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Be^7 Nuclei as Evaporated Particles in High-Energy Reactions*

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The probability for the emission of Be^7 nuclei in an evaporation process has been calculated. These calculations support the hypothesis that the Be^7 produced in high-energy proton irradiation of copper, silver, and gold as described in the previous paper, is formed directly by an evaporation process.

INTRODUCTION

In the preceding paper[1] experimentally determined cross-sections for the production of Be^7 in copper, silver, and gold targets irradiated with 1; 2.2; and 3-Bev protons were presented. In addition, Marquez and and Perlman[2] have published data for the production of Be^7 from the same target elements with 335-Mev protons. Since it was very difficult to explain the Be^7 in the amounts observed as a spallation residue or fission product, the possibility of an evaporation mechanism, i.e., of Be^7 emission competing with emission of neutrons, protons, alpha particles, etc., from highly excited nuclei was investigated.

The over-all reaction between high-energy incident particles and complex nuclei has generally been described, following a suggestion by Serber,[3] as occurring in two stages:

(a) Knock-on cascade—in which, due to the short de Broglie wavelength of the incoming particle, interactions are assumed to occur between individual nucleons. The knock-on cascade results in the emission of relatively high-energy elementary particles and in a residual nucleus which is left in an excited state.

(b) Evaporation process—in which the residual nucleus resulting from the cascade de-excites by the evaporation of neutrons, protons alpha particles, etc., according to the usual compound nucleus picture.

A number of authors[4–6] have studied the evaporation process based on the statistical model of the nucleus and start from Weisskopf's[7] basic equation

$$P_j(T)dT=\gamma_j\sigma T(\rho_f/\rho_i)dT, \tag{1}$$

where $P_j(T)dT$ is the probability per unit time that particle j will be emitted with kinetic energy between T and $T+dT$, $\gamma_j=g_jm_j/\pi^2h^2$, with $g_j=$number of spin states of particle j, and $m_j=$mass of particle j, σ is the cross section for the inverse process, and ρ_f and ρ_i are the level densities of the final and initial nuclei, respectively.

If an explicit model for the nucleus is assumed, for example that of a Fermi gas, the relative probabilities of emission of various particles i, j, k, $\cdots$ from a given nucleus at a given excitation may be calculated, provided an assumption is made about the form of the inverse cross section σ. Usually σ is taken as $\sigma_{\text{geom}}\times(1-V_j/T)$, where σ_{geom} is the geometric cross section for interaction of particle j with the product nucleus, and V_j is the Coulomb barrier for this pair of particles.

Following essentially the notation of LeCouteur,[4] one can write the relative probabilities for evaporation of particles i and j from a highly excited nucleus (initial excitation E_0) in the approximate form

$$\frac{P_i}{P_j}=\frac{\gamma_i}{\gamma_j}\frac{R_i}{R_j}\frac{a_j}{a_i}\exp\{2[(a_iR_i)^{\frac{1}{2}}-(a_jR_j)^{\frac{1}{2}}]\}, \tag{2}$$

where R_i, R_j are the maximum values of the residual excitation energies of the product nuclei, i.e., $R_i=E_0-Q_i-V_i$ and $R_j=E_0-Q_j-V_j$; Q_i, Q_j are the separation energies of particles i and j; V_i, V_j are the Coulomb barriers for emission of i and j; and a_i, a_j are the level density parameters for the product nuclei, according to the expression $\rho=C\exp[2(aE)^{\frac{1}{2}}]$ for the level density at excitation energy E.

Recent Monto Carlo calculations of evaporation cascades by Dostrovsky *et al.*[8] on the basis of LeCouteur's equations take into account at each step the possibilities of evaporation of neutrons, protons, deuterons, tritons, He^3, and He^4. Because evaporation of larger entities is relatively rare, it was neglected in their computations. Its inclusion would have little effect on the average course of the evaporation process, and to obtain, from a complete Monte Carlo evaporation calculation, statistically meaningful results on the cross sections for emission of nuclei of lithium, beryllium, etc., would require the study of a very large

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[1] Baker, Friedlander, and Hudis, preceding paper [Phys. Rev. **112**, 1319 (1958)].

[2] L. Marquez and I. Perlman, Phys. Rev. **81**, 953 (1951).

[3] R. Serber, Phys. Rev. **72**, 1114 (1947).

[4] K. J. LeCouteur, Proc. Phys. Soc. (London) **A63**, 259 (1950).

[5] Y. Fujimoto and Y. Yamaguchi, Progr. Theoret. Phys. Japan **4**, 468 (1950).

[6] G. Rudstam, thesis, Uppsala, 1956 (unpublished).

[7] V. Weisskopf, Phys. Rev. **52**, 295 (1937).

[8] Dostrovsky, Rabinowitz, and Bivins, Phys. Rev. **111**, 1659 (1958).

number of cascades and, even on a high-speed computer, an inordinate amount of time. Therefore a more approximate procedure was adopted for the present study.

