$$\int_F \frac{d^4k}{k^2k^2}(a\cdot k)(b\cdot k)\mathfrak{D}_n=\frac{\pi^2 i}{n}\int dY_{n-1}\int_0^1 dx[D_n(x)]^{-n}\left\{\tfrac{1}{2}(a\cdot b)(1-x)-(1-2x)\frac{(a\cdot\bar{p}_{n-1})(b\cdot\bar{p}_{n-1})}{\bar{p}_{n-1}{}^2}\right\}, \tag{A-4}$$

$$\int_F \frac{d^4k}{k^2k^2}(a\cdot k)(b\cdot k)(c\cdot k)\mathfrak{D}_n=\frac{\pi^2 i}{n}\int dY_{n-1}\int_0^1 dx[D_n(x)]^{-n}\left\{\tfrac{1}{2}x(1-x)[(a\cdot b)(c\cdot\bar{p}_{n-1}) + a_\mu(b\cdot\bar{p}_{n-1})c_\mu+(a\cdot\bar{p}_{n-1})(b\cdot c)]+x(3x-2)\frac{(a\cdot\bar{p}_{n-1})(b\cdot\bar{p}_{n-1})(c\cdot\bar{p}_{n-1})}{\bar{p}_{n-1}{}^2}\right\}. \tag{A-5}$$

The term $a_\mu(b\cdot\bar{p}_{n-1})c_\mu$ is written in this form, rather than $(a\cdot c)(b\cdot\bar{p}_{n-1})$, to insure obtaining the correct expression when any two or more of the a, b, c are, or contain, γ matrices.

$$\int_F \frac{d^4k}{k^2}(a\cdot k)(b\cdot k)(c\cdot k)\mathfrak{D}_n=\frac{\pi^2 i}{n}\int dY_{n-1}\int_0^1 x^3dx[D_n(x)]^{-n}(a\cdot\bar{p}_{n-1})(b\cdot\bar{p}_{n-1})(c\cdot\bar{p}_{n-1}) + \frac{\pi^2 i}{2n(n-1)}\int dY_{n-1}\int_0^1 x^2dx[D_n(x)]^{-(n-1)}[(a\cdot b)(c\cdot\bar{p}_{n-1})+a_\mu(b\cdot\bar{p}_{n-1})c_\mu+(a\cdot\bar{p}_{n-1})(b\cdot c)]. \tag{A-6}$$

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Elimination of Ghosts in Propagators*

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Within the general framework of perturbation theory a method for calculating modified propagators in terms of proper Feynman diagrams is derived. This method differs from previous approaches in that one insists that the propagator have the correct analytical behavior as a function of p^2. As a result one gets an expression for the propagator which is similar to a conventional term-by-term perturbation theory expansion except that it is only necessary to consider proper diagrams and that the iteration of the proper diagrams is represented by a damping factor. As an example, the meson propagator for a pseudoscalar meson coupled to nucleons with a pseudoscalar coupling is approximated by considering only the lowest order proper diagram, a nucleon-antinucleon bubble. The resulting expression for the propagator has the following interesting properties: (1) by construction it has the proper analytical behavior as a function of p^2, (2) the result has a singularity at $g^2=0$ when considered as a function of g^2, and (3) the wave function renormalization is finite. These three properties are intimately connected and when this connection is realized it is easy to understand why the usual methods of expressing propagators in terms of proper Feynman diagrams leads to ghosts. It is the purpose of this paper to understand this connection and to indicate how it is possible to take into account consistently the iteration of proper Feynman diagrams without ever having ghosts appear. It is also found that an asymptotic expansion valid in the region $g^2=0$ is possible and that this asymptotic expansion is identical with the perturbation theory series.

I. INTRODUCTION

FOR definiteness in what follows, and for simplicity in presenting the arguments, we shall consider a pseudoscalar boson field with mass μ represented by the renormalized Heisenberg operator $\varphi(x)$. The dependence on isotopic spin will be suppressed in what follows since for the propagator this dependence is a trivial $\delta_{\alpha\beta}$. The extension of the methods developed in this paper to fields with additional degrees of freedom such as the electromagnetic field, or to fermions, is straightforward and will not be discussed.

