ACKNOWLEDGMENTS

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High-Resolution Electron Spectroscopic Study of the 60-Hour Electron-Capturing Isomer Re^{182}†

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A thorough study of the conversion electron spectrum from electron capture decay of the 60-hr Re^{182} isomer is reported. A very large number of transitions are observed, including most of those reported by previous investigations on the beta decay of Ta^{182} to the same daughter nucleus W^{182}. In addition many new transitions are observed. Relative intensity comparisons from the present study add weight to a Ta^{182} decay scheme published by earlier investigators. Using the Ta^{182} decay scheme as a base, it was possible to fit most of the new transitions into a plausible decay scheme involving several new levels. A tentative grouping of many of the new levels into rotational bands is proposed, mainly on the basis of relative intensities of gamma transitions depopulating various levels.

INTRODUCTION

THE 60-hour electron-capturing isomer of Re^{182} was first identified by Wilkinson and Hicks.[1] Since their investigations no further studies of this isomer have been reported. However, the levels of the daughter nucleus, W^{182}, have been extensively studied from the decay of Ta^{182}. The last and definitive work in these studies was that of Murray *et al.*[2] (hereafter referred to as MBMD). The present paper attempts to present a fairly complete interpretation of the results of a high-resolution electron-spectroscopic study of the transitions arising from the decay of 60-hour Re^{182}.

EXPERIMENTAL PROCEDURES AND RESULTS

The rhenium activities studied were made by alpha particle bombardment of 0.012-inch foils of natural tantalum in both the internal and external beams of the Berkeley 60-inch cyclotron. The rhenium activities were separated carrier-free from the tantalum target by the distillation method proposed by Giles, Garrison, and Hamilton.[3] Bombardments were carried out at 48.6 and 28 Mev, the latter energies to insure correct isotopic assignment of the conversion-electron lines observed.

The instruments used almost exclusively in this study were five permanent-magnet electron spectrographs[4] with fields of 52.6, 99, 160, 246, and 350 gauss.

The sources used in these spectrographs were prepared by the cathodic electrodeposition of the rhenium activity (probably as a basic oxide) on 0.010-inch platinum wires from a $6N$ $(NH_4)_2SO_4$ solution of about pH 2. The plating cell used has been described previously.[4] The plating was carried out with a current of 50 to 75 milliamperes.

In making these sources considerable difficulty was encountered from the deposit of a black substance, probably a basic platinic oxide, on the wire. Since the presence of this material caused a considerable increase in source scattering, with consequent decrease in resolution, studies were made to determine the conditions which minimized this effect. These studies, while not exhaustive, indicated that the optimum conditions were a short plating period ($\leqslant$ a half hour) and about $6N$ concentrations. It was found also that after the cell had been used for a number of hours the amount of deposit decreased. The sources which were used to prepare the several exposures which are the basis of the present study appeared to have little if any of this deposit: hence the resolution of the lines in the spectra was essentially the theoretical resolution of the spectrograph with a 0.010-inch source.

The momentum resolution (full width at half-maximum) that is usually quoted[4] for these spectrographs is about 0.1%. This error is a combination of the error in reading the line and the uncertainty in the determina-

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[1] G. Wilkinson and H. G. Hicks, Phys. Rev. 77, 314 (1950).

[2] Murray, Boehm, Marmier, and DuMond, Phys. Rev. 97, 1007 (1955).

[3] Giles, Garrison, and Hamilton, J. Chem. Phys. 18, 995 (1950).

[4] W. G. Smith and J. M. Hollander, Phys. Rev. 101, 746 (1956).

tions of the magnetic field. In this discussion we assume the radioactive source is an infinitely thin (nonscattering) layer on the 0.010-inch wire, an assumption we have achieved practically.

In the present study we assign an error of 0.05% to the energies of most of the transitions between 100 kev to 250 kev. This is possible because only one plate was used to determine the electron line energies, and a large number of conversion electron lines of transitions whose energies had been measured absolutely by MBMD were present on the plate, allowing accurate calibration of the field. Using these lines as calibration points, the value of the field as a function of linear distance along the plate was interpolated for each of the new electron lines. Because the field is very nearly uniform, the interpolation was not difficult. The error in the precision to which the lines were read was minimized by requiring four people to read each line until an arbitrary small difference in their readings was obtained. The energies of most of the new transitions reported are based on several conversion electron lines, each of which constituted an independent determination of the transition energy. The energy values reported are averages of the energies of the various lines. The agreement among the energies of the conversion lines for all the transitions is consistent with the assignment of 0.05% error to the transition energies between 100 and 250 kev. Larger errors are assigned for higher and lower energy transitions. The electron binding energies used throughout this work are those of Hill.[5]

For the low-energy spectrum the most important results were obtained using the 99-gauss permanent-magnet spectrograph. The results of the high-energy spectrum were obtained almost entirely from the 350-gauss spectrograph.

As an example of the resolution obtained by the 99-gauss spectrograph, and also of the complexity of the electron spectrum, we illustrate in Fig. 1 a densitometer trace of a permanent-magnet plate showing the region around 140 kev.

In electron-spectroscopic studies of a mixture of radioactive isotopes, a sequence of steps is usually followed, namely, the assignment of electron lines to isotopes, the assignment of electron lines to transitions (and, if possible, the assignment of multipolarities to transitions from the relative intensities of L or M subshell conversion), and finally the determination of the decay scheme.

In the study of Re^{182} we were able to identify most of the lines fairly readily by a series of exposures on the permanent magnets and by bombardment above and below the threshold for the $Ta^{181}(\alpha,3n)Re^{182}$ reaction. By these methods we were able to distinguish the 60-hour Re^{182} lines from those of isomeric 13-hour

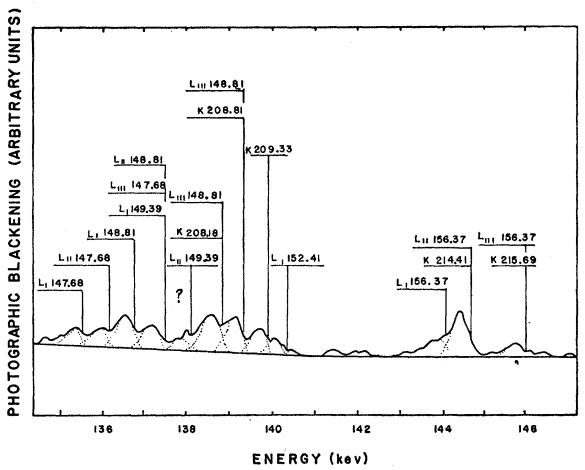

FIG. 1. Densitometer trace of the group of electron lines between 133 and 145 kev observed in the decay of 60-hr Re^{182}. The analysis is indicated. In cases where alternative assignments are possible the assignment preferred is the higher (highest) of the transition energies indicated.

Re^{182}.[6,7] 20-hour Re^{181},[6,8] 50-day Re^{184},[6,9] and 71-day Re^{183}.[6,9,10] The assignment of the electron lines to transitions was much more difficult because of the possibility, especially at low energy, of misassigning lines, and the ever present possibility of the accidental superposition of lines. The resolution of the spectrum, however, allowed us to assign many multipolarities, because the activity levels used in most of the experiments were sufficiently high to enable us to see the L- and higher-shell conversion lines of almost every transition. These transitions, the electron lines observed, and the intensity of these lines are listed in Table I, with the letter designations based on the decay scheme of Fig. 5. For the sake of completeness we have included some W^{182} transitions observed in Ta^{182} decay, but not observed by us, in Table I. These transitions are denoted by the superscript (c) after the energy. We have also quoted several different sets of limits of error in energy values. The transition energies indicated by superscript (b) were assigned by MBMD and are so designated because they were used to provide the field calibration for the new transitions. In the case of the three new high-energy transitions that we report, the energies of Bäckstrom[11] rather than MBMD were used as calibration. Thus in the case of the high-energy transitions we quote Bäckstrom's limits of error rather than MBMD's.

The numerical intensities given in Table I were determined from the photographic films and have

[5] R. D. Hill, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (Interscience Publishers, Inc., New York, 1955), Appendix VI, p. 915.

[6] C. J. Gallagher, Jr., University of California Radiation Laboratory Report UCRL-3928, 1957 (unpublished).

[7] Gallagher, Newton, and Shirley (to be published).

[8] Gallagher, Sweeney, and Rasmussen, Phys. Rev. **108**, 108 (1957).

[9] Gallagher, Strominger, and Unik, Phys. Rev. **110**, 725 (1958).

[10] Thulin, Rasmussen, Gallagher, Smith, and Hollander, Phys. Rev. **104**, 471 (1956).

[11] G. Bäckstrom, Arkiv Fysik **10**, 387 (1956).

estimated probable errors of ±20%. We arrived at this intensity error (low for photographic films) by determining that the intensities calculated from the photographic blackening were consistent with intensities measured by a conventional variable-field spectrometer, and hence were consistent with the error conventionally assigned intensities from such a spectrometer. The actual calculation and comparison of intensities were carried out in the following manner. To convert photographic blackening to numerical intensities it is necessary to know the efficiency of the film as a function of energy. This was determined for the Kodak No-Screen x-ray film used in the present study by Canavan,[12] using the Berkely Svartholm-

TABLE I. Transitions in W^{182} following the decay of 60-hr Re^{182}. Electron intensities are on an arbitrary scale.

