the parameters $\xi$ and $x$ approach

$$\xi \rightarrow 4.23(a/\lambda), \quad (A29)$$

and

$$x \rightarrow 0.66. \quad (A30)$$

Correspondingly, $\Omega^{-1} \ln Q$ and the pressure $p_d$ approach

$$\Omega^{-1} \ln Q \rightarrow [\lambda^{-3}(1.342) - 8\pi a \rho_c^2 \beta - 23.6\lambda^{-3}(a/\lambda)^3], \quad (A31)$$

$$p_d \rightarrow (p_d)_c = \lambda^{-3}\kappa T(1.342) + 8\pi a \rho_c^2 - (36.7)(a/\lambda)^2 \rho_c \kappa T. \quad (A32)$$

(ii) On the gaseous side, the parameter $\xi$ is always given by

$$\xi = 0. \quad (A33)$$

As the density $\rho$ approaches the critical value $\rho_c$, the pressure $p_g$ and $\Omega^{-1} \ln Q$ approach

$$\Omega^{-1} \ln Q \rightarrow [\lambda^{-3}(1.342) - 8\pi a \rho_c^2 \beta - (23.6)\lambda^{-3}(a/\lambda)^3], \quad (A34)$$

and

$$p_g \rightarrow (p_g)_c = \lambda^{-3}\kappa T(1.342) + 8\pi a \rho_c^2 - (7.4)(a/\lambda)^2 \rho_c \kappa T.$$

Thus we find that as we change from the gas phase to the degenerate phase there is a sudden unphysical drop of the pressure of the order of $(a/\lambda)^2$,

$$(\rho_d)_c - (\rho_g)_c = -29.3(a/\lambda)^2 \rho_c \kappa T. \quad (A35)$$

This shows that although to first order of $(a/\lambda)$ the thermodynamical functions are correctly evaluated by using the energy spectrum [Eq. (A1)], the higher order terms of this model do not correspond to any real physical system.

On the other hand, one may use the grand partition function instead of the partition function, to calculate the thermodynamical functions for this model. It is easy to show that the use of the grand partition function leads to the well-known application of Maxwell's rule of equal area on the Van der Waals type isotherm obtained from the partition function. From the previous results for the partition function one finds that the resulting isotherm by using the grand partition function has no discontinuity in pressure, but, instead, a discontinuity of density

$$(\rho_d - \rho_g) \sim O(a/\lambda^4),$$

which occurs at a pressure $p_c$ given by Eq. (A4).

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# Suprathermal Particles. III. Electrons*

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Certain phenomena in nature, such as the 50- to 100-kev electrons of the aurora, suggest that there are plasma-dynamical processes which can quickly transfer the translational energy of the ions in a plasma stream to the electrons (some 20 kev/ion for a 2000-km/sec solar wind). It is shown that two interpenetrating streams of noncolliding and initially neutral plasma can achieve this energy transfer with a characteristic time comparable to $(M/m)^{\frac{1}{2}}$ times the plasma period. The process is closely analogous to the excitation of plasma oscillations by two interpenetrating electron streams, but of course proceeds to much greater electron energies because the ion components of the streams carry so much more kinetic energy than do the electron components. Hence, besides the auroral electrons, it is probably responsible for solar radio emission, rather than the electron streams implied in current theories.

Further, the process is probably the dominant interaction in shock fronts, particularly in astrophysical cases where neither direct collisions nor the existing weak magnetic fields can give sharp fronts. The characteristic thickness of a shock front in the solar wind is of the order of $10^4$ cm due to the above plasma excitation whereas the Larmor radius of the ions, which would otherwise determine the shock thickness in the absence of collisions, is 100 km or more.

## I. INTRODUCTION

WE have previously noted the common appearance in nature of charged particles with energies very much in excess of the general thermal background.[1] Cosmic rays from interstellar space and from solar flares[2] represent the most extreme case of high ion energies. More moderate examples are the primary auroral protons[3] (with energies up to a few hundred kev) and the deuterons responsible for the neutron production observed in laboratory electrical discharges.[4,5] The soft x-rays observed by van Allen[6]

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above the atmosphere in the northern auroral zone, and by Winckler[7] beneath an auroral arc at middle latitudes, indicate that 50- to 100-kev electrons are commonly abundant in the vicinity of Earth. The Crab Nebula[8] appears to be a more extreme case, involving electron energies of perhaps 200 Bev.

