

Mev. The quantity b_1 has been unobserved in lower energy π^- experiments partly because the data were analyzed under the assumption that it is zero. On the other hand, (3.7) predicts, even with an f^2 as low as 0.08, a value of -0.166 ± 0.014 for $b_1 + 2a_{13} + a_{11}$. This would seem to imply, if the dispersion relations are correct, that one or two of the quantities b_1 , a_{13} , and a_{11} , are much larger than has so far been observed. To estimate what values b_1 , a_{13} , and a_{11} should have to be consistent with the dispersion relations, let us take $f^2 = 0.10$. Then

$$\begin{aligned} b_1 + 2a_{13} + a_{11} &= -0.254, \\ f^2 &= 0.10. \end{aligned} \quad (3.9)$$

While present estimates of a_{13} are that it is almost zero, let us follow the suggestion of the Chew-Low theory¹¹ and take $a_{13} = a_{31} = -0.041$. Then (3.9) becomes

$$b_1 + a_{11} = -0.172, \quad a_{13} = -0.041. \quad (3.10)$$

It is interesting that the prediction of the Chew-Low theory that $a_{11} = 4a_{13} = 4a_{31}$ is compatible with (3.9) and (3.10). Reasonable choices for b_1 and a_{11} would be $b_1 = -0.04$ or -0.05 , and $a_{11} = -0.13$ or -0.12 . This choice has two very nice features: First, the relatively large value of a_{11} would lead us to expect a $T = \frac{1}{2}$ cross

¹¹ G. F. Chew and F. E. Low, Phys. Rev. **101**, 1570 (1956).

section of the order of 7 or 8 mb at energies near 150 Mev. Since present measurements in this region¹² find $\sigma_{\frac{1}{2}} \cong 0$, this means a substantial increase in σ_- , at these energies, of about ten percent. A correction to σ_- of this order of magnitude would lower the experimental values of D_-^b , in this energy region, by a significant amount. Secondly, a large, negative effective range for α_1 will keep σ_- small, or even decrease it, at low energies, before the P waves become important. We see, then, that this choice for b_1 , a_{13} , a_{11} to satisfy (3.9) would predict an energy dependence for σ_- which is qualitatively similar to that assumed by Zaidi and Lomon,² and which, in effect, raises the theoretical values for D_-^b while lowering the experimental values at the 150-Mev region. It seems to this author, therefore, that the key to the present difficulties with the π^- dispersion relation lies in the large discrepancy that exists between the value of $b_1 + 2a_{13} + a_{11}$ predicted by the dispersion relations and the value obtained from experiment.

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¹² Ashkin, Blaser, Feiner, and Stern, Phys. Rev. **101**, 1149 (1956).

Charge Symmetry of Weak Interactions*

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The invariance of strong interactions under G , the product of charge symmetry and charge conjugation, has important consequences for strangeness-conserving lepton interactions. According to the G -transformation properties of the strongly interacting "currents," we may divide the primary weak interactions into two classes. The first class includes the conventional nucleon-lepton Fermi interaction, and is the only class that contributes to the β -decay coupling constants. Unambiguous tests for the existence of second-class interactions include: (a) induced scalar term in μ^- absorption, (b) inequality of certain small correction terms in B^{12} and N^{12} , or in Li^8 and B^8 β decay, (c) inequality in rates of $\Sigma^{\pm} \rightarrow \Lambda^0 + e^{\pm} + \nu$. Absence of second-class interactions would indicate a deep relation between isotopic spin and weak interactions; for example, the recent Feynman-Gell-Mann theory predicts that all vector weak interactions are first class. The presence of second-class interactions would mean that the usual Fermi interaction is insufficient, and must be supplemented by terms involving strange particles. Some general remarks are also made about the relations between $(l^-, \bar{\nu})$ and (l^+, ν) processes, and we prove the following useful theorem: no interference between V and A may occur in any experiment which treats both leptons identically and in which no parity nonconservation effects are measured, providing that we may neglect the mass and charge of the leptons.

I. INTRODUCTION

STRONG interactions are charge symmetric and charge conjugation invariant, and therefore also invariant under the product¹ G ,

* This research was supported by the U. S. Atomic Energy Commission.

