Decay Modes of a $(\theta+\bar{\theta})$ System*

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Relations among the different states of a system consisting of a θ meson, a $\bar{\theta}$ meson, and a certain number of pions are discussed. Theorems concerning certain correlations between the production probabilities for charged and uncharged θ mesons and the decay modes (θ_1^0 and θ_2^0) are obtained.

I

THE purpose of this note is to study the decay of a system which consists of a θ meson, a $\bar{\theta}$ meson, and a certain number of pions. Such a system can be produced, e.g., in a collision between an antinucleon and a nucleon. We shall show that by the combined use of the isotopic-spin rotation operator and the charge conjugation operator there exist some interesting correlations, not only in production but also between some of the decay modes of the θ and $\bar{\theta}$. To be specific, let us first consider the reaction

$$\bar{p}+n\rightarrow\theta+\bar{\theta}+m\pi. \qquad (1)$$

We shall denote the final state of the m pions by D_m which represents both a specific charge distribution and a fixed momentum distribution of the pions. The charges and the decay modes[1] of the θ and $\bar{\theta}$ mesons are of interest here. Let us define $P_{ij}(D_m)$ to be[2] the probability of observing the m pions with a distribution D_m together with a θ_i of momentum $\mathbf{k}_a$ and a θ_j of momentum $\mathbf{k}_b$ where i (j) runs over $+$, $-$, 1, or 2 representing the cases in which[3] θ_i (θ_j) is θ^+, θ^-, θ_1^0, or θ_2^0.

We assume that the two decay modes 1 and 2 are given by

$$\theta_1=\frac{1}{\sqrt{2}}(\theta^0+\bar{\theta}^0),\quad \theta_2=\frac{-i}{\sqrt{2}}(\theta^0-\bar{\theta}^0);$$

i.e., (2)

$$\theta^0=\frac{1}{\sqrt{2}}(\theta_1+i\theta_2),\quad \bar{\theta}^0=\frac{1}{\sqrt{2}}(\theta_1-i\theta_2).$$

It is well known that this decomposition corresponds to the two experimental decay modes θ_1 and θ_2 if time-reversal invariance holds in the decay.

II

To study the dependence of P_{ij} on i and j, we consider all the states of a pair of θ and $\bar{\theta}$ of given momenta $\mathbf{k}_a$ $\mathbf{k}_b$. There are eight possible states altogether.

$$\begin{aligned}&\text{Charge}=1: && (+\bar{0}),\ (\bar{0}+);\\ &\text{Charge}=-1: && (-0),\ (0-);\\ &\text{Charge}=0: && (+\,-),\ (-\,+),\ (0\bar{0}),\ (\bar{0}0).\end{aligned} \qquad (3)$$

Here we adopt the obvious notation, e.g., $(+\bar{0})$ means the state with a θ^+ meson having momentum $\mathbf{k}_a$ and a $\bar{\theta}^0$ meson with momentum $\mathbf{k}_b$. For the states with charge $=+1$, we have, by (2),

$$\begin{aligned}(+\bar{0})&=\frac{1}{\sqrt{2}}(+1)-\frac{i}{\sqrt{2}}(+2),\\ (\bar{0}+)&=\frac{1}{\sqrt{2}}(1+)-\frac{i}{\sqrt{2}}(2+).\end{aligned} \qquad (4)$$

A general state of charge $+1$ is a superposition of these two states. One sees that in any superposition the decay modes $(+1)$ and $(+2)$ always have the same probability. Also the modes $(1+)$ and $(2+)$ have the same probability. Similar considerations can be extended to the states with charge $=0$ and -1. One easily proves in this way the following theorem:

Theorem 1.—

$$P_{\pm1}(D_m)=P_{\pm2}(D_m), \qquad (5)$$

$$P_{1\pm}(D_m)=P_{2\pm}(D_m), \qquad (6)$$

$$P_{11}(D_m)=P_{22}(D_m), \qquad (7)$$

$$P_{12}(D_m)=P_{21}(D_m). \qquad (8)$$

In proving this theorem the only assumptions are that the decomposition (2) holds and that the total strangeness of the pair $\theta\bar{\theta}$ is zero (so that, e.g., the pair $\theta^+\theta^0$ is excluded).

III

It is possible to obtain more identities if one uses the conservation of isotopic spin and of the charge conjugation operator in the strong interactions. To do this, one has to study the transformation of the eight states (3) under an isotopic spin rotation and/or charge conjugation. The problem is identical to that of the transformation of the states describing a nucleon-antinucleon

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[1] M. Gell-Mann and A. Pais, Phys. Rev. **97**, 1387 (1955).
[2] If θ should have nonzero spin, then $P_{ij}(D_m)$ should be defined as the probability of finding θ_i with momentum k_a and spin S_a and θ_j with momentum k_b and spin S_b.
[3] Throughout this paper, a $\theta^\pm$ meson is identical with a $K^\pm$ meson, including all the different decay modes of $K^\pm$.

