Experimental Results on the Radiative π-μ Decay

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A total of 93 045 π-μ-e decays have been examined in Ilford G-5 plates, where both the π meson and the μ-meson decays take place in the emulsion, in order to study the $\pi \rightarrow \mu+\nu+\gamma$ decay. From the total number of short muon tracks thus found, all cases possibly due to pion decays in flight were eliminated. Accurate corrections were carried out taking into account the straggling effects on the normal muons, so as to establish a safe upper limit in the anomalous muon range determination. After carefully correcting for the geometrical losses and experimental biases, the value $(1.24\pm0.25)\times10^{-4}$ was obtained for the probability of a pion decaying into a muon whose track is shorter than 435 μ. This result is in fair agreement with the theoretical predictions on the pion radiative decay.

1. INTRODUCTION

THE experimental study of the $\pi \rightarrow \mu+\nu+\gamma$ decay may be made advantageously with the nuclear emulsion technique. After several sporadic observations in nuclear plates of muon tracks shorter than those originating in $\pi \rightarrow \mu+\nu$ decays, the only systematic search for muons of anomalous range was conducted by Fry.[1] From the analysis of 11 841 decays he concluded that not all the short muon ranges could be ascribed to π-meson decays in flight; so the existence of a radiative decay was established.

We have studied this type of decay as part of a series of investigations[2] on the pion and muon decays in emulsion. The object was threefold; (1) to obtain a better value of the ratio $(\pi \rightarrow \mu+\nu+\gamma)/(\pi \rightarrow \mu+\nu)$ previously given by Fry as $(3.3\pm1.3)\times10^{-4}$, (2) to obtain the muon energy spectrum in the radiative pion decay, and (3) to compare our results with the theoretical predictions of several authors.[3] To this end we have analyzed 93 045 π-μ-e decays in nuclear plates exposed to a pion beam.

2. EXPERIMENTAL DETAILS

2.1. Exposure, Scanning, Selection Criteria

Seventy-two Ilford G-5 plates with glass backings, 5 cm $\times$10 cm $\times$600 microns (μ) were exposed to the pion beam of the Liverpool cyclotron, in four stacks of 18 plates at a time. The geometry was chosen so that a pion would travel about 4 cm in emulsion before stopping, thus increasing its chances of being reliably measured.

To simplify the scanning without slowing it down, a mean intensity of 1 π-μ decay per microscope field—over the whole thickness of the emulsion—was chosen. For the same reason, a total magnification of 6×26 was used (field diameter 750 μ), so the normal-range muon could be entirely seen within one field, while both the ionization variation in the π-μ decay and the electron track in the μ-e decay could be easily detected at sight.

The scanners were expected (a) to count all π-μ decays; (b) to count, out of these, all the decays whose muons came to rest in the emulsion; (c) to select all muons whose projected range was $\leqslant 425\pm20$ μ, as measured with calibrated eyepiece micrometers. The 2849 events thus selected were re-examined, and a first-approximation range was assigned. When this range was shorter than 500 μ, the true range was measured with the highest accuracy. The 87 events which survived these selections were considered "of anomalous range", and the π-μ decay angles θ were measured to within 2°.

2.2. Measurements

Range measurements were carried out on a Koristka MS-2 microscope, with a total magnification of 9×100. The MS-2 optics and mechanics are such that the main source of error in range measurements—namely, the determination of the z coordinate—is smaller than in other microscopes. A Leitz eyepiece micrometer was used, calibrated to within 0.3%.

The range was measured by dividing the track into about 8 to 12 segments, depending on the number of large-angle scatterings, in order to reduce the smoothing effects. The horizontal and vertical coordinates of each segment were measured and due account was taken of the shrinkage effect. As just stated, the error originates mainly in the determination of the z coordinate, and varies from 5 μ for nearly flat tracks to 20 μ for tracks at about 40° angle with the plane of the emulsion.

The reproducibility of the results was checked by having different observers repeat the measurement of some events. For flat tracks the agreement was to within 0.5%, and in all cases the results were consistent within the estimated error.