We wish to find an expression for the modified propagator $\Delta_F'(p^2)$ which is defined in terms of the vacuum expectation value of a time-ordered product:

$$\Delta_F'(p^2)=i\int d_4x\, e^{-ip(x-y)}{}_0\langle T(\varphi(x)\varphi(y))\rangle_0. \tag{1}$$

The lowest order approximation to this function is given by $\Delta_F(p^2)$ where $\Delta_F(p^2)=(p^2+\mu^2-i\epsilon)^{-1}$. Historically the first attempts to calculate the corrections terms were based on a term-by-term perturbation expansion in the coupling constant. In electrodynamics the corrections thus obtained were found to be small and reasonable in that, for example, they led to small terms in the Lamb shift which due to the precision of the experiments could be seen to be necessary.

For strong-coupling meson theories it is readily seen that such a simple procedure does not prove to be

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adequate. It is true that $\Delta_F(p^2)$ is an excellent approximation to $\Delta_F'(p^2)$ in the region $p^2 \cong -\mu^2$. However, for values of p^2 which contribute significantly to the results of meson theory calculations it was noted that the corrections to $\Delta_F(p^2)$ can be large compared to the leading term.[1] Dyson[2] and Schwinger[3] found that with very little effort it is possible to iterate the simple perturbation theory result and they obtained an algebraic equation which the propagator must satisfy. Thus

$$\Delta_F'(p^2)=\Delta_F(p^2)+\Delta_F(p^2)K(p^2)\Delta_F'(p^2). \qquad (2)$$

In this equation the kernel $K(p^2)$ is determined by considering only diagrams which do not involve iterations, i.e., only the proper Feynman diagrams.

More recently Lehmann[4] has made a very important contribution to the understanding of the properties of propagators. This contribution is of a somewhat different character from the techniques discussed above and concerns the analiticity properties of $\Delta_F'(p^2)$ as a function of the complex variable p^2. For the meson propagator Lehmann's result can be summed up in the equation

$$\Delta_F'(p^2)=\frac{1}{p^2+\mu^2-i\epsilon}+\int_{(3\mu)^2}^{\infty} dm^2 \frac{\chi(m^2)}{p^2+m^2-i\epsilon}. \qquad (3)$$

In this equation the function $\chi(m^2)$ is real and positive for $m^2>(3\mu)^2$ and vanishes for $m^2=(3\mu)^2$. This equation states that $\Delta_F'(p^2)$ has a pole at $p^2=-\mu^2$ with residue 1, a branch-point going from $p^2=-(3\mu)^2$ to $p^2=-\infty$ and no other singularities. The assumptions made by Lehmann in deriving (3) are such that one would expect the propagator given by any "physically reasonable" local field theory to have this form except for the possibility of the theory having bound states which can virtually go into the one-meson state. Bound states would lead to additional poles with residues not necessarily equal to one. Although the presence of such bound states would not cause any great difficulty in what follows, we shall assume they do not exist.

Lehmann has shown that the term-by-term perturbation series expansion for $\Delta_F'(p^2)$ has the form given in Eq. (3). From this it is possible to deduce that, after renormalization, $K(p^2)$ has the form

$$K(p^2)=(p^2+\mu^2)\int_{(3\mu)^2}^{\infty} dm^2 \frac{\rho(m^2)}{p^2+m^2-i\epsilon}, \qquad (4)$$

with $\rho(m^2)$ real and non-negative. Combining Eqs. (2) and (4), it is then possible to write[5]

$$\Delta_F'(p^2)=(p^2+\mu^2-i\epsilon)^{-1} \times\left[1-(p^2+\mu^2)\int_{(3\mu)^2}^{\infty} dm^2 \frac{\rho(m^2)}{p^2+m^2-i\epsilon}\right]^{-1}. \qquad (5)$$

In the past[1,6,7] attempts to improve the term-by-term expansion for the propagator have consisted of using Eq. (5) and approximating $\rho(m^2)$ by the sum of a small number of terms corresponding to the simplest proper diagrams.

In renormalizable theories where perturbation theory suggests that the wave-function renormalization constant Z^{-1} is logarithmically divergent, such a procedure yields a propagator which does not have the desired form. That is, since $\rho(m^2)$ in such a theory behaves as $1/m^2$ for large m^2, it is found that the denominator in (5) vanishes for some value of $p^2>-\mu^2$.[8] In pseudoscalar meson theory with pseudoscalar coupling and a reasonable value of the coupling constant, it is found that the actual root occurs at $p^2>0$.[8] Hence the doubtful and undesirable explanation that the root represents a stable bound pseudoscalar particle with isotopic spin 1 and a mass less than the meson's mass is not possible. For a nucleon propagator which has a more complicated form, it is possible to have two complex conjugate roots for p^2. Also the wave function renormalization constant Z^{-1}, which can be defined as the limit $p^2\Delta_F'(p^2)$ and which must be greater than one, as can be seen from Eq. (3), approaches zero through negative values. The properties of such approximate solutions have been discussed extensively[9] in connection with the Lee model,[10] for which Eq. (5) provides an exact solution with only one proper diagram contributing. Also Feldman[8] has shown that when one uses such an approximation for the propagators in attempting to calculate the magnetic moment of a nucleon one is led to new infinities in the theory, and he was unable to eliminate these infinities in any consistent way.