¹ T. D. Lee and C. N. Yang, Nuovo cimento **3**, 749 (1956). See also A. Pais and R. Jost, Phys. Rev. **87**, 871 (1952); L. Michel, Nuovo cimento **10**, 319 (1953), etc.

$$G \equiv C e^{i\pi I_2}, \quad (1)$$

$$G \psi_N G^{-1} = i\tau_2 \psi_N, \quad G \phi_\pi G^{-1} = -\phi_\pi, \text{ etc.}$$

This G invariance plays a fundamental part in considering the effects of strong interactions on weak processes, and the rôle of isotopic spin in the primary weak interactions. We will show that all strangeness-conserving lepton interactions may be split into two

classes, with different G -transformation properties. These two classes give rise to different effects in decay processes, and experiments may be devised to separate the effects. In our present state of knowledge there is no evidence for or against the existence of the second class.

One may hope to go further, and state as an "invariance" principle, to be tested experimentally, that only first-class interactions exist in nature. This would imply a deep relation between the weak interactions and isotopic spin, as suggested by the recent Feynman-Gell-Mann theory.² Their theory does in fact predict that only first-class vector interactions exist, and may therefore be tested by some of the experiments suggested below.

On the other hand, if there are second-class interactions in nature, we may use the class distinction as a tool in understanding the effects of virtual strange particles in decay processes.

II. GENERAL THEORY

We shall assume from the beginning that lepton pairs in strangeness-conserving interactions are emitted at a single vertex. The most general Hamiltonian for such interactions may then be written

$$H_{\text{int}} = \sum_i J_i^{(+)} \bar{\psi}_l O^i \psi_n + J_i^{(-)} \bar{\psi}_l O^i \gamma_5 \psi_n + \text{H.c.}, \quad (2)$$

$$O_i \equiv 1, \gamma_\lambda, (1/\sqrt{2})\sigma_{\lambda\mu}, i\gamma_5\gamma_\lambda, \gamma_5, \quad l \equiv e^- \text{ or } \mu^-.$$

The $J_i^{(\pm)}$ are 10 functions of the strongly interacting fields, with Lorentz transformation properties of a scalar, vector, etc. We use H_{int} as a primary, not effective, Hamiltonian so that the same 10 $J_i^{(\pm)}$ generate all strangeness-conserving lepton interactions. Almost all successful predictions for β decay and related processes are actually independent of the detailed form of the $J_i^{(\pm)}$, resting only upon Lorentz invariance and, for β decay, on the smallness of the momentum transfer.

We may split each of the $J_i^{(\pm)}$ into first- and second-class terms, defined by

$$J_i^{(\pm)} = J_{i1}^{(\pm)} + J_{i2}^{(\pm)},$$

$$J_{i1}^{(\pm)} \equiv J_i^{(\pm)} - \xi_i G J_i^{(\pm)} G^{-1}, \quad (3)$$

$$J_{i2}^{(\pm)} \equiv J_i^{(\pm)} + \xi_i G J_i^{(\pm)} G^{-1},$$

$$\xi_i \equiv +1 \text{ for } S, A, P, \text{ and } -1 \text{ for } V, T.$$

The sign factor ξ_i is inserted purely for convenience. Since $G^2 = (-)^{N+S}$, and N and S commute with $J_i^{(\pm)}$, we have

$$G J_{i1}^{(\pm)} G^{-1} = -\xi_i J_{i1}^{(\pm)}, \quad (4)$$

$$G J_{i2}^{(\pm)} G^{-1} = +\xi_i J_{i2}^{(\pm)}.$$

Equation (4), together with the G invariance of strong interactions, places a powerful restriction on the parts

of the matrix element generated by first- and second-class interactions, respectively.