[1] W. F. Fry, Phys. Rev. **91**, 130 (1953).
[2] Castagnoli, Franzinetti, and Manfredini, Suppl. Nuovo cimento **6**, 441 (1957); Nuovo cimento **5**, 684 (1957). C. Castagnoli, International Conference on Mesons, Padova, Venezia, 1957; Castagnoli, Manfredini, and Merrison, Nuovo cimento **9**, 186 (1958); Castagnoli, Ferro-Luzzi, and Manfredini, Ricerca sci. **28**, 1644 (1958).
[3] H. Primakoff, Phys. Rev. **84**, 1255 (1951); Nakano, Nishimura, and Yamaguchi, Progr. Theoret. Phys. Japan **6**, 1028 (1951); T. Eguchi, Phys. Rev. **85**, 943 (1952); G. E. A. Fialho and J. Tiomno, Anais acad. brasil. cienc. **24**, 245 (1952); B. Joffe and A. Rudik, Doklady Akad. Nauk S.S.S.R. **82**, 359 (1952); S. Kamefuchi and S. Oneda, Nuclear Phys. **6**, 114 (1958); K. Huang and F. E. Low, Phys. Rev. **109**, 1400 (1958).

Shrinkage was taken equal to 2.0±0.1. It is to be noted that since $\Delta R/R=(\sin^2\varphi)\Delta S/S$, a 5% change in the mean emulsion thickness causes a 2% change, or less, in the mean range of tracks inclined with $\varphi\leqslant 40°$.

Ionization measurements were carried out by means of a semiautomatic apparatus described elsewhere.[4] The ionization in different plates was calibrated by measuring the tracks of 50 pions coming to rest in the emulsions, and analyzing the data by the method of least squares. The variable $\bar{x}$—total gap-length per 50-μ cell—was used in the analysis; in the energy region of interest, it is the variable most sensitive to changes in velocity.

Multiple-scattering measurements were carried out on the Koristka MS-2 microscope. The usual constant-sagitta cell-scheme for pions was used.

3. EXPERIMENTAL PROCEDURE

Once the 87 π-μ decays of anomalous range had been selected, the following problems had to be solved:

(1) to determine the number of anomalous ranges which could have been due to straggling effects;

(2) to eliminate all cases which could be ascribed to a pion decay in flight with the muon emitted backwards in the center-of-mass system;

(3) to eliminate all cases in which a muon belonging to the beam background was scattered, simulating an anomalous π-μ decay;

(4) to determine the experimental bias.

We shall now discuss separately each one of the methods adopted to deal with the foregoing problems.

3.1 Straggling Effects

In order to determine the maximum acceptable range for an anomalous event, it is very important to examine the straggling effects closely.

The total range straggling for muons is made up from the sum of several instrumental terms besides the Bohr straggling. Since our plates were practically distortion-free, we may suppose the distortion contribution to the straggling equal to zero. There remain the terms related to the microscopic distortions and heterogeneity of the material; the corresponding variance[5] in (microns)^2 is $\sigma_p{}^2=0.05\bar{R}$ where $\bar{R}$ is the mean range. Finally, the observer introduces a variance $\sigma_m{}^2$ which we estimated by comparing several measurements of the same track, as explained before; we obtained $\sigma_m{}^2\cong 5\times 10^{-3}\ \bar{R}$ (microns)^2.

From the physical point of view, the range straggling of nonrelativistic particles in homogeneous materials, as calculated by Bohr on the basis of the central limit theorem, has been re-examined by Lewis.[6] He observes that most of the straggling is due to a small number of violent collisions. The range distribution will not be Gaussian in that case, sensibly favoring ranges shorter than the mean. One finds—taking into account also the electron binding, neglected by Bohr—an explicit formula for the mean range and for the higher moments of the range distribution. Integration of this function gives the probability that a particle belonging to a monochromatic beam with mean range $\bar{R}$ will have a range shorter than R, as a function of R; as follows:

