Theory of Cyclotron Resonance in Metals*

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The surface impedance of a metal, in the extreme anomalous skin effect region and in the presence of a magnetic field $\mathbf{H}_0$ parallel to its surface, is calculated assuming specular reflection and spherical energy bands. We give the surface impedance in the cases in which the microwave electric field is parallel and at right angles with $\mathbf{H}_0$, which we call longitudinal and transverse, respectively. The position, intensities, and width of the cyclotron resonance lines are the same for longitudinal and transverse cyclotron resonance.

I. INTRODUCTION

WHEN an electromagnetic wave impinges on a metal, the electric and magnetic fields are rapidly damped inside the metal by the surface currents arising from the influence of the fields on the conduction electrons. The fields decrease with a characteristic length δ as we go into the metal surface. This length δ is called the skin depth. In the derivation of the result quoted above, the validity of Ohm's law is assumed. However, at liquid helium temperatures, for a relatively pure sample and in the microwave region, we often find ourselves in a situation in which the mean free path l of the electrons is larger than the penetration depth δ. If this is the case, the electric field $\mathbf{E}$ varies considerably during an electron mean free path. In this condition, the current at a certain point within the metal does not depend upon the electric field $\mathbf{E}$ at that point alone but also on its value within a sphere centered at the point under consideration and of a radius equal to the electron mean free path. When $l \gg \delta$, we say that we are in the extreme anomalous skin effect region.

Under the conditions that occur in the ordinary skin effect, the power absorbed by the metal from the electromagnetic wave is proportional to the square root of the product of the frequency and the resistivity. However, in the extreme anomalous skin effect region, the power absorption varies as the two-thirds power of the frequency and is independent of the resistivity. To explain these experimental facts, Pippard[1] introduced a rather crude but useful idea known as the "ineffectiveness concept." According to this model, only those electrons that remain in the skin depth during most of their mean free path can contribute to the surface currents. With this assumption, the experimental results just described can be qualitatively explained.

A theoretical treatment of the anomalous skin effect has been given in great detail by Reuter and Sondheimer.[2] Their work gives the relation of the current density $\mathbf{J}$ to the microwave field $\mathbf{E}$ as obtained from the solution of the Boltzmann transport equation for the conduction electrons in the metal. This relation, in conjunction with Maxwell's equations, gives an integro-differential equation governing the variation of the electric field $\mathbf{E}$ inside the sample. The solution of this equation permits them to obtain the power absorbed in a variety of conditions. In the anomalous skin effect region, the result agrees with that obtained by Pippard.

The extension of the results of Reuter and Sondheimer to a sample that is in the presence of both a constant magnetic field $\mathbf{H}_0$ and of a microwave electric field has been made by various authors. In all the cases, the microwave electric field is polarized in the plane of the sample. Azbel' and Kaganov[3] and Chambers[4] have considered the geometry in which the constant magnetic field is normal to the surface of the metal. The case in which the field $\mathbf{H}_0$ is contained in the plane of the sample has been treated by Azbel',[5] Azbel' and Kaner,[6] Mattis and Dresselhaus,[7] and Heine.[8] The latter situation is the most important to us, because, if the field $\mathbf{H}_0$ is chosen in such a way that the electron cyclotron frequency ω_c is equal to an integral submultiple of the microwave frequency ω, we obtain a peak in the absorption that is caused by cyclotron resonance, as we shall soon explain. Here, we shall limit our considerations to the extreme anomalous skin effect region ($l \gg \delta$) and to magnetic fields $\mathbf{H}_0$ such that the radius of the cyclotron orbit is much larger than the skin depth (this condition can be written in the form $\omega_c \tau \delta \ll l$, where τ is the relaxation time of the electrons).

We now explain how resonances arise when $\omega_c = \omega/n$, n being an integer. In the metal, there are some electrons whose cyclotron orbits have a portion inside the skin depth. If the condition $\omega_c = \omega/n$ is satisfied, these

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[7] D. C. Mattis and G. Dresselhaus, Phys. Rev. **111**, 403 (1958).

