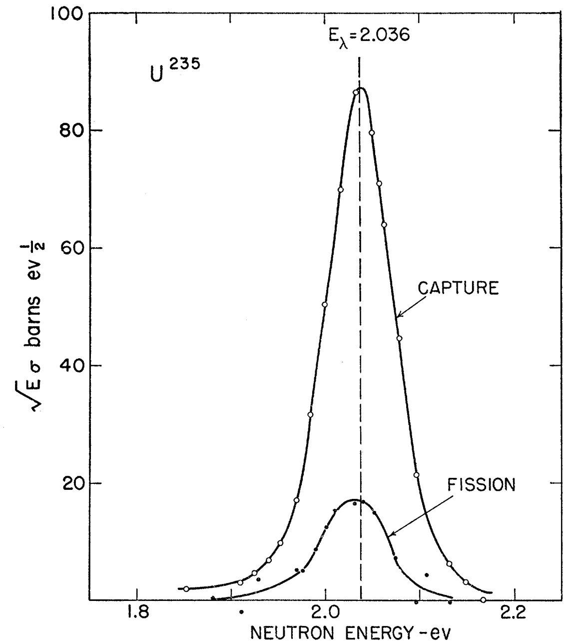

FIG. 6. Fission and capture cross sections for the 2.036-ev level. The capture curve is symmetrical about E, whereas the fission curve is not.

resonances were postulated at 0.42 and 0.91 ev, would now require that these "unresolved resonances" be practically pure fission resonances. A similar situation would have to exist at the other resonances as well, which is extremely unlikely.

Because of the uncertainty in the normalization of the fission cross section to 582 barns at thermal neutron energy, an analysis similar to those of Figs. 4 and 5 was done using $\sigma_{n,f}(0.0253)=614$ barns, and also 556 barns. In all cases the qualitative results were the same, for the 0.29- and 1.14-ev resonances the capture part was symmetrical; whereas the fission part was not.

Since the arguments which have been adduced depend on the assumption that each resonance, including the bound level, is fit by a single-level Breit-Wigner formula, and since the asymmetry in the fission cross section is a contradiction, one is forced to the conclusion that the single level formula is inadequate for fitting the U^{235} resonances. Recent evidence from the study of other fissile nuclei indicate a similar situation. Bollinger[29] has shown for Pu^{239} that the variation of η, the number of neutrons emitted per neutron absorbed, for the 10.9- and 11.9-ev resonances requires a multilevel formula. Moore *et al.*[30] have shown that it is necessary to invoke a multilevel formula in order to fit their fission data for U^{233}.

IV. ATTEMPTS AT ANALYSIS

A. Multilevel Formula

The Wigner-Eisenbud theory,[31] which describes the energy dependence of reaction and scattering cross sections in a very general way, cannot be applied directly to practical cases. However, formulas can be derived from the general theory using various approximations and restrictions. Two such practical formulas have been derived which appear to suffer little loss of generality for cases involving fission and capture. One formula, obtained by Reich and Moore,[12] is valid for (1) one fission channel plus very many capture channels, and (2) many levels. This is similar to the earlier result of Krotkov[32] which was applied to the case of a single channel for scattering and many channels for radiative capture. Reich and Moore have recently extended their formula to include the case of two fission channels. The second formula, derived by Vogt in the accompanying paper,[13] applies to the case of (1) many fission channels plus very many capture channels and (2) few levels.

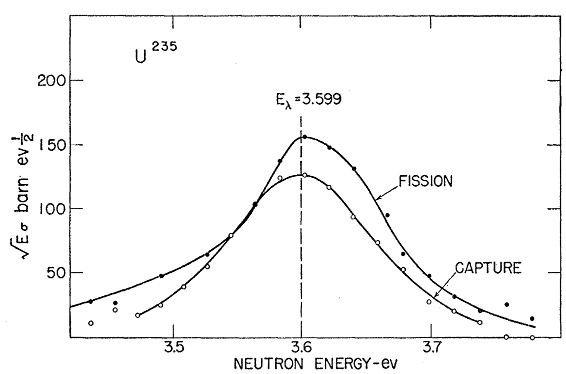

FIG. 7. Fission and capture cross sections for the 3.599-ev level. The capture curve is symmetrical about E, whereas the fission curve is not.