$$F(x)=\int_{-\infty}^{x}\varphi(x)dx+\varphi(x)\{\gamma_3H_2(x)-[\gamma_4H_3(x)+\tfrac{1}{2}\gamma_3{}^2H_5(x)]-[\gamma_5H_4(x)+\gamma_3\gamma_4H_6(x)+\tfrac{1}{6}\gamma_3{}^3H_8(x)]+\cdots\},\quad(1)$$

where $\varphi(x)$ is the normal distribution with respect to the variable $x=(R-\bar{R})/\sigma$, σ^2 being the range distribution variance. $H_i(x)$ are the Hermite polynomials of ith degree, and the γ_i are coefficients depending on the physical conditions of the problem (absorbing material and mean ionization potential, mass, energy, and charge of the particle), and related to the momenta β_i of the range distribution by means of $\gamma_i=\beta_i/i!\beta_2{}^{\frac{1}{2}}$. For nuclear emulsions, the mean ionization potential, I, was taken equal to 335 ev. Similarly, the number of electrons per cm^3 was taken equal to 1.052×10^{24}. For σ the values 25 μ and 27 μ were adopted: the first one is the theoretical prediction with the aforementioned choice of parameters and agrees with the experimental determinations by several authors[7]; the second one, more conservative, has also been obtained in some experimental determinations.[5] On the basis of measurements performed by various authors and by ourselves, we assumed $\bar{R}=602\ \mu$.

Table I shows the result of these calculations. It is clear that, over the total number of decays examined by us, we should expect, in the more conservative case, 16 events with ranges between 475 and 500 μ due to straggling effects, while the normal distribution predicts only 6. Between 450 and 475 μ we should expect 0.6 event, while the normal distribution predicts just 0.1.

TABLE I. Integral straggling and normal distributions.

Muon range R (in μ)	Integral straggling distribution $F((R-\bar{R})/\sigma)$		Integral normal distribution $G((R-\bar{R})/\sigma)$	
	$\bar{R}=602\ \mu$, $\sigma=27\ \mu$	$\bar{R}=600\ \mu$, $\sigma=25\ \mu$	$\bar{R}=602\ \mu$, $\sigma=27\ \mu$	$\bar{R}=600\ \mu$, $\sigma=25\ \mu$
450	5.5×10^{-8}	1.1×10^{-8}	9.0×10^{-9}	1.0×10^{-9}
475	6.0×10^{-6}	1.5×10^{-6}	1.3×10^{-6}	2.8×10^{-7}
500	1.7×10^{-4}	8.2×10^{-5}	7.8×10^{-5}	3.0×10^{-5}

[4] G. Baroni and C. Castagnoli, Suppl. Nuovo cimento **12**, 364 (1954).

[5] Barkas, Smith, and Birnbaum, Phys. Rev. **98**, 605 (1955).

[6] H. W. Lewis, Phys. Rev. **85**, 20 (1952).

[7] Bacchella, Berthelot, di Corato, Goussu, Levi-Setti, René, Revel, Scarsi, Tomasini, and Vanderhaege, Nuovo cimento **4**, 1548 (1956).

On account of these calculations, it was decided to consider "anomalous" all muons with range shorter than 475 μ.

3.2 Ionization Measurements

In order to establish the true nature of a presumably anomalous decay on the basis of ionization measurements, the following procedure was adopted. From the π-μ junction, provided the dip angle was less than 30°, the total gap-length per 50-μ cell, $\bar{x}$, was determined. The values thus obtained, corrected for the inclination of the track, were plotted *versus* R/M (in μ/m_π) as abscissa. This was done firstly assuming the track to be due to a pion at rest and secondly assuming it to be due to a muon just before a large-angle scattering. Typical cases in which the first or second hypothesis, respectively, was verified on account of the goodness of fit to the mean-square calibration straight line, are shown in Figs. 1(a) and (b). It is evident that the method can be used, in some cases, to distinguish a pion decay at rest from a decay in flight. However, the method is not always sensitive enough to elucidate conclusively the nature of an event. A typical dubious case is shown in Fig. 1(c) for the two hypotheses, π meson at rest and π meson in flight with 80-μ residual range.