[8] V. Heine, Phys. Rev. **107**, 431 (1957).

electrons will always see a microwave field in the same direction while they are in the skin depth. Thus, the electrons under consideration will acquire a drift velocity in the direction of the microwave field, and this gives rise to power absorption. This phenomenon has been observed in tin by Kip *et al.*,[9] and in bismuth by Aubrey and Chambers.[10]

Two different geometries will be considered here. In both, the constant magnetic field $\mathbf{H}_0$ is contained in the plane of the sample and the microwave field propagates in the direction of the normal to the surface of the metal. In the first, the microwave electric field $\mathbf{E}$ is parallel to $\mathbf{H}_0$, while in the second, it is at right angles with $\mathbf{H}_0$. We shall call these two cases longitudinal and transverse cyclotron resonance geometries, respectively.

In the longitudinal case, the only nonvanishing component of the surface current density $\mathbf{J}$ is in the direction of $\mathbf{E}$ and therefore there will be no Hall field produced. However, in the transverse situation, the resonant electrons will give rise to a current normal to the metal surface, which, in turn, will produce a space charge (see reference 8). When the radius of the cyclotron orbit is much larger than the skin depth, the nonresonant electrons are able to reduce effectively the production of space charge. In fact, the increase of the space charge is opposed by two forces. The first is proportional to the space charge and its constant of proportionality is ω_p^2, where ω_p is the plasma frequency of the electron gas. The second force is of a viscous nature and is inversely proportional to the conductivity relaxation time.[11] These two forces are equivalent to an impedance which limits the production of space charge. In this paper the effect of the space charges is treated exactly.

In Sec. II, we calculate the surface impedance of a metal in the extreme anomalous region and in the presence of a constant magnetic field, for both the longitudinal and the transverse cases described above. The surface impedance is the pertinent magnitude to calculate, as the power absorption is proportional to its real part. The method of calculation consists in the solution of the Boltzmann transport equation in conjunction with Maxwell's equations for the microwave field. The electrons are assumed to have an isotropic effective mass m and to constitute a degenerate Fermi gas. It will be seen that, in the extreme anomalous skin region, the surface impedances for longitudinal and transverse cyclotron resonance are the same.

II. THEORY

Consider a semi-infinite sample of metal with a plane surface. Take a system of Cartesian coordinates x, y, z, with the xz plane in the plane of the sample and the y axis normal to this plane and pointing into the metal. The external field $\mathbf{H}_0$ will be taken as parallel to the z axis. The eflect of the collisions of the electrons with the surface is now considered. The scattering of the electrons may be diffuse (*i.e.*, the electrons are thermalized after colliding with the surface) or specular with a wide range of intermediate possibilities. Reuter and Sondheimer[2] have shown that, when $\mathbf{H}_0=0$, the surface impedance, if the scattering is specular, differs from the result obtained in the case of diffuse scattering by a factor of 8/9. Therefore, we shall consider here that we have specular reflection, because the treatment is mathematically simpler.

We notice that if we fill the remaining half of space ($y<0$) with another piece of the same metal, and if we imagine that somehow it is possible to produce a microwave field in the xz plane, the situation for each of the two samples is the same as if the electron reflection at the boundary were specular.

The power absorbed by each half is proportional to the real part of the surface impedance Z, which is defined by

$$Z_z=\frac{4\pi}{c}\frac{E_z(0)}{H_x(0)}, \tag{1}$$

$$Z_x=-\frac{4\pi}{c}\frac{E_x(0)}{H_z(0)}, \tag{2}$$

for the two cases of longitudinal and transverse cyclotron resonance. Here $\mathbf{H}$ is the microwave magnetic field, and c the velocity of light. The argument zero indicates that the fields are to be evaluated at $y=0$. From Maxwell's equations we obtain

$$\partial^2\mathbf{E}/\partial y^2+(\omega^2/c^2)\mathbf{E}=(4\pi i\omega/c^2)\mathbf{J}, \tag{3}$$

assuming that the time variation of the field is of the form $\exp(i\omega t)$ with angular frequency ω. $\mathbf{J}$ is the electron current density. It is convenient to work, not with the fields and currents themselves, but with their Fourier transforms. Therefore, we define the quantities $\boldsymbol{\mathcal{E}}(s)$ and $\mathbf{j}(s)$ by the expressions

$$\boldsymbol{\mathcal{E}}(s)=(2\pi)^{-\frac{1}{2}}\int_{-\infty}^{\infty}\mathbf{E}(y)\exp(isy)dy, \tag{4}$$

$$\mathbf{j}(s)=(2\pi)^{-\frac{1}{2}}\int_{-\infty}^{\infty}\mathbf{J}(y)\exp(isy)dy. \tag{5}$$

