Discrepancy between π^--Proton Scattering and a Dispersion Equation*

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The π^--proton scattering is reanalyzed following Puppi and Stanghellini. The use of more recent data in evaluating the very sensitive principal value integral and a more detailed error analysis lead, with the same coupling constant, $f^2=0.08$, and the same S-wave scattering lengths, $a_1=0.165$ and $a_3=-0.105$, to much less disagreement than they found. However, a residual discrepancy of the same general character remains. It is pointed out that the analysis is very sensitive to the experimental charge exchange scattering, and two alternatives to using this data are suggested. The importance of D waves, in extending the analysis to energies above 300 Mev, is demonstrated. It is shown that contributions from energies above $\sim$2 Bev, and reasonable changes in f^2, a_1 and a_3, cannot contribute significantly towards resolution of the discrepancy.

INTRODUCTION

DURING the last few years dispersion equations for a variety of scattering processes have been derived. Frequently a definitive comparision with experimental data is not possible because of the occurrence in the equations of "unphysical" terms. Contributions from such terms always occur for non-forward angles, and can also occur for forward angles, as happens, for example, in nucleon-nucleon scattering.

It is therefore of particular interest to test such equations against the experimental data whenever possible, and thereby to test the validity of the rather small set of assumptions that enter in their derivation, particularly the assumption of microscopic causality. The first test we know of that led to apparently serious disagreement with the data was carried out by Puppi and Stanghellini[1] (PS) for forward angle elastic scattering of negative pions by protons. If we write for the π^- forward scattering amplitude in the barycentric system,

$$F_-^b=D_-^b+iA_-^b, \tag{1}$$

then the discrepancy observed by PS is shown in Fig. 1, in which the real part of the forward amplitude, D_-^b, in units of the pion Compton wavelength, is plotted as a function of the laboratory energy of the meson. Dimensionless units are used throughout, in which $\hbar$, c, and the π-meson Compton wavelength, $\hbar/m_\pi c\,[=1/m_\pi]$, are set equal to unity.

The experimental points in Fig. 1 are obtained directly from the fits to the nuclear differential cross sections for elastic and charge exchange scattering with the formulas,

$$\sigma_-\equiv d\sigma(\pi^-\rightarrow\pi^-)/d\Omega=a_-+b_-\cos\theta+c_-\cos^2\theta+\text{higher powers of }\cos\theta, \tag{2}$$

$$\sigma_0\equiv d\sigma(\pi^-\rightarrow\pi^0)/d\Omega=a_0+b_0\cos\theta+c_0\cos^2\theta+\cdots,$$

by use of the equations

$$|D_-^b|=\{|F_-^b|^2-(A_-^b)^2\}^{\frac{1}{2}}, \tag{3a}$$

$$|F_-^b|^2=a_-+b_-+c_-+\cdots, \tag{3b}$$

$$A_-^b=(k_b/4\pi)\sigma_-^{\text{total}}\approx k_b(a_-+\tfrac{1}{3}c_-+\cdots+a_0+\tfrac{1}{3}c_0+\cdots), \tag{3c}$$

where k_b is the barycentric pion momentum and the approximate equality is indicated in Eq. (3c) to account for the presence, in principle, of other processes besides elastic and charge exchange scattering. Only S and P waves are assumed to enter significantly in Eqs. (2), (3b), and (3c).

The continuous curve is obtained from the dispersion equation

$$D_-^b(\omega)=\frac{-2kk_b}{\omega+(1/2M)}f^2+\frac{1}{2}\left(\frac{k}{k_b}\right)_{\omega=1}\frac{k_b}{k}\{(\omega+1)D_-^b(1)+(\omega-1)[-D_+^b(1)]\}+\frac{kk_b}{4\pi^2}\text{P.V.}\int_1^\infty\frac{\sigma_-^{\text{tot}}(\omega')}{\omega'-\omega}\frac{d\omega'}{k'}+\frac{kk_b}{4\pi^2}\int_1^\infty\frac{\sigma_+^{\text{tot}}(\omega')}{\omega'+\omega}\frac{d\omega'}{k'}, \tag{4}$$

where k is the laboratory pion momentum, $\omega=(1+k^2)^{\frac{1}{2}}$, M is the proton rest mass (in units of m_π), f^2 is the unrationalized, renormalized P-wave coupling constant, the subscript on $D_\pm^b(1)$ denotes $\pi^\pm$-proton scattering, P.V. denotes the principal value integral, and $\sigma_\pm^{\text{tot}}(\omega')$ is the total $\pi^\pm$-p cross section at energy ω'. The right-hand side is determined by specifying the parameters f^2, $D_-^b(1)$, and $D_+^b(1)$, and by evaluating the integrals as well as the experimental values of the total cross sections permit.

If charge independence is assumed, the parameters $D_\pm^b(1)$ are related to the S-wave scattering lengths at zero kinetic energy a_1 and a_3 for isotopic spin $\frac{1}{2}$ and $\frac{3}{2}$, respectively, by

$$D_+^b(1)=a_3,\quad D_-^b(1)=\tfrac{2}{3}a_1+\tfrac{1}{3}a_3.$$

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[1] G. Puppi and A. Stanghellini, Nuovo cimento 5, 1256 (1957).

We obtain the continuous curve of Fig. 1 with

$$f^2=0.08, \quad a_1=0.165, \quad a_3=-0.105,$$

the same values used by PS, and with the total π^- cross section shown (in the important energy region) in Fig. 2.[2] The experimental points and the dashed curve in Fig. 1 are transcribed directly from PS. The accuracy of the curves is discussed in Sec. 1.