TABLE I. Eigenfunctions for which strangeness $S=0$. Notations are explained in the text.

$Q=I_3$	I	G	Wave function Form a	Wave function Form b
1	1	-1	$2^{-\frac{1}{2}}[(+\bar{0})+(\bar{0}+)]$	$=\frac{1}{2}[(+1)-i(+2)+(1+)-i(2+)]$
1	1	1	$2^{-\frac{1}{2}}[(+\bar{0})-(\bar{0}+)]$	$=\frac{1}{2}[(+1)-i(+2)-(1+)+i(2+)]$
-1	1	-1	$-2^{-\frac{1}{2}}[(0-)+(-0)]$	$=-\frac{1}{2}[(1-)+i(2-)+(-1)+i(-2)]$
-1	1	1	$-2^{-\frac{1}{2}}[(0-)-(-0)]$	$=-\frac{1}{2}[(1-)+i(2-)-(-1)-i(-2)]$
0	1	-1	$\frac{1}{2}[-(+-)+(0\bar{0})+(\bar{0}0)-(-+)]$	$=\frac{1}{2}[-(+-)-(-+)+(11)+(22)]$
0	1	1	$\frac{1}{2}[-(+-)+(0\bar{0})-(\bar{0}0)+(-+)]$	$=\frac{1}{2}[-(+-)+(-+)-i(12)+i(21)]$
0	0	-1	$\frac{1}{2}[-(+-)-(0\bar{0})+(\bar{0}0)+(-+)]$	$=\frac{1}{2}[-(+-)+(-+)+i(12)-i(21)]$
0	0	1	$\frac{1}{2}[-(+-)-(0\bar{0})-(\bar{0}0)-(-+)]$	$=\frac{1}{2}[-(+-)-(-+)-(11)-(22)]$

pair.[4,5] Following the arguments used in reference 4 for studying the latter problem, we first consider the four states[6] of a single θ or a single $\bar{\theta}$:

$$\begin{bmatrix} \theta^+ \\ \theta^0 \\ \bar{\theta}^0 \\ -\theta^- \end{bmatrix}. \tag{9}$$

The isotopic spin operators which operate on these states are

$$I_1=\frac{1}{2}\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix},\quad I_2=\frac{1}{2}\begin{bmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{bmatrix},$$

$$I_3=\frac{1}{2}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}. \tag{10}$$

Instead of discussing the charge conjugation operator C, we define, as in reference 4,

$$G=C\exp[i\pi I_2], \tag{11}$$

and obtain

$$G=\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}. \tag{12}$$

The operator G commutes with I. The strangeness number operator is diagonal in this representation,

$$S=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}. \tag{13}$$

We notice that

$$SG+GS=0. \tag{14}$$

[4] T. D. Lee and C. N. Yang, Nuovo cimento **3**, 749 (1956).

[5] See also L. Michel, Nuovo cimento **10**, 319 (1953); A. Pais and R. Jost, Phys. Rev. **87**, 871 (1952); D. Amati and B. Vitale, Nuovo cimento **2**, 719 (1955); C. Goebel, Phys. Rev. **103**, 258 (1956).

[6] Note the minus sign in front of the θ^- state. The convention used here is similar to that discussed in footnote 1 of reference 4.

For a system of two particles with momenta $\mathbf{k}_a$ and $\mathbf{k}_b$, each described by a four-component wave function (9), the operators I and S are additive, whereas G is multiplicative. Of the sixteen states of the two-particle system, eight belong to $S=0$ and are given in (3). For these states, the charge

$$Q=I_3 \tag{15}$$

and I^2, G, and S all commute and can be simultaneously diagonalized. The eight eigenstates can be easily constructed and are displayed in Table I in the column headed by "Form a." In the next column "Form b," these eigenfunctions are rewritten by the use of (2) so that their decay modes are explicitly exhibited.