The field $\mathbf{E}(y)$ is a continuous function of the coordinate y, but its gradient along y has a cusp at $y=0$, because there the field is damped in both the direction of the positive and of the negative y axis. If we denote derivatives with respect to y by primes, we shall have, by symmetry,

$$\mathbf{E}'(+0)=-\mathbf{E}'(-0), \tag{6}$$

[9] Kip, Langenberg, Rosenblum, and Wagoner, Phys. Rev. **108**, 494 (1957).
[10] J. E. Aubrey and R. G. Chambers, J. Phys. Chem. Solids **3**, 128 (1957).
[11] C. Kittel, Conference on Radio-Frequency Spectroscopy, Durham, North Carolina, November, 1957 (unpublished).

where the arguments $+0$ and -0 indicate that the limits are to be evaluated for positive and negative values of y, respectively. From Eqs. (3), (4), (5), and (6), we obtain

$$-(2/\pi)^{\frac{1}{2}}\mathbf{E}'(+0)-s^2\boldsymbol{\mathcal{E}}(s)+(\omega/c)^2\boldsymbol{\mathcal{E}}(s) = (4\pi i\omega/c^2)\mathbf{j}(s). \quad (7)$$

Another expression connecting $\mathbf{j}(s)$ and $\boldsymbol{\mathcal{E}}(s)$ is found from the solution of the transport problem. We shall assume, for simplicity, that the collisions of the electrons with the lattice imperfections and the thermal phonons can be taken into account by assuming the existence of a relaxation time τ, which is a function of the electron energy alone. The electron distribution function f satisfies the well-known Boltzmann equation,

$$\partial f/\partial t+\mathbf{v}\cdot\nabla f+\frac{e}{\hbar}\left[\mathbf{E}+\frac{1}{c}\mathbf{v}\times\mathbf{H}_0\right]\cdot\nabla_{\mathbf{k}}f + (1/\tau)(f-f_0)=0, \quad (8)$$

where $\mathbf{v}=\hbar\mathbf{k}/m$ is the velocity of the electrons, e and m their charge and mass, and $\mathbf{k}$ their wave vector. The function f_0 is the Fermi distribution function. In Eq. (8) we have neglected the force exerted by the microwave magnetic field on the electrons, because it is v_0/c times smaller than the force exerted by the electric field $\mathbf{E}$. Here, v_0 denotes the Fermi velocity of the electrons, which is of the order of 10^8 cm/sec for the monovalent metals.

We shall assume a solution of Eq. (8) of the form

$$f=f_0+f_1, \quad (9)$$

where the deviation f_1 of the electron distribution function from its thermal equilibrium value is proportional to the electric field $\mathbf{E}$. If we linearize the Eq. (8) in the usual way, and if we assume that f_1 has the same time periodicity as $\mathbf{E}$, we get

$$(1+i\omega\tau)f_1+\omega_c\tau\frac{\partial f_1}{\partial\varphi}+v\tau\sin\theta\sin\varphi\frac{\partial f_1}{\partial y} = -\frac{df_0}{d\epsilon}e\tau v\mathbf{E}\cdot\mathbf{n}, \quad (10)$$

where $\mathbf{n}$ is a unit vector in the direction of $\mathbf{v}$ and θ and φ are the polar angles of $\mathbf{v}$, with the z axis as polar axis.

Equation (10) is a linear partial differential equation with the independent variables y and φ, where θ appears only as a parameter. Its solution can most easily be found if we introduce the function Φ_1, defined by

$$\Phi_1(s,\theta,\varphi)=(2\pi)^{-\frac{1}{2}}\int_{-\infty}^{\infty}f_1\exp(isy)dy. \quad (11)$$