Now what sort of term enters in J_1 and J_2 ? We note that the conventional currents,

$$J_i^{(+)\text{NN}} = C_i \bar{\psi}_p O_i \psi_n, \quad J_i^{(-)\text{NN}} = C_i' \bar{\psi}_p O_i \psi_n, \quad (5)$$

are of the first class. For this reason one often encounters the implicit assumption that second-class interactions are automatically excluded.³ We believe this point of view to be fallacious. There is no evidence that (5) is more fundamental in decay processes (even in nucleon β decay) than a term like

$$J_i^{(\pm)\Sigma\Sigma} = a_i^{(\pm)} \bar{\psi}_\Sigma O_i \psi_\Sigma + b_i^{(\pm)} \bar{\psi}_\Sigma O_i \psi_\Sigma^-. \quad (6)$$

This is of the first class if $a = -b$, and of the second if $a = +b$. There is certainly no *a priori* reason to choose the first rather than the second possibility, or rather than a mixture ($|a| \neq |b|$) of the two. In fact, setting $a = -b$ is equivalent to the assumption that $J_i^{\Sigma\Sigma}$ gives vector isospin selection rules, with no $\Delta I = 2$. [The principle that first-order effects of the weak interactions be renormalizable leads to the requirement that terms like (6) must exist, since they are needed as counter-terms.⁴]

Many other examples may be given. The only simple (i.e., renormalizable) V , T , A , or P second-class currents involve strange particles in the bilinear combinations ($\Sigma\Sigma$), ($\Sigma\Lambda$), and (KK). Thus, if second-class effects are observed in a process like μ -meson absorption, they might give information about virtual strange-particle pairs in the physical nucleon.

Finally, we may note that the Feynman-Gell-Mann current is given by²

$$J_\mu^{(\pm)} = C_V \{ \bar{\psi}_p \gamma_\mu \psi_n + \sqrt{2} [\phi_\pi \partial_\mu \phi_\pi^0 - \phi_\pi^0 \partial_\mu \phi_\pi]$$

$$+ \sqrt{2} [\bar{\psi}_\Sigma \gamma_\mu \psi_\Sigma - \bar{\psi}_\Sigma^+ \gamma_\mu \psi_\Sigma^0] + \bar{\psi}_\Sigma \gamma_\mu \psi_\Sigma^-$$

$$+ \phi_K \partial_\mu \phi_K^* - \phi_K^* \partial_\mu \phi_K^0 \}. \quad (7)$$

All these terms are of the first class. This is just a consequence of the fact that this current is generated by an infinitesimal isospin rotation, $\delta\varphi \sim [\varphi, I_1 + iI_2]$.

We will not make any assumptions about the form of the $J_i^{(\pm)}$, but will use (4) directly. This gives us two different sorts of information. Let us consider a process $\alpha \rightarrow \beta$ where α and β are states (of equal strangeness) of the strongly interacting particles, and a lepton and neutrino are emitted and/or absorbed. Because strong interactions conserve G , we may split the matrix element M for this process into parts M_1, M_2 generated by first- and second-class interactions, respectively. Equation (4), together with the TCP theorem, tells how to relate M_1 and M_2 for $\alpha \rightarrow \beta$ to the corresponding M_1 and M_2 for $\bar{\alpha}^c \rightarrow \bar{\beta}^c$, $\bar{\beta}^c \rightarrow \bar{\alpha}^c$, $\bar{\alpha} \rightarrow \bar{\beta}$, $\bar{\beta} \rightarrow \bar{\alpha}$, where $\bar{\alpha}$ denotes the charge mirror $\bar{\alpha} \equiv e^{i\pi I_2} \alpha$ of α . Furthermore, there are special cases where $\alpha \equiv \bar{\alpha}^c$, $\beta \equiv \bar{\beta}^c$, or $\alpha \equiv \bar{\beta}$, $\beta \equiv \bar{\alpha}$. In these cases (4) relates $\alpha \rightarrow \beta$ to itself, i.e.,

² R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958). See also S. S. Gerstein and J. B. Zeldovich, Zhur. Eksptl. i. Teoret. Fiz. **29**, 698 (1955).

³ See, for example, references 4, 8, and 11.

⁴ S. Weinberg, Phys. Rev. **106**, 1301 (1957).

gives a restriction on the forms of M_1 and of M_2 . Thus, second-class interactions may be detected either by searching for certain terms in the matrix element in these special cases, or in general by comparing a process $\alpha \rightarrow \beta$ with $\bar{\alpha}^c \rightarrow \bar{\beta}^c$, $\bar{\beta} \rightarrow \bar{\alpha}$, $\bar{\alpha} \rightarrow \bar{\beta}$, or $\bar{\beta}^c \rightarrow \bar{\alpha}^c$. We discuss the former in Sec. III and the latter in Sec. IV.