It is possible to arrive at any one of several, not all consistent, conclusions from an examination of the above results. (1) It is possible to suppose that the unphysical features of the approximate solution, which are collectively known as ghosts, are a characteristic

[1] K. A. Brueckner, Phys. Rev. **91**, 761 (1953).
[2] F. J. Dyson, Phys. Rev. **76**, 1 (1949).
[3] J. Schwinger, Proc. Natl. Acad. Sci. U. S. **37**, 452 (1951).
[4] H. Lehmann, Nuovo cimento **11**, 342 (1954).

[5] Equations (4) and (5) are characteristic of perturbation theory. The author does not believe that they can be proven in a more general sense. However, if the exact propagator is such that it does not vanish in the region p^2 real and $-(\mu)^2>p^2>-(3\mu)^2$, then the arguments used later in this paper can be reversed to prove Eq. (5) and hence Eq. (4) as a consequence of Eq. (3), which of course can be established generally.
[6] Ning Hu, Phys. Rev. **80**, 1109 (1950).
[7] S. Kamefuchi and H. Umezawa, Progr. Theoret. Phys. Japan **9**, 429 (1953).
[8] G. Feldman, Proc. Roy. Soc. (London) **A223**, 112 (1954).
[9] G. Källen and W. Pauli, Kgl. Danske Vidensdab. Selskab, Mat.-fys. Medd. **30**, No. 7 (1955).
[10] T. D. Lee, Phys. Rev. **95**, 1329 (1954).

of the exact solution if such an exact solution exists and that the ghosts cannot be eliminated. In this case it would be necessary to abandon local field theory.[11–14] (2) It is also possible to surmise that the field theory does not in itself completely define the solution but that one can in some self-consistent way impose additional restrictions corresponding to additional physical assumptions which permit one to infer results which can be compared with experiment. (3) Finally, one can assume that an exact solution exists which does not have any "unphysical properties."

We shall find that the assumption of a well-behaved exact solution, i.e., a solution with no ghosts, permits us to rewrite Eq. (5) in the form given by Eq. (3) and to determine $\chi(m^2)$ in terms of $\rho(m^2)$. Although these two forms are equivalent for the exact solution, the effect of making approximations to $\rho(m^2)$ in the two solutions can be quite different. The difference between the two approximations thus obtained gives considerable insight into why the ghosts appear when one approximates to Eq. (5). We believe that the insight thus gained gives a strong argument that an exact solution does indeed exist and that many of its properties can be inferred.

It should also be remarked, however, that the weaker postulate numbered 2 could also be adopted and would yield the same results. The equations would remain the same but the text would have to be changed.

II. ARGUMENT

We shall assume therefore that for the exact solution the denominator in Eq. (5) never vanishes. (That is there are no additional bound states and no ghosts.) This implies that the integral

$$\int_{(3\mu)^2}^{\infty} dm^2\, \rho(m^2)$$

is finite, since otherwise there would be a real root for some $p^2 > -\mu^2$. The function $\Delta_F'(p^2)$ can then be continued for complex p^2 and it has the following properties:

1. It has a pole at $p^2 = -\mu^2$ with residue 1.
2. It has a branch line from $p^2 = -(3\mu)^2$ to $p^2 = -\infty$.
3. It is of the order $1/p^2$ as $p^2 \to \infty$.
4. It has no other singularities.