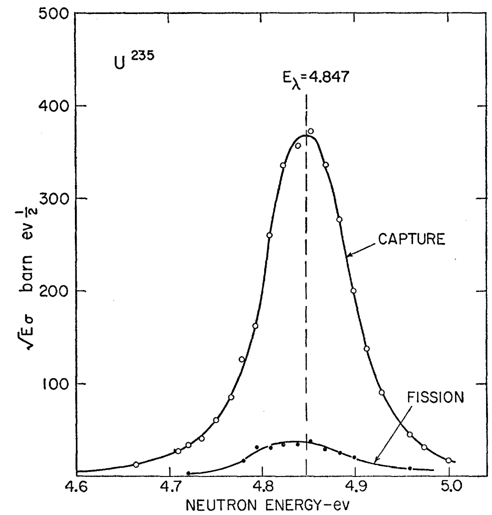

FIG. 8. Fission and capture cross sections for the 4.847-ev level. The capture curve is symmetrical about E, whereas the fission curve is not.

Unfortunately, we have no way of knowing which formula is best suited for application to the cross section of U^{235}. However, Vogt[13] has applied his "many-channel, few-level" formula to our U^{235} data and achieved what appears to be an excellent fit. Therefore, we have tested the Reich-Moore "one-channel, many-level" formula against the same data to see if an equally good fit can be obtained.

For this case of one fission channel and very many capture channels Reich and Moore obtained the following equations for the fission and capture cross section:

$$\sigma_{n,f}(E)=\sum_J \frac{4\pi\lambda\!\!\!^{-2}g}{|\Delta|^2}(a_{12}^2+b_{12}^2), \tag{5}$$

and

$$\sigma_{n,\gamma}(E)=\sum_J \frac{4\pi\lambda\!\!\!^{-2}g}{|\Delta|^2}[b_{11}+a_{22}(b_{11}a_{22}-a_{12}b_{12}) + (2+b_{22})(b_{11}b_{22}-b_{12}^2)+a_{12}(a_{12}b_{22}-a_{22}b_{12})]. \tag{6}$$

Note that the summations are made over all possible values of the angular momentum, J, of the compound state, which for s-wave neutrons consist of $J=I+\frac{1}{2}$ or $I-\frac{1}{2}$, where I is the spin of the target nucleus. The a_{ij}^2, etc., contain cross-product terms between the amplitudes of the various levels, which are called interference terms. Only levels of the same J can interfere with each other.

In Eqs. (5) and (6),

$$a_{ij}\equiv\sum_\lambda \frac{4\beta_{\lambda i}\beta_{\lambda j}(E_\lambda-E)}{4(E_\lambda-E)^2+\Gamma_{\lambda\gamma}^2}, \tag{7}$$

[29] L. M. Bollinger, Proceedings of Tripartite Conference on Cross Sections of Fissile Nuclei, Atomic Energy Research Establishment, Harwell Report AERE *NP/R*-2076-Rev., 1957 (unpublished), p. 22.

[30] Moore, Miller, and Reich, Bull. Am. Phys. Soc. Ser. II, **1**, 327 (1956); Miller, Fluharty, Brugger, and Moore, Bull. Am. Phys. Soc. Ser. II, **2**, 70 (1957).

[31] E. P. Wigner and L. Eisenbud, Phys. Rev. **72**, 29 (1947); T. Teichman and E. P. Wigner, Phys. Rev. **87**, 123 (1952); Feshbach, Porter, and Weisskopf, Phys. Rev. **96**, 448 (1954).

[32] R. Krotkov, Can. J. Phys. **33**, 622 (1955).

$$b_{ij}\equiv\sum_{\lambda}\frac{2\beta_{\lambda i}\beta_{\lambda j}\Gamma_{\lambda\gamma}}{4(E_\lambda-E)^2+\Gamma_{\lambda\gamma}{}^2},\tag{8}$$

$$|\Delta|^2=[(1+b_{11})(1+b_{22})-a_{11}+a_{22}a_{12}{}^2-b_{12}{}^2]^2+[(1+b_{11})a_{22}+(1+b_{22})a_{11}-2a_{12}b_{12}]^2,\tag{9}$$

where $\beta_{\lambda i}=\pm(\Gamma_{\lambda i}/2)^{\frac{1}{2}}$. It should be noted that the signs of the $\beta_{\lambda i,j}$ can be either positive or negative. This effectively introduces an additional parameter to be determined for each level, i.e., the sign of the product $\beta_{\lambda i}\beta_{\lambda j}$ relative to any other resonance. The E_λ in the above equations denotes the resonant energy of level λ and is equivalent to the E_0 used in other sections.