It is possible to raise the continuous curve of Fig. 1 by decreasing f^2; in fact, as shown by PS, the $f^2=0.04$ curve is in agreement with the 150- and 170-Mev points.[3] However, the coefficient of f^2 is approximately a linear function of ω in the range of interest, $1\leqslant\omega\leqslant 4$, and a decrease in f^2 therefore acts roughly like a counterclockwise rotation about the intercept at $\omega=1$. Any such improvement between the curve and the data in the 150-Mev region destroys the agreement at energies above 200 Mev. Moreover, a smaller value than $\sim$0.08 is argued against by the agreement of the π^+-p dispersion equation with experiment.[1]

We have re-examined this dilemma in order to see

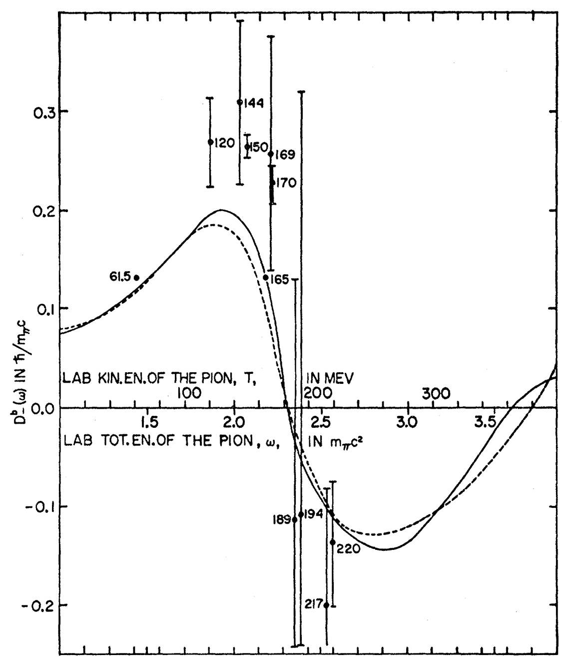

FIG. 1. Character of the original discrepancy. The solid curve is the dispersion theory prediction for $D_-^b(\omega)$ obtained with $f^2=0.08$, $a_1=0.165$, $a_3=-0.105$, and the σ_+^{tot} and σ_-^{tot} described in reference 2. The dashed curve and the experimental points are transcribed from the paper of Puppi and Stanghellini, reference 1. Although a smaller coupling constant, $f^2=0.04$, gives agreement with the 150- and 170-Mev data, it destroys the agreement at 220 Mev. There was no value of f^2 consistent with the π^- data alone.

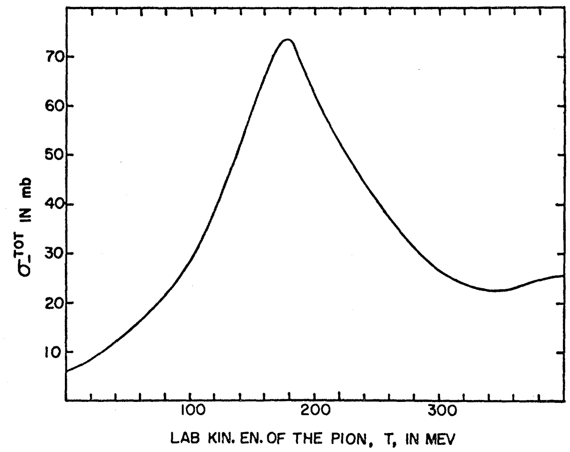

FIG. 2. The π^- total cross section described in reference 2, which is used to obtain the solid curve, D_-^b, of Fig. 1. Above 200 Mev this σ_-^{tot} and the more recent curve of Fig. 5 are almost identical.

how firmly it is established. The items considered are the following:

1. The sensitivity of the principal value (P.V.) integral in Eq. (4) to small changes in σ_-^{tot}.
2. The error analysis and the sensitivity of $|D_-^b|$ in Eq. (3a) to small changes in σ_-^{tot}.
3. Use of charge independence as a test of the data in the peak cross section region.
4. The presence of higher partial waves in the scattering.
5. Possible changes in f^2, a_1, a_3, and in the contributions to the integrals of Eq. (4) from energies ω' above $\sim$2 Bev.

1. THE PRINCIPAL-VALUE INTEGRAL

The most striking feature of the curve in Fig. 1 is the steep drop from a maximum in the neighborhood of 130 Mev to a minimum in the neighborhood of 250 Mev. This behavior can come from only one term of Eq. (4), the P.V. integral. In Fig. 3 is shown the energy dependence of each of the terms of Eq. (4), with the parameters $D_-^b(1)$ and $D_+^b(1)$ replaced by a_1 and a_3. Each of the parameters f^2, a_1, $-a_3$, is of the order of 0.1. Therefore their coefficients multiplied by 0.1 are shown, and the algebraic sum of the five curves is approximately the curve $D_-^b(\omega)$. Each of the terms is quite linear in the range up to 400 Mev, except the P.V. integral, whose behavior is due to the fact that the contribution to the integral

$$\text{P.V.}\int_1^\infty \frac{\sigma_-^{\text{tot}}(\omega')}{\omega'-\omega}\frac{d\omega'}{k'}$$

is negative for $\omega'<\omega$ and positive for $\omega'>\omega$. Thus as ω increases through the peak value of σ_-^{tot} the major contribution changes, rather abruptly, from positive to negative. This makes evaluation of the integral

[2] The values of σ_-^{tot} in Fig. 2 are taken, as are those of σ_+^{tot}, from Anderson, Davidon, and Kruse, Phys. Rev. **100**, 339 (1955), up to 350 Mev. Those from 350 Mev to 1.9 Bev are from Cool, Piccioni, and Clark, Phys. Rev. **103**, 1082 (1956). At all higher energies $\sigma_-^{\text{tot}}=\sigma_+^{\text{tot}}=30$ mb is assumed.

[3] When not otherwise specified, an energy given in Mev refers to the laboratory pion kinetic energy, T.