Initial and final states	Transition energy (kev)	Electron intensities K	L_I	L_{II}	L_{III}	M_I	$M_{II}M_{III}$	N_I	$N_{II}N_{III}$	Total electron intensity	Multipolarity assignments First confidence	Second confidence
RQ	19.86±0.05					a	a	a	a	···		(*E*1+*M*2)
ED	33.36±0.01[b,e]									···	*E*1	
NM	39.10±0.05		13	a		a				13	(*M*1)	
HG	42.71±0.01[b,e]									···	(*E*1)	
KJ′	52.96±0.05		a			a				···		(*M*2)
RP	60.51±0.05		4			a				4	(*M*1)	
KJ	65.71±0.01[b]		97	15	34[d]	24				140	*M*1+*E*2	
FD	67.74±0.01[b]		34[d]	11	12	17	a a	1.2		68	*E*1	
MK	68.10±0.08			a,e						···		(*E*2)
IG	74.41±0.05		a	f	g					···		(*E*1)
HF	84.67±0.02[b]	250	49	17	13	26	a a	a	a	360	*M*1+*E*2	
BA	100.09±0.02[b]	130		∼1080			∼260		h	1470	*E*2	
NK	107.13±0.05	29	5.9	8.8	6.0					50	(*M*1+*E*2)	
PN	108.57±0.05	19	4.4			0.8		<0.6		24	(*M*1)	
JH	113.66±0.02[b]	170	34	5.8	58[i]	12	3.4	a		220	*M*1+*E*2	
IF	116.40±0.02[b]	7.2[j]	k	l						···		(*M*1+*E*2)
MJ′	120.94±0.06	a	m							···		(*E*1)
J′H	126.40±0.06	n										
SR	130.76±0.07	220	40	15	7.3	12		6.0		290	(*M*1+*E*2)	
TR	131.30±0.07	a,o								···		···
MJ	133.78±0.07	80[g]	100[p]			5.3				98	(*M*1)	
PM	147.68±0.07	15	5.6	3.7	<7.5[q]					27	(*M*1+*E*2)	
PL	148.81±0.07	46	9.2	<7.5[q]	15[r]					<78		(*E*1+*M*2)
QN	149.39±0.07	16	<7.5[q]	≤2.6[s]	t					<26		(*E*1)
TQ	151.19±0.08	a								···		···
HD	152.41±0.03[b]	>12[u]	3.6							>16	*E*1	
JG	156.37±0.04[b]	>4.0[u]	4.9	13[v]	3.5[w]					>8.9	*E*1	
NJ′	160.09±0.08	>6[u]	3.4							>9.4		(*E*1)
RN	169.18±0.08	256[h]	43	3.5	x	13		1.3		315	(*M*1)	
NJ	172.78±0.09	58[i]	10			2.5				71	(*M*1)	
KH	179.36±0.05[b]	31	7.2	3.3	1.2	1.2				44	*M*1+*E*2	
OK	181.63±0.09	3.2								3.2		(*M*1)
QL	189.48±0.10	6.2								6.2		(*M*1)
SP	191.31±0.10	100[p]	16							<116	(*M*1)	
JF	198.31±0.06[b]	19		85[y]	6.1		z			34	*E*2	
RM	208.18±0.10	15[r]	<3			aa				<18	(*M*1)	
RL	209.33±0.10	5.5								5.5		(*E*1)
LI	214.41±0.11	13[v]	aa			aa				<13		···
PK	215.69±0.11	3.5[w]		aa	aa					3.5	(*E*2)	
(bb)	221.60±0.11	9.5								9.5		···
KG	222.05±0.07[b]	7.9	aa							7.9	*E*1	
TO	226.10±0.11	30	6.9			aa				37		···
CB	229.27±0.08[b]	96		aa	aa					96	*E*2	
MH	247.43±0.12	19	aa	aa	aa					19	(*E*2)	
QK	256.37±0.13	85[y]	13			aa				<98	(*M*1 or *M*2)	
KF	264.09±0.10[b]	12	a	a	a					12	*E*2	
RK	276.30±0.14	15	aa	aa	aa					15	(*E*2)	
PJ	281.42±0.14	8.6	aa	aa	aa					8.6	(*E*2)	
NH	286.52±0.14	37	aa	aa	aa					37	(*E*2)	
SN	299.88±0.15	2.5[cc]	dd	aa	aa					2.5	(*E*2)	

[12] F. L. Canavan, 1956 (unpublished).

TABLE I.—(*Continued*).

Initial and final states	Transition energy (kev)	Electron intensities K	L_I	L_{II}	L_{III}	M_I	$M_{II}M_{III}$	N_I	$N_{II}N_{III}$	Total electron intensity	Multipolarity assignments First confidence	Second confidence
TN	300.49±0.15	3.0[cc]	dd							3.0		...
SM	338.98±0.17	(4.8)[ee]	aa	aa	aa					4.8	(*E*2)	
C'C	351.02±0.18	(2)[ee]	aa	aa	aa					2	(*E*2)	
EC	927 ±1[b]	c									(*E*3)	
FC	960 ±1[b]	c									(*E*3)	
GC	1003 ±1[b]	c									(*M*1+*E*2)	
IC	1076.7 ±0.6	aa										(*E*1+*M*2)
DB	1121.6 ±0.2[ff]	aa	aa								*M*1+*E*2	
EB	1155 ±1[b]	c									*E*1+*M*2	
JC	1158 ±0.6	aa										(*E*1+*M*2)
FB	1189.3 ±0.2[ff]	aa									*E*1+*M*2	
DA	1221.8 ±0.2[ff]	aa									*E*2	
GB	1231.3 ±0.5	aa									*M*1+*E*2	
EA	1254[ff]	gg										(*E*1)
HB	1273[ff]	gg										(*E*1+*M*2)
FA	1289 ±1[b]	c									(*M*2)	
HA	1375 ±2[b]	c									(*E*3)	
?	1437 ±4[b]	hh										
KB	1454 ±4[b]	c									(*M*2+*E*3)	

[a] Intensity too weak to be obtained from densitometer trace.
[b] These transition energies and limits of error were assigned by MBMD. Conversion lines of these transitions were used to provide the field calibrations for the new transitions.
[c] Not seen in 60-hr Re^{182} decay.
[d] L_I 67.74, L_{II} 65.71 superimposed.
[e] The assignment of this line is questionable.
[f] L_{II} 74.41, M_I 65.71 superimposed.
[g] L_{III} 74.41, K 133.78 superimposed.
[h] K 169.18, $N_{II}N_{III}$ 100.09 superimposed.
[i] K 172.78, L_{III} 113.66 superimposed.
[j] K 116.40, KL_IL_{III} superimposed.
[k] L_I 116.40, M_I 107.13 superimposed.
[l] L_{II} 116.40, M_{III} 107.13 superimposed.
[m] Seems too intense to be L_I 120.94, but because K is on a dark background, it is difficult to be sure.
[n] KL_IN_I–N_{III} would be superimposed on K 126.40.
[o] This line observed only weakly on one plate. It may belong to Re^{181}.
[p] K 191.31, L_I 133.78 superimposed.
[q] L_I 149.39, L_{II} 148.81, L_{III} 147.68 superimposed.
[r] K 208.18, L_{III} 148.81 superimposed.
[s] This line visible only on densitometer trace as a low-energy tail of K 208.81 (Re^{183}) line.
[t] K 208.81, L_{III} 149.39 superimposed. A comparison of intensities of Re^{183} transitions in sample with the intensities of the Re^{183} lines in a pure Re^{183} sample indicates a very low intensity for the L_{III} 149.39.
[u] On very dark background.
[v] K 214.41, L_{II} 156.37 superimposed.
[w] K 215.69, L_{III} 156.37 superimposed.
[x] K 229.27, L_{III} 169.18 superimposed.
[y] K 256.37, L_{II} 198.31 superimposed.
[z] L_I 208.81 (Re^{183}), $M_{II}M_{III}$ 198.31 superimposed.
[aa] Weak line observed on higher-field, less-accurately-calibrated magnets.
[bb] Transition not assigned in decay scheme. K 221.60, L_{III} 162.33 (Re^{183}) superimposed.
[cc] The sum of the intensities of the lines was observed as equal to 5.6 on the exposure used to calculate intensities, but the lines were not resolved. The intensity ratio was estimated visually from another exposure.
[dd] This line was diffuse and difficult to read.
[ee] These intensities were estimated from a second plate.
[ff] Limits of error assigned by Bäckstrom.
[gg] Lines reported by Bäckstrom and not by MBMD in the decay of Ta^{182}. Not observed in Re^{182} decay.
[hh] This transition was assigned as *IA* by MBMD. Because we have reassigned level *I*, it no longer fits into the decay scheme. It should be noticed that the K 1437 and L_I 1375 would be superimposed.

Siegbahn spectrometer and a standard beta-ray source. A Dietart ARL recording photometer was used to provide a tracing of the photographic intensities. The method of Mladjenović and Slätis[13] was then used to correct the peak heights on the photometer tracing to numerical intensities. The most difficult part of this method is to determine the relationship between observed peak height (i.e., photographic blackening) and numerical intensity, because the scale is not linear. This was done by assuming the validity of MBMD's electron intensities for the transitions depopulating state K. Since there are a number of transitions, depopulating this level, each with several conversion electron lines, this provided lines in all energy regions (and consequently in regions with a wide range of background intensity). The correction curve was then adjusted to yield the MBMD intensities for these transitions. (It should be remarked that the K line of the 222.05-kev transition appears more intense than MBMD report.) To check this curve, the intensities of the stronger lines calculated from it were compared to the intensities of the same lines obtained by integrating the areas of the conversion lines in a spectrum taken at about 0.3% resolution in the Svartholm-Siegbahn spectrometer and dividing the intensity by the $H\rho$ of the line. All of the intensities so compared agreed to within the 20% limit of error and gave us confidence

[13] M. Mladjenović and H. Slätis, Arkiv Fysik 8, 65 (1954).

both in our own and the MBMD electron intensities. The reason for not using the Svartholm-Siegbahn spectrometer to obtain intensities for all transitions was because many groups of lines were not resolved by it.

It should be noted here that the correction curve used was used for only one plate from which all the reported transitions and transition intensities below 250 kev are taken. We believe that the 20% limit of error quoted (which applies only to the deviation about the correction curve used, and does not apply to any error in the correction curve itself) is internally consistent with all the results we have obtained. We feel, however, that as with any intensities determined by photographic methods, these results should be checked with high-resolution variable-field spectrometers.

It is immediately apparent from Table I that the extreme complexity of the gamma spectrum precluded meaningful scintillation studies of this isomer. However, gamma scintillation studies were carried out using a 1 in.$\times 1\frac{1}{2}$ in. diameter NaI(Tl) scintillation spectrometer with 50-[14] and 100-channel[15] differential pulse-height analyzers. A typical NaI scintillation spectrum of 60-hour Re^{182} taken with the 100-channel analyzer is illustrated in Fig. 2. From this we obtained the ratio of the composite 1122–1222 kev peak intensity to the K x-ray peak. Using a K-fluorescence yield of 0.95 and this measurement, we obtained a ratio of K x-rays to 1122–1222 gammas of 2.3.