The quantity $\lambda\!\!\!^{-}$ is the neutron wavelength divided by 2π. The λ indices refer to levels while the ij refer to channels. In this case, i, $j=1$ is the neutron channel corresponding to elastic scattering, i, $j=2$ is the fission channel, and i, $j=3,\cdots,n+2$ (which have disappeared from the equations) are the radiative capture channels.

For s-wave neutrons the statistical weight factor has two possible values, $g=\frac{1}{2}[1\pm(2I+1)^{-1}]$, corresponding to the two possible values of J. Since we have no knowledge of the J-value of each resonance, we absorb g into the $\Gamma_{\lambda n}$ of each resonance. However, in U^{235} ($I=\frac{7}{2}$), $g=\frac{7}{16}$ or $\frac{9}{16}$, so for most purposes the value $g\approx\frac{1}{2}$ is a good approximation.

It is curious that the radiation width $\Gamma_{\lambda\gamma}$ appears in the denominator of a_{ij} and b_{ij} instead of the total width Γ_λ; however, it is readily shown that the equations reduce to the single-level Breit-Wigner form when only one level is present, and that Γ_λ replaces $\Gamma_{\lambda\gamma}$ in the denominator when the small terms in $|\Delta|^2$ are neglected.

The equations are awkward to use, and it appears that their properties are best studied by making numerical calculations on hypothetical cases. This can be done by exact computation on high-speed computing machines, or by disposing of the terms which prove to be negligible when numbers are substituted. We have used both methods to study resonances which have large separation compared to their widths, and which have both fission and capture components. The following results emerge: (1) The fission component of each resonance shows marked asymmetries, whereas, the capture component is approximately symmetrical; (2) the peak of the fission component is markedly shifted from E_λ, whereas the peak of the capture is at almost exactly E_λ; (3) the total width of the capture component is approximately equal to the input total width Γ_λ. In other words, within the limitations stated above, the Reich-Moore capture cross section is only slightly different from the Breit-Wigner capture cross section.

One additional important feature should be noted: (4) at E_λ the ratio

$$\sigma_{n,\gamma}(E_\lambda)/\sigma_{n,f}(E_\lambda)\equiv\alpha(E_\lambda)\approx\Gamma_{\lambda\gamma}/\Gamma_{\lambda f}\equiv\alpha_\lambda$$

for resonance λ providing that the background due to resonances of the opposite spin state is subtracted out. These conclusions, of course, would not be expected to apply to pathological cases, e.g., when two resonances lie very close together.

In addition, Reich and Moore[33] have shown numerically, and we have verified, that when capture is the only process occurring, the multilevel formula reproduces the sum of single-level contributions to a very high degree of accuracy. However, the capture cross section is slightly different when fission is present than when fission is not present.

B. Analysis of the Resonances

1. *First Approximation Parameters*

In the previous section it was indicated that a rough set of parameters for each resonance can be obtained by analyzing the capture component in terms of the Breit-Wigner single-level formula. This analysis yields provisional values of E_λ, Γ_λ, and $g\Gamma_n{}^0\Gamma_{\lambda\gamma}/\Gamma_\lambda$. Then assuming that the ratio $\sigma_{n,\gamma}/\sigma_{n,f}\approx\alpha_\lambda$, at $E=E_\lambda$, provisional values of all the remaining parameters, $\Gamma_{\lambda f}$, $\Gamma_{\lambda\gamma}$, and $g\Gamma_{\lambda n}{}^0$ can be derived. More exact values, including the relative signs of the $\beta_{\lambda i,j}$, must be obtained by laborious trial-and-error adjustments followed by numerical computing with Eqs. (5) and (6).