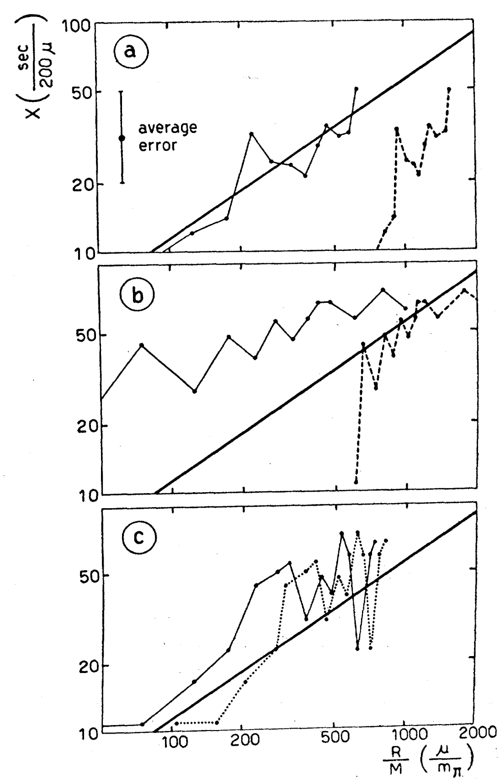

FIG. 1. Ionization analysis. (a), (b): Pion at rest (——) or muon scattering (– – –). (c): Pion at rest (——) or decay in flight (· · ·). The straight line is the least-squares calibration on 50 normal pions at rest.

TABLE II. Multiple scattering measurements.

(a) Possible decays in flight

Event No.	Pion residual range R (if in flight) (μ)	X (μ) $X_0=0\ \mu$	X (μ) $X_0=500\ \mu$	Remarks
40	25	−16	−96	π at rest
49	25	7	−91	may be decay in flight
30	200	3	−500	π at rest
38	200	1	−103	π at rest
65	320	−7	−685	π at rest

(b) Possible μ-meson scatterings

Event No.	Muon residual range R (if μ-meson scattering) (μ)	X (μ) $X_0=0\ \mu$	X (μ) $X_0=500\ \mu$	Remarks
60	253	28	66	ambiguous result
63	314	47	744	ambiguous result
75	329	25	418	ambiguous result
2	383	18	−335	not μ-meson scattering
22	472	−34	52	not μ-meson scattering
Probable error		±20	±200	

3.3. Multiple-Scattering Measurements

In some cases, constant-sagitta multiple-scattering measurements of the presumed π meson could give an indication of whether the track belonged to a particle at rest or not. To this end, the measurements were analyzed according to a method devised in our laboratory.[8] Suppose that the particle decays in flight with a residual range X and that, ignoring this fact, a constant-sagitta measurement is performed using the usual pion cell scheme. Then, the residual range at the ith cell will be R_i+X, so that instead of $D_0=kf(M)$, the theoretical set of second differences will be given by

$$D_i(X)=D_0\left[\left(\frac{R_i}{R_i+X}\right)^{1.16}+\frac{\epsilon^2}{D_0^2}\right]^{\frac{1}{2}}, \qquad (2)$$

where ϵ is the noise. Now, assigning a first-approximation residual range X_0 to the track, and putting $X=X_0+x$, it is possible to find x by adjusting the experimental set of second differences Δ_i to the theoretical set by means of the method of least squares. The result is

$$x\pm\sigma(x)=\frac{\sum \Delta_i D_i' D_i^{-2}-\sum D_i' D_i^{-1}}{\sum (D_i' D_i)^2} \pm 0.75\left[\sum (D_i' D_i^{-1})^2\right]^{-\frac{1}{2}}, \qquad (3)$$

where $D_i'=dD_i/dX\,|\,_{X=X_0}$.

The first-approximation residual range X_0, in our case, could assume one of the following values: (a) zero; (b) a value given by the dynamics of the decay in flight; (c) the range of the decay muon. In this latter case, of course, D_0 will be the constant second difference

[8] Baroni, Cortini, and Manfredini, Nuovo cimento 1, 473 (1955).