The gamma scintillation studies were used to check the half-life of the isomer. The decay of the 1122–1222 peaks was followed for five half-lives after all the 13-hour Re^{182} had decayed away, and the decay was a simple exponential with $t_{\frac{1}{2}}=60\pm4$ hours, in reasonable agreement with the value of 64 hours reported by Wilkinson and Hicks.[1]

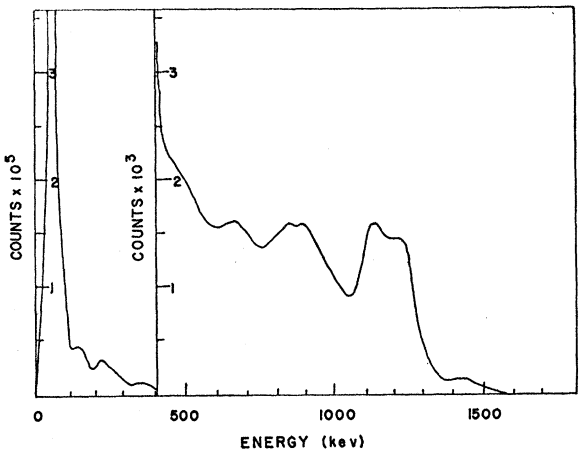

FIG. 2. Gamma-ray spectrum of Re^{182} obtained with a NaI(Tl) scintillation spectrometer.

[14] A. Ghiorso and A. E. Larsh, University of California Radiation Laboratory Report UCRL-2647, July, 1954 (unpublished).
[15] 100-channel gamma analyzer, manufactured by the Pacific Electro-Nuclear Company, Culver City, California.

DISCUSSION OF RESULTS

Decay Scheme

The good energy resolution together with multipolarity assignments for the more intense transitions enables us to build on the existing level scheme of W^{182} proposed by MBMD. Many transitions, assigned on the basis of a number of their conversion lines, were found to fit into this scheme. The scheme also predicted weak transitions, whose K- or L-conversion lines were observed but were unassigned. Only 8 electron lines assigned to Re^{182} are not assigned definitely to transitions in the decay scheme. These eight, with probable assignments, are listed in Table II. All of these lines were extremely weak.

Because much of our interpretation assumes the correctness of the decay scheme of MBMD, we shall review other experiments (Coulomb excitation of W^{182} and studies of the decay schemes of Ta^{182}) which support it.

Coulomb excitation[16–20] of W^{182} has verified level B. Mihelich[21] has performed gamma-gamma coincidences that indicate that DB (1122) and BA (100.09) are in coincidence, while DA (1222) and BA are not. More recently, very thorough coincidence studies by Fröman and Ryde[22] have shown the complete consistency of the MBMD decay scheme. The gamma intensities that they obtain are not in good agreement with those reported by MBMD, however, but indicate the need for a linear correction to be applied to the MBMD data. The calculations that we have made using MBMD intensities must therefore be considered tentative, but the conclusions we have reached are not changed. (See, e.g., the notes to Table V.) Williams and Roulston have performed gamma-gamma angular correlation experiments on the cascades F–D–A, H–D–A, and F–D–B, and their results agree with spin assignments

TABLE II. Unassigned internal-conversion electron lines of transitions of W^{182} following Re^{182} decay.

Energy (kev)	Possible assignment
54.99	KL_IM Auger electrons
56.64	L_{II} 68.10 kev
58.86	$KL_{III}N$ Auger electrons
66.28	KMN Auger electrons
74.86	L_{II} 86.40 kev
76.16	L_{III} 86.36 kev
108.99	L_I 120.94 kev
187.44	Probably film imperfection (PM IV, plate 397)

[16] T. Huus and J. H. Bjerregaard, Phys. Rev. **52**, 1579 (1953).
[17] McClelland, Mark, and Goodman, Phys. Rev. **93**, 904 (1954).
[18] P. H. Stelson and F. K. McGowan, Phys. Rev. **99**, 112 (1955).
[19] E. M. Bernstein and W. H. Lewis, Phys. Rev. **99**, 617(A) (1955).
[20] Clark, DuMond, Gordon, and Mark, Bull. Am. Phys. Soc. Ser. II, **2**, 69 (1957).
[21] J. W. Mihelich, Phys. Rev. **95**, 626(A) (1954).
[22] P. O. Fröman and H. Ryde, Arkiv Fysik **12**, 399 (1957).

to these levels of 2–2–0, 3–2–0, and 2–2–2, respectively.[23] Bäckstrom[11] (hereafter referred to as Bä) has carried out high-resolution electron spectroscopy on the transitions *DB*, *FB*, and *DA* and has determined the energies 1121.6±0.2 kev, 1189.3±0.2 kev, and 1221.8±0.2 kev, respectively. He has also observed, for the first time, conversion lines that correspond to transitions *EA* (1254) and *HB* (1273). Because of the high-resolution gamma spectroscopy employed by MBMD to study the low-energy transitions in Ta^{182} decay, the energies and arrangement of levels *D* through *K* (excepting *I*) seem excellent. The careful studies mentioned above gave confidence for building upon this level scheme for interpretation of the highly complex decay of 60-hr Re^{182}.

In order to determine whether the levels of W^{182} populated by Ta^{182} decay were also populated by Re^{182} decay we first determined whether we saw all of the low-energy transitions (<300 kev) reported by MBMD. We found that all except the 33.36- and 41.72-kev transitions were seen. Because these two are very weak

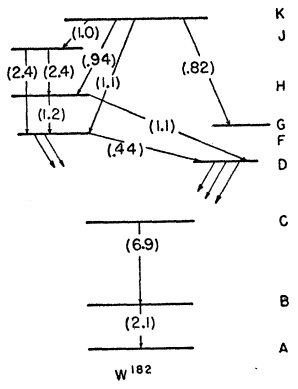

FIG. 3. Comparison of electron intensities of transitions observed in both Ta^{182} and 60-hr Re^{182} decay. The ratios indicated on the transitions are normalized so that the ratio of the electron intensity of the L_I 65.71-kev transition observed following Re^{182} decay to that observed following Ta^{182} decay is 1.

in Ta^{182} decay, their absence in no way effects the levels proposed by MBMD.

Because of the low transmission of our high-resolution instruments, we were able to see the electron lines of only six high-energy transitions. Two of these, the 1158- and 1076.6-kev transitions, have not been reported previously. The 1158 was found to fit the energy difference *JC* very well and has been so assigned; the 1076.6 has been assigned as transition *IC*. Although we have made this assignment because of the energy agreement, we think it is necessary to indicate that the transitions feeding level *I* are apparently less intense than those depopulating it, and hence this part of the decay scheme deserves further study.

Another useful item of information, adding confidence to the MBMD scheme, was obtained by comparing the relative intensities of transitions common to both decay schemes. We did this by comparing our electron intensities with the electron intensities of MBMD calcu-

[23] R. C. Williams and R. I. Roulston, Can. J. Phys. 34, 1087 (1956).

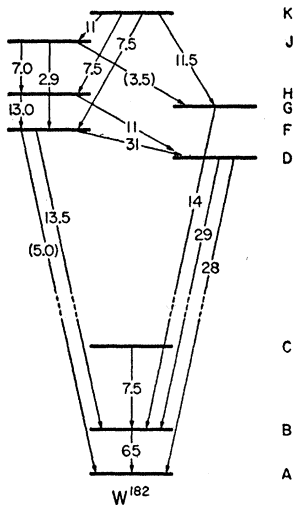

FIG. 4. Transition intensities reported by MBMD for the transitions of W^{182} following Ta^{182} decay. The intensities are normalized so that a total of 100 populates the ground state.

lated by multiplying their gamma intensities by the experimental conversion coefficients they report. The ratios of our electron intensities to MBMD's for these transitions are illustrated in Fig. 3, which is a schematic diagram of the MBMD decay scheme. The numbers on each transition are the ratio of the electron intensities of the conversion lines of the transition in the Re^{182} spectrum to those in the Ta^{182} spectrum. The ratio for each transition is the average of the ratios for all the conversion lines seen in both spectra. All ratios are normalized by assuming that the ratio for the L_I of the 65.71-kev transition is 1.

This comparison is valid, although we did use MBMD's election intensities for branching from level *K* to calibrate our photographic blackening curve, because this curve was checked against intensities we obtained in the Svartholm-Siegbahn spectrometer. The particular order for calibrating and checking the curve was chosen because it was easier and quicker than the alternate order.

A positive check on the MBMD level scheme is provided by these ratios. If their scheme is correct, the ratios that we calculate for the transitions depopulating one of their levels will be equal, within experimental error, whereas this would probably not be so if they have misassigned some of the transitions. From the ratios shown in Fig. 3 it can be seen that our results are consistent with the MBMD level scheme, because the deviation in the ratios is within the probable error of the electron intensities. In order to facilitate comparison with the results of MBMD, we illustrate in Fig. 4 a schematic drawing of the MBMD level scheme, similar to Fig. 3. The numbers on each transition in this case, however, are their observed decay fractions carried by the transition, normalized so that an intensity of 100 feeds the ground state. From a comparison of Fig. 3 and Fig. 4 we can readily draw some conclusion about differences in level populations between Ta^{182} and Re^{182} decay. It is clear from the comparison that a large per-