The procedure we have used to derive the approximate parameters follows:

1. The contribution of the bound level is subtracted from the observed absorption and fission cross-section curves.
2. The capture cross section is obtained from $\sigma_{nX}-\sigma_{n,f}$.
3. The residual capture component is analyzed as if each resonance were a simple Breit-Wigner curve, yielding E_λ, Γ_λ and $g\Gamma_{\lambda n}{}^0\Gamma_{\lambda\gamma}/\Gamma_\lambda\propto(\sigma_0\Gamma_\lambda)_\gamma$. A process of "shape" analysis is used which corrects for spectrometer resolution and Doppler broadening.
4. The quantity α_λ is determined from the limiting value of the ratio of the area under the capture curve

TABLE I. First approximation parameters for U^{235} resonances. Uncertainties have not been listed because of the provisional nature of these values. Values of $\Gamma_{\lambda\gamma}$ listed in parentheses have been assumed, and hence the values of all other partial widths reflect this assumption.

E_λ (ev)	α_λ	Γ_λ (10^{-3} ev)	$\Gamma_{\lambda\gamma}$ (10^{-3} ev)	$\Gamma_{\lambda f}$ (10^{-3} ev)	$2g\Gamma_{\lambda n}{}^0$ (10^{-6} ev)
−1.45	0.1475	256	(33)	223	3036
−0.02[a]	0.54	97	(34)	63	0.72
0.282	0.388	99.7	27.9	71.8	4.49
1.138	0.455	134	41.9	92.1	12.9
2.036	5.1	41.4	34.6	6.8	5.30
3.599	0.82	81.4	37	45	24.3
4.847	11.0	27.8	25.5	2.3	25.0
6.40	~3.8	~42	(33)	~9	~100
8.795	~0.55	~93	(33)	~60	~257

[a] The original parameters derived by J. A. Harvey (reference 26) have been slightly modified to give a better fit.

[33] C. W. Reich and M. S. Moore (private communication).