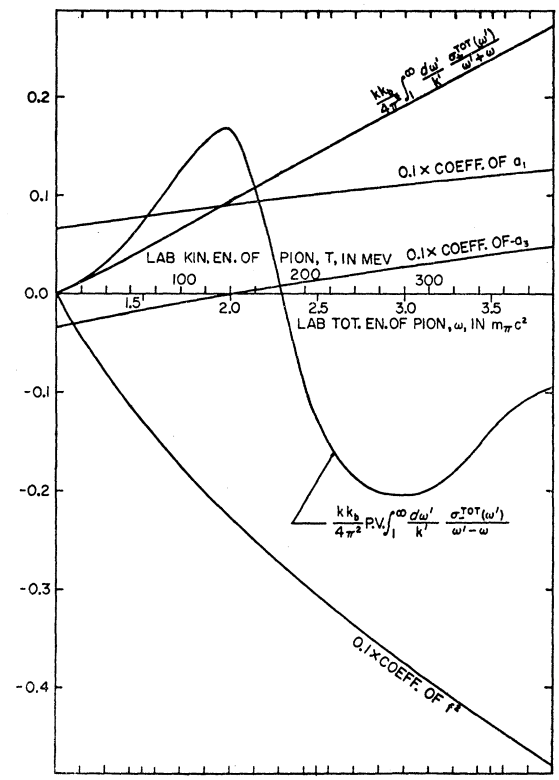

FIG. 3. The energy dependence of the various terms in the equation for $D_-^b(\omega)$. The algebraic sum of the five terms approximately reproduces the curve of Fig. 4, which is also almost given by the principal value integral term alone.

somewhat critical and for comparison, the integral curve of PS is shown (the dashed curve) in Fig. 1. The two curves agree reasonably well in the significant energy range, that is, up to about 230 Mev, with our curve somewhat above that of PS in the peak region. The maximum difference here is roughly at 160 Mev and amounts to about 22%.

To provide another computational check we have calculated $D_-^b(\omega)$ from the *ad hoc* π^- cross section of Zaidi and Lomon[4] (ZL), (Fig. 1, Curve A in their paper), and compared it to the function $D_-^b(\omega)$ obtained by them from the same cross section. The two curves agree closely up to 230 Mev, with our curve slightly below the ZL curve in the peak region. The maximum difference, at about 150 Mev, amounts to 9%, and is probably accounted for by the sensitivity of the principal value integral to small errors introduced in reading the numerical values of σ_-^{tot} from the published curve of ZL.

This sensitivity of the P.V. integral, which is also noted by ZL, emphasizes the desirability of having improved measurements of σ_-^{tot} in the region of the resonant peak. However, it seems unnecessary to modify the total cross section to the extent of Curve B of ZL. The agreement between the dispersion curve and the experimental points improves considerably from use of the total cross section values obtained in the more recent phase shift analysis of Anderson and Metropolis,[5] and from more realistic calculation of the uncertainties attached to the experimental values of $D_-^b(\omega)$. The latter is discussed in Sec. 2. The results of these improvements are shown in Fig. 4,[6–9] where the disagreement at 150 and 170 Mev is much less than in Fig. 1.

The experimental points in Fig. 4 are obtained as follows: The 41.5-Mev point is the work of Barnes *et al.*[6] at the University of Rochester; the 98-Mev point is

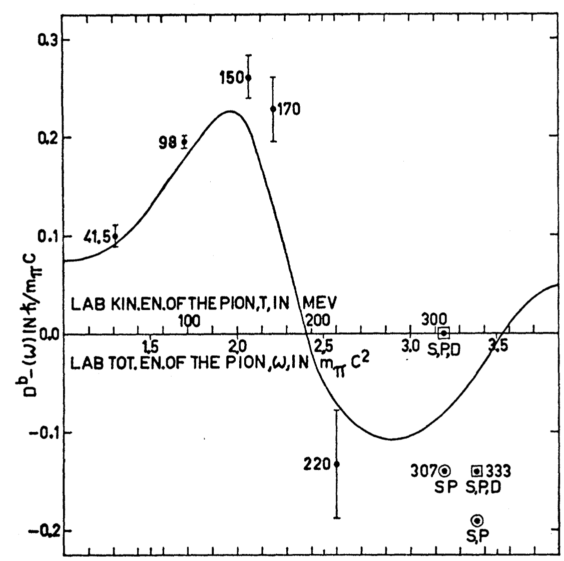

FIG. 4. Present status of the discrepancy. The solid curve is the dispersion theory prediction for $D_-^b(\omega)$ obtained with $f^2=0.08$, $a_1=0.165$, $a_3=-0.105$, with the σ_+^{tot}, and the σ_-^{tot} above 350 Mev, described in reference 2, and with the σ_-^{tot} up to 350 Mev of reference 5. The experimental points at 150, 170, and 220 Mev are derived from published work of Ashkin *et al.* (reference 8) with the errors recalculated as described in Sec. 2. The 41.5-Mev point and its error are transcribed from a preprint of Barnes *et al.* (reference 6). The 98-Mev point and its error are taken from a private communication from Holt (reference 7). The 307- and 333-Mev points are derived, as described in Sec. 4, from a preprint of Korenchenko and Zinov (reference 9).

[4] M. H. Zaidi and E. L. Lomon, Phys. Rev. **108**, 1352 (1957).

[5] H. L. Anderson and N. Metropolis, *Proceedings of the Sixth Annual Rochester Conference on High-Energy Nuclear Physics, 1956* (Interscience Publishers, Inc., New York, 1956), Sec. I, p. 20.

[6] Barnes, Rose, Giacomelli, Ring, and Miyake, University of Rochester Report NYO-2170 (unpublished). This point is from a phase shift analysis of 29 elastic π^+ and π^- data at energies from 24.8 Mev to 65 Mev, in which the Chew-Low dependence of α_{33} and the usual η and η^3 dependences of the S-wave and other P-wave phases, respectively, are assumed.

