moment—we might put it crudely in terms of incomplete cancellation[13] of small r and large r effectively (see reference 12). We do not mean simply a rediagonalization after applying an E field and/or an H field, having started from some set of zero-field levels split according to J, L, and S, since such computation clearly yields no $\boldsymbol{\sigma}\cdot\mathbf{E}$ effect. One wonders whether some new calculation made, as it were, in the presence of the applied E might be fruitful.

[13] Numerically one would need a magnitude of $(137)^2\xi$ times fractional noncancellation to be about unity, to be in a position to account for the observed value of Δ.

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Multiple Scattering of Protons*

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Multiple scattering has been studied experimentally for protons of energy between 0.7 and 4.8 Mev in several metals. Agreement with Molière's theory of multiple scattering is found for both the shape of the distribution function and its width within the errors of the experiment (3–5%), except possibly below 1 Mev. A Gaussian curve deviates considerably from the observed distribution at large angles, and therefore does not appear to be suitable for the description of multiple scattering.

I. INTRODUCTION

SINCE multiple scattering is of importance in proton range-energy measurements,[1] it seemed desirable to test experimentally Molière's theory[2] of multiple scattering for protons. While several measurements for electrons have been published,[3,4] none seem to exist for protons.

Monoenergetic protons from the Van de Graaff accelerator of The Rice Institute were used to bombard Al, Ni, Ag, and Au foils of several thicknesses. The protons were detected in nuclear track emulsions.

II. EXPERIMENTAL METHODS

The momentum of the protons from the electrostatic accelerator was analyzed in a 90° magnet which was calibrated with the Li(p,n) threshold. The magnetic field was measured with a nuclear induction probe, and energies could be determined with better than 1% accuracy. The horizontal path of the protons after they leave the magnet was defined by two slits: (a) a slit about 1 mm high at a distance of 12 cm from the edge of the magnet, and (b) a square opening of 0.2×0.2 mm at a distance of 3.6 meters from the magnet.

The proton beam then traveled through the multiple-scattering camera (Fig. 1). It was 62 cm long with 6.93 cm inside diameter. The "stray-beam absorber" was a brass disk with a hole of 5 mm diameter, located 30 cm from slit (b) and at the front end of the camera. The scattering foil was mounted on the face of a movable cylinder 10 cm long with a diameter of 6.87 cm. Its distance, d_0, from the point where the center of the scattered proton beam hit the track plate was measured with an accuracy of better than 1 mm. After traversing the foil, the beam hit the nuclear track plate at the rear end of the camera. Ilford $E1$ emulsions with a thickness of 25 microns were used, and were developed in the usual manner.

No attempts were made to measure experimentally the energy of the protons after they have traversed the foil. It was assumed that their energy loss could be determined from stopping-power values.[5] In the scanning of the photographic plates energy discrimination

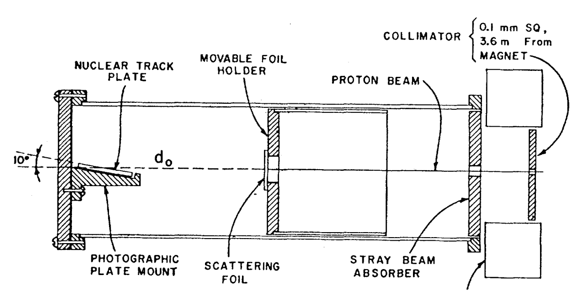

FIG. 1. Multiple-scattering camera, not drawn to scale.

* Supported in part by the U. S. Atomic Energy Commission.
[1] Bichsel, Mozley, and Aron, Phys. Rev. **105**, 1788 (1957).
[2] G. Molière, Z. Naturforsch. **3a**, 78 (1948); see also W. T. Scott and H. S. Snyder, Phys. Rev. **76**, 220, 1949.
[3] Hanson, Lanzl, Lyman, and Scott, Phys. Rev. **84**, 634 (1951).
[4] L. V. Spencer and C. H. Blanchard, Phys. Rev. **93**, 114 (1954).

[5] H. Bichsel (to be published); and R. Fuchs and W. Whaling, "Stopping Cross Sections," mimeographed tables from Kellogg Radiation Laboratory, California Institute of Technology (unpublished).

was used qualitatively; less than 5% of tracks were found to be 30% longer or shorter than average. Further discrimination was possible through consideration of the direction of the tracks in the plate.

Very short exposure times were necessary with this arrangement. They were obtained by sweeping within about 2 seconds the accelerating voltage of the protons from 30% above the proton energy selected by the analyzing magnet to 30% below it. At energies below 1 Mev, the molecular beam HH^+ was used. A small number of contamination deuterons which were then registered simultaneously on the track emulsion could be excluded because their range was twice the range of the protons.