III. RESTRICTIONS ON MATRIX ELEMENTS

Let us first consider the nucleon process $n \rightarrow p$. This is a special case since $\bar{n} = \bar{p}$ and $\bar{p} = -n$. The matrix element may be written in terms of 10-vertex functions

$$\Gamma_i^{(\pm)}(k_p, k_n),$$

as

$$M = \sum_i [\bar{u}_p \Gamma_i^{(+)}(k_p, k_n) u_n \mathcal{L}^i + \bar{u}_p \Gamma_i^{(-)}(k_p, k_n) u_n \mathcal{L}'^i], \quad (8)$$

where $\mathcal{L}^i = \bar{u}_l O_i u_\nu^c$, $\mathcal{L}'^i = \bar{u}_l O_i \gamma_5 u_\nu^c$ for $n \rightarrow p + l + \nu$, etc. These vertex functions are usually drawn as a black box. We can relate them to the "currents" $J_i^{(\pm)}$ by the exact equation⁵

$$S_{F'}(k_p | p) \Gamma_i^{(\pm)}(k_p, k_n) S_F(k_n | n) = -4 \int d^4x d^4y \times e^{ik_p(x-z)} e^{-ik_n(y-z)} \langle T \{ \psi_p(x), \bar{\psi}_n(y), J_i^{(\pm)}(z) \} \rangle_0. \quad (9)$$

Applying the G invariance of strong interactions, we find

$$S_{F'}(k | p \text{ or } n) = B^{-1} S_{F'}^T(-k | n \text{ or } p) B, \quad (10)$$

and

$$-\langle T \{ \psi_p(x), \bar{\psi}_n(y), J_i^{(\pm)}(z) \} \rangle_0 = B^{-1} \langle T \{ \psi_n(y), \bar{\psi}_p(x), G J_i^{(\pm)}(z) G^{-1} \} \rangle_0^T B, \quad (11)$$

where

$$B^{-1} O_i^T B = \xi_i O_i, \quad B^T = -B. \quad (12)$$

Thus, if we split $\Gamma_i^{(\pm)}$ into terms $\Gamma_{i1}^{(\pm)}$ and $\Gamma_{i2}^{(\pm)}$ generated by $J_{i1}^{(\pm)}$ and $J_{i2}^{(\pm)}$, respectively, and insert (10) and (11) back into (9), we obtain

$$\Gamma_{i1}^{(\pm)}(k_p, k_n) = +\xi_i B^{-1} \Gamma_{i1}^{(\pm)T}(-k_n, -k_p) B, \quad (13)$$

$$\Gamma_{i2}^{(\pm)}(k_p, k_n) = -\xi_i B^{-1} \Gamma_{i2}^{(\pm)T}(-k_n, -k_p) B. \quad (14)$$

The same relations also hold for $p \rightarrow n$ processes, and hold on or off the mass shell.

To see how this rule works, let us consider β decay where we may generally use

$$\Gamma_i^{(+)}(k', k) \simeq C_i O_i, \quad \Gamma_i^{(-)}(k', k) \simeq C_i' O_i. \quad (15)$$

Using (12) we find that (15) satisfies Eq. (13) automatically, so *no second-class interaction makes any contribution to the effective nucleon β -decay coupling constants*. If we neglect the possibility of important "exchange" or cooperative effects, we may extend this statement to hold also for β decay in light complex nuclei. (However, for high Z , Coulomb effects destroy G invariance, so

⁵ See, for example, F. E. Low, Phys. Rev. **97**, 1392 (1955).

that both first- and second-class interactions may contribute to C_i, C_i' . Therefore the absence of Fierz interference in Cl^{34} ,⁶ S^{35} , and Cu^{64} ⁷ decays is evidence that no S, T couplings of either class exist. However, the evidence is weaker for the second class, and very weak for second-class S coupling.)