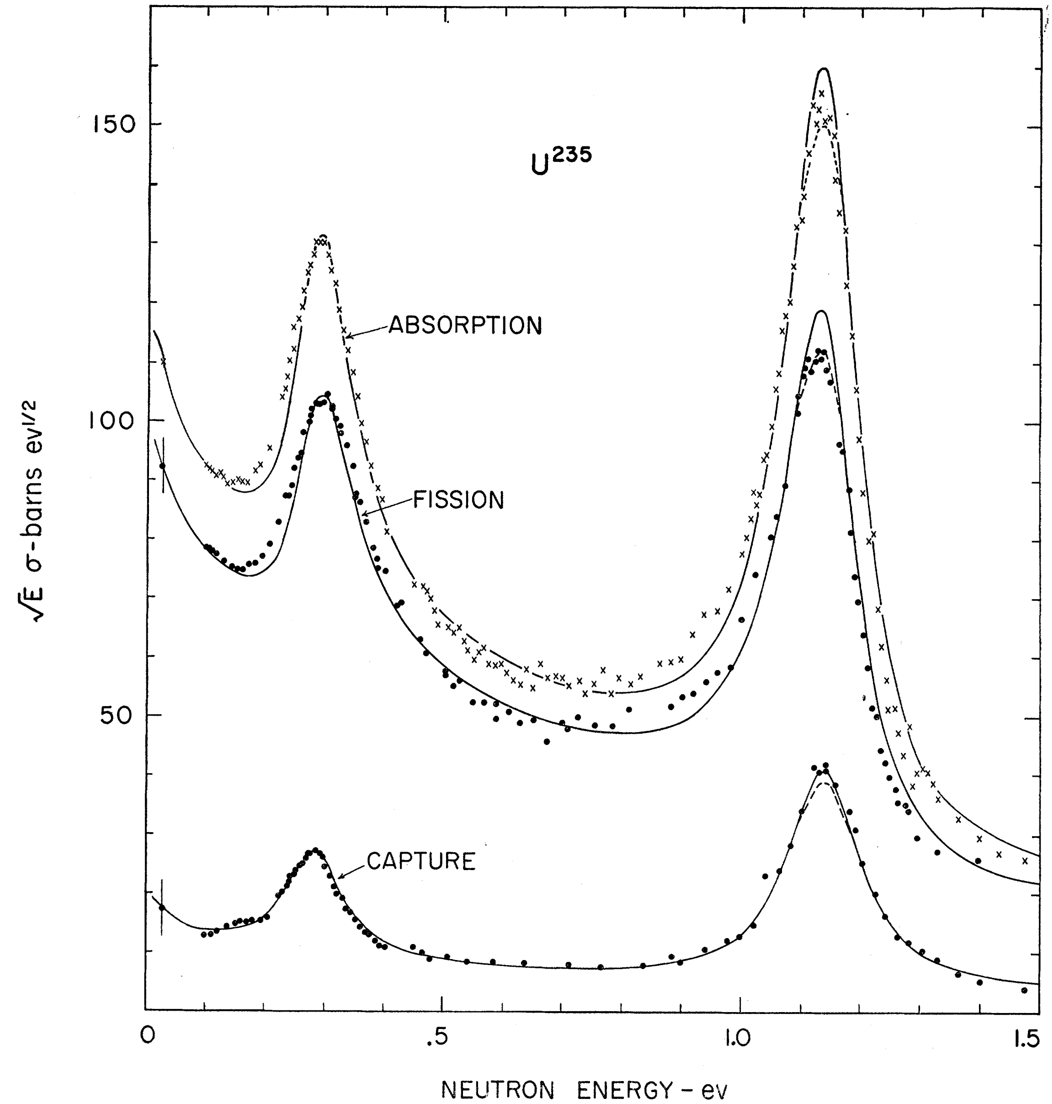

FIG. 9. Single-fission-channel multilevel fit to the U^{235} partial cross sections. Solid curves for absorption, fission, and capture are derived from the formula of Reich and Moore allowing interference between levels at $E=0.282$, 1.138, 3.599, 6.400, and 8.795 ev. A noninterfering background is included which is ascribed to assumed levels at -1.45 and -0.02 ev. Experimental points for the fission cross section are shown, whereas the points for the absorption and capture cross sections are derived. Dashed curves at the peak of the 1.138-ev resonance are Doppler-corrected calculated curves. Points at 0.0253 ev are the "world values" with a $\pm5\%$ uncertainty indicated for the fission cross section normalized to 582 barns.

to that under the fission curve according to

$$\alpha_\lambda \approx \lim_{\Delta E\to 0}\int_{E_1}^{E_2} E^{\frac{1}{2}}\sigma_{n,\gamma}dE \Big/ \int_{E_1}^{E_2} E^{\frac{1}{2}}\sigma_{n,f}dE, \qquad (10)$$

where $E_1=E_\lambda-\Delta E$ and $E_2=E_\lambda+\Delta E$.

Approximate parameters obtained in this way for several of the resonances are listed in Table I. The results for the 6.40- and 8.795-ev resonances are very crude and depend on the assumption that $\Gamma_{\lambda\gamma}\approx 33\times 10^{-3}$ ev. They have been included in Table I only because they are used in the multilevel analysis which follows and for this purpose crude parameters are adequate.

2. *Adjustment of Parameters*

If we insert the first approximation parameters (Table I) into the multilevel formulas [Eqs. (5) and (6)] and generate fission and capture cross-section curves, we find regions in which the computed fission and the observed fission disagree. We shall attempt to make slight modifications of the parameters in Table I and obtain a better fit. Note that to generate the theoretical curves it is necessary to assume the sign of the amplitude factor, $\beta_{\lambda 1}\beta_{\lambda 2}$ for each resonance relative to any other resonance. We have arbitrarily assigned a positive sign to the $\beta_{\lambda 1}\beta_{\lambda 2}$ of the 0.282-ev resonance, and attempt to

TABLE II. U^{235} resonance parameters. These values were used for generating the curves shown in Fig. 9. They were obtained by making trial-and-error adjustments to the values in Table I. The resulting curves give a good fit to the data; however, these parameters cannot be regarded as a unique solution. In the computing it was assumed that the two "negative-energy" resonances did not interfere with each other or with the other five resonances. The latter five resonances were assumed to belong to the same spin state J, and to interfere mutually.