The equation

$$(1+i\omega\tau-isv\tau\sin\theta\sin\varphi)\Phi_1+\omega_c\tau\frac{\partial\Phi_1}{\partial\varphi} = -\frac{df_0}{d\epsilon}ev\tau\boldsymbol{\mathcal{E}}(s)\cdot\mathbf{n}, \quad (12)$$

is equivalent to (10). The solution of (12) is

$$\Phi_1=\delta(v-v_0)(e/m\omega_c)[\exp(2\pi\gamma)-1]^{-1}\int_{\varphi}^{2\pi+\varphi}d\varphi' \times\boldsymbol{\mathcal{E}}(s)\cdot\mathbf{n}(\theta,\varphi')\exp[\gamma(\varphi'-\varphi) +ia\gamma\sin\theta(\cos\varphi'-\cos\varphi)], \quad (13)$$

where

$$\gamma=(1+i\omega\tau)/(\omega_c\tau), \quad (14)$$

$$a=(sl)/(1+i\omega\tau), \quad (15)$$

with $l=v_0\tau$. Here we have used the fact that at temperatures much smaller than the Fermi temperature,

$$-df_0/d\epsilon=\delta(\epsilon-\epsilon_0),$$

where ϵ_0 is the Fermi energy and δ is the Dirac δ function.

The current density is given by

$$\mathbf{J}=\frac{e}{4\pi^3}\int f_1\mathbf{v}\,d\mathbf{k}, \quad (16)$$

which is equivalent to

$$\mathbf{j}(s)=\frac{e}{4\pi^3}\int\Phi_1\mathbf{v}\,d\mathbf{k}. \quad (17)$$

From (17) and (13), we obtain the relation

$$\mathbf{j}(s)=\tfrac{3}{4}(Ne^2/\pi m\omega_c)[\exp(2\pi\gamma)-1]^{-1}\int_0^{\pi}d\theta \times\sin\theta\int_0^{2\pi}d\varphi\,\mathbf{n}(\theta,\varphi)\int_{\varphi}^{2\pi+\varphi}d\varphi' \times\boldsymbol{\mathcal{E}}(s)\cdot\mathbf{n}(\theta,\varphi')\exp[\gamma(\varphi'-\varphi) +ia\gamma\sin\theta(\cos\varphi'-\cos\varphi)]. \quad (18)$$

Here N is the number of conduction electrons per unit volume.

Equation (18) defines a conductivity tensor which relates the components of $\mathbf{j}(s)$ to those of $\boldsymbol{\mathcal{E}}(s)$. We can write, after some manipulation,

$$j_x=\sigma_{xx}\mathcal{E}_x+\sigma_{xy}\mathcal{E}_y, \quad (19)$$

$$j_y=\sigma_{yx}\mathcal{E}_x+\sigma_{yy}\mathcal{E}_y, \quad (20)$$

$$j_z=\sigma_{zz}\mathcal{E}_z. \quad (21)$$

The components of the tensor are

$$\sigma_{xx}=\frac{\sigma}{1+i\omega\tau}\Big[\frac{3\pi}{4a}\coth(\pi\gamma)-\frac{3}{a^2}+\frac{3\pi}{4a^3}\coth(\pi\gamma)(1+\tfrac{3}{4}\gamma^{-2})$$
$$-3\sum_{r=0}^{\infty}\frac{(-1)^r a^{-2r-4}}{(2r+1)(2r+3)}\{1+2(r+1)^2(r+2)\gamma^{-2}\}$$
$$\times\prod_{n=0}^{r}\{1+(n\gamma^{-1})^2\}\Big], \quad (22)$$

$$\sigma_{xy}=-\sigma_{yx}=\frac{\sigma}{1+i\omega\tau}\Big[-\frac{3\pi}{4a^3}\frac{\coth(\pi\gamma)}{\gamma}$$
$$+\frac{3}{\gamma}\sum_{r=0}^{\infty}\frac{(-1)^r a^{-2r-4}(r+1)}{2r+1}\prod_{n=0}^{r}\{1+(n\gamma^{-1})^2\}\Big], \quad (23)$$

$$\sigma_{yy}=\frac{\sigma}{1+i\omega\tau}\Big[-\frac{3\pi}{2a^3}\coth(\pi\gamma)+\frac{3}{a^2}$$
$$+3\sum_{r=0}^{\infty}\frac{(-1)^r a^{-2r-4}}{2r+1}\prod_{n=0}^{r}\{1+(n\gamma^{-1})^2\}\Big], \quad (24)$$

$$\sigma_{zz}=\frac{\sigma}{1+i\omega\tau}\Big[\frac{3\pi}{4a}\coth(\pi\gamma)+\frac{3\pi}{4a^3}(1+\tfrac{1}{4}\gamma^{-2})\coth(\pi\gamma)$$
$$+3\sum_{r=0}^{\infty}\frac{(-1)^r a^{-2r-2}}{(2r+1)(2r-1)}\prod_{n=0}^{r}\{1+(n\gamma^{-1})^2\}\Big]. \quad (25)$$