It is clear that in order to detect the second-class weak interactions, we must turn to a case like μ^- absorption in which (15) is not a good approximation, i.e., in which induced nonlocalities in $\Gamma_i^{(\pm)}$ appear because of a large momentum transfer. For any $\frac{1}{2} \rightarrow \frac{1}{2}$ process we may use relativistic covariance to define 12 form factors,

$$\begin{aligned} \Gamma^{(\pm)}(k', k) &= f_S^{(\pm)} 1, \\ \Gamma_\lambda^{(\pm)}(k', k) &= f_V^{(\pm)} \gamma_\lambda + g_V^{(\pm)} \sigma_{\lambda\eta} q^\eta + i h_V^{(\pm)} q_\lambda, \\ \Gamma_{\lambda\eta}^{(\pm)}(k', k) &= f_T^{(\pm)} (1/\sqrt{2}) \sigma_{\lambda\eta} + g_T^{(\pm)} [q_\lambda \gamma_\eta - q_\eta \gamma_\lambda] \\ &\quad + i h_T^{(\pm)} [\gamma_\lambda q^\mu \gamma_\mu \gamma_\eta - \gamma_\eta q^\mu \gamma_\mu \gamma_\lambda] \\ &\quad + i j_T^{(\pm)} [k_\lambda k'_\eta - k_\eta k'_\lambda], \\ \Gamma_{5\lambda}^{(\pm)}(k', k) &= i f_A^{(\pm)} \gamma_5 \gamma_\lambda + g_A^{(\pm)} q_\lambda \gamma_5 + i h_A^{(\pm)} \sigma_{\lambda\eta} \gamma_5 q_\eta, \\ \Gamma_5^{(\pm)}(k', k) &= f_P^{(\pm)} \gamma_5, \quad (q \equiv k - k'). \end{aligned} \quad (16)$$

The form factors $f_S^{(\pm)}, f_V^{(\pm)}$, etc., are functions of q^2 , and are real if T invariance holds. Equation (15) follows for $q \sim 0$ if we set $C_i = f_i^{(+)}(0)$, $C_i' = f_i^{(-)}(0)$.

Now applying (13) and (14) we see that for nucleon processes h_V, h_A , and h_T can arise only from second-class interactions while all other form factors f_S, f_V, g_V , etc., can arise only from first class interactions. Using the lepton Dirac equations and conservation of momentum, we see that the h_V term contributes to μ^- absorption an induced scalar coupling $-m_\mu h_V^{(\pm)}$, while the h_A contributes an induced Konopinski-Uhlenbeck tensor coupling $i \sigma_{\lambda\eta} (P_\mu + P_\nu)^\eta \gamma_5 h_A^{(\pm)}$. Experiments to distinguish the former would be particularly interesting, since the Feynman-Gell-Mann theory excludes second-class V interactions. [Goldberger and Treiman⁸ have calculated the other functions f_V, g_V, f_A, g_A but since they use (5) as the total current they naturally obtain $h_V = h_A = 0$. A rough guess would be $h_V, A \sim f_V, A/M$ so that the induced scalar would have magnitude $\sim (m_\mu/M) f_V$.] Since the momentum transfer in μ absorption is large, it seems reasonable to assume that the same form factors are measured in μ absorption in complex nuclei and in hydrogen. (It may also be possible to observe the nucleon form factors in very energetic β decay. This will be discussed in more detail in Sec. IV.)

For completeness, we may note that the pion decay is another "special case". Since $\pi \rightarrow -\pi$ under G , we have only first-class S, A, P , and second-class V, T coupling

⁶ J. B. Gerhart and R. Sherr, Bull. Am. Phys. Soc. Ser. II, **1**, 195 (1956).

⁷ H. M. Mahmoud and E. J. Konopinski, Phys. Rev. **88**, 1266 (1952); J. P. Davidson and D. C. Peaslee, Phys. Rev. **91**, 1232 (1953).

⁸ M. L. Goldberger and S. B. Treiman, Phys. Rev. **111**, 354 (1958).

contributing to $\pi \rightarrow l + \nu$, while only first-class V , T and second-class S , A , P contribute to $\pi \rightarrow \pi^0 + e + \nu$. However, Lorentz invariance implies that only A , P coupling contributes to the former and S , V , T to the latter. Furthermore S , T coupling is probably absent. Thus, probably only first-class interactions may contribute to either pion-decay mode.