The foils used were commercial Al, Ni, Ag, and Au foils. Several pieces were cut, weighed, and their area

TABLE I. Experimental data and results. E_p=incident proton energy in Mev; E=approximate average energy of proton inside foil; σ=foil thickness in mg/cm²; θ_0=measured scattering angle in degrees; χ_c="Winkelkonstante" of Molière's theory, in degrees; B=auxiliary function (Table II); $\theta_{th}=\chi_c\sqrt{B}$.

Element	E_p	E	σ	θ_0	χ_c	B	θ_0/θ_{th}
Al	0.766	0.619	1.37	4.50	1.73	7.87	0.93
	0.766	0.619	1.37	4.38	1.73	7.87	0.90
	1.017	0.894	1.37	3.25	1.19	7.85	0.97
	1.254	1.002	2.40	3.81	1.32	8.47	0.99
	1.584	1.350	3.42	3.57	1.24	8.84	0.97
	1.984	1.787	3.42	2.89	0.94	8.82	1.04
	2.02	1.82	3.42	2.71	0.91	8.82	1.00
	2.02	1.92	1.37	1.45	0.532	7.68	0.98
	2.355	2.18	3.42	2.37	0.761	8.78	1.05
	2.38	2.20	3.42	2.40	0.755	8.78	1.07
	2.78	2.62	3.42	1.936	0.633	8.76	1.03
	3.99	3.87	3.42	1.30	0.428	8.66	1.03
	4.37	4.26	3.42	1.126	0.390	8.63	0.98
	4.76	4.66	3.42	1.105	0.356	8.63	1.06
Ni	1.20	1.07	2.18	4.77	1.81	6.95	1.00
Ag	2.02	1.57	12.4	10.4	3.70	7.85	1.00
Au	1.34	1.21	4.29	7.98	3.44	5.43	0.99
	2.02	1.92	4.29	5.46	2.18	5.43	1.07
	4.15	4.06	4.29	2.54	1.03	5.43	1.06
Error	±0.005	±0.010	2%	3–5%	···	···	5%

measured. The accuracy of the measurements of the surface density was better than 2%.

Scanning of Nuclear Track Plates

It will be convenient to call the direction of the incident beam in the track plate the y direction, the direction perpendicular to it in the plane of the plate the x direction.

The evaluation was done in the following way: under a microscope with a magnification of 250, the number $f(x)$ of proton tracks in strips $\Delta x=0.1$ mm wide and Δy between 2 and 10 mm long (depending on the track density) were counted. Between 30 and 240 of these strips were measured per plate, over a distance of $\sim$30 mm in the x direction.

Δx was defined by two parallel cross-hairs in the eye piece of the microscope. Their projected separation was 100 μ. The width of the tracks was about 0.6 μ.

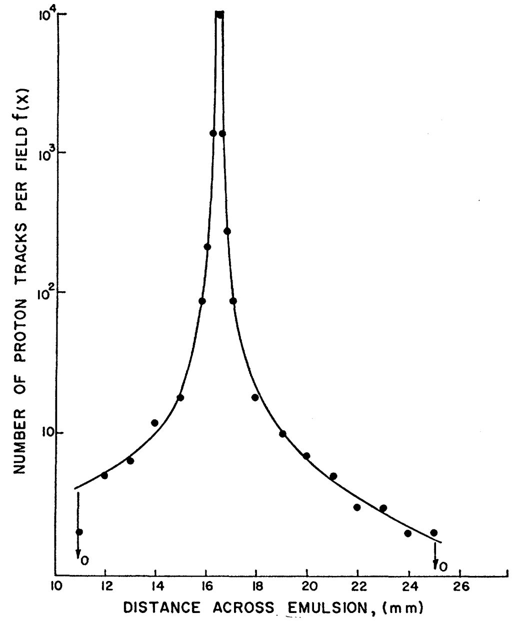

FIG. 2. Unscattered proton beam hitting track plate. The number of tracks in the center was estimated from the blackening of the plate and is probably even larger than indicated. The tails are probably produced by slit-edge scattering at slit (b) (see Fig. 1). The sharp cutoff near 11 and 25 mm corresponds to the geometrical shadow of the stray-beam absorber. Obviously, the center of the beam did not coincide with the center of the stray-beam absorber.

Δy was defined by stops and was reproducible to better than 0.1 mm. x was measured to an accuracy of ±0.02 mm.