We have collectively called[1] such ultra-speed particles *suprathermal particles* to indicate their appearance as a high-energy non-Maxwellian tail on the general thermal velocity distribution function. To understand their origin we have started with the simplest idealized case first.[9] We have regarded the electrically conducting gases filling all of space, except for planetary atmospheres and interiors, as an ohmic-conducting fluid, so that the usual hydromagnetic condition obtains,

$$\mathbf{E} = -\mathbf{v}\times\mathbf{B}/c, \tag{1}$$

relating the electric, magnetic, and gas velocity fields. Then the equation of motion of a particle with mass $M$, charge $q$, and velocity $\mathbf{w}$ reduces to

$$\begin{aligned} d\mathbf{w}/dt &= (q/M)[\mathbf{E}+\mathbf{w}\times\mathbf{B}/c] \\ &= (q/Mc)(\mathbf{w}-\mathbf{v})\times\mathbf{B}. \end{aligned}$$

Forming the scalar product with $\mathbf{w}$ yields the energy equation

$$d(\tfrac{1}{2}Mw^2)/dt = (q/c)\mathbf{v}\cdot(\mathbf{w}\times\mathbf{B}), \tag{2}$$

which tells us that in the presence of hydromagnetic fields defined by (1), the kinetic energy of a charged particle can be increased only by the fluid motion $\mathbf{v}$ working against the Lorentz force on the particle. This is the Fermi acceleration mechanism, of which the betatron effect is a special case.[10]

Consequently we began our investigation by including the Fermi acceleration in the general Fokker-Planck equation for the velocity distribution function.[1,9,11] It seems, on the basis of our crude theoretical models, that Fermi acceleration can account for the high-speed ions of solar and galactic cosmic rays, for the primary auroral protons, and for the general heating and non-Maxwellian velocity distribution of high-density electrical discharges in laboratory plasmas. In order to establish experimentally the effectiveness of suprathermal particle generation in the simplest (hydromagnetic) case, we have suggested[11] that laboratory discharges can be probed with high-speed particles to ascertain in a quantitative way the effectiveness of Fermi acceleration under various conditions of macroscopic gas motions.

But while it seems not unreasonable to suppose on theoretical grounds that the suprathermal *ions* can be accounted for in the hydromagnetic approximation, it is certainly clear that the observed suprathermal *electrons* cannot be so accounted for. An electron is handicapped in at least two ways. The Larmor radius of a thermal electron of mass $m$ is smaller by the factor $(m/M)^{\frac{1}{2}}$ than the Larmor radius of a thermal ion of mass $M$. Therefore, the electron generally does not penetrate very far across the gradients in the gas velocity field to experience "collisions" with the relatively moving magnetic field, with the result that the electron gains velocity by Fermi acceleration only very slowly, if at all. Also, apart from the difficulty of penetration across the magnetic field there is the fact that the Fermi acceleration process is essentially a velocity-increasing mechanism. A head-on "collision" of a particle, with a magnetic field moving with velocity $v$, results in a particle velocity increase of $2v$ regardless of whether the particle is an electron or an ion. Thus at best we could hope to find electrons with the same velocity as the suprathermal ions, and this yields trivial electron energies.

As a matter of fact, in auroral phenomena we find electrons with energies (up to 100 kev, or $\beta \cong 0.3$) comparable to the ion energies. Therefore, we believe that there is some mechanism other than Fermi acceleration which is available for suprathermal electron generation. The mechanism lies outside the hydromagnetic approximation of (1).

The fact that the suprathermal electron energies are comparable to the ion energies suggests an electrostatic interaction, though obviously a truly *static* electric field is ruled out by the high electrical conductivity of the gas. What we have in mind, then, are plasma oscillation effects, and it is the purpose of this paper to show that two interpenetrating streams of neutral plasma (such as the solar wind[1,12–14] blowing against the outer terrestrial atmosphere) produces suprathermal electrons with energies comparable to the streaming kinetic energy of the individual ions.

## II. INTERPENETRATING PLASMA STREAMS

It is well known[15–17] that two interpenetrating electron streams will convert the kinetic energy of their relative motions to plasma oscillations. It has further been pointed out[18] that a single stream of electrons passing through a cloud of ions will also yield plasma oscillations, and Buneman[19] has suggested that this may be of interest in current-carrying plasmas. But these processes can do no more than convert the initial kinetic energy of the electron stream into plasma

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[17] D. Bohm and E. P. Gross, Phys. Rev. **75**, 1851, 1864 (1949); **79**, 992 (1950).
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oscillations, and we know from the astrophysical examples already cited that electrons actually achieve very much more energy than this. In particular it appears that the electrons have as much energy as the initial translational motion of the ions: Solar winds of the order of 2000 km/sec are responsible for the aurora in middle latitudes, with which are associated[7] the bremsstrahlung of 50–100 kev electrons; the kinetic energy of a 2000-km/sec hydrogen ion is 20 kev (a 2000-km/sec electron is only 10 ev) so that the observed electron energies prove to be of the same order of magnitude. With this situation in mind we shall now undertake an investigation of the dynamics of two interpenetrating streams of initially neutral plasma.