IV. COMPARISON BETWEEN MIRROR PROCESSES

We shall prove two theorems that are useful in comparing a process $\alpha \rightarrow \beta$ (e.g., $\alpha \rightarrow \beta + e^- + \bar{\nu}$, $e^+ + \alpha \rightarrow \beta + \bar{\nu}$, etc.) with its mirror process $\bar{\alpha} \rightarrow \bar{\beta}$ (e.g., $\bar{\alpha} \rightarrow \bar{\beta} + e^+ + \nu$, $e^- + \bar{\alpha} \rightarrow \bar{\beta} + \nu$, etc.).

Theorem 1

The differential transition probabilities for strangeness-conserving lepton processes $\alpha \rightarrow \beta$ and $\bar{\alpha} \rightarrow \bar{\beta}$ are identical, except for a difference in sign in those terms which arise from the following:

(A) interference between $J^{(+)}$ and $J^{(-)}$, i.e., an *observed* parity nonconservation effect like a $\sigma \cdot \mathbf{K}$ correlation;

(B) interference between time-reversal conserving and nonconserving parts of the currents, i.e., an *observed* time reversal noninvariance effect like a $\mathbf{J} \cdot (\sigma \times \mathbf{K})$ correlation;

(C) interference between S , A , P , and V , T coupling (defined by the *lepton* fields); and

(D) interference between first- and second-class interactions.

Proof.—The *TCP* theorem tells us that $\alpha \rightarrow \beta$ and $\alpha^c \rightarrow \beta^c$ are identical except for a sign change in interference terms of type (A) or (B). If *T* invariance is violated we should add the restriction that the states α , β are eigenstates of the strong *S* matrix.⁹ The *G* invariance of strong interactions tells us that $\alpha^c \rightarrow \beta^c$ and $\bar{\alpha} \rightarrow \bar{\beta}$ are identical except for a sign change in interference between currents which transform under *G* with different sign, i.e., in interference terms of type (C) and (D). Thus the theorem holds, if we may neglect electromagnetic effects which violate *G* invariance.

This theorem is well known¹⁰ in the special case of low-energy β decay where, as we have seen in Sec. III, second-class interactions do not enter. In order to apply the theorem in cases where second-class interactions may be important, we must face the difficulty of deciding when an experiment involves a type (C) interference. This is not easy when the momentum transfer is so large that multipole expansions are useless; in these cases we may use the following theorem.

⁹ Lee, Oehme, and Yang, *Phys. Rev.* **106**, 340 (1957).

¹⁰ H. A. Tolhoek and S. R. de Groot, *Physica* **17**, 81 (1951).

Theorem 2

If we perform any experiment with arbitrarily large momentum transfer, which does not distinguish between the lepton and neutrino (e.g., a measurement of total decay rate or average $l - \nu$ angular correlation, but *not* measurement of electron energy spectrum), and if we may neglect the mass and charge of the lepton, then there can be no interference between coupling types S , A , P , S' , V' , P' and V , T , T' , A' . (Here a prime means parity-nonconserving coupling.) This theorem holds for strangeness-violating decays (like K_{e3} , $\Lambda \rightarrow N + e + \nu$, etc.) well as for strangeness-conserving decays.

Proof.—The interaction Hamiltonian (2) is invariant under the following formal transformation:

$$\begin{aligned} \psi_l &\rightarrow \psi_l^c, & \psi_\nu &\rightarrow \psi_\nu^c, & J_i^{(+)} &\rightarrow \xi_i J_i^{(+)}, \\ J_i^{(-)} &\rightarrow \xi_i \eta_i J_i^{(-)}, \end{aligned} \quad (17)$$

where $O_i \gamma_5 = \eta_i \gamma_5 O_i$. The product $\eta_i \xi_i$ is $+1$ for S , V , P , and -1 for A , T . Thus if we assume only V , A coupling, and if we perform a “parity-conserving” experiment symmetric between e and ν , we can get no interference of type (C).

An immediate application of these two theorems would be to the decay modes $\Sigma^\pm \rightarrow \Lambda^0 + e^\pm + \nu$, expected to occur with branching ratio $10^{-4} \sim (\Sigma \rightarrow \Lambda + e + \nu) / (\Sigma \rightarrow N + \pi)$. The momentum transfer is sufficiently large for induced nonlocal effects to be important. Measurement of the total decay rate does not involve interference of type (A), (B), or (by Theorem 2) (C), so any inequality in the rates for the Σ^+ and Σ^- modes would be evidence for the existence of second-class interactions. The subsequent Λ^0 decay should make these modes easily recognizable, and may also serve as an analyzer of the Λ^0 polarization.