Care was taken that the center of Δy coincided with the place where the proton beam would hit the plate in the absence of a scattering foil. In all measurements the maximum spread in the y direction covered less than 0.3° of the scattering cone, and the error in $f(x)$

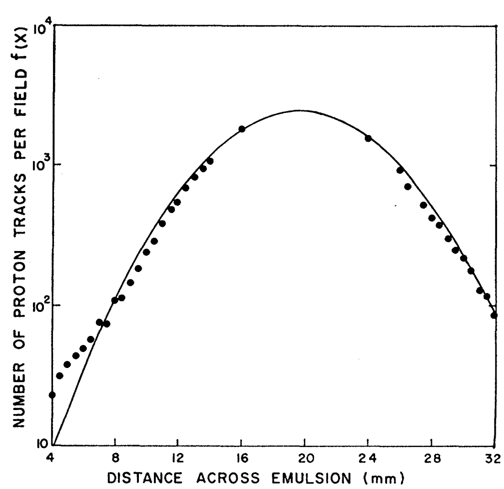

FIG. 3. Experimentally measured distribution of proton tracks in nuclear track plate. Foil thickness σ=3.42 mg cm^{-2} Al, average proton energy 2.18 Mev, d_0=160 mm. A Gaussian curve is drawn for comparison.

TABLE II. Molière's auxiliary function.

$\log_{10}\Omega$	1	2	3	4	5	6	7	8	9
B	3.36	6.29	8.93	11.49	13.99	16.46	18.90	21.32	23.71

due to this was less than 1%. On the other hand, the distance d from the point where the proton entered the track plate to the scattering foil could vary around d_0 as much as 5 mm. The distribution $f_1(x)$ for a strip with $dy \ll \Delta y$ would have a width proportional to d, and the measured $f(x)$ would be the sum of distributions of various widths. Since $\Delta y/2d_0$ was always smaller than 5%, the first approximation $f(x)=f_1(x)$ at d_0 was used. One can show that for a Gaussian curve this approximation would give the correct result to about 1% in the angles considered, and since $f(x)$ is almost Gaussian it is believed that the above approximation for $f(x)$ is justified.

Experimental Measurements

Table I lists the foils and proton energies used.

For every series of exposures a plate was exposed to the proton beam without scattering foil. A typical distribution without a foil is given in Fig. 2. Since this distribution is very narrow it was assumed that its shape did not contribute measurably to the shape of the $f(x)$ measured with multiple scattering foils in place.

Because the average angle θ_0 of scattering was approximately proportional to $1/E$, it was convenient to place the scattering foil at a distance d_0 from the plate approximately proportional to the proton energy. For example, for 1.6-Mev protons scattered in Al, $d_0 \sim 11$ cm.

A typical experimental distribution of protons multiply scattered in an Al foil is shown in Fig. 3. Relatively few points have been measured close to the center since the track density was too high; also they contribute very little to the evaluation of the data. For $x<8$ mm, a significant deviation from a Gaussian shape can be seen. Small deviations are apparent as well in other parts of the curve.

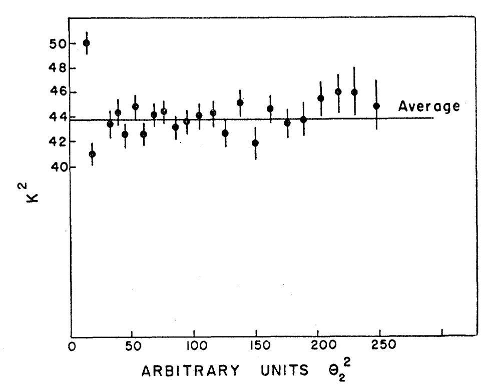

FIG. 4. Ratio of experimental and theoretical proton distribution functions. Note that on the ordinate the zero is suppressed. The points for small angles are unreliable because the center of $f(x)$ can be determined with only limited accuracy. The points for large angles are unreliable because the number of proton tracks in each field is small.

III. RESULTS AND COMPARISON WITH THEORY

Molière's theory[2] gives the distribution function $f(\theta)$ for charged particles that have suffered multiple scattering through a total angle θ in a foil of thickness σ for angles $\theta < 20°$:

$$f(\theta)2\pi\theta d\theta = \left[2\exp(-\delta^2) + \frac{f^{(1)}(\delta)}{B} + \frac{f^{(2)}(\delta)}{B^2} + \cdots\right] 2\pi\delta d\delta,$$

with

$$\delta = \frac{\theta}{\theta_c} = \frac{\theta}{\chi_c\sqrt{B}}, \quad \text{and} \quad \chi_c{}^2 = \int_{E_0}^{E_f} \chi_c'^2(E') \frac{dE'}{-dE/dx},$$