Suppose that we have a stream of ionized hydrogen moving in the positive $x$ direction with velocity $U$. We assume that the stream is initially of uniform density, being composed of $N_0$ electrons and $N_0$ protons per unit volume. Suppose that a similar stream is moving in the negative $x$ direction also with velocity $U$. If $U$ is sufficiently large, then the collisions by Coulomb interaction may be ignored.

We shall use the subscripts 1 and 2 to refer to the streams moving in the positive and negative $x$ directions, respectively, and we shall consider small deviations from the initial uniform streaming. Thus we let the ion velocities be $U+u_1$, $-U+u_2$, and the electron velocities be $U+v_1$, $-U+v_2$. The ion densities are $N_0+N_1$ and $N_0+N_2$, and the electron densities $N_0+n_1$ and $N_0+n_2$. Then if the electric field is $E$, we have the Poisson equation

$$\partial E/\partial x=4\pi e(N_1+N_2-n_1-n_2). \tag{3}$$

The linearized equations of motion are

$$\partial u_1/\partial t+U\partial u_1/\partial x-(e/M)E=0, \tag{4}$$

$$\partial u_2/\partial t-U\partial u_2/\partial x-(e/M)E=0, \tag{5}$$

$$\partial v_1/\partial t+U\partial v_1/\partial x+(e/m)E=0, \tag{6}$$

$$\partial v_2/\partial t-U\partial v_2/\partial x+(e/m)E=0, \tag{7}$$

and the linearized equations of continuity are

$$\partial N_1/\partial t+U\partial N_1/\partial x+N_0\partial u_1/\partial x=0, \tag{8}$$

$$\partial N_2/\partial t-U\partial N_2/\partial x+N_0\partial u_2/\partial x=0, \tag{9}$$

$$\partial n_1/\partial t+U\partial n_1/\partial x+N_0\partial v_1/\partial x=0, \tag{10}$$

$$\partial n_2/\partial t-U\partial n_2/\partial x+N_0\partial v_2/\partial x=0. \tag{11}$$

As is well known, the large ion mass maintains the initially steady flow of the ions, with $u_1=u_2=N_1=N_2=0$, while the electrons quickly convert the kinetic energy of their relative streaming motion into plasma oscillations (amounting to 10 ev per electron for a 2000-km/sec solar wind). The halting of the relative motion of the electron streams is illustrated, from (3), (6), (7), (10), and (11), in the appendix. The characteristic time is of the order of a few electron plasma periods, $(\pi m/N_0e^2)^{\frac{1}{2}}$. Thus, before the ions have time to respond, the two electron streams have come to rest to form a static (except for plasma oscillations of a few volts energy) electron gas of density $2N_0$. We ignore the plasma oscillations and consider only the smoothed-out electron density $2N_0$ to see if it will eventually be perturbed by the ion motions. If the perturbed electron density is $2N_0+n$ and the velocity is $v$, then in place of (3), (6), (7), (10), and (11) we have

$$\partial E/\partial x=4\pi e(N_1+N_2-n), \tag{12}$$

$$\partial v/\partial t+(e/m)E=0, \tag{13}$$

$$\partial n/\partial t+2N_0\partial v/\partial x=0. \tag{14}$$

We now consider the simultaneous solution of (4), (5), (8), (9), (12), (13), and (14) to determine whether the two oppositely moving ion streams will induce plasma oscillations in the electron gas in which the individual electrons receive a nontrivial fraction of the ion energy $\frac{1}{2}MU^2$; thus in the case of the solar wind we are looking for 20-kev, rather than 10-ev, electron motions.

We represent the solutions involving $u_1$, $u_2$, $v$, $N_1$, $N_2$, $n$, $E$ in the form $\exp(i\omega t+ikx)$ with amplitudes $A_1$, $A_2$, $A$, $B_1$, $B_2$, $B$, $C$, respectively. We put $\Omega=kU$ and obtain the seven equations

$$A_1i(\omega+\Omega)-Ce/M=0, \tag{15}$$

$$A_2i(\omega-\Omega)-Ce/M=0, \tag{16}$$

$$Ai\omega \qquad +Ce/m=0, \tag{17}$$

$$A_1N_0k+B_1(\omega+\Omega)=0, \tag{18}$$

$$A_2N_0k+B_2(\omega-\Omega)=0, \tag{19}$$

$$A2N_0k+B\omega \qquad =0, \tag{20}$$

$$B_1+B_2-B-Cik/4\pi e=0. \tag{21}$$

The determinant of the coefficients must vanish, yielding the dispersion relation

$$0=(\omega^2-\Omega^2)^2(\omega_e^2-\omega^2)+\epsilon\omega_e^2\omega^2(\omega^2+\Omega^2), \tag{22}$$

where $\epsilon=m/M$ and $\omega_e$ is the plasma frequency for the electron gas of density $2N_0$,

$$\omega_e^2=8\pi N_0e^2/m. \tag{23}$$

The dispersion relation is readily solved for $\Omega$, yielding the roots

$$k^2U^2=\omega_e^2-\epsilon\omega_e^2\omega^2/2(\omega_e^2-\omega^2) \pm i[8\epsilon\omega_e^2\omega^4(\omega_e^2-\omega^2)-\epsilon^2\omega_e^4\omega^4]^{\frac{1}{2}}/2(\omega_e^2-\omega^2).$$