[7] J. R. Holt (private communication). We are indebted to Professor Barnes for access to this early report of this work. The point is from a 3 parameter fit, Eq. (2), to $\sigma_-(\theta)$ at 13 angles, and from a transmission measurement of σ_-^{tot}.

[8] Ashkin, Blaser, Feiner, and Stern, Phys. Rev. **101**, 1149 (1956); **105**, 724 (1957). Calculation of the errors attached to these points is described in Sec. 2.

[9] S. M. Korenchenko and V. G. Zinov, Joint Institute for Nuclear Research, Dubna, U.S.S.R. (to be published). Calculation of these points is described, and the reason for omitting errors given, in Sec. 4.

that of Holt and collaborators[7] at the University of Liverpool; the 150-, 170-, and 220-Mev points are from Ashkin *et al.*[8] at the Carnegie Institute of Technology; and the 307- and 333-Mev points are from Korenchenko and Zinov[9] at Dubna. The curve is obtained from Eq. (4), again with the same parameter values f^2, a_1, a_3 as in Fig. 1, but with the more recent total π^- cross section.[5] This cross section, shown in Fig. 5, differs from that of Fig. 2 mainly in its greater curvature in the energy region below the resonant peak.

2. THE ERROR ANALYSIS

In addition to the continuous curve of Fig. 4 lying closer to the center values of the experimental points at 150 and 170 Mev, the experimental error attached to each of these points is approximately twice as large as in Fig. 1. This is because the error calculation includes

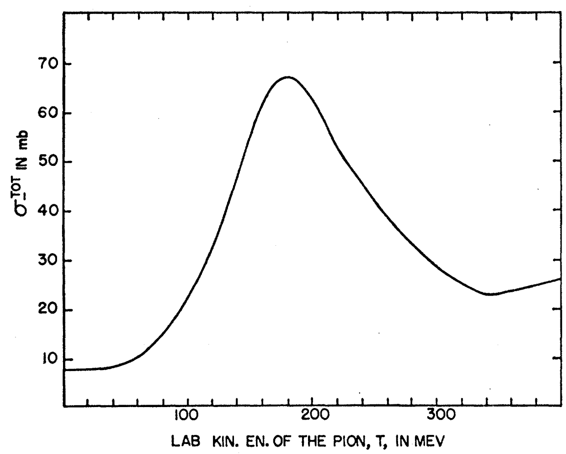

FIG. 5. The π^- total cross section of reference 5, which is used to obtain the curve, D_-^b, of Fig. 4. This $\sigma_-^{\rm tot}$ differs from that of Fig. 2 mainly in its greater curvature up to $\sim$150 Mev and in its broader peak.

the over-all uncertainty in the absolute value of the differential cross sections. Specifically, the three parameter formulas [keeping terms up through $\cos^2\theta$ in Eq. (2)] to which Ashkin *et al.*[8] make least-squares fits of their differential cross sections, are of the form

$$\sigma\equiv d\sigma/d\Omega=(1.00\pm\Delta)[(a+\delta a)+(b+\delta b)\cos\theta+(c+\delta c)\cos^2\theta],$$

where the normalization factor $(1\pm\Delta)$ collects all uncertainties not affecting the angular distribution. For the π^- cross sections at 150, 170, and 220 Mev Δ is estimated to be 0.05.[8] The errors shown at these energies in Fig. 4 take into account, in addition to Δ, the correlations among δa, δb, and δc.[10] Also, the transmission value of $\sigma_-^{\rm tot}$ rather than that from integration of $\sigma_-+\sigma_0$ is used for A_-^b. This leads to a negligible change in the center value of D_-^b. The correlations tend to reduce the size of the error, but their effect is negligible compared to that of the 5% over-all uncertainty, except at 220 Mev, where the net uncertainty is slightly less than that of PS.

Still, each of the experimental points of Fig. 4, up to 220 Mev, has a greater magnitude than that of the solid curve at that energy. This systematic disagreement suggests the possibility that in the evaluation of D_-^b from Eq. (3a), either the imaginary part, A_-^b, of the elastic forward amplitude, F_-^b, is consistently too small, or $|F_-^b|$ is consistently too large. The first of these possibilities is examined in the remainder of this section, the second in Sec. 4.

An error in charge exchange scattering would affect A_-^b, through the coefficients a_0 and c_0 in Eq. (3c), without changing F_-^b, which involves only the elastic scattering. The charge exchange cross section is more difficult to measure than the elastic[8] because it requires detection of the photons from the π^0 decay, which involves γ-ray detection efficiencies at various angles and energies, and the indirect determination of σ_0 from the experimentally determined $\sigma_{\pi^-\rightarrow\gamma}$. The γ-detection efficiencies at 150 and 170 Mev have an over-all uncertainty of about 6%.[8] Furthermore, the charge exchange cross section is a large fraction of the total. Specifically, from 150 Mev to 233 Mev, it is more than one and one-half times the elastic cross section.

At some energies, both F_-^b and A_-^b are large compared to D_-^b, and the calculation of D_-^b from Eq. (3a) is sensitive to small changes in A_-^b. At 220 Mev, the experimental value of D_-^b is given by

$$|D_-^b|=(0.16-0.14)^{\frac{1}{2}}=0.13.$$

An increase of A_-^b by 4.5%, which would be caused by a 7.2% increase in the charge exchange cross section, serves to halve $|D_-^b|$ and place it directly on the curve of Fig. 4. The lower energy Carnegie Tech. points, and the Dubna (U.S.S.R.) points at higher energy are not as sensitive to small fractional changes in the charge exchange cross section; roughly 19% increments are needed to lower the 150- and 170-Mev points so that they coincide with the curve, and 25 and 22% increments are needed to raise the 307- and 333-Mev points to the curve. Increments of this magnitude seem unlikely, however, at the Russian energies, as shown in Sec. 4, the discrepancy is not serious. In addition an increase in $\sigma_-^{\rm tot}$ in the region of the peak also moves the solid curve towards the experimental points.[11,12] Since such an increment in $\sigma_-^{\rm tot}$ moves both the experimental points and the curve towards each other, a smaller increase of σ_0 might suffice.