TABLE III. Data for some decays in flight.

(a) Probable decays in flight

Event No.	R (μ) $\pm 10\,\mu$	θ $\pm 2°$	Pion residual range R_π (μ)	Probability of decay within R_π ($\times 10^5$)
50	434	82°	80_{-25}^{+20}	$10\ \pm 2$
51	439	88°	34_{-14}^{+24}	5.5_{-2}^{+3}
53	472	84°	45_{-15}^{+20}	7.0 ± 2
54	473	90°	18_{-7}^{+9}	3.5 ± 1

(b) Dubious cases

Event No.	Muon range (μ)	Remarks
59	208	ambiguity in ionization measurement
60	253	ambiguity in ionization and scattering measurements
61	286	nonmeasurable pion
62	293	ambiguity in ionization measurement
64	451	ambiguity in ionization measurement
65	466	π-μ junction not clearly identifiable

for a muon—and not a pion—at rest. Table II summarizes the data of the events thus treated.

3.4. Pion Decays in Flight

A π-μ decay in flight may simulate an anomalous decay if the muon is emitted backwards in the center-of-mass system. By comparing the kinematically expected value of the pion velocity at decay with the value obtained from ionization measurements of the pion track, it is often possible, when the pion residual range is not too small, to distinguish the decay in flight from that at rest. Since ionization measurements cannot detect range differences under 100 μ, it is well to calculate the probability of a pion decaying within the last 100 μ of its expected range. This probability is 1.2×10^{-4}. Considering the angular distribution of the muons, for a population of 93 045 events one expects about 6 short muon tracks due to π-meson decays in flight. We have indeed found 7 such events, four of which are listed in Table III, being of range shorter than 475 μ.

On the other hand, when the residual pion range R_π is very short, it is possible to distinguish the two cases by considering the probability of the decay in flight. Indeed, the probability of a pion decay in flight with $R \leqslant 15\,\mu$ is less than 3×10^{-5}, rather small compared to the probability of the anomalous decay.

It is indicative, and a simple calculation shows, that with a tolerance of 5° on the decay angle θ and of 10 μ on R, and assuming the present theories on the pion radiative decay to be valid, the probability that an actually anomalous event simulates a decay in flight with residual range $R \leqslant 15\,\mu$ is less than 6×10^{-9}; no such case is to be expected within our population of $\sim 10^5$ events.

3.5. Geometrical Losses and Experimental Bias

In order to calculate the branching ratio of radiative to normal pion decay, and to obtain the form of the muon range spectrum, it is necessary to examine carefully the various geometrical losses and experimental biases. We have considered the following factors.

(1) *Finite dimensions of the emulsion.*—Very simple geometrical arguments lead to the following formula for the fraction $g_d(R)$ of muons of range $R \leqslant d$ entirely contained within a plate of thickness d:

$$g_d(R) = 1 - \frac{1}{2d}\int_0^d \left\{ \left(1-\frac{z}{R}\right)\delta(z) + \left(1-\frac{d-z}{R}\right)\delta(d-z) \right\} dz = 1 - \frac{R}{2d}, \quad (4)$$

$g_d(R) = 1-(1/d)\int_0^R [1-(z/R)]dz = 1-(R/2d)$. For the normal muon range and $d = 600\,\mu$, $g_d(R) = 0.5$. Our 93 045 events in emulsion have been selected from a total of 186 046 decays, the ratio of these two numbers being indeed 0.5.

(2) *Scanning and measurement cutoffs.*—Since the scanners were told to pick an event if its projected range was $\leqslant 425\,\mu$, it is evident that an event with actual range R will be picked only if it is sufficiently inclined to satisfy such condition. Furthermore, the range of any given event can be reasonably determined only if the dip angle is not larger than about 65°. Then the probability of recording and measuring an event recognized as a π-μ-e series of decays is given by

$$g_\varphi(R) = 0.9[1-\delta(\alpha)\cos\alpha], \quad (5)$$

where $\alpha = \arcsin(425/R)$, and $\delta(\alpha) = 1$ for $425 \leqslant R \leqslant 600$ and zero otherwise.