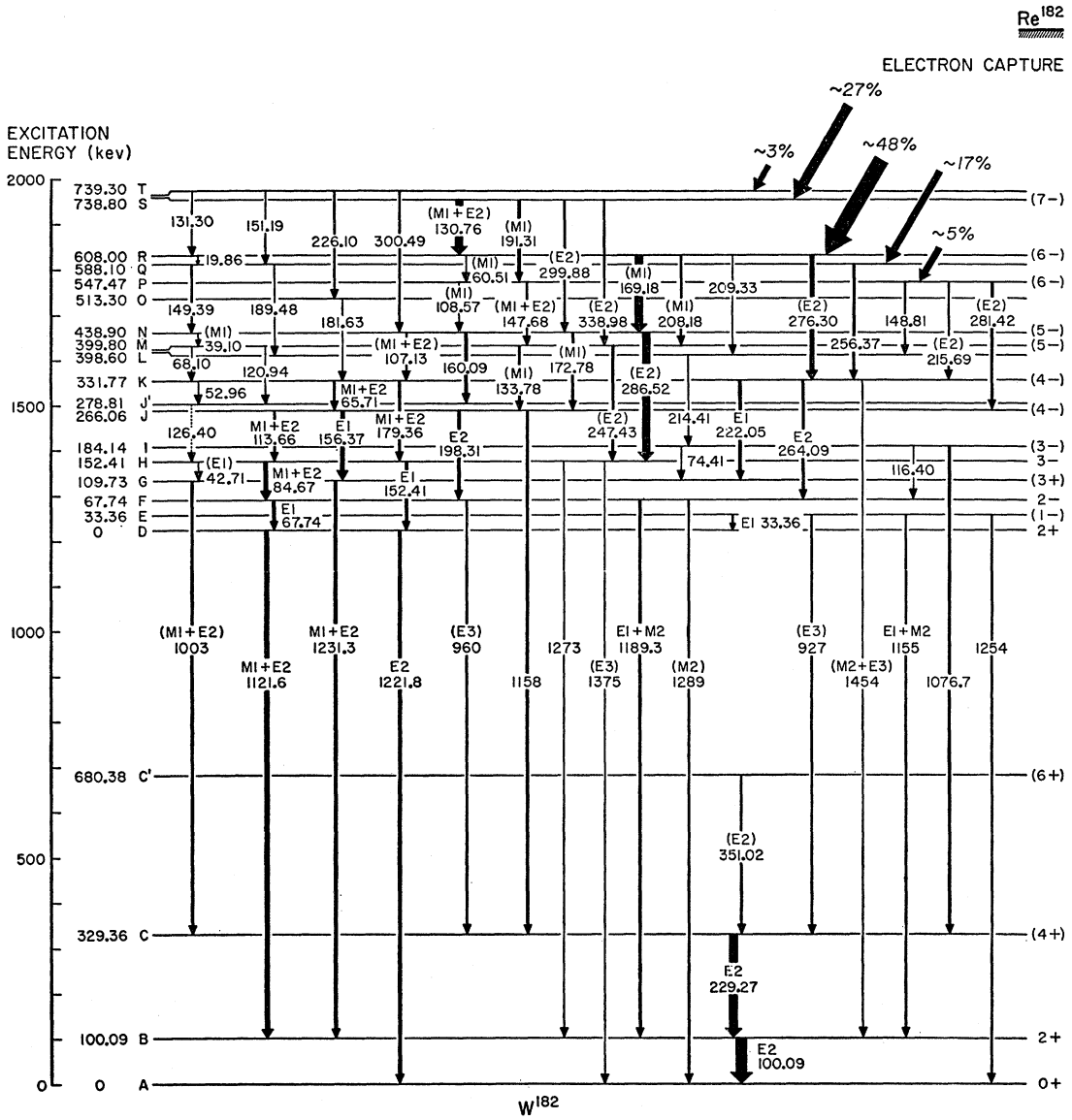

FIG. 5. Decay scheme of 60-hr Re^{182}.

centage of depopulation in Re^{182} decay cascades through states J and K, whereas in Ta^{182} decay a large part of the primary beta decay goes directly to level F and H. It also indicates a much larger population of level C than in the former case, thus indicating that states of higher spin must receive more direct population in Re^{182} decay.

A further check has been provided on the MBMD level scheme by the decay of 13-hour Re^{182} which has been observed to populate only the W^{182} levels populated by Ta^{182} decay. However, these results will be reported elsewhere.[7]

Figure 5 illustrates our proposed decay scheme for Re^{182}. It includes all the transitions reported by MBMD (including those unobserved by us) and the new transitions reported by Bä (also unobserved by us). This level scheme is based primarily on agreement of energy sums of pairs of transitions being equal to the energy of a third transition. Because of the complexity of this decay there are many accidental energy sums; thus there are several alternative decay-scheme possibilities if the energies alone are considered. Fortunately, the intensities and multipolarities of the observed transitions often guided the choice of alternative level schemes. The new parts of the scheme of Fig. 5 are based solely on conversion-electron-spectroscopic results. We will discuss the sums that constitute the basis of the decay scheme later.

Discussion of the New Levels in W^{182}

As previously stated, the energies of the levels were based primarily upon sums and differences. The levels may be considered to fall into three "confidence" groups. The first is those levels that were placed from differences equal to differences between levels established by MBMD. The second is those placed by differences corresponding to differences between states established by MBMD and our "primary" states, and the third arises from differences not involving any levels of MBMD. Levels I, M, and N, are in the first group; J', P, Q and R, are in the second group; and L, O, S, and T, in the third. The agreement of the sums for the various groups is shown in Table III. We do not include all possible differences in Table III, but rather only those that directly support each state. Level C' is so placed because $C'C$ is $E2$, and its energy fits the theoretically predicted rotational spacing for the 6+ state of the ground rotational band of W^{182}. Because all of the levels except C' and O have at least three transitions tying them into the decay scheme (and some have as many as seven), we feel that the statistical significance of the scheme is quite large. Before discussing the level spin assignments we shall digress first to discuss the method of determining multipolarities and then the transitions for which multipolarities are not very well established. This will then allow us to discuss the spin assignments of the levels. The multipolarities were determined primarily by L-subshell conversion ratios. Because of the high Z dependence of internal conversion, this method is fairly reliable for $Z=74$ and low energies (<350 kev). The L_{I}, L_{II}, and L_{III} conversion coefficients for $M1$, $M2$, $E1$, and $E2$ transitions were interpolated from Rose's theoretical values.[24] Although Rose's values uncorrected for finite size were used in most of the calculations, when we calculated the mixing ratios in Table IV the newer values corrected for finite size were used.[25] We have not considered higher multipole orders for the low-energy transitions because their much longer half-lives were expected to prevent their competing favorably with the dipole and quadrupole radiations.

At low energies the presence of L_{II} conversion comparable to L_{I} is usually indicative of electric radiation. L_{III} conversion stronger than L_{II} conversion is usually indicative of $E1$ or $M2$ radiation at the lower energies.

The L_{II} and L_{III} conversion is greatest for the electric quadrupole, for which the L_{I} conversion only becomes equal to the L_{II} at about 350 kev. In the $E1$, however, the $L_{\mathrm{I}}/L_{\mathrm{II}}$ ratio is unity at about 25 kev, three at 100 kev, and about seven and a half at 350 kev. In both cases the $L_{\mathrm{III}}/L_{\mathrm{II}}$ ratio decreases slowly. From these considerations it is clear that for transitions between 250 and 350 kev the presence of L_{II}–L_{III} conversion in intensity greater than that of the L_{I} is indicative of $E2$ character. New transitions which were assigned $E2$ character on this basis are PK, MH, RK, PJ, NH, SN, SM, and $C'C$. We assigned these transitions pure $E2$ character, although $M1$ mixing cannot be excluded. The amount of $M1$ admixture is very small if present; the $L_{\mathrm{I}}/L_{\mathrm{II}}$ ratio, wherever it was possible to observe it, was in good agreement with (if not somewhat smaller than) the theoretical $L_{\mathrm{I}}/L_{\mathrm{II}}$ ratio for $E2$.

In the case of the three new transitions assigned as $M1$–$E2$ mixtures, transitions NK, SR, and PM, the $L_{\mathrm{I}}/L_{\mathrm{II}}/L_{\mathrm{III}}$ ratios ruled out $E1$. The remaining transitions in Table I with $M1$–$E2$ mixing were so assigned on the basis of MBMD's results.

Above 100 kev it really becomes difficult, on the basis of electron spectroscopic results alone, to distinguish between $M1$, $M2$, and $E1$ transitions. Sometimes, however, it is possible to differentiate between them by remembering that the total conversion of magnetic transitions is much higher than that of electric transitions. Therefore, if we know that a certain transition in this energy range is a pure $E1$ of total intensity comparable to strong $M1$ and $E2$ transitions in the

TABLE III. Energy differences (in kev) supporting the proposed W^{182} level scheme.

Transition 1	Transition 2	Difference	Transition 3	Energy of state above D	State designation
		Group I			
1076.7	74.41	1002.3	1003	184.14	I
1076.7	116.40	960.3	960	184.14	I
247.43	133.78	113.65	113.66	399.84	M
172.78	107.13	65.65	65.71	438.84	N
286.52	172.78	113.74	113.66	438.93	N
		Group II			
160.09	120.94	39.15	39.10	278.81	J'
160.09	52.96	107.13	107.13	278.81	J'
281.42	215.69	65.73	65.71	547.48	P
281.42	147.68	133.74	133.78	547.48	P
281.42	108.57	172.85	172.78	547.48	P
256.37	149.39	106.98	107.13	588.14	Q
276.30	169.18	107.12	107.13	608.07	R
208.18	169.18	39.00	39.10	607.98	R
		Group III			
215.69	148.81	66.88	66.83[a]	398.60	L
209.33	189.48	19.85	19.86	398.67	L
209.33	148.81	60.52	60.51	398.67	L
338.98	299.88	39.10	39.10	738.78	S
338.98	191.31	147.67	147.68	738.78	S
338.98	130.76	208.22	208.18	738.78	S
191.30	130.76	60.54	60.51	738.78	S
151.19	131.30	19.89	19.86	739.28	T
300.49	169.18	131.31	131.30	739.34	T

[a] The line supporting this transition can also be assigned as the L_{II} 68.10.

[24] M. E. Rose (privately circulated tables). Also, with G. H. Goertzel, in *Beta- and Gamma-ray Spectroscopy*, edited by K. Siegbahn (Interscience Publishers, Inc., New York, 1955), Appendix IV, p. 905.

[25] M. E. Rose (privately circulated tables, to be published).

TABLE IV. Comparison of theoretical and experimental conversion coefficients of transitions in W^{182}.