It may also be possible to observe second-class effects in very energetic β decays, like B^8 , Li^8 , B^{12} , N^{12} ($E_\alpha \sim 15$ Mev). Let α and β be initial and final nuclei, with momenta zero and $-\mathbf{q} = -(\mathbf{P}_e + \mathbf{P}_\nu)$, respectively. We may expand the matrix elements of the currents in (2) up to first order in \mathbf{q}/M and obtain (omitting the “ \pm ”) for allowed transitions

$$\begin{cases} V \left\{ \begin{aligned} \langle \beta | \mathbf{J} | \alpha \rangle &= \frac{1}{2M} \mathbf{q} \times \mathbf{A}^{(V)} + \frac{1}{2M} \mathbf{q} S^{(VII)} + \dots, \\ \langle \beta | J_0 | \alpha \rangle &= S^{(VI)} + \dots, \end{aligned} \right. \\ A \left\{ \begin{aligned} \langle \beta | \mathbf{J}_5 | \alpha \rangle &= \mathbf{A}^{(AI)} + \dots, \\ \langle \beta | J_{50} | \alpha \rangle &= \frac{1}{2M} \mathbf{q} \cdot \mathbf{A}^{(AII)} + \dots. \end{aligned} \right. \end{cases} \quad (18)$$

Here the S and \mathbf{A} are matrix elements of various three-dimensional scalars and axial vectors. The $S^{(VI)}$ and $\mathbf{A}^{(AI)}$ are just the usual $iC_V \int 1$ and $C_A \int \sigma$, while the terms proportional to \mathbf{q} produce small distortions of the

spectrum shape and $e-\nu$ correlation¹¹ and may also be observed in $\beta-\alpha$ correlations.¹² One would expect all the S and \mathbf{A} to be of roughly the same order of magnitude, but of course the expansion parameter \mathbf{q}/M is quite small.

Now, how are we to assign a "class" to these matrix elements? In general, each of the S and \mathbf{A} may be split into first- and second-class contributions $S_1, S_2, \mathbf{A}_1, \mathbf{A}_2$. However, we suspect that some of these matrix elements get their main contribution from one or the other of the two classes. Suppose we write the vertex function for nonrelativistic nucleons in the form (18). Then, referring back to (16), we find for the nucleon,

$$\begin{aligned} S^{(VI)} &= if_V \langle 1 \rangle, & \mathbf{A}^{(AI)} &= f_A \langle \boldsymbol{\sigma} \rangle, \\ S^{(VII)} &= i(f_V + 2Mh_V) \langle 1 \rangle, & \mathbf{A}^{(AII)} &= (f_A - 2Mh_A) \langle \boldsymbol{\sigma} \rangle, \\ \mathbf{A}^{(V)} &= (f_V + 2Mg_V) \langle \boldsymbol{\sigma} \rangle. \end{aligned} \quad (19)$$

Thus, to the extent that the main contribution to the S and \mathbf{A} comes from the structure of the nucleon, rather than the nucleus, we may expect that $\mathbf{A}^{(V)}, S^{(VI)}, \mathbf{A}^{(AI)}$ are mainly first-class matrix elements, while $S^{(VII)}$ and $\mathbf{A}^{(AII)}$ have large contributions from second-class interactions (h_V, h_A). (This assumption is analogous to ignoring all orbital or exchange currents in γ decay; Gell-Mann¹¹ shows that this works well in the $\Delta I=1$ C^{12} decay.)