For negative values of s, the components of the tensor have the same values as for $-s$. The symbol σ denotes the electrical conductivity of the sample. The procedure for obtaining these expressions has been given elsewhere.[12]

When $\mathbf{E}$ is parallel to the z axis, the only nonvanishing component of $\mathbf{j}$ is in the z direction. Then, from (7) and (21), we find

$$\mathcal{E}_z(s)=-(2/\pi)^{\frac{1}{2}}E_z'(+0)[s^2-(\omega/c)^2$$
$$+(4\pi i\omega/c^2)\sigma_{zz}(s)]^{-1}. \quad (26)$$

Combining (26) with the inverse transform of (4), we get

$$E_z(y)=-(2/\pi)E_z'(+0)\int_0^{\infty}[s^2-(\omega/c)^2$$
$$+(4\pi i\omega/c^2)\sigma_{zz}(s)]^{-1}\cos(sy)ds. \quad (27)$$

From Maxwell's equations, we observe that the surface impedance in this case can be written in the form

$$Z_z=-\frac{4\pi i\omega}{c^2}\frac{E_z(+0)}{E_z'(+0)}. \quad (28)$$

Z_z can now be easily calculated in the extreme anomalous skin effect region ($l\gg\delta$) and for moderate magnetic fields such that $\omega_c\tau\delta\ll l$, if we observe that a is of the order of l/δ. Thus, only the first term in the asymptotic expansion (25) needs to be taken into account. Also, the term ω^2/c^2 coming from the displacement current can be neglected. The argument to show that the displacement current is negligible in this case is similar to that given by Reuter and Sondheimer[2] and will not be repeated here. Therefore,

$$Z_z=(8i\omega/c^2)\int_0^{\infty}[s^3+(3\pi^2 i\omega\sigma/c^2 l)\coth(\pi\gamma)]^{-1}sds. \quad (29)$$

Integration of (29) yields

$$Z_z=\frac{8}{9}\Big(\frac{\sqrt{3}\pi\omega^2 l}{c^4\sigma}\Big)^{\frac{1}{3}}(1+i\sqrt{3})\tanh^{\frac{1}{3}}(\pi\gamma). \quad (30)$$

We notice that, when $\mathbf{H}_0=0$, Eq. (30) coincides with the result obtained by Reuter and Sondheimer.[2] The integral at the right of Eq. (29) is performed by a method similar to the one described in Appendix I of reference 2.

We now consider the case when the microwave electric field is directed along the x axis at right angles with $\mathbf{H}_0$. Here, a field E_y in the direction of the normal to the surface of the sample is present. From the Maxwell equation relating the divergence of the electric field to the charge density, and using the continuity equation, we obtain

$$j_y(s)=-(i\omega/4\pi)\mathcal{E}_y(s). \quad (31)$$

From Eqs. (20) and (31) we get

$$\mathcal{E}_y(s)=\frac{\sigma_{xy}}{\sigma_{yy}+(i\omega/4\pi)}\mathcal{E}_x(s), \quad (32)$$

so that

$$j_x(s)=\sigma_x\mathcal{E}_x(s), \quad (33)$$

where

$$\sigma_x=\sigma_{xx}+\frac{\sigma_{xy}^2}{\sigma_{yy}+(i\omega/4\pi)}. \quad (34)$$

The surface impedance,

$$Z_x=-\frac{4\pi i\omega}{c^2}\frac{E_x(+0)}{E_x'(+0)}, \quad (35)$$

is obtained exactly as before with the difference that σ_x plays the role of σ_{zz}. In the extreme anomalous region, only the term containing s^{-1} in the expansion of σ_x is important. The asymptotic expansion of σ_x is[13]

[12] S. Rodriguez, Phys. Rev. **112**, 80 (1958).