3. CHARGE INDEPENDENCE TEST OF OF THE DATA

Because of the persistent discrepancy at 150 and 170 Mev, and also because the Carnegie Tech. data are

[10] The correlation matrices were kindly provided by Professor Ashkin.

[11] H. Y. Chiu, Phys. Rev. **110**, 1140 (1958).
[12] J. Hamilton, Phys. Rev. **110**, 1134 (1958).

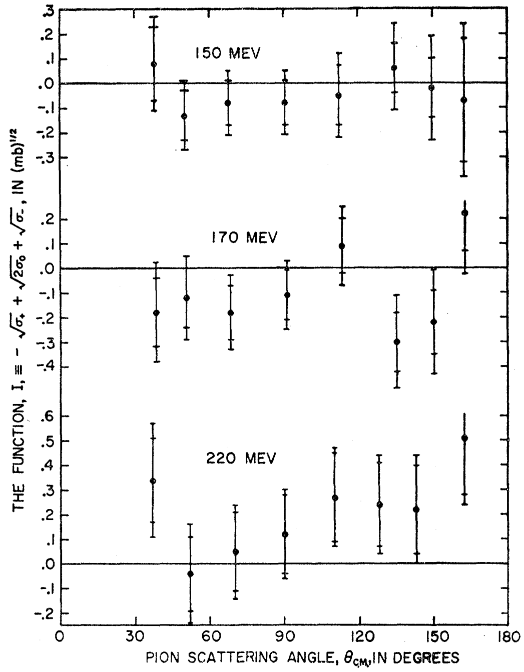

FIG. 6. Charge independence test of the Carnegie Tech. data. The inner errors are lower limits. The outer errors are obtained with the assumption that each σ_0 datum, if available, would have an error of about 9%.

the most complete (σ_+, σ_-, and σ_0) and most crucial, we have tested these data with one of the triangular inequalities that follows from the assumption of charge independence,[13]

$$I \equiv -(\sigma_+)^{\frac{1}{2}}+(2\sigma_0)^{\frac{1}{2}}+(\sigma_-)^{\frac{1}{2}} \geq 0. \quad (5)$$

In addition to this inequality there are the two others obtained by permuting the minus sign among the terms of I, however, Eq. (5) is expected to be the most sensitive of the three because σ_+ is large, and, as indicated by the discussion in Sec. 2, σ_0 may be too small. Both of these factors would combine to help violate $I \geq 0$.

The function I is calculated at each experimental angle and the results are shown in Fig. 6 for each of the three energies. Due to uncertainty, two errors are assigned each point, as is explained in the following.

Of the three cross sections σ_+, σ_-, and σ_0 that enter in Eq. (5), only σ_- and σ_0 are correlated. This occurs because one of the determinations of the absolute value of the γ-ray detection efficiency is made by comparison of the transmission value of σ_-^{tot} with that from integration of $\sigma_-+\sigma_0$.[8,14] This correlation is expected to be small because two other methods were used for independent determinations. In this analysis it is assumed that there is no correlation among σ_+, σ_-, and σ_0. Therefore the error in I, δI, is given by

$$\delta I = \tfrac{1}{2}\{(\delta\sigma_+)^2/\sigma_+ + (\delta\sigma_-)^2/\sigma_- + 2(\delta\sigma_0)^2/\sigma_0\}^{\frac{1}{2}}. \quad (6)$$

[13] D. Feldman, Phys. Rev. **89**, 1159 (1953); **103**, 254 (1956).
[14] J. Ashkin (private communication).

Further, we write

$$(\delta\sigma_\pm)^2 = (\delta_1\sigma_\pm)^2 + (\delta_2\sigma_\pm)^2, \quad (7)$$

where $\delta_1\sigma_\pm$ is the error attached to each datum $\sigma_\pm(\theta,E)$, the angle dependent part, and $\delta_2\sigma_\pm$ is the over-all, angle independent part of the error. From reference 8 we have

$$\begin{aligned}\delta_2\sigma_+ &= 0.03\ \sigma_+ \text{ at 150 and 170 Mev,}\\ &= 0.04\ \sigma_+ \text{ at 220 Mev,}\\ \delta_2\sigma_- &= 0.05\ \sigma_- \text{ at each of the three energies,}\\ \delta_2\sigma_0 &= 0.05\ \sigma_0 \text{ at each of the three energies.}\end{aligned}$$

Since $\delta_1\sigma_0$ is not similarly available, δI is evaluated for each of the two cases $\delta\sigma_0/\sigma_0 = 0.00$, 0.10. The inner error shown on each of the points in Fig. 6 is that due to the uncertainties in σ_+ and σ_- alone, and is a lower limit. The outer error, obtained with $\delta\sigma_0/\sigma_0 = 0.10$ is reasonable, as it corresponds to $\delta_1\sigma_0/\sigma_0 \sim 0.09$.

Because of the uncertainty in the errors, it cannot be said that a statistically significant violation of charge independence exists, even at 170 Mev. However the center values of I are rather systematically negative at 150 and at 170 Mev. We would be more inclined to attribute any difficulty to σ_0 rather than to the breakdown of charge independence.

4. *D* WAVES

In an effort to see whether D waves are present, and if so to examine their effect on the forward amplitude, three-, four-, and five-parameter fits of the form of Eq. (2) have been made to σ_- for the several energies from 150 to 307 Mev. The results of this analysis do not show a systematic behavior with energy of the coefficients of powers of $\cos\theta$ greater than $\cos^2\theta$. This is consistent with the results of Korenchenko and Zinov,[9] who have made a preliminary least-squares phase shift analysis of their data, and find that the π^- data at 307 Mev can be fitted satisfactorily without D waves.