From these properties we can rewrite $\Delta_F'(p^2)$ as a Cauchy integral, deforming the path of integration so that it goes along both sides of the branch line, and closing the contour with an infinite circle. Thus one arrives at an equation of the same form as Eq. (3):

$$\Delta_F'(p^2) = \frac{1}{p^2+\mu^2} + \int_{(3\mu)^2}^{\infty} dm^2 \frac{\chi(m^2)}{p^2+m^2}. \tag{6}$$

For $p^2 < -(3\mu)^2$ the Feynman function is defined as the limit obtained when $p^2 \to m^2 - i\epsilon$. The function $\chi(m^2)$ can be expressed in terms of $\rho(m^2)$ by using the relationship

$$\pi\chi(m^2) = \text{imaginary part} \lim_{p^2 \to -m^2 - i\epsilon} \Delta_F'(p^2). \tag{7}$$

Thus

$$\chi(m^2) = \rho(m^2)/D(m^2), \tag{8}$$

where

$$D(m^2) = R^2(m^2) + \pi^2(m^2-\mu^2)\rho^2(m^2), \tag{9}$$

and

$$R(m^2) = 1 - (m^2-\mu^2)\,\mathrm{P}\int_{(3\mu)^2}^{\infty} dl^2 \frac{\rho(l^2)}{m^2-l^2}, \tag{10}$$

where P indicates that the principal value of the integral is to be taken. The fact that $\chi(m^2)$ is obviously real and non-negative completes the identification of Eqs. (3) and (6).

Therefore the form of $\Delta_F'(p^2)$ we wish to discuss is

$$\Delta_F'(p^2) = \frac{1}{p^2+\mu^2-i\epsilon} + \int_{(3\mu)^2}^{\infty} dm^2 \frac{\rho(m^2)}{D(m^2)(p^2+m^2-i\epsilon)}. \tag{11}$$

III. PROPERTIES OF THE RESULT

In order to illustrate the difference between Eq. (5) and Eq. (11) when an approximation to $\rho(m^2)$ is inserted in the respective equations, we shall consider the contribution from the nucleon-antinucleon bubble. This approximation we shall denote with a subscript 0, and it is found that

$$\rho_0(m^2) = \frac{g^2}{4\pi^2}\frac{m(m^2-4M^2)^{\frac{1}{2}}}{(m^2-\mu^2)^2}, \quad m^2 > 4M^2 \tag{12}$$

$$= 0, \quad m^2 \leqslant 4M^2$$

where M is the nucleon's mass. Then

$$R_0(m^2) = 1 - \frac{g^2}{4\pi^2}(m^2-\mu^2)\mathrm{P} \times \int_{4M^2}^{\infty} dl^2 \frac{l(l^2-4M^2)^{\frac{1}{2}}}{(l^2-\mu^2)^2(m^2-l^2)}. \tag{13}$$

When $m^2 \to \infty$ it is easily seen that $R_0(m^2) \sim \ln m^2$. We shall also verify this by calculating $R_0(m^2)$ in what follows. Therefore $D_0(m^2)$ behaves like $(\ln m^2)^2$ for large m^2.

The general expression for the wave-function re-

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[11] Landau, Abrikosov, and Khalatnikov, Doklady Akad. Nauk S.S.S.R. **95**, 1177 (1954); **96**, 261 (1954).

[12] Abrikosov, Galanin, and Khalatnikov, Doklady Akad. Nauk S.S.S.R. **97**, 793 (1954).

[13] J. C. Taylor, Proc. Roy. Soc. (London) **A234**, 296 (1956).

[14] For a contrary opinion, see N. N. Bogolyubov and D. V. Shirkov, Nuovo cimento **3**, 845 (1956).

normalization constant is

$$Z^{-1}=1+\int_{(3\mu)^2}^{\infty} dm^2\,\frac{\rho(m^2)}{D(m^2)}. \qquad (14)$$

When one substitutes $\rho_0(m^2)$ and $D_0(m^2)$ in Eq. (14), one finds that the damping factor $D_0(m^2)$ provides an additional convergence factor which is just sufficient to make the integral converge. We thus arrive at the surprising result that by considering only the single proper diagram corresponding to a nucleon-antinucleon bubble, one gets a finite wave function renormalization which is in the physically sensible region.

However, if $\rho_0(m^2)/D_0(m^2)$ is expanded in powers of g^2, a series of divergent terms for Z^{-1} results.