We see that we have complex roots when $\omega^2<\omega_e^2$, indicating exponentially growing oscillations. We note that the rate of growth over distance becomes infinite as $\omega^2\rightarrow\omega_e^2$. It is shown in the appendix that inclusion of the thermal, or oscillatory, motion of the electrons eliminates the singularity as $\omega^2\rightarrow\omega_e^2$ and does not alter the general unstable character of the solutions. Therefore we shall pass over this singularity in the cold plasma model by considering the rate of growth in time, which remains finite.

Since $\epsilon \ll 1$, it is possible to compute the roots of (22) by expanding $\omega^2$ in ascending powers of $\epsilon$. Discarding all terms third order in $\epsilon$, we obtain the two real roots

$$\omega = \pm\omega_e\left\{1+\epsilon\frac{1+\Pi}{2(1-\Pi)^2} - \epsilon^2\frac{(1+\Pi)(1+12\Pi+3\Pi^2)}{8(1-\Pi)^5}+O(\epsilon^3)\right\}, \quad (24)$$

and the four complex roots

$$\omega = \pm\Omega\left\{1-\epsilon\frac{1}{2(1-\Pi)^2}+\epsilon^2\frac{\Pi^3+\Pi^2+31\Pi-1}{16(1-\Pi)^5} \pm\frac{i\epsilon^{\frac{1}{2}}}{2^{\frac{1}{2}}(1-\Pi)^{\frac{1}{2}}}\left[1+\epsilon\frac{(\Pi-11)(\Pi+1)}{16(1-\Pi)^3}\right]+O(\epsilon^{\frac{3}{2}})\right\}, \quad (25)$$

where $\Pi \equiv \Omega^2/\omega_e{}^2$.

The expansion in powers of $\epsilon$ converges rapidly unless $(1-\Pi)^2$ is small compared to one. When $\Pi=1$ it is obvious that (24) and (25) are completely invalid. We obtain instead

$$\omega = \pm\Omega[1+e^{1/3}/2^{2/3}+\epsilon^{2/3}/2^{7/3}+\epsilon/24+O(\epsilon^{4/3})],$$

and the complex roots

$$\omega = \pm\Omega[1-\epsilon^{1/3}/2^{5/3}-\epsilon^{2/3}/2^{10/3} \pm i\epsilon^{1/3}(3^{1/2}/2^{5/3})(1-\epsilon^{1/3}/2^{2/3})+O(\epsilon)] \quad (26)$$
$$\cong \pm\Omega[0.97\pm i0.042].$$

We see from (25) and (26) that we have complex roots provided $\Pi \leq 1$. To a useful degree of approximation we may write the unstable roots of (25) as

$$\omega = \pm\Omega - i\Omega\epsilon^{\frac{1}{2}}/2^{\frac{1}{2}}(1-\Pi)^{\frac{1}{2}}+O(\epsilon), \quad (27)$$

noting from (26) that even at $\Pi=1$ the real part of $\omega$ is very nearly $\Omega$. Thus the amplitude grows as $\exp\{\Omega t[\epsilon/2(1-\Pi)]^{\frac{1}{2}}\}$. The ratio of the imaginary part to the real part of $\omega$ is shown in Fig. 1 for $\Pi \leq 1$. The ratio reaches a maximum value of 0.044 at $\Pi=1$ and a minimum of 0.016 at $\Pi=0$. The approximate form (27) is invalid for $\Pi \gtrsim 0.8$, as may be seen from Fig. 1, and numerical methods were used in $0.8<\Pi<1.0$.