E_λ (ev)	Spin state	Γ_λ (10^{-3} ev)	$\Gamma_{\lambda\gamma}$ (10^{-3} ev)	$\Gamma_{\lambda f}$ (10^{-3} ev)	$2g\Gamma_{\lambda n}^0$	Relative sign of $\beta_{\lambda 1}\beta_{\lambda 2}$
−1.45	Not J	259	(33)	223	3056	
−0.02	Not J	97	(34)	63	0.72	
0.282	J	114.7	32.2	82.5	5.16	+
1.138	J	148	42	106	14.3	−
3.599	J	81.4	37	45	24.3	−
6.400	J	42	(33)	9	100	−
8.795	J	42	(33)	60	257	−

determine the sign of all other resonances relative to this.

In all subsequent analysis we have confined our attention to the range between 0.1 and 1.5 ev. Of course, the resonances outside this region contribute slightly, but only a few of the more important must be included in the calculations. The effects of the 2.04-, 2.86-, 3.14-, 4.84-, and 7.09-ev resonances are negligible, while the 3.599-, 6.40-, and 8.795-ev resonances contribute very small amounts. Hence, we have included only the latter three in our calculations.

Slight modification of the first approximation parameters yield a better fit. Our best fit is shown in Fig. 9 and the parameters used for generating the theoretical curve are listed in Table II. As can be seen, the fit is good but not perfect. We have been unable to find a completely satisfactory set of input parameters and combination of amplitude signs using only the resonances between 0 and 10 ev. The effects of resonances above 10 ev can be estimated from the parameters of Simpson *et al.*,[9] and it is found that they cannot remove the remaining discrepancies, because their contributions below 1.5 ev are too weak. In fact, it is expected that the effects of these resonances would tend to cancel out since the signs of the amplitude factor $\beta_{\lambda 1}\beta_{\lambda 2}$ should be random.

We have attempted to eliminate the remaining discrepancies in Fig. 9 by assuming that a small fraction of the "background" cross section is due to a bound level in the same spin state as the 0.282- and 1.138-ev resonances and hence must be included in the multilevel calculation. Unfortunately this type of contribution does not appear to be capable of eliminating all of the remaining troubles. If the region around 0.8 ev is brought into better agreement, serious disagreement appears in the region below 0.25 ev, and vice versa. For example, a bound level located at -1.0 ev with relatively average strength $\Gamma_{\lambda n}^0=0.03\times10^{-3}$ ev, does violence to the curves in Fig. 9, regardless of which sign is selected for the amplitude. Thus, if the Reich-Moore single-fission-channel formula is used, we conclude that the negative-energy resonances must be in the opposite spin states from the 0.282- and 1.138-ev, resonances.

Using the parameters from Table II the scattering cross section was computed for the energy range 0.1 to 2.0 ev. The solid curve of Fig. 10 shows this result assuming $\sigma_p=10.3$ barns determined on the basis of $\sigma_p=4\pi R^2$, where $R=1.47A^{\frac{1}{3}}\times10^{-13}$ cm. The points are the experimental data of Foote,[25] and are seen to lie approximately 0.8 barn lower than the calculated curve; i.e., a value of $\sigma_p=9.5$ barns would give a better fit. The cross section at 0.28 and 1.14 ev have slight dips due to the interference between potential and resonance scattering. The resonance scattering for these two resonances is so small that it cannot be seen on this scale. Because of uncertainties in the parameters for the negative-energy resonance, and in the experimental data, the 0.8-barn difference is not considered significant.

V. DISCUSSION OF THE RESULTS

A. Agreement between Computed and Observed Curves

It can be seen in Fig. 9 that our computed curves give a good but not perfect fit to the data. This probably indicates that the *one*-fission-channel formula is too restrictive. The fit obtained by Vogt[13] using the many-channel formula and the same data shows similar discrepancies of similar magnitude. It is important to note, however, that Vogt's results give a more reasonable interpretation of the negative-energy resonances than do ours. There are three points in favor of Vogt's interpretation; (1) Vogt's level at $E_0=-1.0$ ev has a significantly smaller Γ_n^0 than does our level at -1.45 ev, which brings it into more reasonable agreement with the neutron width distribution observed at higher energies.[9] (2) Vogt's curve fits the thermal cross sections without the necessity of the $E_0=-0.02$-ev resonance (although, it is not clear whether the computed curve will fit the data at the very lowest energies in the region