Gamma-ray energy (kev)	Source	K	L_I	L_{II}	L_{III}	Multipolarity and mixing ratio $M1/E2$
			(a) $M1+E2$ Mixtures			
65.71	This work		1.8	(0.17)	a	>99/<1
	Theoretical[b]		1.85	0.17		
	MBMD[c]		2.2	0.32	a	
84.67	This work	d	(0.76)	(0.26)	0.25	89/11
	Theoretical		0.76	0.26	0.20	
	MBMD		1.44	0.48	0.4	
113.66	This work	1.8	(0.35)	(0.56)	0.02	96/4
	Theoretical	2.6	0.35	0.56	0.02	
	MBMD	1.4	0.32	0.056		
179.36	This work	0.3	(0.07)	(0.03)	0.01	64/36
	Theoretical	0.57	0.07	0.03	0.02	
	MBMD	0.33	0.12		0.005	
107.13	This work	1.3	(0.26)	(0.39)	0.3	52/48
	Theoretical	1.9	0.26	0.39	0.35	
130.76	This work	1.0	(0.20)	(0.07)	0.05	82/18
	Theoretical	1.6	0.20	0.07	0.04	
147.68	This work	0.76	(0.12)	(0.08)		61/39
	Theoretical	0.93	0.12	0.08		
			(b) $E1$ Transitions			
67.74	This work		a	(0.034)	0.037	
	Theoretical		0.077	0.034	~0.04	
	MBMD		0.14[a]	0.056	0.056	
152.41	This work	~0.035[e]	(0.011)			
	Theoretical	0.11	0.011			
	MBMD	0.056				
156.37	This work	~0.008[e]	(0.010)			
	Theoretical	0.11	0.010			
160.09	This work	~0.02[e]	(0.009)			
	Theoretical	0.09	0.009			

[a] L_{III} 65.71, L_I 67.74 superimposed.
[b] The theoretical K-shell coefficients are those of Sliv and Band,[26] corrected for finite nuclear size. The L-conversion coefficients are the new values of Rose,[25] corrected for finite nuclear size. Unfortunately the latter are subject to about 5% error in interpolation for $Z=74$.
[c] The published values of these authors have been corrected for finite size by the ratio α_K(Rose, point nucleus)/α_K(Sliv, finite size) =0.80, where the α_K is that of the 246.05-kev $M1$ transition in W^{183} which was used to normalize the original data.
[d] The K line of this transition was observed but the energy corresponds to an energy at which the photographic efficiency curve is not reliable.
[e] These intensities may be 2 or 3 times too small because the lines are on an intense background. Unless something completely unexpected is affecting the photographic blackening of the plate here, however, the K/L_I ratio is still much too small.

sample, we can use its electron-conversion lines to serve as a sort of internal intensity standard. We can do this if we say that all transitions with electron lines three or more times more intense than those of the standard cannot be $E1$ because then the total intensity of this transition would be too large to be consistent with the decay scheme. In W^{182} there are three transitions, JF, HD, and KF, whose multipolarities were established by MBMD, which we used in this way. The decay scheme was used where possible to differentiate between $M1$ and $M2$.

We have not found it possible to differentiate between weak $M1$ or $M2$ and strong $E1$ transitions on the basis of subshell conversion alone.

From subshell-conversion ratios alone it has not been necessary to assign to any electric transition a multipole order higher than $E2$. In three or four cases there appeared to be $M2$ character in the transitions. In at least one of these cases (PL) this may be $E1$–$M2$ mixing.

Discussion of Transitions

In the interpretation of as complex a spectrum as that of Re^{182}, it is probably not possible to be entirely correct in the analysis, especially because single lines in the low-energy spectrum can have alternative assignments. For the most part, however, we have been able to avoid basing assignments on single lines, the majority being based on two or more. In order to provide a classification for the degree of confidence we have in the transitions, we have arbitrarily decided to divide them into three groups. The first confidence group comprises most of the transitions in Table I, and are based upon the observation of two or more electron lines. The second confidence group, which consists of transitions IG, TR, TQ, OK, QL, RL, IC, JC, and the 221.60-kev unassigned transition, are transitions which are based on the observation of one reproducible line. The third confidence group consists of only two transitions, MK and $J'H$. The assignment of MK is based on only one weak line which can be assigned as the L_{II} line of such a transition. $J'H$ is probably nonexistent, but it is impossible to rule it out, because the decay scheme predicts such a transition and the K line would be superimposed on the K N_I–N_{III} Auger lines. From the intensity of these lines, however, this transition must be very weak.

Spin Assignments

The spin assignments of five of the new levels in W^{182} were based upon the $E2$ transitions. Negative parity was established for these five states since the $E2$ transitions populate the negative parity states assigned by MBMD. The levels so fixed are M, N, P, R, and S. The spin assignments are illustrated in Fig. 5. Most of these levels are also connected by transitions consistent with $M1$ and $M1$–$E2$ assignments, serving as a further check on the spins. We assume that the $E2$ transitions connect states with $\Delta I=2$ in order to assign the spins shown in Fig. 5, although it is possible that a predominantly $E2$ transition may connect states with $\Delta I=0$ or 1.

Besides these five levels, we place six other levels in the decay scheme. These are levels C', I, J', L, O, and T. On the basis of our experimental data we are unable to make definite spin or parity assignments to these levels; the spin and parity assignments that agree best with the observed data are $C'=6+$, $I=3-$, $J'=4+$, and $L=5+$. The branching from O and T is consistent with spins of 5 and 6, respectively, but the parity is uncertain.

It should be noted that the level we have assigned as I is not the MBMD level I. In their decay scheme they suggested that the 116.40-kev transition depopulated level K and populated a level at 215.3 kev above state D. Because their data were not definitive they regarded their assignment as tentative. The two other transitions that now establish our level I make it appear that the assignment of the 116.40-kev transition as IF is better justified than its previous assignment.

We have assigned the 1076.6-kev transition as IC in spite of the fact that neither MBMD nor Bä observed this transition. Because it should be observed in Ta^{182} decay if our assignment is correct, such studies should be made to check our assignment.

In Fig. 5 we only indicate possible spins and parities for the states connected by $E2$ radiation. In cases where the multipolarity of a transition is in doubt it is not indicated on the transition in Fig. 5. As stated previously, proposed assignments for these transitions are given in the "Second confidence" multipolarity column of Table I. We defer until later our analysis of the decay scheme, which we consider to be the most reasonable interpretation of the data. It is from this analysis, however, that the "Second confidence" multipolarities of the weaker transitions were deduced.

Conversion Coefficients and Mixing Ratios

Let us discuss the method of determining the mixing ratios listed in Table I. If we could assume that the theoretical conversion coefficients could be used it would be quite easy to obtain these ratios from our data. Such a calculation is possible because in most cases we have observed K, L_{I}, L_{II}, and L_{III} intensities whereas only two parameters, the mixing ratio and normalization constant (necessary because we have only measured electron intensities) are needed.

Unfortunately, from the point of view of such a calculation, evidence has recently been observed that indicates finite nuclear size[25–27] effects are especially important for $M1$ transitions and $S_{\frac{1}{2}}$ and $P_{\frac{1}{2}}$ electrons. Dynamic nuclear structure effects are expected to be important only in slow $E1$ or $M1$ transitions.[27–31] The problem is to choose the theoretical conversion coefficients which most closely correspond to actuality. Sliv and Band have calculated K-shell conversion coefficients corrected for finite size effects[26]; recently Rose has calculated K, L_{I}, and L_{II} subshell conversion coefficients corrected for these effects.[25] We have compared Rose's and Sliv's K-shell values for $Z=75$ and they are identical. We have therefore used Sliv's K-shell coefficients for $Z=74$, and interpolated between Rose's L_{I}, L_{II}, and L_{III} values for $Z=65$ and $Z=75$ to obtain L-subshell values for $Z=74$. Rose's L_{III} coefficients[24] were not corrected for finite size, as this effect is expected to be negligible for $P_{\frac{1}{2}}$ electrons.

In Table IV(a) we present a comparison of the theoretical conversion coefficients calculated as described above and our electron intensities normalized to the theoretical values for the $M1$–$E2$ mixture indicated. The normalization points are indicated by parentheses. We have also included the measured absolute conversion coefficients of MBMD for those transitions common to both Re^{182} and Ta^{182} decays. The MBMD values have been reduced by the ratio $\alpha_K(\text{Sliv and Band}^{26})/\alpha_K(\text{Rose}^{24})$ for the 246.05 $M1$ transition in W^{183} to renormalize their absolute values to the theoretical values for finite size rather than for the point nucleus approximation.

It is apparent from Table IV(a), that in every case where $M1$–$E2$ mixing occurs our experimental results indicate that the K conversion of these transitions is low. We at first suspected a systematic error in our intensities; however, after completing the comparison of our normalized coefficients with the corrected absolute experimental conversion coefficients of MBMD shown in Table IV, it became apparent that the experimental results are consistent and both are at variance with the theoretical values, even when the latter have been corrected for finite nuclear-size effects. These intensities should be checked again because the consistency of the experimental results does not eliminate the possibility of a systematic error in both sets of electron data. However, the general direction of the

[26] L. A. Sliv and I. M. Band, Leningrad Physico-Technical Institute Report, 1956 [translation: Report 57 ICCK1, issued by Physics Department, University of Illinois, Urbana, Illinois (unpublished)].
[27] A. H. Wapstra and G. J. Nijgh, Nuclear Phys. 1, 245 (1956).
[28] E. L. Church and J. Weneser, Phys. Rev. 104, 1382 (1956).
[29] S. G. Nilsson, University of California Radiation Laboratory Report UCRL-3803, June, 1957 (unpublished).
[30] S. G. Nilsson and J. O. Rasmussen, Nuclear Phys. 5, 617 (1958).
[31] Asaro, Stephens, Hollander, and Perlman (to be published).

present experimental results indicates that the theoretical K-conversion coefficients are still too high. Such a conclusion has also been reached by Wapstra and Nijgh.[27]

In part (b) of Table IV, we show the results for transitions that we believe to be $E1$. The 67.74-, 152.41-, and 156.37-kev transitions have been definitely assigned $E1$ multipolarity by MBMD. The 169.09 is assigned $E1$ multipolarity from the decay scheme. It is apparent from Table IV that, at least for the transitions around 150 kev, the K/L_{I}-conversion ratio seems low. Although this discrepancy may reflect the large experimental difficulty involved in obtaining the K intensities of this group of transitions because they lie on a very intense background, the fact that a similar result was obtained for the 152.41-kev transition in 13-hour Re^{182},[7] where the conditions are more favorable, tends to support these conclusions.