In reaction (1) the process in general goes through many channels of I and G which produce interference effects. However, the interference between different G values disappears if we *average* over the collisions $(\bar{p}n)$ and $(n\bar{p})$, where the states are defined in a manner similar to Eq. (3). To see this, one writes

$$(\bar{p}n)=\tfrac{1}{2}[(\bar{p}n)+(n\bar{p})]+\tfrac{1}{2}[(\bar{p}n)-(n\bar{p})],$$
$$(n\bar{p})=\tfrac{1}{2}[(\bar{p}n)+(n\bar{p})]-\tfrac{1}{2}[(\bar{p}n)-(n\bar{p})].$$

The term $(\bar{p}n)+(n\bar{p})$ belongs to $G=-1$ and the term $(\bar{p}n)-(n\bar{p})$ to $G=+1$. The interference terms between $G=\pm 1$ therefore have opposite signs in $(\bar{p}n)$ and $(n\bar{p})$ and cancel exactly upon taking the average. In other words, after the average one may consider (1) as going through incoherent noninterfering channels with definite G values. Since the pions are[4] eigenstates of G, for any given distribution D_m of the pions, there are therefore no interferences between the states of the $(\theta\bar{\theta})$ system with different G values. Table I shows then that for charge $Q=1$, the first two rows cannot interfere and one obtains, in addition to (5) and (6), the identity

$$P_{+1}(D_m)=P_{1+}(D_m).$$

These considerations are easily extended to states with $Q=0$ and -1.

Theorem 2.—Consider $\bar{p}+n$ and $n+\bar{p}$ which are related by the substitution of a $\bar{p}$ by an n with equal momentum and spin and vice versa. After summation over these two initial states, the partial cross sections satisfy

$$P_{\pm 1}(D_m)=P_{1\pm}(D_m)=P_{\pm 2}(D_m)=P_{2\pm}(D_m), \quad (16)$$

$$P_{11}(D_m)=P_{22}(D_m), \quad (17)$$

$$P_{12}(D_m)=P_{21}(D_m), \quad (18)$$

and

$$P_{11}(D_m)+P_{22}(D_m)+P_{12}(D_m)+P_{21}(D_m) = P_{+-}(D_m)+P_{-+}(D_m). \quad (19)$$

We thus have the interesting result that in those collisions where θ^0 and $\bar{\theta}^0$ are produced, their decay modes (θ_1^0 and θ_2^0) are in general not independent of each other but are related in a manner given by Eqs. (17)–(19).

In the capture of an antiproton by a neutron, both at rest, it is clear that theorem 2 applies.

A simple consequence of (16)–(19) is that in a collision $\bar{p}+n$, one has

$$\tfrac{1}{2}(n_+ + n_-)=n_1=n_2, \quad (20)$$

where n_i is the total number of θ_i produced ($i=1, 2, +,$ or $-$) integrated over all angles.

IV

It is easy to see that the same identities (theorems 1 and 2) are also valid for the reaction

$$\bar{n}+p \rightarrow \theta+\bar{\theta}+m\pi. \quad (1')$$

Furthermore, for both reactions (1) and (1′) additional equalities and inequalities may be obtained when m takes on some specific small values. In all cases, we found it convenient to use the eigenstates tabulated in Table I. They are useful also in discussing

$$p+\bar{p} \rightarrow \theta+\bar{\theta}+m\pi,$$

and

$$n+\bar{n} \rightarrow \theta+\bar{\theta}+m\pi.$$

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Unusual Cosmic-Ray Intensity Fluctuations Observed at Southern Stations during October 21–24, 1957

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Observations are reported from four southern stations of cosmic-ray intensity fluctuations during the period October 21–24, 1957. These fluctuations exhibit unusual features which suggest the existence of a short-lived and highly directional anisotropy of the primary radiation during the period immediately preceding a Forbush-type decrease.

I. INTRODUCTION

We report here observations made at four southern hemisphere stations of cosmic-ray intensity fluctuations which occurred during the period October 21–24, 1957.

The main feature of the fluctuations is an intensity decrease of the well-known Forbush type, but other features make the observations of unusual interest. In particular, a transient, longitude-dependent decrease prior to the Forbush event suggests the existence of a direction of reduced primary intensity, which also provides a plausible explanation for the observed differences in onset times of the main Forbush event.

II. OBSERVATIONS

Figure 1 displays the cosmic-ray intensity records obtained at the network of four southern stations (Lae, Hobart, Macquarie Island, and Mawson) on the days October 21–24, 1957. The locations of these stations and details of the equipment used are included with Fig. 1.

The stations at Hobart and Lae are maintained by the cosmic-ray research group at the University of Tasmania, and those at Macquarie Island and Mawson by the Australian National Antarctic Research Expeditions, for the conduct of whose cosmic-ray program the University group is responsible.

The hourly intensities plotted in Fig. 1 are expressed as percentage deviations from the mean level during the first 12 hours of October 21. Except in the case of the underground records at Hobart, adjustments have been made for the effects of barometric pressure variations. For the data from both vertical and inclined meson telescopes, which are all near sea level, a theoretical total barometer coefficient obtained from Fig. 1 of a paper by Trefall[1] has been used. The use of such coefficients has been discussed elsewhere by Parsons.[2] In the present case coefficients of -2.31% per cm Hg

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[2] N. R. Parsons, Australian J. Phys. **10**, 387 (1957).