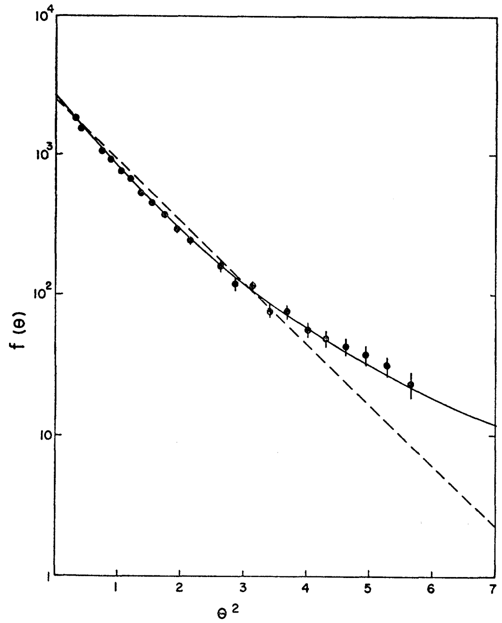

FIG. 5. Comparison of experimental data from Fig. 3 with theory. The solid line represents a normalized Molière function (adjusted with only one parameter: the absolute cross section). The dotted line represents a Gaussian curve, the zero-order term of the Molière function.

where E_0 is the incident proton energy, E_f the average energy of the protons after leaving the foil, $-dE/dx$ the stopping of the material, and $\chi_c' = 22.7Z/(pc\beta\sqrt{A})$, with Z and A the atomic number and weight of the material, p the momentum of the proton in Mev/c, and $\beta = v/c$ its velocity. For the present purposes numerical integration was used. $f^{(1)}$ and $f^{(2)}$ are tabulated in reference 2. B is given in tabular form by Molière; it can also be found in Segrè's book.[6] For convenience it is reproduced in Table II, where

$$\log_{10}\Omega = 8.215 + \log_{10}\left(Z^{-\frac{4}{3}} \times \frac{\sigma}{A} \times \frac{\alpha^2}{1.13+3.76\alpha^2}\right)$$

[6] E. Segrè, *Experimental Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), Vol. I, p. 287.

with $\alpha = Z/137\beta$. The function inside the logarithm is rather insensitive to changes in energy and taking the average energy $E_1 = (E_0 + E_f)/2$ will give B quite accurately. To obtain very accurate results, it would be necessary to use formula (6,10) of Molière's article.

A number of theoretical functions $f(\theta)$ were computed for various values of B. For the evaluation of the experiment, the experimental functions $f(x)$ were normalized to the theoretical $f(\theta)$ at the center, giving a function $f_2(x')$ which had the same maximum as $f(\theta)$. On a graph of $f(\theta)$ it was then possible to determine the angle θ_2 for which $f(\theta_2) = f_2(x')$ for each value $f_2(x')$. If Molière's theory is correct, one would expect to find that the ratio $K = \theta_2/x'$ is a constant, except for statistical fluctuations in $f(x)$.

In Fig. 4 the values of K^2 for the same distribution $f(x)$ as in Fig. 3 are plotted *versus* θ_2^2. The experimental curve $f(x)$ of Fig. 3 is plotted in Fig. 5 on a logarithmic scale *versus* x^2. Both a Gaussian curve $2\exp(-\delta^2)$ and Molière's function $f(\theta) = 2\exp(-\delta^2) + f_1(\delta)/B + f_2(\delta)/B^2$ are plotted for comparison.

The angle θ_0, given by $\tan\theta_0 = K/d_0$, should be equal to $\chi_c\sqrt{B}$. Values are given in Table I. The comparison between theory and experiment can be best shown on a plot of $\theta_0/(\chi_c\sqrt{B'})$ *versus* the average proton energy (Fig. 6).

The uncertainty in θ_0 was estimated to be about $\pm 3\%$. Uncertainties in the values of stopping powers (which enter the determination of χ_c and B) accounted probably for another 3% or 5%.

The uncertainty of the ratio $\theta/(\chi_c\sqrt{B})$ is therefore about $\pm 5\%$, and the deviation for Al values at low energies *may* be significant. For the other energies and elements the experimental results agreed with the theory within the stated errors.

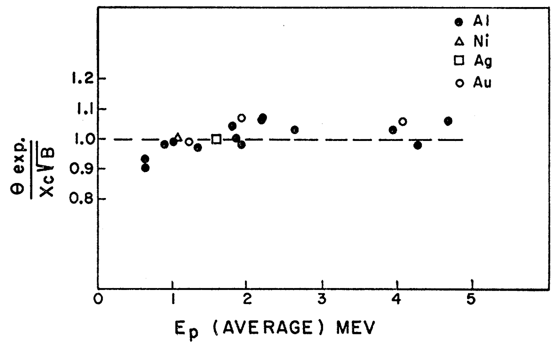

FIG. 6. Compilation of experimental data. Note the suppressed origin of the ordinate.

Conclusion

Multiple scattering of protons at low energies is described correctly by Molière's theory. It is seen that a Gaussian distribution deviates by 10–20% from the observed distribution at small angles ($\theta < 1.7$), and by very large amounts at larger angles.

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