However, they find from a combined phase shift analysis that their 307-Mev data and the π^+ data of Mukhin and Pontecorvo[15] at the same energy are not reasonably fitted without D waves. They also have made three- and five-parameter fits to σ_- at 307 Mev and at 333 Mev, using, instead of Eq. (2), the equivalent form

$$\sigma_- = A_- + B_- P_1(\cos\theta) + C_- P_2(\cos\theta) + \cdots. \quad (8)$$

Although their analysis is preliminary, the changes in the coefficients A_-, B_-, C_- induced by making a five- instead of a three-parameter fit are similar at the two energies, as is shown in Table I, where all quantities are in units of mb/steradian.

In each case, each of the three coefficients is reduced, and the additional two coefficients are negative. Since the modulus squared of the forward amplitude is the

[15] A. I. Mukhin and B. M. Pontecorvo, J. Exptl. Theoret. Phys. (U.S.S.R.) **31**, 550 (1956) [translation: Soviet Phys. JETP **4**, 373 (1957)].

TABLE I. Three- and five-parameter fits of σ_- at 307 and 333 Mev.

	307 Mev 3 parameters	307 Mev 5 parameters	Difference	333 Mev 3 parameters	333 Mev 5 parameters	Difference
A_-	0.88 ± 0.05	0.85 ± 0.05	-0.03	0.85 ± 0.05	0.83 ± 0.05	-0.02
B_-	0.34 ± 0.09	0.21 ± 0.09	-0.13	0.35 ± 0.09	0.29 ± 0.12	-0.06
C_-	0.65 ± 0.09	0.43 ± 0.15	-0.22	0.60 ± 0.09	0.51 ± 0.12	-0.09
D_-		-0.24 ± 0.15			-0.11 ± 0.15	
E_-		-0.18 ± 0.13			-0.05 ± 0.16	

sum $A_-+B_-+\cdots$, each of these changes serves to reduce the magnitude of D_-^b. At 307 Mev the experimental point (see Fig. 4) is carried all the way to the axis, with an enormous but meaningless error.[16] At 333 Mev the experimental point is changed from -0.19 to -0.14, also with a large error.[17,18] These numbers are obtained with the values of σ_-^{tot} from transmission measurements rather than from the integrated differential cross sections because of lack of knowledge of σ_0. Errors for these points are not shown in Fig. 4 because these data are not sufficiently refined to be considered on the same footing as the other points shown.

5. VERY-HIGH-ENERGY CONTRIBUTIONS AND CHANGES OF f^2, a_1, AND a_3

Some uncertainty in the integral curve is due to lack of knowledge of $\sigma_\pm^{\text{tot}}(\omega')$ at energies $\omega'\gtrsim 2$ Bev. The contribution of the high-energy "tails" of the integrals in Eq. (4) are estimated on the assumption that for $\omega'\geq 2$ Bev, $\sigma_+^{\text{tot}}=\sigma_-^{\text{tot}}=30$ mb.[19] It has been remarked that resolution of the discrepancy may lie partly in the incorrect evaluation of these high-energy contributions.[20]

The analysis described in this section was undertaken in order to test the possibility that such contributions, combined with reasonable changes in f^2, a_1, and a_3, may help eliminate the disagreement. It is assumed that the cross sections do not behave pathologically, and of course that the integrals exist. The conclusion is negative, the main points being: first, that without pathological behavior the contributions to the integrals from the range 2 Bev $\lesssim\omega'<\infty$ cannot have sufficient curvature in the range 240 Mev $\lesssim\omega\lesssim 360$ Mev to resolve the π^- discrepancy; and second, corrections of the required magnitude for the π^- curve are such as to destroy completely the good agreement of the π^+ curve with the experimental points.[1]

To show this explicitly, we write the cross sections as

$$\sigma_\pm^{\text{tot}}(\omega')=30\text{ mb}+\delta\sigma_\pm(\omega')$$

for $\omega'\gtrsim 14$ ($14\, m_\pi c^2\simeq 2$ Bev). The corrections to the tails of the integrals may then be written as

$$\frac{kk_b}{4\pi^2}\int_{14}^{\infty}\left[\frac{\delta\sigma_-(\omega')}{\omega'\mp\omega}+\frac{\delta\sigma_+(\omega')}{\omega'\pm\omega}\right]\frac{d\omega'}{k'}=\frac{kk_b}{4\pi^2}\sum_{n=1}^{\infty}\int_{14}^{\infty}\left(\frac{\pm\omega}{\omega'}\right)^{n-1}\times[\delta\sigma_-(\omega')+(-1)^{n-1}\delta\sigma_+(\omega')]\frac{d\omega'}{k'\omega'},$$

where the upper (lower) sign refers to the $\pi^-(\pi^+)$ dispersion equation. A measure of the convergence rate of this series is given by the "expansion parameter" (ω/ω'), which for the energies in question satisfies the condition

$$\omega/\omega\leqslant 360\text{ Mev}/2\text{ Bev}=0.18.$$

The third and fourth terms ($n=3,4$) are thus expected to have magnitudes less than $\sim 4\%$ of the first and second terms, respectively. This is not assured because the integrands are not of definite sign; however, it is probable, particularly because ω/ω' is smaller than 0.18 over most of the integration range. In what follows, it is assumed that no significant corrections come from terms with $n>2$. The corrections $\delta D_\mp^b(\omega)$ to the functions $D_\mp^b(\omega)$ may then be written as

$$\delta D_\mp^b(\omega)=\mp\frac{2kk_b}{\omega\pm(1/2M)}\delta(f^2)\pm\frac{1}{3}\left(\frac{k}{k_b}\right)_{\omega=1}\frac{k_b}{k}[(\omega\pm1)\delta a_1+(\omega\mp2)\delta(-a_3)]+kk_bc\mp kk_b\omega d,\quad(9\mp)$$

where

$$c\equiv\frac{1}{4\pi^2}\int_{14}^{\infty}[\delta\sigma_-(\omega')+\delta\sigma_+(\omega')]\frac{d\omega'}{k'\omega'},$$

$$d\equiv\frac{1}{4\pi^2}\int_{14}^{\infty}[-\delta\sigma_-(\omega')+\delta\sigma_+(\omega')]\frac{d\omega'}{k'\omega'^2}.$$