Chew has pointed out that the approximations (12) and (5) to $\Delta_{F}'(p^2)$ are simply related, for g^2 real and positive. Thus

$$\frac{1}{\mu^2+p^2}=\int_{4M^2}^{\infty} dm^2\,\frac{\rho_0(m^2)}{D_0(m^2)(p^2+m^2)}=\frac{Z^{-1}}{p^2-K^2}$$
$$+(p^2+\mu^2)^{-1}\left[1-(p^2+\mu^2)\int_{4M^2}^{\infty} dm^2\,\frac{\rho_0(m^2)}{p^2+m^2}\right]^{-1}, \qquad (15)$$

where K^2 is the value of p^2 at which the ghost occurs in the approximation (5). The residue at the pole in the approximation to (5), $(-Z^{-1})$, is obtained by comparing the behavior of both sides of Eq. (15) when $p^2\rightarrow+\infty$. Therefore it is possible to note that if an approximation is made in Eq. (5) and the ghost pole is subtracted with the correct residue, that a positive-definite density function results.

The proof of Eq. (15) is straightforward. After the pole is removed, the right-hand side of the equation is sufficiently bounded that one may perform the Cauchy integral and neglect the semicircle at infinity. The extra term does not contribute to the imaginary part and the pole has been eliminated so the result immediately follows.

The relationship given by Eq. (15) is of course also valid for any other approximation to $\rho(m^2)$ for which the term in square brackets on the right-hand side of Eq. (5) has only one zero. In particular it is true for any approximation which satisfies the conditions $\rho_0(m^2)>0$ for $m^2>m_0^2\geq(3\mu)^2$; $\rho_0(m^2)=0$ for $m^2\leq m_0^2$, and $\rho_0(m^2)\sim m^{-2}$ for large m.

The above result provides a convenient tool for examining the relationship between the two approximation methods. If one makes the very valid approximation that $m^2\gg\mu^2$ in the region of integration $m^2>4M^2$, then the right-hand side of Eq. (15) is readily evaluated. Thus[15]

$$\Delta_F'(p^2)=(p^2+\mu^2-i\epsilon)^{-1}\Bigg\{1-(p^2+\mu^2)\frac{g^2}{4\pi^2}$$
$$\times\left[-\frac{2}{p^2}+\frac{1}{p^2}\left(\frac{p^2+4M^2}{p^2}\right)^{\frac{1}{2}}\right.$$
$$\left.\times\ln\frac{1+[p^2/(p^2+4M^2)]^{\frac{1}{2}}}{1-[p^2/(p^2+4M^2)]^{\frac{1}{2}}}\right]\Bigg\}^{-1}$$
$$+\frac{Z^{-1}}{p^2-K^2};\quad g^2>0,\quad p^2>0. \qquad (16)$$

This equation lends itself readily to the computation of K^2 and Z^{-1}. For values of $g^2/4\pi<25$ one finds that $K^2\gg\mu^2$ and this permits further simplification of the equation. One then finds the following relations by locating the pole and finding its residue

$$\frac{g^2}{4\pi}=\pi\Big/\left(\frac{1}{y}\ln\frac{1+y}{1-y}-2\right), \qquad (17)$$

$$Z^{-1}=\left(\frac{1}{y}\ln\frac{1+y}{1-y}-2\right)\Big/\left(1-\frac{(1-y^2)}{2y}\ln\frac{1+y}{1-y}\right), \qquad (18)$$

and y is defined by

$$K^2/4M^2=y^2/(1-y^2). \qquad (19)$$

By varying y it is possible to determine K^2 and Z^{-1} as functions of g^2. Thus for $y=0.5(g^2/4\pi)=16$, $Z=0.894$ and $K^2=4M^2(0.33)$, and for $y=0.6(g^2/4\pi)=10.1$, $Z=0.845$ and $K^2/4M^2=0.56$.

It is much more interesting, however, to examine the behavior of these constants in the region $g^2\approx0$ and hence $y\approx1$. Making these approximations one readily finds that

$$Z\cong g^2/4\pi^2-\exp(-4\pi^2/g^2), \qquad (20)$$

$$K^2/4M^2\cong\exp(4\pi^2/g^2). \qquad (21)$$

If these results are substituted into Eq. (16), we see immediately that the result we have obtained using only a slight modification of perturbation theory has an essential singularity at $g^2=0$.[16]

A more precise description of the singularities in the approximation to $\Delta_F'(p^2)$ can be obtained by inspecting Eq. (11). The function is a singular function of g^2 for all values of g^2 for which $D_0(m^2)=0$ and for which m^2 is in the region of integration. It is readily seen that the curve of singularities is a closed curve enclosing that part of the real axis for which $-g_0^2<g^2<0$. $-g_0^2$ is the

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[15] In spite of its appearance this expression has no singularity at $p^2=0$. The form given is suitable for $p^2>0$ and its continuation to other regions presents no difficulties.