Another transition, the 120.94-kev, which was assigned an $E1$ multipolarity from the decay scheme, also appears to have too small a K/L_{I} ratio. In this case the intensities of both lines were too weak to be calculated from the densitometer trace, but because both were visible the K/L_{I} ratio must be about the same as those of the transitions listed, if the line assignments are correct. These transitions might be the first observation in this region of the periodic table of anomalous $E1$ conversion coefficients similar to those which have been observed in the heavy-element region.[30,31]

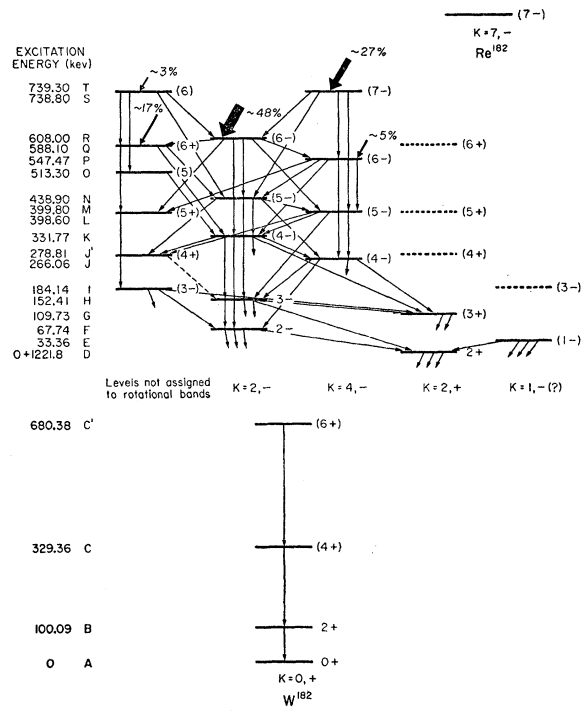

FIG. 6. Analysis of the level spectrum of W^{182}.

Primary Electron Capture Population

Although the complexity of the decay scheme prevented accurate determination of primary electron-capture branching from electron data alone, we attempted to estimate primary branching by assuming that the total decay proceeds through states Q, P, R, S, and T. This assumption is probably not strictly correct, but the intensities populating and depopulating the lower energy states are consistent with it. Furthermore, because we have no high-resolution gamma-intensity data, the percentage primary populations we calculated are dependent upon our multipolarity assignments and theoretical conversion coefficients. Using the electron intensities and assigned multipolarities shown in Table I, we calculated roughly that the percentage primary populations to states Q, P, R, S, and T, are, respectively, 17%, 5%, 27%, 48%, and 3%.

Because most of the observed decay of Re^{182} is to levels assigned spin 6 and 7, although levels assigned spin 5 are present, it seemed reasonable to assign a spin 7 to Re^{182}.

ANALYSIS AND DISCUSSION OF THE LEVELS W^{182}

The presence of large numbers of low-energy magnetic-dipole and electric-quadrupole transitions between states of over 1 Mev of excitation energy is rather remarkable and cannot be understood in terms of single-particle transitions. W^{182} clearly lies within the region of stable spheroidal deformation. Alaga *et al.*[32] (hereafter referred to as AABM) have previously analyzed the levels of W^{182} populated by Ta^{182} into rotational bands. We have attempted to extend this analysis into rotational bands to include the new levels from the Re^{182} study.

In Fig. 6 we present a possible analysis into rotational states for the levels in W^{182} above 1 Mev. This interpretation assigns the eighteen levels into four rotational bands and six extra levels.

In making this analysis we have made considerable use of other data besides our own. These data, of course, are subject to the experimental errors quoted by the authors of the data. We feel that in some cases we have used the data in a manner which implies smaller errors than those quoted. We have done this to try to test the validity of some of the aspects of the Bohr-Mottelson theory which are susceptible to measurement. As is widely known, intensity measurements are extremely difficult to obtain accurately, so that we feel strongly that more careful measurements should be made. In the meantime, however, we have used the present data to obtain as many checks as possible.

Because the explanation of our analysis requires some discussion in addition to that already given, we

[32] Alaga, Alder, Bohr, and Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **29**, No. 9 (1955).

shall begin by discussing the experimental evidence supporting the assignment of the base states of the four bands. In particular we attempt assignment of the Bohr-Mottelson K quantum numbers by the simple Clebsch-Gordan coefficient branching relations (see Alaga *et al.*[32] and Rasmussen *et al.*[33]).

Level D is assigned $K=2+$ because of the agreement of the observed branching ratio from this state to levels B and A (and C) by $E2$ radiation with that predicted theoretically. The assignment of $K=2$ to this level was originally proposed by AABM, who calculated the experimental value

$$B(E2;\ 2,2\rightarrow 2,0)/B(E2;\ 2,2\rightarrow 0,0)=1.61,$$

from the data of MBMD, assuming transition DB was pure $E2$. Using the 10% to 90% $E2$–$M1$ mixing ratio determined by Williams and Roulston from angular-correlation studies involving transitions DB,[23] we recalculate this value to be 1.42, in excellent agreement with the theoretical value for $K_i=2$, of 1.43. The theoretical ratio for $K_i=0$ is the same, but the choice $K_i=2$ is clearly made in view of the weakness of the transition to level C. The spin and parity of this stage suggest that this is a gamma vibrational state. The rotational band based upon this level can then be considered to be the first excited gamma vibrational band.

No evidence supporting level E was obtained in this study, but the data of MBMD and Bä support this level quite conclusively. The 1− assignment was made by MBMD. In order to determine which of the two possible K-quantum numbers (0 or 1) is the most likely for level E, we again compare the experimental branching ratio from this state to states B and A with the theoretical branching ratio for $E1$ transitions. Because we did not observe the transitions supporting this level, we had to use the data of MBMD and Bä which fortunately were sufficient. The theoretical reduced transition probabilities for depopulation of states with $K=0$ and $K=1$ are, respectively,

$$B(E1;\ 1,1\rightarrow 2,0)/B(E1;\ 1,1\rightarrow 0,0)=0.50,$$

and

$$B(E1;\ 1,0\rightarrow 2,0)/B(E1;\ 1,0\rightarrow 0,0)=2.0.$$

We have used the algebraic tables of Clebsch-Gordan coefficients prepared by Sears and Radtke,[34] to calculate Clebsch-Gordan coefficients throughout this paper. Transition EB was seen by MBMD and assigned $M2$ multipolarity. However, because their conversion coefficients are at least 20% too high (the percentage difference between Sliv and Band's and Rose's α_K for the 246.05 kev they used for normalization), we reduced their α_K for this transition by 20%. Their corrected α_K is consistent with an 83% $E1$–17% $M2$ mixture. We used Sliv and Band's conversion coefficients of 0.0012 ($E1$) and 0.013 ($M2$) to calculate the mixing ratio. Using MBMD's gamma intensity of 6.5 for this transition, we found that the total amount of $E1$ radiation is, on the same scale, 5.4. An estimate of the gamma intensity of the 1254 photon is somewhat more uncertain because it was not observed by MBMD although it was observed by Bä. Fortunately, Bä has published the electron spectrum containing the K line of the 1255-kev (EA) and the $L_{\rm I}$ lines of the 1189-kev (FB) and 1222-kev (DA) transitions. Using MBMD's conversion coefficient and K/L ratio for FB, and Bä's K line intensity for the 1255, reduced by 22% to account for the $L_{\rm I}$ line of the 1155 which he shows as superimposed, we obtain a ratio of the intensities of the $L_{\rm I}$ lines of the 1155- and 1255-kev transitions. By assuming that the 1255 is pure $E1$, and $\alpha_K=0.001$ (Sliv and Band's value), we obtained a gamma intensity of 10.5 for the 1255, on the same scale as above. From these data, we calculated the experimental reduced transition probability ratio,

$$B(E1);\ (1155)/B(E1);\ (1255)$$
$$=(5.4/10.5)(1255/1155)^3=0.79,$$

which agrees more closely with the theoretical value for $K=1$ than for $K=0$. Its intensity indicates that probably it has been previously included in the 1231 peak, which would account for the fact that it has not been reported by MBMD. In this calculation we did not attempt to account for any possible anomalies in the conversion coefficients of the 1155- and 1255-kev transitions. State E is therefore tentatively assigned $K=1$ rather than $K=0$.

The assignment of $K=1$ to state E suggests the possibility that level F, which was assigned by AABM as the base state of a $K=2-$band might actually be the second member of the $K=1-$rotational band. Using the results of MBMD and Bä, in order to obtain experimental data to compare with the theoretical predictions, we were able to show that AABM's assignment is correct. Because F has spin 2,[23] it will be expected to decay to levels A and B by $M2$ radiation. The ratio of reduced transition probabilities for $M2$ radiation from state F to states B and A is, for $K=1$

$$B(M2;\ 2,1\rightarrow 2,0)/B(M2;\ 2,1\rightarrow 0,0)=0.345,$$

and for $K=2$

$$B(M2;\ 2,2\rightarrow 2,0)/B(M2;\ 2,2\rightarrow 0,0)=1.43.$$

Although MBMD did not see the photons of the 1289-kev (FA) transition, they did measure the gamma intensity, K/L ratio, and absolute K-conversion coefficient for transition FB. They assigned the transition as an $M2$–$E3$ mixture. However, after their value was reduced by the 20% correction previously discussed, the only possible interpretation for this transition (assuming only two components) was that it is a 69%

[33] Rasmussen, Stephens, Strominger, and Åstrom, Phys. Rev. 99, 47 (1955).

[34] B. J. Sears and M. G. Radtke, Chalk River Report TPI-75, August, 1954 (unpublished).

$E1$–31% $M2$ mixture. Sliv's conversion coefficients $\alpha_K(M2)=0.013$ and $\alpha_K(E1)=0.00113$ were used to determine the mixture. From this mixing ratio we calculated an $M2$ gamma intensity of 14 on the MBMD gamma-intensity scale. Since the $E1$ component of the 1189 is K-forbidden, it is not expected[30] that the $E1$ conversion coefficients of this transition will be anomalous.