(3) μ-meson scatterings. An important question to be settled is how to fix the minimum range detectable (for the maximum one, the considerations made on the straggling effects apply). A background μ-meson scattering may simulate a very short-range anomalous μ meson; in that case, the changes in ionization involved are too small to elucidate its nature. On the other hand, the number of μ-meson scatterings with angle greater than a certain minimum cutoff is, for such short residual ranges, far greater than the expected number of anomalous events. The estimate is as follows. The probability of an anomalous decay with $50 \leqslant R \leqslant 150\,\mu$ is, according to theory, of the order of 0.6×10^{-5}. The probability of a muon scattering with the same residual range, experimentally determined by us on 870 π-μ-e events (cutoff at 25°, minimum residual range 50 μ; total number of scatterings found, 77; see Fig. 2), is of the order of 3.7×10^{-3}. Since the background muon contamination of the pion beam, as measured by us, is nearly 4%, the probability over the same population of events is 1.5×10^{-3}. So practically all the events of this type which have been observed may be attributed to μ-meson scatterings. We have therefore assumed as a lower limit for our experimental range spectrum of the anomalous muons the value $R = 150\,\mu$.

For $150 \leqslant R \leqslant 475\ \mu$ there remains the problem of estimating the number and ranges of anomalous events lost in the scanning on account of their resemblance to μ-meson scatterings. We have empirically estimated this correction as a function of the residual μ-meson range, $g_s(R)$. It varies from 0.4 at $R=150\ \mu$ to 1.0 at $R=450\ \mu$.

The over-all geometrical and experimental bias correcting function will then be given by

$$g(R)=g_d(R)g_\varphi(R)g_s(R).$$

4. CONCLUSIONS

4.1. Experimental Results

An event of anomalous range was defined to be "of dubious nature" when (a) the measurements performed on it gave ambiguous results; or (b) the measurement of two different parameters (like ionization and multiple scattering) gave conflicting results; or (c) the decay in flight of the pion could not be excluded; or (d) contingent reasons did not allow reliable measurements to be performed. The rest of the events of anomalous range shorter than $475\ \mu$ were attributed, according to the considerations of the preceding section, to radiative pion decays. The following 26 ranges (in μ) belong to the muons originating in radiative pion decays found by us: 342, 383, 396, 418, 418, 421, 425, 425, 441, 445, 445, 446, 448, 457, 461, 461, 465, 467, 468, 469, 471, 471, 471, 472, 473, and 474. Table III lists the 6 dubious cases found.

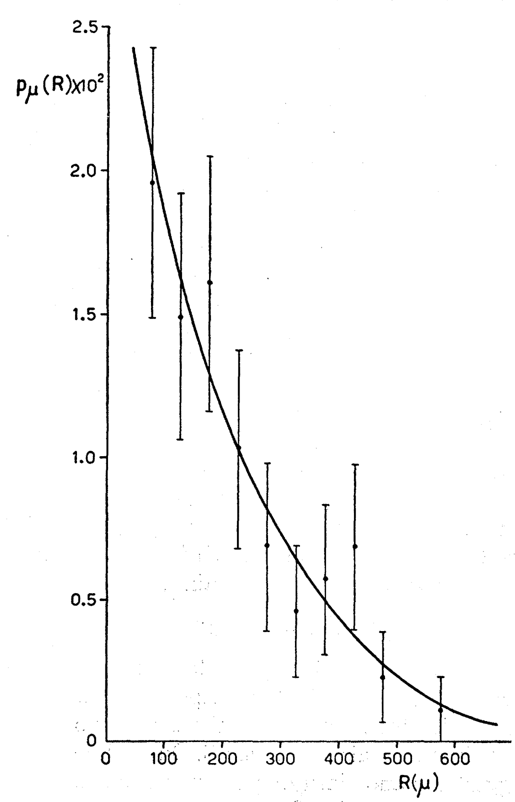

FIG. 2. Muon scattering probability in emulsions *versus* residual range ($\theta \geqslant 25°$; $R \geqslant 50\ \mu$).