Since none of the matrix elements in (18) can be positively identified as a pure second-class term, the only unambiguous test for the existence of second-class interactions here would be the comparison of mirror transitions, like $B^{12}, N^{12} \rightarrow C^{12}$ ($\Delta J=1$, no) or $B^8, Li^8 \rightarrow Be^8$ ($\Delta J=0$, no). Using Theorem 1, if there are no second-class interactions we should have a sign change, in going from $\alpha \rightarrow \beta$ to $\bar{\alpha} \rightarrow \bar{\beta}$, in the interference between $\mathbf{A}^{(AI)}$ and $\mathbf{A}^{(V)}$ (Gell-Mann's "a"), but not for the interference between $\mathbf{A}^{(AI)}$ and $\mathbf{A}^{(AII)}$ (Gell-Mann's "b"), or between $S^{(VI)}$ and $S^{(VII)}$ ("c"). According to our previous analysis, if there are second-class interactions they are most likely to show up strongly in $S^{(VII)}$ or $\mathbf{A}^{(AII)}$, so that if b is found not equal for B^{12} and N^{12} , or if b and c are found not equal for B^8 and Li^8 , we may certainly conclude that second-class interactions exist. The inequality in b or c would produce inequalities of a few percent in the ft values for these mirror transitions. Unfortunately, experiments are not yet accurate enough to check this prediction,¹³ and in any case Coulomb effects may also

give a 1% inequality in ft values.¹⁴ The inequality of $b(B^{12})$ and $b(N^{12})$ would also interfere with Gell-Mann's proposed experiment,¹¹ which was to measure a .

There is one point about the measurement of these small corrections that deserves mention. As shown in Theorem 2, there can be no V/A interference in a (P,T) -conserving experiment which does not distinguish between e and ν . This means that the $\mathbf{A}^{(V)}$ term in (18) [which must occur in interference with $\mathbf{A}^{(AI)}$] can have no effect on the decay rate or average $e-\nu$ angular correlation, though it may appear in the spectrum shape. This may be verified directly by integrating over electron energy in Gell-Mann's formulas, and letting $m_e \rightarrow 0$; the "a" term then drops out. (Also, interference between an \mathbf{A} and an S can only occur in $\Delta J=0$ transitions with oriented nuclei.)

Our discussion has been restricted to the G defined in (1). However, the CTP theorem tells us how the $J_i^{(\pm)}$ transform under CT , so that (3) might equivalently be written in terms of an operator $G' \equiv T e^{i\pi I_2}$. If T invariance holds, we then have

$$\begin{aligned} e^{i\pi I_2} J_{i1}^{(\pm)} e^{-i\pi I_2} &= -\xi_i \omega_i J_{i1}^{(\pm)\dagger} \\ e^{i\pi I_2} J_{i2}^{(\pm)} e^{-i\pi I_2} &= +\xi_i \omega_i J_{i2}^{(\pm)\dagger}, \\ \omega_i &= +1 \text{ for } S, V, A \text{ and } -1 \text{ for } P, T, \end{aligned} \quad (20)$$

so that the distinction between first and second class could then be derived from charge symmetry alone.

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Note added in proof.—The example presented in (6) of a possible second-class current is badly chosen; since $J_{i2}^{\Sigma\Sigma}$ has $\Delta I=2$ isospin transformation properties, it cannot contribute to an $I=1 \rightarrow I=0$ transition, such as $B^{12} \rightarrow C^{12}$ or $\Sigma \rightarrow \Lambda$. (I wish to thank Professor G. Feldman for pointing this out.) However, we can find examples of second-class currents with pure $\Delta I=1$ behavior, such as

$$J_{i2}^{\Sigma\Lambda} \sim \bar{\psi}_{\Sigma^+} O_i \psi_{\Lambda^0} - \bar{\psi}_{\Lambda^0} O_i \psi_{\Sigma^-}. \quad (21)$$

It is also possible to have second-class currents using only nucleon fields, if we introduce direct derivative terms corresponding to h_V, h_T, h_A in (16).

¹¹ M. Gell-Mann, Phys. Rev. **111**, 362 (1958). Also J. Bernstein and R. Lewis (to be published).

¹² M. Morita and M. Yamada, Progr. Theoret. Phys. (Japan) **13**, 114 (1955).

¹³ Experiments quoted by Cook, Fowler, Lauritsen, and Lauritsen, Phys. Rev. **107**, 508 (1957), and by T. Ajzenberg and T. Lauritsen, Revs. Modern Phys. **27**, 77 (1955).

¹⁴ This estimate is based on the values of isotopic spin impurities in light nuclei, as known from the rates of $\Delta I=0$ $E1$ γ transitions in self-conjugate nuclei. See D. H. Wilkinson, *Proceedings of the Rehovoth Conference on Nuclear Structure* (North-Holland Publishing Company, Amsterdam, 1958), p. 175.