Now consider the mode for which $\pm$ is $-$ in (27). Then we have a wave propagating in the positive $x$ direction with velocity $U$. From (15)–(17) we find that

$$A_1 = A(m/M)[\omega/(\omega+\Omega)]$$
$$= -iA(m/M)^{\frac{1}{2}}2^{\frac{1}{2}}(1-\Pi)^{\frac{1}{2}}[1+O(\epsilon^{\frac{1}{2}})],$$

and

$$A_2 = A(m/M)[\omega/(\omega-\Omega)]$$
$$= A(m/2M)[1+O(\epsilon^{\frac{1}{2}})].$$

We note first that the ion stream moving in the negative $x$ direction is hardly perturbed at all, its amplitude being smaller by $(m/M)^{\frac{1}{2}}$ than the stream moving in the positive $x$ direction,

$$|A_2/A_1| \cong [m/8M(1-\Pi)]^{\frac{1}{2}}.$$

Thus, in this mode the exponential growth $\exp\{\Omega t \times[\epsilon/2(1-\Pi)]^{\frac{1}{2}}\}$ affects principally the electrons and the stream moving in the positive $x$ direction. Note that the velocity amplitude of the electron oscillation is larger by the factor $[M/2m(1-\Pi)]^{\frac{1}{2}}$ than the velocity amplitude of the positive moving ions. Thus the electrons receive the fraction $1/[2(1-\Pi)]$ of the initial kinetic energy of the ion stream, and this demonstrates that the electrons can be accelerated to energies comparable to the initial ion energies in two interpenetrating streams of plasma.

Hence we have shown that in a tenuous plasma there is rapidly (characteristic time comparable to the ion plasma period) achieved an approximate equipartition of energy between the electrons and the kinetic energy of the relative motion of the ions of two interpenetrating streams. This equipartition is expected in any irregular plasma motion in which transverse magnetic fields and/or small collision lengths do not prevent the interpenetration of plasma clouds.

We expect that the solar wind, impinging upon the tenuous gas[20] moving with Earth, would automatically yield ~20-kev electrons, which we would associate with the high-energy electrons of the aurora.[6,7]

We suggest that in any tenuous "turbulent" body of ionized gas, such as the solar chromosphere and corona, or such as the Crab Nebula, the interpenetration of relatively moving bodies of plasma may yield significantly non-Maxwellian electron velocity distributions. The effect may perhaps be important in the theoretical analysis of the observed optical spectra of such objects. Finally, we suggest that because it is the ions, rather than the electrons, in a stream which carry

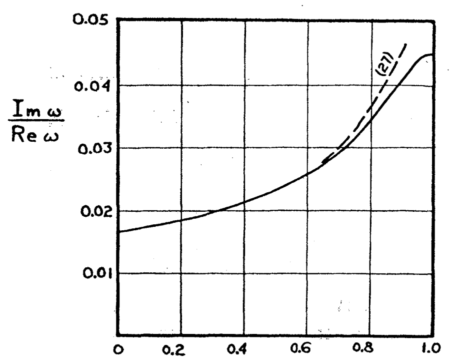


FIG. 1. The solid curve represents the ratio of the imaginary to the real part of $\omega$ for two interpenetrating plasma streams. The broken curve represents the approximate form (27) where it deviates significantly from the correct value.

[20] L. R. O. Storey, Trans. Roy. Soc. (London) A246, 113 (1954).

the kinetic energy to excite plasma oscillations, it is high-speed ion streams which are responsible for much of the observed solar nonthermal radio emission.

Going back to our remark concerning transverse magnetic fields, it is, of course, obvious that a magnetic field parallel to the direction of motion will have little effect on the plasma oscillations. It turns out that even a transverse magnetic field generally does not alter the over-all dynamical situation, because the principal effect of a weak transverse magnetic field is only to change the effective plasma frequency from $\omega_e$ to $\omega_e(1+\omega_c^2/\omega_e^2)^{\frac{1}{2}}$ where $\omega_c$ is the electron cyclotron frequency.[21] In most cases of astrophysical interest the electron frequency is smaller even than the ion plasma frequency, and very much smaller than the electron plasma frequency. For instance at the interface between the geomagnetic field and the solar wind, some 6 or 8 Earth's radii distant from Earth, the geomagnetic field is about $10^{-3}$ gauss, yielding an electron cyclotron frequency of only $1.8\times10^4$/sec. On the other hand, the ion plasma frequency in an enhanced solar wind of $N_0=10^4/\text{cm}^3$, such as is presumed to be responsible for the energetic aurora, is $1.5\times10^5$/sec. In interstellar space where $N_0=1/\text{cm}^3$ and $B=10^{-5}$ gauss, the electron cyclotron frequency is $1.8\times10^2$/sec and the ion plasma frequency is $1.3\times10^3$/sec.

We see then that in most cases the plasma interaction dominates the effects of transverse magnetic fields. Suprathermal particle production is unhindered by the presence of the magnetic fields. The thickness of shock fronts is determined by the plasma interaction, which has a characteristic exponential rise time comparable to the ion plasma period, $(M/m)^{\frac{1}{2}}/\omega_e$. Thus the characteristic scale of a shock front is $(U/\omega_e)\times(M/m)^{\frac{1}{2}}$. For the solar wind with $N_0=10^4$ ions/cm$^3$ the thickness of a shock front may be of the order of a few times 50 meters.