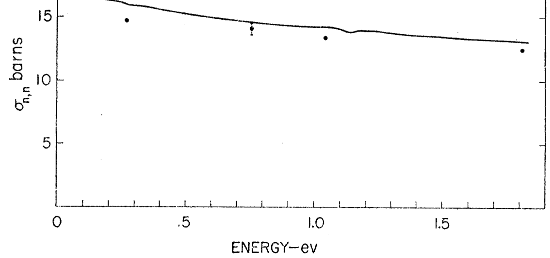

FIG. 10. Scattering cross section. The solid curve is calculated from the parameters of Table II for the -1.45, -0.02, 0.282, and 1.138 ev resonances. A value of 10.3 barns is assumed for the potential scattering. Note the small dips in the curve at 0.282 and 1.138 ev due to interference between the potential and resonance scattering. The solid points are the data of Foote, reference 25.

where the -0.02-ev resonance has the most pronounced effect). (3) If our interpretation were correct and the very strong level at -1.45 ev really existed, this level would badly distort the fission component of all nearby resonances of the same spin state. We see no such effects in the resonances below 10 ev and it seems unreasonable to assume that none of these resonances belong to the same spin state as the -1.45 ev resonance.

Aside from the points listed above, it appears that neither analysis is strongly favored. In both analyses, the final fitting was done by trial-and-error adjustment of parameters. Such a trial-and-error process of fitting always fails to answer the question as to the uniqueness of the analysis and the situation in U^{235} is aggravated by the uncertainty as to the origin of the low-energy "background" cross section. More complete data of better precision will be required for a unique solution. It would be particularly helpful if the angular momentum of the various resonances and the bound level were known.

Despite the imperfect data, and the doubts concerning the uniqueness of either of the two analyses, it is quite certain that the U^{235} slow-neutron resonances require some form of multilevel formula for analysis. The data appear to favor the assumption of more than *one* fission channel for the spin state of the 0.282- and 1.138-ev resonances, but definitely indicate that the number of fission channels which make important contributions is small.

B. The Partial Widths

In the absence of specific information, it has become customary to assume that the radiation width is essentially constant from resonance to resonance. The values of $\Gamma_{\lambda\gamma}$ in Tables I and II scatter over a relatively large range. Two cases, the 2.04- and 4.8-ev resonances have very weak fission components which makes the uncertainties in the capture analysis due to fission normalization, interference terms, and background relatively small. Therefore, the large difference in radiation widths $\Gamma_{\lambda\gamma}$ for these two levels (34.6 compared to 25.5×10^{-3} ev) is disturbing. One notices an even larger variation in the $\Gamma_{\lambda\gamma}$ for the 0.282- and 1.138-ev levels. In this case, the analysis of the bound level has important influence on the results; hence it would be imprudent to emphasize the difference. Note, however, that Vogt[13] obtains essentially the same radiation widths for these two levels, even though he has treated the bound level from an entirely different point of view. Thus there is evidence for small variations in $\Gamma_{\lambda\gamma}$ from resonance to resonance in U^{235}. It appears that the average is about $\bar{\Gamma}_{\lambda\gamma}=33\times10^{-3}$ ev with possible variations from average of $\pm10\times10^{-3}$ ev.

The neutron widths $\Gamma_{\lambda n}{}^0$ have the usual characteristic distribution common to all isotopes and have been discussed elsewhere.[9,10] The fission widths, $\Gamma_{\lambda f}$, listed in Tables I and II show large fluctuations from level to level and in this respect appear more similar to the neutron width than to the radiation width distribution.

C. Relation to Fission Theory

In the various neutron cross sections a definite relationship exists between the shape of the resonances, the distribution in size of the partial widths, and the physical process by which the compound nucleus decays. It is useful to discuss these related features in terms of the reaction exit channels. Let us examine in more detail the meaning of the term "exit channel" and in particular the term "fission channel."

Insofar as the multilevel formula is concerned, the meaning of the term "channel" is quite clear. The particular form which one obtains for the multilevel formula depends on the rank which one assumes for the R matrix. In this matrix there will be one row and one column for each reaction channel. Thus if the R matrix is allowed to have only one row and one column pertaining to fission, the resulting formula is valid for the case of one fission channel. For processes involving a large number of channels, e.g., radiative capture, the multilevel formula reduces to the single-level formula.