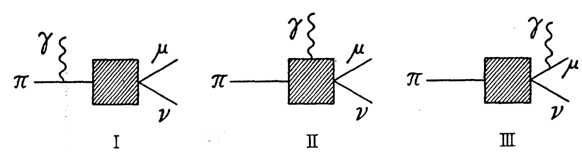

FIG. 3. Feynman diagrams for radiative pion decay.

4.2. Theoretical Discussion

The $\pi \to \mu+\nu+\gamma$ process has been treated theoretically by various authors.[3] The results of the complete theory, allowing for a general structure of the $\pi \to \mu+\nu$ vertex, are reported by Kamefuchi and Oneda in terms of many unknown functions of the energies and angles. Comparison with such a theory is of course impossible. However, we show in the following that it is reasonable to expect that the results of the complete theory are fairly accurately approximated by those of Joffe and Rudik, and of Fialho and Tiomno, which are calculated for a $\pi \to \mu+\nu$ point interaction of gradient or nongradient type.

The diagrams contributing to $\pi \to \mu+\nu+\gamma$ are of three different kinds (Fig. 3). The black box is that for the process $\pi \to \mu+\nu$. The contribution of diagrams I is proportional to $(\varphi_\mu{}^{(\pi)}\epsilon^\mu)$, where $\varphi_\mu{}^{(\pi)}$ is the pion momentum and ϵ^μ the polarization vector of the γ ray. These contributions may be made to vanish if one chooses the gauge for which $(\varphi_\mu{}^{(\pi)}\epsilon^\mu)=0$. Diagrams of group II cannot be evaluated without a detailed knowledge of the structure of the black box. In such diagrams the photon is emitted from the virtual currents which are effective in the $\pi \to \mu+\nu$. Diagrams III are diagrams of internal bremsstrahlung, and can be expressed as products of the $\pi \to \mu+\nu$ amplitude by a standard electrodynamical factor. If we indicate the set of Fermi couplings by S, P, T, V, and A, it is apparent that only those terms which contribute to $\pi \to \mu+\nu$, namely A and P, can contribute to the internal bremsstrahlung diagrams III. Furthermore it can be shown that only V, A, and T can contribute to the structure-dependent diagrams II, while S does never contribute to the decay. Cutoff calculations show that the structure-dependent contributions from II are presumably smaller than the contributions from III, the largest term being that coming from T, which, however, is still negligible as compared to the internal bremsstrahlung terms in the limit of a large cutoff. Contributions from diagrams II are effective only in the lower-energy portion of the spectrum, since the upper portion of the spectrum is dominated by the internal bremsstrahlung terms. We therefore expect that most of the possible deviations from the predictions from III alone would be appreciable only below our cutoff at $150\ \mu$, discussed in Sec. 3.3. The relevant point is that the internal bremsstrahlung contributions

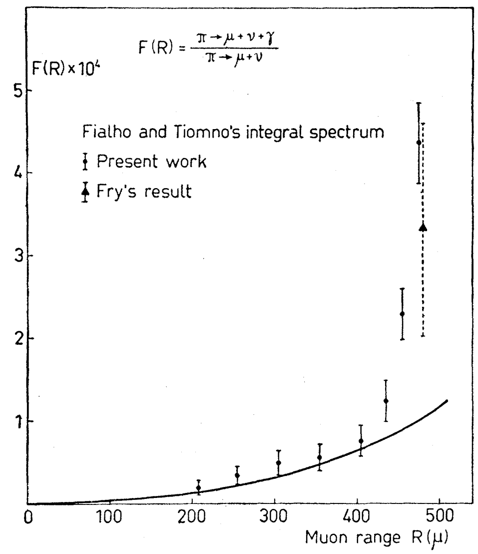

FIG. 4. Theoretical and experimental values of the $(\pi\rightarrow\mu+\nu+\gamma)/(\pi\rightarrow\mu+\nu)$ ratio.