### III. CRITICAL CONDITIONS

We can expect that when the collision period of the electrons is less than the characteristic time of the exponential growth of the plasma oscillation, the nature of the oscillations will be modified.

Spitzer[22] gives the characteristic time $t_s$ in which a test particle of mass $m$ and velocity $w$ will be slowed down while moving through a region containing $n_1$ field particles per unit volume, of mass $m_1$ and thermal velocity $C_1$,

$$t_s=m^2w/(1+m/m_1)8\pi e^4n_1\ln\Lambda\; l_1^2G(l_1w),$$

where $\ln\Lambda$ is the usual factor giving the increased collision rate due to encounters of small deflection, discussed below. The function $l_1^2G(l,w)$ is related to the error integral; we are interested in the case where the velocity $w$ of the test particle is large compared to the thermal velocity of the gas (2000-km-sec motion through an ion field at $10^5$°K for which $C_1\cong50$ km/sec) in which case $l_1^2G(l_1w)\sim1/2w^2$, and

$$t_s\cong m^2w^3/(1+m/m_1)4\pi e^4n_1\ln\Lambda.$$

We see immediately that the collision damping occurs first for the electrons, whose collision period is less than that of the ions by the factor $(m/M)^2$. The problem of the modifications in the plasma oscillation which occur when electron-electron and electron-ion collisions are frequent is taken up in the appendix, where it is shown that, while we do not expect suprathermal electron production, we do still expect the ions to interact with each other to give shock thicknesses of $(U/\omega_e)\times(M/m)^{\frac{1}{2}}$. Only when the densities become so high that the ion slowing-down time becomes comparable to the characteristic time of growth do we lose the plasma interaction, and the shock thicknesses become determined principally by the mean free path of the ions.

In order that the electrons may oscillate sufficiently freely to become suprathermal, then, we must require that the electron slowing-down time $t_s$ exceed the characteristic time of exponential growth, the ion plasma period $(\pi M/N_0e^2)^{\frac{1}{2}}$. Hence we must require that

$$U^6>16\pi^3e^6MN_0(\ln\Lambda)^2/m^4,$$

upon equating $n_1$ to $N_0$ and $w$ to $U$. Since

$$\Lambda=(3/2e^3)(k^3T^3/\pi N_0)^{\frac{1}{2}}=1.24\times10^4T^{\frac{3}{2}}/N_0^{\frac{1}{2}},$$

we have the numerical requirement

$$U^6>1.5\times10^{31}N_0\ln^2[1.24\times10^4T^{\frac{3}{2}}/N_0^{\frac{1}{2}}]\ \text{cm}^6/\text{sec}^6$$

for electrons streaming with velocity $U$ through protons of density $N_0$ and temperature $T$.

Consider the solar wind with density $N_0=10^3$ per cm$^3$ and $T_0=10^5$°K. Then we must have $U$ in excess of 14 km/sec. Since $U$ is actually of the order of $10^3$ km/sec, so that $t_s$ is larger by a factor $3.5\times10^5$ than the plasma period, we conclude that collision damping will be entirely negligible so far as the initial exponential growth is concerned. And once the exponential growth has progressed very far, the electron velocity will exceed $U$, vastly increasing $t_s$.

Similar conclusions may be drawn for jets of gas in the solar corona where $N_0<10^6/\text{cm}^3$ and $T\cong10^6$°K.

But now consider the solar chromosphere where $N_0\cong10^{12}/\text{cm}^3$ and $T$ is of the order of a few times $10^4$°K. We require that $U$ be in excess of 300 km/sec in order to excite the plasma oscillations. There are observed jets of gas with such velocities, but they are not commonly occurring.

Finally, consider the solar photosphere, or a laboratory shock tube, where $N_0$ may be of the order of $10^{15}$

[21] For a more complete discussion of the effect of a transverse magnetic field, and of the role of the plasma interaction in shock fronts, see a forthcoming paper by E. N. Parker, Astrophys. J. **129**, (1959).

[22] L. Spitzer, *Physics of Fully Ionized Gases* (Interscience Publishers, Inc., New York, 1956), p. 79.

and $T$ may be $10^4$°K. Then we must have $U > 10^3$ km/sec, and conditions do not appear favorable for the generation of suprathermal electrons in such gases. As is observed, the principal effect is a classical shock wave in which collisions predominate over the other forms of interaction.

We conclude, then, that suprathermal electrons are to be expected in the regions around, but not inside, stars and around any tenuous atmospheres subjected to the impact of high velocity stellar or solar winds. We suggest that the 50-kev auroral electrons originate in this way. It is probably high-speed plasma streams, rather than electron streams, which are responsible for much of the observed solar radio emission, since the ions in a plasma stream, with their greater translational kinetic energy, can put $10^3$ times more energy into the plasma oscillations.