[13] In the conditions encountered in practice σ_{yy} is much larger than $\omega/4\pi$. In fact, for a sample of copper with a ratio of conductivity at 4°K to conductivity at room temperature of 1000 and $\omega=10^{11}$ sec^{-1}, we have $4\pi|\sigma_{yy}|/\omega\sim 8\times 10^3$. Here l/δ is of the order of 10^3. When l/δ is about 10^6, σ_{yy} becomes comparable with $\omega/4\pi$. In any event the asymptotic expansion (36) is correct up to and including terms of the order of a^{-3}.

$$\sigma_x=\frac{\sigma}{1+i\omega\tau}\left[\frac{3\pi}{4a}\coth(\pi\gamma)-\frac{3}{a^2}+\frac{3\pi}{4a^3}\coth(\pi\gamma)(1+\tfrac{3}{4}\gamma^{-2})+\cdots\right]. \quad (36)$$

As this expression differs from the expansion of σ_{zz} only in the third term, the surface impedance $Z_x=Z_z$, in the approximation considered in this note.

The power absorbed by the metal turns out to have a series of maxima and minima of the same nature as that predicted in Sec. I. As $Z_x=Z_z$, we have shown that the position and intensity of the peaks and the width of the resonant lines are identical in both longitudinal and transverse cyclotron resonance, provided that the metal under investigation is in the extreme anomalous skin effect region.

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Some Low-Temperature Properties of the R, M, and N Centers in KCl and NaCl

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Temporary changes in the absorption spectrum of a colored NaCl crystal can be produced at 77 and 4°K by irradiating the crystal with a strong auxiliary light. The changes in the absorption spectrum indicate that the R, M, and N centers are not randomly distributed but are located near each other. It is concluded that previous arguments based upon less complete observations are not valid and that one cannot decide from this type of experiment whether the R_1 and R_2 bands arise from transitions in two distinct centers or from two transitions in the same center. Measurements of the degree of polarization of the emission excited with R_1- or R_2-band light indicate that the concentration of other centers probably affects this number. It is concluded that this measurement may not determine the symmetry of these centers. Measurements of the temperature dependence of the half-widths of the absorption and emission bands of the M center indicates that the optical transition is primarily influenced by local interactions of the center with neighboring ions and not by interactions with the long-range phonons of the lattice.

INTRODUCTION

TWO prominent optical absorption bands lie between the F and M bands in the alkali halides. These have been called the R bands and the centers giving rise to these absorptions are called R_1 and R_2. The most usual treatments for the production of these bands consist of optically bleaching the F band in additively colored crystals or of extensively irradiating the crystals with ionizing radiation. Both operations are done at room temperature.

Although the mechanism of production of these centers is not well established it has been suggested that these centers result from the coagulation of F centers. A consideration of the kinetics of the coagulation led Seitz to suggest that the R_1 center consists of an electron trapped at a pair of negative-ion vacancies and the R_2 center consists of two electrons trapped at a pair of negative-ion vacancies.[1]

The observation that the ratio of the peak heights of the R_1 and R_2 bands is nearly constant for a large variation in the concentration of the centers led Herman, Wallis, and Wallis to suggest that both absorption bands arise from transitions in the same center.[2] They suggested that the center consists of two electrons trapped at a pair of negative-ion vacancies. The R_2 band results from the electronic transition from the ground state to the first excited state while the R_1 band results from the electronic transition from the ground state to the second excited state. These authors found reasonable agreement between the energies of the peak absorption of these bands and the transition energies calculated on the basis of their model.

Lambe and Compton have argued that both absorption bands cannot arise from transitions in the same center.[3] This was based upon the observation that irradiation at 77°K with an auxiliary R_1-band light bleached the R_1 band but did not affect the R_2 band. When the auxiliary R_1-band light was removed, the absorption in the R_1 band returned to its original value. It was argued that both bands should be bleached by the auxiliary R_1 light if the same center is responsible for both bands.

[1] F. Seitz, Revs. Modern Phys. **26**, 7 (1954).

[2] Herman, Wallis, and Wallis, Phys. Rev. **103**, 87 (1956).

[3] J. Lambe and W. D. Compton, Phys. Rev. **106**, 684 (1957).