are independent of the structure of the black box for $\pi\rightarrow\mu+\nu$. They can be calculated, for instance, by replacing the black box by an effective local interaction. Such interaction can be of gradient or nongradient type, or a mixture of both. However, a nongradient interaction may be reduced to a gradient interaction by the equivalence theorem. Therefore the theoretical prediction will be unique (apart from an undetermined constant expressing the rate of $\pi\rightarrow\mu+\nu$) and the spectrum obtained will be exactly the same as that given by Joffe and Rudik and by Fialho and Tiomno. Alternatively, one may take advantage of the present indications of the absence of T in a universal Fermi interaction, and thus again exclude the only appreciable contribution from diagrams II. We have therefore compared our experimental results with the spectra given by Joffe and Rudik and by Fialho and Tiomno.

4.3. Discussion of Results

Joffe and Rudik, and Fialho and Tiomno, carry out a perturbation treatment of the pion radiative decay and obtain[9]

$$f(E)=\frac{e^2}{2\pi}f_0\frac{P}{P_0{}^2}\left[\frac{4P_0}{E_0-E}\frac{E}{P}\left(\ln\frac{E+P}{\mu}-\frac{P}{E}\right)+\frac{E_0-E}{P}\ln\frac{(E+P)M-\mu^2}{(E-P)M-\mu^2}\right], \quad (6)$$

where E and P are, respectively, the energy and momentum of the muon, with maxima E_0 and P_0 corresponding to the normal decay; and f_0 is the probability of the normal decay. This differential energy spectrum was transformed into an integral range spectrum $F(R)$ (Fig. 4), by integrating graphically and using the usual range-energy relation. The experimental spectrum, on the other hand, including the 6 dubious events, was modified according to the bias-correcting function $g(R)$.

The errors shown in Fig. 4 are only statistical. In the same figure we have included the experimental point obtained by Fry.[1]

The following conclusions may be drawn from the figure.

(1) Considering only those events with $R\leqslant 435\,\mu$, the agreement of experiment with theory is fairly good. Indeed the theoretical branching ratio $F(R=435\,\mu)$ is in this case 0.79×10^{-4}, and our result is

$$(\pi\rightarrow\mu+\nu+\gamma)/(\pi\rightarrow\mu+\nu)=(1.24\pm0.25)\times10^{-4}.$$

Also the general form of the experimental spectrum up to $435\,\mu$ may be considered in agreement with the theory, in view of the uncertainties on the bias-correcting function $g(R)$.

(2) If the cutoff value for R is taken as $475\,\mu$, a discrepancy arises between theory and experiment which, we believe, may be ascribed essentially to three causes: firstly, a possible underestimation of the straggling effects (as is seen in Table I, a small change in the cutoff value of R in this region causes a relatively large change in the straggling contribution to the spectrum); secondly, the fact that our correction for the experimental and geometrical bias, and especially the function $g_\varphi(R)$ of Sec. 3, begin to have a rather large influence at these values of R, just where the spectrum $f(R)$ rapidly increases. These two effects, on the other hand, are not felt at values of $R\leqslant 455\,\mu$; thirdly, an actual physical effect. If this were the case, the disagreement, both in the shape of the spectrum and in the total probability, would suggest the possibility that the structure-dependent terms contribute appreciably. Furthermore, any deviation from the theoretical spectrum chosen for our comparison would allow a conclusion to be reached on whether the $\pi\rightarrow\mu+\nu$ interaction is to be regarded as direct, or as indirect via the formation and subsequent annihilation of baryon-antibaryon pairs.

In view of the aforementioned alternatives, we think it is difficult at present to reach such conclusions. If the interaction is indeed of the indirect type, a reliable observation would require a much improved statistics, which seems very difficult to obtain at present.

5. ACKNOWLEDGMENTS

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Thanks are also due Mr. G. Bertucci and Mr. A. Guspini for their help throughout the measurements.

[9] The spectrum given by Fialho differs appreciably from that reported by Joffe and Rudik. If a couple of misprints in the former, and a missing factor $\frac{1}{2}$ in the latter are taken into account, and the same values of the masses are used, the two coincide.