We probably cannot produce the suprathermal electron in the laboratory because of the difficulty in achieving sufficiently high-speed streams of neutral plasma. The experiment has essentially been done in reverse,[18] of course, exciting ions to plasma oscillation with high-speed electron streams; just a few volts will yield the necessary $10^3$-km/sec electrons.

## APPENDIX

### A. Deceleration of Interpenetrating Electron Streams

Consider the well-known solution for the excitation of plasma oscillations in two interpenetrating streams of electrons. We have equations (6), (7), (10), (11), and the Poisson equation

$$\partial E/\partial x = -4\pi e(n_1+n_2).$$

Assuming solutions of the form $\exp(i\omega t+ikx)$, the dispersion relation is

$$(\omega^2-\Omega^2)^2-\omega_e^2(\omega^2+\Omega^2)=0,$$

where $\omega_e$ is given by (23). Two roots are real and, if $\Omega^2<\omega_e^2$, two are imaginary,

$$\omega_I = \pm i(\omega_e/\sqrt{2})[(1+8\Omega^2/\omega_e^2)^{\frac{1}{2}}-1-2\Omega^2/\omega_e^2]^{\frac{1}{2}},$$

giving exponential growth of the oscillations, $\exp(|\omega_I|t)$. If $\Omega^2 \ll \omega_e^2$, so that the growth takes place over many plasma periods as our linearized equations tacitly assume, then

$$\omega_I \cong \pm i\Omega,$$

and the growth is $\gtrsim \exp\Omega t$.

It is readily shown, then, that a solution of the initial equations is

$$E=+E_0 \exp kUt \cos kx,$$
$$v_1=-E_0(e/\sqrt{2}mkU)\exp kUt \sin(kx+\tfrac{1}{4}\pi),$$
$$v_2=+E_0(e/\sqrt{2}mkU)\exp kUt \sin(kx-\tfrac{1}{4}\pi),$$
$$n_1=+N_0(E_0e/2mkU^2)\exp kUt \cos kx,$$
$$n_2=-N_0(E_0e/2mkU^2)\exp kUt \cos kx,$$

to the order we are considering.

The mean kinetic energy density of the plasma oscillation is

$$\tfrac{1}{2}N_0m(\langle v_1^2\rangle+\langle v_2^2\rangle)=[N_0E_0^2e^2/4mk^2U^2]\exp 2kUt,$$

and its time derivative is just $[N_0e^2E_0^2/2mkU]\exp 2kUt$. On the other hand, the work done by the streaming velocity $U$ against $E$ has a mean value

$$e\{U\langle E(N_0+n_1)\rangle - U\langle E(N_0+n_2)\rangle\} = [N_0e^2E_0^2/2mkU]\exp 2kUt,$$

which is, of course, just the rate at which the kinetic energy of the plasma oscillations is increasing. Thus the two electron streams are decelerated according to

$$2d[\tfrac{1}{2}N_0mU^2]/dt \cong -(N_0e^2E_0^2/2mkU)\exp 2kUt.$$

### B. Effect of Electron Velocity Dispersion

It is physically obvious that inclusion of an electron velocity dispersion, such as thermal motions, will result in a finite rate of growth over distance as $\omega^2\to\omega_e^2$, because each electron sees $\omega$ Doppler-shifted by its own thermal motion. Only an infinitesimal fraction of the electrons can have their effective $\omega$ exactly equal to $\omega_e$. To demonstrate this formally we suppose that initially there is an electron velocity dispersion $\Psi(v')$ so that the number of electrons per unit volume with velocities in $(v+v', v+v'+dv')$ is $\Psi(v')dv'$. $\Psi(v')$ is normalized so that

$$2N_0=\int_{-\infty}^{+\infty} dv'\Psi(v').$$

This distribution will be perturbed by the plasma oscillations so that subsequently it will be of the form $\Psi(v')[1+f(v',x,t)]$. The first-order equation of continuity, or Liouville equation, is

$$\partial f/\partial t+v'\partial f/\partial x+\partial v/\partial x=0,$$

and the equation of motion is

$$\partial v/\partial t+v'\partial v/\partial x-Ee/m=0.$$

Assuming solutions of the form

$$v=A\exp(i\omega t+ikx),$$
$$f=B\exp(i\omega t+ikx),$$
$$E=C\exp(i\omega t+ikx),$$

we have

$$Ai(\omega+kv')-Ce/m=0,$$
$$B(\omega+kv')+Ak=0.$$

Thus

$$B=-iCek/m(\omega+kv')^2,$$

and the electron density is

$$\int_{-\infty}^{+\infty} dv'\Psi(v')[1+f(v',x,t)]=2N_0-iC\frac{ek}{m}\int_{-\infty}^{+\infty}\frac{dv'\Psi(v')}{(\omega+kv')^2}.$$

The Poisson equation for $E$ becomes

$$ikC=4\pi e\left\{N_1+N_2+iC\frac{ek}{m}\int_{-\infty}^{+\infty}\frac{dv'\Psi(v')}{(\omega+kv')^2}\right\},$$

which we now must solve simultaneously with (15), (16), (18), and (19).