[16] The first suggestion that field theories might be singular in the region $g^2=0$ was made by F. J. Dyson, Phys. Rev. **85**, 631 (1952). Also see W. Thirring, Helv. Phys. Acta **26**, 33 (1953).

double root of the equation $D_0(4M^2)=0$. In the region $-g_0^2<g^2<0$ there is no ghost and Eq. (15) is modified in that Z^{-1} must be set identically equal to zero. It is clear that the curve of singularities is a natural boundary with Eq. (15) holding for all points outside of the closed curve and that for all points inside the curve of singularities the ghost removing term does not appear.

Since, as is easily seen, the left-hand side of Eq. (15) exists at the origin and all its derivatives exist at $g^2=0$, it is possible to define an asymptotic expansion for the function. By evaluating the derivatives in the region $g^2<0$, it is immediately proven that this asymptotic expansion is identical with the usual perturbation series expansion. [Note that by using the approximate Eqs. (20) and (21) it is seen that the extra term which occurs in the region $g^2>0$ is dominated by $\exp(-4\pi^2/g^2)$ which vanishes and has only vanishing derivatives at the origin. Therefore, to within the approximations used in deriving (20) and (21), the result that the asymptotic expansion coincides with the perturbation series is again confirmed.]

IV. DISCUSSION OF RESULTS

The essential point to consider, when one tries to understand the significance of the fact that Eq. (5) yields ghosts when $\rho(m^2)$ is approximated by a finite power-series expansion in g^2, is that the wave-function renormalization is finite. The basis for believing that the wave-function renormalization is finite is already present in some work by Lehmann, Zimmermann, and Symanzik[17] where they suggest that the vertex function must go to zero as its argument goes to ∞ instead of remaining finite. If the vertex function goes to zero sufficiently rapidly, then one can show that at least a contribution to the wave-function renormalization, which in perturbation theory is infinite, will become finite. Equation (5) itself, if one accepts the fact that it yields a $\Delta_F'(p^2)$ with the correct analytical form, implies quite transparently that the limit of $p^2\Delta_F'(p^2)$ as $p^2\rightarrow\infty$ is finite. If the conclusion that the wave-function renormalization is finite is accepted, this must be reconciled with the fact that when one attempts to expand Z^{-1} in a power series in g^2, one gets a series all of whose coefficients are infinite. The reconciliation is simple; $Z^{-1}(g^2)$ is not regular at $g^2=0$.[18]

When one substitutes into Eq. (5) an approximate expression for $\rho(m^2,g^2)$ which is a power series in g^2, it must be realized that one is doing the following: one is approximating $\Delta_F'(p^2,g^2)$ by a function which cannot have the correct analyticity properties as a function of g^2, and which does not necessarily have the correct analytical properties as a function of p^2. It should not be considered surprising then if it fails to reproduce either of the analyticity properties correctly.

V. CONCLUSIONS

Experience has shown that in field theory one must be extremely careful when one attempts to go beyond a term-by-term series expansion in the coupling constant. As Feldman has shown, the most straightforward extension of perturbation theory when applied to simple propagators leads to the appearance of "nonphysical" infinities in the theory. As is well known, the correct physical behavior of the functions of field theory is intimately connected with their analyticity properties. Since field-theoretical computations involve integrations in the complex plane it is extremely important, when one makes an approximation to a function which is to be used in such a calculation, that the approximation have the correct analyticity properties as a function of the momenta involved.

It is found that when one imposes the correct analyticity properties on the propagator when considered as a function of p^2, it is possible to draw some conclusions about its behavior with respect to the coupling constant. One also obtains a form which is suitable for further calculations.

VI. ACKNOWLEDGMENTS

The author wishes to acknowledge the benefit he received from conversations with Dr. G. F. Chew, Dr. F. E. Low, and Dr. J. L. Uretsky.

[17] Indications that perturbation theory overestimates the magnitude of the functions of field theory are expressed in Lehmann, Symanzik, and Zimmermann, Nuovo cimento 2, 425 (1955), where they discuss the vertex function.

[18] One should note that this does not necessarily imply that $\Delta_F'(p^2,g^2)$ has a singularity at $g^2=0$ for finite p^2. We have shown only that one can expect $\lim_{p^2\rightarrow\infty}p^2\Delta_F'(p^2,g^2)$ to be singular at $g^2=0$. However, the form we have obtained implies that $\Delta_F'(p^2,g^2)$ is singular at $g^2=0$ for all p^2.