Bä resolved the L lines of the 1289- (FA) and 1189-kev (FB) transitions. Without making any assumptions about the multipolarity of FB, we calculated a gamma intensity from these data, assuming an $M2$ multipolarity for FA. From the assignment of 2− to level F the multipolarity can only be $M2$. To calculate the gamma intensity of FA, we first calculated the absolute L_I-shell conversion coefficient of 0.00074 for FB from the MBMD α_K (corrected) and the K/L_I ratio for this transition. The theoretical α_{L_I} for an $M2$ transition of 1289 kev is, from Rose's tables,[25] 0.00135. The gamma intensity of FA (on the MBMD intensity scale) calculated from these data and Bä's experimental L line intensities is 12.9. The experimental value of the reduced transition probability is

$$B(M2);\ (FB)/B(M2);\ (FA) = (14/12.9)(1289/1189)^5 = 1.61,$$

which agrees quite well with the theoretical value, 1.43, for a state with $K=2$. It thus appears that level F is the base state of a $K=2$ band and is not the second state of a $K=1$ band.

Because state F fails to satisfy the branching expected from the $I=2$ member of the $K=1-$ rotational band, and because we have not been able to find evidence for a state close in energy to state F that might be so assigned, we are led to question the $K=1-$ assignment of state E, and suggest that a $K=0-$ assignment is preferable. We shall later suggest other reasons why a $K=0-$ assignment would be preferred for state E. However, because the present branching-ratio data agree with the $K=1-$ assignment to level E, this assignment is illustrated in Fig. 6.

From these calculations we conclude that there are at least three rotational bands to be expected in the W^{182} level spectrum above 1000 kev. Besides these three, a fourth, with $K=4$, was postulated by AABM in their analysis of the W^{182} levels observed from Ta^{182} decay. We have attempted to analyze the observed levels in terms of rotational states based upon these four states. Our analysis is shown in Fig. 6.

Before discussing our analysis of the rotational states we would like to point out that, because it is based upon comparison of theoretical and experimental $E2$ reduced transition probabilities, the possibility exists that other assignments can also be made which will be in agreement with other theoretical values. In order to avoid this possibility we calculated reduced transition probabilities for all the values of K less than 4, for all the states (but keeping the present spin assignments) and found that the best over-all fit of the data was given by the assignments illustrated.

In Fig. 6 we have assigned twelve of the eighteen levels as levels of four rotational bands. Five have been assigned to the $K=2-$ band, four to the $K=4-$ band, two to the $K=2+$ band, and only one, the base state, to the $K=1-$ band. We have also assigned tentative spins and parities to the levels unassigned in Fig. 5. The multipolarities shown in the "Second confidence" multipolarity column are deduced from the level scheme in Fig. 6.

It is immediately clear from Fig. 6 that our proposed assignment does not show the expected rotational-energy spacing between levels. Although we attempted at first to analyze the levels by energy relationships (which ended by assigning levels K, N, and R as states of the $K=4-$ band, J, M, P, and S as members of the $K=2-$ band, all other levels being unchanged) from Fig. 6 it became clear almost immediately that the energy levels of the observed states do not fall into well developed rotational patterns. This is not excessively surprising, since the interactions acting to perturb the rotational spacing at this excitation energy must be quite large.

The analysis shown in Fig. 6 is based on the comparison of the reduced transition probabilities for $E2$ transitions from the rotational levels to other members of their same rotational bands, and also for the cross-over radiations to rotational states in the other bands. The comparison of experimental and theoretical reduced transition probabilities is shown in Table V. The agreement of the experimental with the theoretical values is remarkable, considering the amount of configuration interaction that must be responsible for such large perturbations of the level energies. In making the analysis we have given the greatest weight to the agreement of relative intensities of the intraband transitions.

States J ($K=4-$) and K ($K=2-$) have been changed from the previous assignment given by AABM, on the basis of the results in Table V, although the energy spacings favor the earlier assignment. State N ($K=2-$) was so assigned largely because the ratio of the reduced transition probabilities from N to K and H is in excellent agreement with that for $K=2$ for all these levels. The $E2$ radiations from M are so weak that reduced transition probabilities cannot be calculated with much certainty, but the data are consistent with the assignment. M and P are assigned to the $K=4-$ band because the ratio of the reduced transition probabilities from P to M and J is in excellent agreement with the theoretical value for $K=4$.

The assignment of levels S and R, although not dotted in Fig. 6, must be considered somewhat tentative, because the radiations populating and depopulating R and S show anomalous behavior. The evidence

supporting the present assignment for R is that RN is an extremely strong $M1$, and the crossover $E2$ goes to K and not to J. The $E2$ component of RN is also predicted to be small, which it is. On the other hand, the large primary population of R relative to that observed to state P, which has been assigned the same spin and parity would suggest (assuming that K forbiddenness will be a factor in inhibiting primary electron capture) that R has $K=4$. It might be possible to explain this if we postulate that R has a very large admixture of higher-K wave functions. However, in the present state of knowledge this is a moot point.

Level S is observed to decay by strong $M1$ radiation to P and by strong $E2$ radiation to M and N, SM being much the stronger of the two, which also supports the assignment $K=4$. The anomalous features are the strength of transition SR, which is about twice as strong as SP, and the ratio of reduced transition probabilities (which is much larger than that predicted theoretically) from S to R and to N, which seems well outside the experimental limits of error on the intensities.

We have not been able to assign any states definitely to a rotational band based on E. Of the unassigned states there are three that might possibly be assigned as members of this band. Of these, state I is the most likely, and we have tentatively assigned it in Fig. 6 (dotted level) to this band. We have previously discussed the reasons why we believe a spin of 3 is most probable for this level. That it has $K=1$, rather than $K=2$, or 3, is supported by the observation that it is populated only weakly from Ta^{182} decay, and very weakly, if at all, from 13-hr Re^{182} decay, both isotopes probably having $K=3$. It also decays at least partially to ground, which suggests a low K value.

The possibility that state I is the spin-3 member of the $K=1$ band, whereas no state has been observed with $I=2$, $K=1$, suggests that this latter is only weakly populated, if at all. Because this level must be present if the K-assignments of states E and I are correct, it would be very interesting to attempt more detailed studies of Ta^{182} and 13-hr Re^{182} to see if evidence for such a state could be found.

Only two states of the eighteen observed have definitely been assigned positive parity. These were both assigned by MBMD. We have observed three states which we have tentatively assigned (dotted levels in Fig. 6) to the $K=2+$ band. The reason for such an assignment is that the observed branching to these states occurs with the same branching pattern from the high-spin, negative-parity levels as is observed from the low-spin, negative-parity levels to the 2+ and 3+ states. Furthermore, no strong radiations are observed to depopulate levels J' and L, which is consistent with the interpretation that they would decay directly to the high-spin members of the ground-state rotational band. Such an explanation could account for the increased intensity of population of the spin-4 member of the ground-state band in Re^{182} decay over that observed in Ta^{182} decay (see Fig. 3). State Q is assigned to this band although the observed radiation pattern from this state differs from that of the lower states, which might possibly indicate that states with higher K are mixed with this state.

TABLE V. Comparison of theoretical and experimental reduced transition probabilities for de-excitation of some of the levels in W^{182}.

K, π (initial)	Transitions compared	Reduced transition probability calculated	Theoretical	Experimental
1−	*EB/EA*	$B(E1\ 1,1—2,0)/B(E1\ 1,1—0,0)$	0.50	0.80[a]
2+	*DB/DA*	$B(E2\ 2,2—2,0)/B(E2\ 2,2—0,0)$	1.43	1.42[b]
2−	*FB/FA*	$B(M2\ 2,2—2,0)/B(M2\ 2,2—0,0)$	1.43	1.61[a]
2−	*RN/RK*	$B(E2\ 6,2—5,2)/B(E2\ 6,2—4,2)$	0.36	···[c]
2−	*NK/NH*	$B(E2\ 5,2—4,2)/B(E2\ 5,2—3,2)$	1.00	2.32[d]
2−	*KH/KF*	$B(E2\ 4,2—3,2)/B(E2\ 4,2—2,2)$	2.24	1.80[e]
2−	*RP/RM*	$B(E2\ 6,2—6,4)/B(E2\ 6,2—5,4)$	5.9	···[c]
2−	*NM/NJ*	$B(E2\ 5,2—5,4)/B(E2\ 5,2—4,4)$	10.	···[c]
4−	*SP/SM*	$B(E2\ 7,4—6,4)/B(E2\ 7,4—5,4)$	2.6	···[c]
4−	*PM/PJ*	$B(E2\ 6,4—5,4)/B(E2\ 6,4—4,4)$	5.5	4.3[f]
4−	*SR/SN*	$B(E2\ 7,4—6,2)/B(E2\ 7,4—5,2)$	1.6	~58.[g]
4−	*PN/PK*	$B(E2\ 6,4—5,2)/B(E2\ 6,4—4,2)$	1.35	···[c]
4−	*RK/RH*	$B(E2\ 5,4—4,2)/B(E2\ 5,4—3,2)$	1.00	···[c]
4−	*JH/JF*	$B(E2\ 4,4—3,2)/B(E2\ 4,4—2,2)$	0.56	0.78[h]

[a] Calculated from the data of MBMD and Bäckstrom.
[b] Calculated from MBMD gamma intensities assuming a 10%/90% $M1$-$E2$ mixture in FB (deduced by Williams and Roulston from angular-correlation data).
[c] The $E2$ component of the $\Delta I=1$ transition is too small to be detected from L-subshell conversion, in agreement with the theoretical prediction.
[d] Calculated from electron intensity data assuming a 52%/48% $M1$-$E2$ mixture in SP.
[e] Calculated from MBMD gamma intensities and 64%/36% $M1$-$E2$ mixture in KH. The value using the Fröman-Ryde intensities is 2.64.
[f] Calculated from electron intensities assuming a 61%/39% $M1$-$E2$ mixture in PM.
[g] Calculated from electron intensity data assuming an 82%/18% $M1$-$E2$ mixture in SR. Although the uncertainty in the intensity of SR is quite large, we do not believe the intensity data are in error by an order of magnitude.
[h] Calculated from MBMD gamma intensities assuming a 96%/4% $M1$-$E2$ mixture in JH. The value using the Fröman-Ryde intensities is 1.04.