The dispersion relation is

$$0=(\omega^2-\Omega^2)^2\left\{\frac{4\pi e^2}{m}\int_{-\infty}^{+\infty}\frac{dv'\Psi(v')}{(\omega+kv')^2}-1\right\}+\omega_e^2(m/M)(\omega^2+\Omega^2).$$

Regarding the electron velocity dispersion as the thermal velocity due to some small temperature $T$, so that $kv'\ll\omega$, we expand the integrand in a power series, discarding all terms third order and higher in $kv'/\omega$. If $\langle v'^2\rangle$ is the mean square thermal velocity in the $x$ direction, then $m\langle v'^2\rangle=kT$ and we have

$$0=(\omega^2-\Omega^2)^2[\omega_e^2-\omega^2+3\Omega^2\omega_e^2kT/\omega^2mU^2]+(m/M)\omega_e^2\omega^2(\omega^2+\Omega^2).$$

We see immediately that the coefficient of $(\omega^2-\Omega^2)^2$ does not vanish, and hence $\Omega^2$ remains finite, as $\omega^2\rightarrow\omega_e^2$. If we put $\omega^2=\omega_e^2$, we obtain the finite roots

$$\Omega^2\sim\pm i\omega_e^2mU/(3MkT)^{\frac{1}{2}}$$

for small electron temperatures, and for sufficiently large electron temperatures that $kT\gtrsim mU^2$ we have the roots

$$\Omega^2=-\omega_e^2mU/(3MkT)^{\frac{1}{2}}$$

and

$$\Omega^2=\omega_e^2[1\pm i(2m^2U^2/3MkT)^{\frac{1}{2}}].$$

Thus high electron temperatures do not destroy the unstable character of the solutions.

### C. Collision Damping

When the electron collision frequency becomes comparable to, or greater than, the ion plasma frequency, then the electrons will make at least one collision in the time that the oscillations would otherwise grow by a factor of $e$. We expect, therefore, that the oscillations will not proceed according to the free-electron equation (13). To obtain some rough idea as to what we might expect we represent the effect of electron-electron collisions as a viscosity term $\delta\partial^2v/\partial x^2$ in the electron equation of motion (13). We represent ion-electron collisions by a drag term $\kappa(u_1+u_2)$, so that in place of (13) we have

$$\partial v/\partial t+Ee/m=\delta\partial^2v/\partial x^2+\kappa(u_1+u_2),$$

to be solved simultaneously with (4), (5), (8), (9), (12), and (14). The vanishing of the determinant of the coefficients yields the dispersion relation

$$0=(\omega^2-\Omega^2)^2(\omega_e^2-\omega^2)+\epsilon\omega_e^2\omega^2(\omega^2+\Omega^2)+ik\delta\omega[(\omega^2-\Omega^2)^2-\epsilon\omega_e^2(\omega^2+\Omega^2)]+i2\epsilon\kappa\omega_e^2\omega(\omega^2-\Omega^2),$$

which reduces to (22), of course, when the damping constants $\delta$ and $\kappa$ go to zero.

Consider the case where $\delta$ and/or $\kappa$ are exceedingly large. Then we have just

$$0\cong k^2\delta[(\omega^2-\Omega^2)^2-\epsilon\omega_e^2(\omega^2+\Omega^2)]+2\epsilon\kappa\omega_e^2(\omega^2-\Omega^2).$$

Solving for $\omega^2$, we obtain

$$\omega^2=\Omega^2+\tfrac{1}{2}\epsilon\omega_e^2(1-2\kappa/\delta k^2)\pm\tfrac{1}{2}[8\epsilon\Omega^2\omega_e^2+\epsilon^2\omega_e^4(1-2\kappa/\delta k^2)^2]^{\frac{1}{2}}.$$

We find that $\omega^2<0$, so that we have exponentially growing oscillations, for

$$\Omega^2<\omega_e^2\epsilon(1+2\kappa/\delta k^2).$$

Since $k$ and $\delta$ are both positive quantities, we see that it is sufficient to require that

$$\Omega^2<\omega_e^2\epsilon.$$

Friction and viscosity of the electron component do not alter the growth of the ion oscillations, though obviously it prevents the electrons from sharing the ion energy. The effect of $\delta$ is to immobilize the electron gas so that the ion streams interact in a manner analogous to two interpenetrating electron streams.