For each energy ω at which a correction $\delta D_-^b(\omega)$ is specified, Eq. (9$-$) gives an equation linear in the

[16] The expression for the error in D_-^b has D_-^b as a factor in the denominator. Therefore, when D_-^b is ~ 0, this measure of the error is not meaningful. This is clearly connected with the fact that D_-^b, as given by Eq. (3a), becomes imaginary if complete cancellation occurs, as in fact happens with the five parameter fit to σ_-(307 Mev). In such a case it would be more appropriate to consider, instead of D_-^b, the function $(D_-^b)^2$, whose error remains finite at a zero of D_-^b.

[17] H-Y Chiu and J. Hamilton [Phys. Rev. Letters 1, 146 (1958)] also reach the same conclusion regarding these data.

[18] H. J. Schnitzer and G. Salzman [Bull. Am. Phys. Soc. Ser. II, 2, 353 (1957). D waves are not included in analyzing data below 300 Mev, although they are included in reference 19.

[19] Cool, Piccioni, and Clark, Phys. Rev. **103**, 1082 (1956) give 30 mb at 2 Bev. J. O. Clarke and J. V. Major, Phil. Mag. 2, 37 (1957), with 4.2-Bev π^-, and F. A. Brisbout *et al.*, Phil. Mag. 1, 605 (1956), with 1 to 300 Bev charged pions, find no indication that the cross sections are not essentially constant. We are indebted to Dr. A. Engler for calling our attention to these very-high-energy measurements.

[20] S. J. Lindenbaum, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Stanford, 1957), Vol. 7, p. 317.

parameters $\delta(f^2)$, δa_1, $\delta(-a_3)$, c, d, and is thus a basis for examining phenomenologically whether reasonable values of these parameters can produce the changes $\delta D_-{}^b(\omega)$ needed to bring the curve of Fig. 4 into agreement with the experimental points.

The required correction, as may be seen in Fig. 4, must be small and positive at 98 Mev, must increase as 150 Mev is approached, must then decrease to zero and become negative as 220 Mev is approached. The correction is thus roughly of parabolic shape, with a peak near 150 Mev. However, each of the five terms of Eq. (9−) is monotonic with almost linear behavior in the region 98 Mev $\leqslant T \leqslant$ 220 Mev. Since the sum of a number of straight lines is still a straight line, such a phenomenological fit can only give agreement in a limited energy region.

To give an idea of the sizes of the terms involved, let us ask for corrections to the solid curve of Fig. 3 that make it go through the lower ends of the errors at 98 and 150 Mev, and the upper end of the 220-Mev error. This gives three equations, and if one takes $\delta(f^2)=\delta(-a_3)=0$, then the following results are obtained:

$$\delta a_1=-0.107, \quad c=+0.159, \quad d=+0.053.$$

The value of a_1 is about 0.165. We regard the change δa_1 as unacceptable. Also, the numbers c and d represent very large changes in the high-energy integrals. To gauge their size, we note that if $\delta\sigma_\mp(\omega')$ were constants, then these values of c and d would correspond to

$$\delta\sigma_-\simeq 70 \text{ mb}, \quad \delta\sigma_+\simeq 1700 \text{ mb}.$$

The curve produced by these "corrections" drops sharply at energies $T<98$ Mev and for $T>220$ Mev, going to zero at $T=0$ and to -0.240 at $T=290$ Mev. One might be willing to consider this curve as not completely ruled out by the π^- data, however the corresponding correction $\delta D_+{}^b(\omega)$ to the π^+ curve is totally unacceptable. This is because the c and d terms, which are large but mostly cancel each other in the π^- correction, add together in the π^+ case, and give a "corrected" curve that is already hopelessly positive at $T=150$ Mev. If the π^- curve is corrected to go through the center values of the 98-, 150-, and 220-Mev points each of the corrections δa_1, c, and d is about tripled, and the disagreements, of the π^- curve at energies outside the range 98 Mev to 220 Mev, and of the π^+ curve, are made extreme.

One may ask whether a satisfactory fit might be given with reasonable parameter values if all five parameters $\delta(f^2)$, δa_1, $\delta(-a_3)$, c, and d are used. From the following considerations we infer that this is not so. In the range 98 Mev $\leqslant T\leqslant$ 220 Mev the coefficients of δa_1 and $\delta(-a_3)$, multiplied by 0.1, have small slopes compared to 0.1 times the coefficient of $\delta(f^2)$ and to kk_b and $kk_b\omega$. Reasonably small changes of a_1 and $-a_3$ thus cannot give significant terms in Eq. (9−) and may be omitted from further consideration. Of the three "large" terms, the coefficients of $\delta(f^2)$ and of d are both negative for π^- and positive for π^+, while that of c is positive for both π^- and π^+. The "parabolic" behavior of the correction is generated, in the example of the previous paragraph, by large cancellations between the c and d terms, the positive c term first increasing more rapidly than the negative d term, then at higher energies being overtaken by the d term. Even if some (positive) $\delta(f^2)$ is admixed so that d is not as large and more reasonable values of $\delta\sigma_-$ and $\delta\sigma_+$ result, the $\delta(f^2)$ and d terms both switch sign for π^+ and will add to the c term, destroying the π^+ fit.