The size distribution of the partial widths is closely related to the number of channels for the given process. Porter and Thomas[34] have shown that the size distributions of the partial widths can be fitted by chi-squared distributions of various degrees of freedom. It is reasonable to associate the number of degrees of freedom ν with the number of channels for the given process. According to Porter and Thomas "the number of degrees of freedom ν will in general be smaller than the actual number of channels if the average widths for the various channels are unequal and if there are correlations in the distribution."[34]

Thus the neutron widths, which are clearly related to a single exit channel, can be fit by a chi-squared distribution for which $\nu=1$. This is a distribution in which there is a wide variation in size, with the frequency of occurrence decreasing rapidly with size. The only case for which several resonances with strong scattering components has been observed, i.e., manganese, requires a multilevel formula for fitting the curve.[32,35]

At the other extreme, for ν very large, the size distribution of partial widths is very narrow. In the nonfissile isotopes, the radiation widths are believed to be of this type, i.e., the radiation widths are relatively uniform in size from resonance to resonance. This is a consequence of the large number of possibilities for the first gamma-ray transition. Each first transition defines a separate exit channel. The large value of ν is consistent with the fact that the capture component of resonances ordinarily reduces to a simple Breit-Wigner shape.

[34] C. E. Porter and R. G. Thomas, Phys. Rev. **104**, 483 (1956).
[35] Bollinger, Dahlberg, Palmer, and Thomas, Phys. Rev. **100**, 126 (1955).

Porter and Thomas, using highly provisional values of the fission widths, concluded that the fission width distribution for U^{235} was fitted by $\nu=2.3$; i.e., the distribution was broad and more nearly resembled the neutron widths rather than the radiation widths. A new computation based on fifteen values of $\Gamma_{\lambda f}$ from Table I and from Simpson *et al.*[9] gives $\nu\approx2.0$. The important point here is not the actual value obtained for ν since better data will in time force a revision, but rather the fact that ν is small. Thus, the spread in values of $\Gamma_{\lambda f}$ obtained for the U^{235} resonances is consistent with the need for a multilevel formula to fit the fission cross section.

Since both the shapes of the resonances and the size distribution of the fission widths strongly imply that the slow-neutron fission of U^{235} is a process defined by one, or at most, a few reaction channels, it is interesting to reconcile fission theory with these facts. At first sight this appears to be difficult, because it is natural to assume that each pair of fission fragments in each possible state of excitation constitutes a separate fission channel. If this is indeed the correct interpretation, then the broad mass distribution of fission fragments would indicate a large number of channels, which is in contradiction with the above facts. However, the interesting paper of Bohr at the last Geneva conference[36] has afforded a different concept of the fission channel. Bohr points out that before fissioning the highly excited compound nucleus passes through a transition state in which much of the excitation energy has been converted into potential energy of deformation. At this moment, the elongated nucleus is relatively "cold" and resembles the typical elongated nuclei which have been so successfully treated by the unified model. If the original excitation energy was not excessive, the transition nucleus will have a limited number of well-defined quantum states available to it. The original compound nucleus can pass through only those transition states which have the proper total angular momentum and parity, and these might be very limited in number, depending on the magnitude of the neutron binding energy relative to the required deformation energy. It is apparent that the term "fission channel" must be associated with these intermediate states and not with pairs of fission fragments. Each of these transition states or fission channels emits a complete spectrum of mass fragments. This concept of fission channel has been discussed more fully by Wheeler.[37]

Each fission channel will have a threshold energy determined by the height of the potential barrier. Wheeler[38] has pointed out that even when the excitation energy of the compound nucleus lies below the threshold for a given channel, the channel can still contribute a small amount of fission because of barrier penetration. It appears likely that the lack of a perfect fit in U^{235} with a one-fission-channel formula can be attributed to a small admixture of several fission channels whose thresholds lie considerably above the binding energy of U^{235} plus a neutron.

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[36] A. Bohr, *Proceedings of the International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1955* (United Nations, New York, 1956), Vol. 2, p. 151.

[37] J. A. Wheeler, Physica **22**, 1103 (1956).

[38] J. A. Wheeler, Oak Ridge National Laboratory Report ORNL-2309, 1956 (unpublished), p. 165.