Although the experimental data are inconclusive about many aspects of the interpretation presented, it is interesting to speculate upon a possible interpretation of the very strongly perturbed rotational-band energy spacings that appear as a consequence of the assignments. It must be emphasized that what is written below is in the line of speculation and will require much additional data to prove.

If we consider the positive parity states first, including the levels only tentatively assigned, it is very interesting to observe that the level spacings in the gamma vibrational band ($K=2$) are alternately large and small. Such an alternation is especially interesting because the unified model predicts the near presence of a beta-vibrational band ($K=0$), whose characteristic rotational pattern is 0+, 2+, 4+, etc. If there is an interaction mixing these beta and gamma vibrational bands, such an interaction could produce a rotational pattern similar to that observed if the beta vibrational band (unobserved here) lay somewhat below the gamma-vibrational band. This of course implies that the beta and gamma vibrational states are not completely orthogonal as they would be if the simple picture of $\lambda=2$ vibrations were strictly true.

An explanation of the negative-parity states is more difficult, because of the greater number of negative-parity states observed. It is interesting to note, how-

ever, that the effect of a Coriolis interaction, as suggested by Kerman,[35] (which would couple rotational state bands with $K=1-$ and $K=2-$) would not be expected to produce the alternation in level spacing observed for the $K=2-$ band. However, if state E does not have $K=1$ but perhaps has $K=0$, then the alternation of the energy-level spacing could be due to a similar unspecified interaction as that postulated to couple the $K=0+$ and $K=2+$ rotational-band states, because the $K=0-$ band also has an incomplete spin sequence, in this case 1, 3, 5, etc. It must be admitted, however, that the very large spacing KH suggests that there are additional forces at work, if state I is really the spin-3 member of a $K=0-$ band.

If we try to estimate whether the levels of the $K=4-$ band are perturbed, we can only conclude that level P appears to be about 13 kev lower than the energy calculated from the moment of inertia calculated from spacing MJ, whereas spacing JS is approximately that calculated. The moment of inertia calculated from spacing MJ is about 20% greater than that of the ground-state band. It seems qualitatively, however, that the states of the $K=4-$ band are not as seriously perturbed as those assigned to the other bands.

At this point we might speculate a little on some of the newer features of nuclear energy levels brought to light in the present investigation. Probably the most striking aspect of the level scheme is the appearance of many states, apparently of rotational nature, based upon levels at 1 Mev. An interesting feature of these states is that the decay characteristics of the negative and positive parity states seem remarkably different. The negative parity states appear to deexcite by cascade through the rotational band to the base state, which then deexcites to the excited positive parity states, or they de-excite directly to the positive parity states. On the other hand, the high-energy positive parity states show almost no decay to the negative parity states of low energy, and decay directly to the ground rotational band states.

This feature can be expressed more simply in terms of transition probabilities by saying that the observed intensity of the electric-dipole transitions which connect the positive and negative parity states are smaller than single particle estimates for $E1$ transition probabilities, and the $M1$ and $E2$ transitions compete favorably with them.

An interesting explanation for such delayed $E1$'s perhaps follows from the suggestion[36,37] that these negative parity states, especially the $I=2-$, $K=2$ and $I=1-$, $K=0$ levels are first-excited octupole ($\lambda=3$) vibrational levels corresponding to projections $\nu=2$, and $\nu=0$ of the $\lambda=3$ vector on the nuclear symmetry axis. The systematic occurrence of $2-$ levels in the regions of large nuclear deformation[9,38–40] and the well-known $1-$, $K=0$ levels in the heavy element region[41] tend to support such a conclusion.

The possible explanation is a consequence of a theoretical check on the octupole nature of these negative parity states recently proposed by Bohr and Mottelson.[42] They have said, in essence, that the presence of octupole and quadrupole deformations in a nucleus can cause a nuclear dipole moment. This dipole moment will have frequencies lower than the main "giant" dipole frequencies and will have matrix elements considerably ($\sim 10^{-3}$) smaller[40] than the single-particle matrix elements. Furthermore, their final expression allows a calculation of the average octupole deformation, providing $B(E1)$ is known. Furthermore, although the article of Bohr and Mottelson did not attempt to interpret any data from the decay of the $2-$ states, it seems worthwhile to evaluate the transition probabilities in this case to see if they are in quantitative agreement with theory. We were able to do this in one case, because the half-lives of two excited states in W^{182} have been measured by Sunyar.[43] Mihelich,[21] interpreting these data, assigned one of the measured half-lives to state F. The measured half-life is 1.03×10^{-9} sec.

Using the branching ratio data of MBMD we were able to calculate the half-lives of the transitions depopulating level F. The partial radiative half-lives we calculate are $t_{\frac{1}{2}}(E1; 67.74) = 2.2\times10^{-9}$ sec; $t_{\frac{1}{2}}(E1; 1189) = 5.5\times10^{-9}$; $t_{\frac{1}{2}}(M2; 1189) = 1.21\times10^{-8}$; $t_{\frac{1}{2}}(M2; 1289) \approx 1.0\times10^{-8}$; $t_{\frac{1}{2}}(960) \approx 1\times10^{-7}$. The calculation of the $E1$–$M2$ mixture in transition FB (1189) and the gamma intensity of FA (1289) has already been discussed. We can now compare these half-lives with the single-particle estimates.[44] We find that the 1289 and 1189 $M2$ transition rates are lower than the single-particle estimates by factors of 2.0×10^2 and 1.6×10^2, respectively. These factors represent the retardation generally observed over single-particle rates. The weak transition FC of 960 kev was observed in the photon spectrum but not in electron spectra of MBMD. They tentatively assigned an $E3$ multipolarity. $M2$ is also allowed by selection rules although it would be expected to be highly depressed by the vector addition relations, i.e., $\langle 222\text{-}2 | 2240\rangle^2 = 0.029 \ll 1$. If transition FC were pure $M2$, it would be slower than single particle by $\sim 4.6 \times 10^2$. If the transition were pure $E3$, it would be

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[44] S. A. Moszkowski, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (Interscience Publishers, Inc., New York, 1955), Chap. XIII, p. 391.

occurring 2.5 times as fast as the single particle rate. If the MBMD assignment of $E3$ is correct to the extent that there is at least half $E3$, then this represents a rather fast rate for $E3$, and lends weight to the speculation that the $K=2-$ band is a collective octupole surface excitation. It would seem an important point for future work to determine accurate fractions of $E3$ radiation in the 960- and 1189-kev transitions. The $E1$ component of FB has a delay of $\sim 6\times 10^{7}$, much of which is to be attributed to the K forbiddenness of this transition, albeit a somewhat larger retardation than is usual for such a class. The 67.74-kev transition, FD, which is not K forbidden, has a delay of 4.5×10^{3} over single particle estimates. Using the half-life above for FD, we calculated $B(E1)=6.75\times 10^{-30}e^2$ cm^2. From the equation corresponding to Bohr and Mottelson's[40,45]‡ we calculate the root-mean-square dipole moment associated with the transition $\langle D^2\rangle_{Av}{}^{\frac{1}{2}}$ as $6.5\times 10^{-15}e$-cm. This value is to be compared with $\langle D^2\rangle_{Av}{}^{\frac{1}{2}}$ values for the $1-$ to ground transition in Sm^{152} and Th^{228} of $5.2\times 10^{-14}e$-cm and $1.8\times 10^{-14}e$-cm, respectively.[38,40]

Another interesting feature of excited nuclear states, which can be seen if the analysis shown in Fig. 6 is assumed to be correct, pertains to the moments of inertia of these excited states. From the highly perturbed rotational level spacing observed it becomes apparent that the measurement of moments of inertia of excited vibrational bands may not be the useful aid in the determination of the nuclear potential surface that similar measurements in molecular spectroscopy are in the determination of the potential associated with the atomic coordinates. Furthermore it may indicate the danger in attempting to draw simple conclusions about the character of excited states on the basis of moments of inertia. This seems particularly sharply borne out by a comparison of the moments of inertia of the excited $K=2+$ bands calculated from the $2+\rightarrow 3+$ spacing in W^{182} and W^{184}.[9] In W^{182} the spacing indicates a smaller moment of inertia for this band than for the ground band; in W^{184} the moment of inertia calculated is larger than that of the ground band. The same type of effect may be present in the $2-$ states, although the $2-$, $3-$ spacing in W^{184} is not known with sufficient accuracy to be compared.[9]

We must emphasize that many aspects of our decay scheme deserve further experimental study. As stated previously, the new levels in the scheme shown in Fig. 5 are based solely on conversion electron spectroscopic results. High-resolution gamma-ray spectroscopy and coincidence studies would be of the greatest value toward confirming or revising our proposed scheme.

Further studies also need to be carried out on the high-energy electron spectrum, the results of which are needed to verify some of the present assignments, especially those of J' and L. Furthermore, such studies would be of great interest, because the new decay scheme predicts many weak high-energy transitions that the low transmission of our instruments prevented us from measuring.

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[45] S. A. Moszkowski (private communication) has shown that Bohr and Mottelson's formula[42] can be written for $K=2$ states

$$\langle D^2\rangle_{Av}=\frac{10}{9}\frac{1}{(8\pi)^2}\frac{(81)^2}{(35)^3}\left(\frac{e^2A}{R_0c_1}\right)^2(ZeR_0)^2\langle\alpha_{32}{}^2\alpha_{22}{}^2\rangle$$

providing the orthogonality of the $\mu=0, 1, 2, 3$ components of the $\lambda=3$ vibration is assumed. This form differs by 10/9 from the form given by Bohr and Mottelson. The interpretation of $\langle\alpha_{32}{}^2\alpha_{22}{}^2\rangle$ is much more difficult than is that of the product $\langle\alpha_{30}{}^2\alpha_{20}{}^2\rangle=\beta_2{}^2\langle\beta_3{}^2\rangle_{Av}$ considered by Bohr and Mottelson. We have, therefore, compared only the values of $\langle D^2\rangle_{Av}{}^{\frac{1}{2}}$.

‡ *Note added in proof.*—Recent additional considerations by Bohr and Mottelson [Nuclear Phys. (to be published)], based upon some results of Strutinski [Atomnia Energia 4, 150 (1956)] cast some doubt on the basic value of this expression as a guide to the interpretation of spectra. The conclusions reached in this paragraph must therefore be viewed with some reservation.