Only if the $\delta(f^2)$ and d terms are of opposite sign, the d term considerably reduced, and the c term much reduced, so that the major cancellation is between the $\delta(f^2)$ and d terms and is thus maintained for the π^+ correction as well, might a reasonable fit be possible. However, a fit with these parameters through the inner extremities of the errors at 98, 150, and 220 Mev yields

$$\delta(f^2)=+0.168, \quad c=+0.349, \quad d=+0.087,$$

which corresponds approximately to tripling $f^2=0.08$, and to

$$\delta\sigma_-\simeq 590 \text{ mb}, \quad \delta\sigma_+\simeq 3200 \text{ mb}.$$

The π^+ agreement is completely destroyed by such corrections. Thus even this possibility, although not eliminated *a priori*, is ruled out because of the magnitudes involved.

6. SUMMARY AND CONCLUSIONS

The π^- dispersion theory integrals have been recalculated with more recent experimental $\sigma_\pm{}^{\text{tot}}$ values.[5] In agreement with previous investigators,[4,11] we find the principal-value integral very sensitive to $\sigma_-{}^{\text{tot}}$. Considerable improvement over the original curve[1] for $D_-{}^b(\omega)$ (Fig. 1) results from this alone (compare Fig. 4). Further improvement comes from reanalysis of the errors in the experimental points at $T=150$, 170, and 220 Mev. Inclusion of the over-all angle-independent error in σ_-, and of the correlations among a_-, b_-, and c_-, leads to doubling the original errors[1] shown in Fig. 1 at 150 and 170 Mev (compare Fig. 4), but to no change in the error at 220 Mev.

Even with both corrections a residual discrepancy remains, in which each of these three experimental values of $|D_-{}^b|$ is greater than the magnitude of the curve and $D_-{}^b$ ($T=98$ Mev) is also above the theoretical curve. This reduced discrepancy presists with the same sign over a considerable range of energy. If the curve is correct,[11] then this suggests the possibility that in the evaluation of $|D_-{}^b|$ from Eq. (3a), either the imaginary part, $A_-{}^b$, of the elastic forward amplitude, $F_-{}^b$, is always too small, or $|F_-{}^b|$ is always too large. Particularly near the resonance, where $|D_-{}^b|$ is small and each of $|F_-{}^b|$ and $A_-{}^b$ is large, the experimental value of $D_-{}^b$ is extremely sensitive to small changes in either

$|F_-{}^b|$ or $A_-{}^b$. Low values of σ_0, due perhaps to overestimates of γ-ray detection efficiencies, would give too small values of $A_-{}^b$.

A charge independence test (triangle inequality) which is particularly sensitive to a low value of σ_0 has been applied to the differential cross section data at 150, 170, and 220 Mev. No statistically significant violation exists (Fig. 6), but systematically negative center values of the function $I(T,\theta)$ at 150 Mev and particularly at 170 Mev, may be indicative of difficulty. Use of the experimental σ_0 values can be avoided in the evaluation of $|D_-{}^b|$ either by making a phase shift analysis of σ_- (or of σ_- and σ_+) and obtaining $D_-{}^b$ from the phases, which assumes charge independence, or by making use of a transmission measurement of $\sigma_-{}^{\text{tot}}$, which is preferable because it involves no theoretical assumptions.

The σ_- data have been analyzed at $T=150$, 165, 170, 217, 220, and 307 Mev to see whether D waves are present, and if so, to see whether they reduce $|F_-{}^b|^2$, and thereby $|D_-{}^b|$. No systematic indication of D waves is found. However, above 300 Mev the value of $|F_-{}^b|^2$ is extremely sensitive to small amounts of D waves, and a *combined* analysis of σ_- and σ_+ data at 307 Mev by Korenchenko and Zinov[9] has shown that it is necessary to take D waves into account. Therefore we cannot say that the 307- and 333-Mev data disagree significantly with the theoretical curve.

A phenomenological analysis has been made to determine the effect on the curve $D_-{}^b$ of changes in f^2, a_1, a_3, and in unknown high-energy ($\omega' \gtrsim 2$ Bev) contributions to the dispersion integrals. Such changes are found to be incapable of producing the needed corrections. Changes which fit the π^- data from 98 to 220 Mev give disagreement of $D_-{}^b$ with the π^- data outside of this energy range, and give complete disagreement of $D_+{}^b$ with the π^+ data. Directly related to this result is the fact that the only term of the dispersion equation responsible for the resonance-like behavior of $D_-{}^b(\omega)$ is the P.V. integral (Fig. 3).

The effects of the n-p mass difference, of the $\pi^\pm$-π^0 mass difference, of Coulomb corrections, of the radiative capture process, $\pi^-+p\rightarrow\gamma+n$, to the dispersion integral in the unphysical region, and of K mesons and hypersons have been investigated by other authors,[21] and found to be negligible.[22]

In conclusion, we find that a small discrepancy remains between the dispersion theory curve, $D_-{}^b$, calculated with best fits to the experimental cross sections $\sigma_-{}^{\text{tot}}$ and $\sigma_+{}^{\text{tot}}$, and the experimentally determined values of $D_-{}^b$. However, in view of present experimental uncertainties, it is not sufficient to raise serious doubt about the validity of the dispersion relation. Because of the extreme sensitivity of the P.V. integral to $\sigma_-{}^{\text{tot}}$, it would be very helpful to have improved measurements in the region from 100 to 300 Mev.

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[21] A. Agodi and M. Cini, Nuovo cimento **5**, 1256 (1957); Agodi, Cini, and Vitale, Phys. Rev. **107**, 630 (1957); G. F. Chew and H. P. Noyes, Phys. Rev. **109**, 556 (1958).

[22] R. Sorenson (to be published) finds that, in the fixed source model, the Coulomb interaction and the π^+-π^0 mass difference effects are not individually negligible, but that their combined effect in the resonance region is small [D. A. Geffen (to be published)].