EVAPORATION CALCULATION

The starting point for the present evaporation calculation is the spectrum of excited nuclides left after the completion of the knock-on cascades. A Monte Carlo calculation of the knock-on phase of nuclear reactions in the energy region of interest has been published,[9] and results are available for Cu^{64}, Ru^{100}, and Bi^{209} targets irradiated with 460-, 940-, and 1840-Mev protons. Time limitations made it impossible to follow each excited nuclide, left after the knock-on cascade, down the evaporation path. Instead, the spectrum of excited nuclides was represented by a small number of average excited nuclides and these average nuclides were used as starting points for the evaporation calculation. The averaging procedure was as follows:

(a) Knock-on residues were grouped into j bins according to their excitation energy and the members of each group were assigned the energy of the midpoint of the interval; 100–200 Mev ($\bar{E}=150$ Mev), 200–300 Mev ($\bar{E}=250$ Mev), etc.

(b) The most probable mass ($\bar{A}$) was found for each energy bin.

(c) The values of Z for nuclides of $\bar{A}$, $\bar{A}+1$, and $\bar{A}-1$ were tabulated and the most probable value $\bar{Z}$ was found.

(d) This averaging procedure was carried out on the excited nuclides left after the knock-on cascades of Cu^{64}, Ru^{100}, and Bi^{209} irradiated with 1840-Mev protons. The values of $\bar{A}$ and $\bar{Z}$ corresponding to $\bar{E}$ in the 1840-Mev calculations were also used for the 460- and 940-Mev calculations, since the values of $\bar{A}$ and $\bar{Z}$ for a given $\bar{E}$ change only slightly with incident energy.[9]

(e) The values of $\bar{A}$ and $\bar{Z}$ obtained from the calculations for Ru^{100} and Bi^{209} targets were adjusted to correspond to Ag^{108} and Au^{197} targets.

Results of the Monte Carlo knock-on cascades were thus reduced to a small group of average nuclides $(\bar{A},\bar{Z},\bar{E})_j$ for each target. There were 5 such representative excited nuclides for Cu, 7 for Ag, and 7 for Au. Evaporation cascades, approximately 20 per $(\bar{A},\bar{Z},\bar{E})_j$, were performed on the Maniac I electronic computer with the Monte Carlo routine developed by Dostrovsky *et al.*[8] The actual course of the evaporation path was plotted for each group of 20 cascades. Approximately 100-Mev energy intervals were taken along the evaporation path and the average values of A and Z $(\bar{A}^*,\bar{Z}^*)_i$ in each energy interval $(\bar{E}^*)_i$ were determined. There were 15 such average nuclides $(\bar{A}^*,\bar{Z}^*,\bar{E}^*)_i$ for copper, 29 for silver, and 32 for gold targets. In addition, the evaporation cascades yielded the average number of neutrons and protons emitted for each $(\bar{A},\bar{Z},\bar{E})_j$ starting point.

Before the ratio P_{Be^7}/P_n could be calculated for each of the $(\bar{A}^*,\bar{Z}^*,\bar{E}^*)_i$ nuclides, Eq. (2) had to be put in more explicit form. According to LeCouteur, the parameter a in Eq. (2) depends on the neutron excess of the nucleus and is formulated by him as

$$a_n^{\frac{1}{2}}=a^{\frac{1}{2}}(1-1.3\theta/A), \tag{3}$$

where a_n is the level density parameter of the daughter nucleus resulting from neutron emission, $a=A/10$, A = mass of parent nucleus, and $\theta=(N-Z)/A$.

Based on LeCouteur's formulation, the corresponding equation for the level density parameter of a nucleus resulting from Be^7 emission is

$$a_{Be^7}^{\frac{1}{2}}=a^{\frac{1}{2}}(1-3/A+1.3\theta/A). \tag{4}$$

With the use of Eqs. (3) and (4) and the definition $R_j=E_0-Q_j-V_j$, Eq. (2) may now be rewritten for the ratio of emission probabilities of Be^7 and neutron:

$$\frac{P_{Be^7}}{P_n}=\frac{\gamma_{Be^7}}{\gamma_n}\frac{R_{Be^7}}{R_n}\frac{a_n}{a_{Be^7}}\exp\left\{\frac{2a^{\frac{1}{2}}}{(R_{Be^7,n})^{\frac{1}{2}}}\times\left[\frac{Q_n-Q_{Be^7}-V_{Be^7}}{2}+R_{Be^7,n}\left(2.6\frac{\theta}{A}-\frac{3}{A}\right)\right]\right\}, \tag{5}$$

where $(R_{Be^7,n})^{\frac{1}{2}}=(R_{Be}^{\frac{1}{2}}+R_n^{\frac{1}{2}})/2$.

To compute values for the separation energies Q, Cameron's[10] mass tables were used. Coulomb barrier calculations were based on the usual relationship:

$$V=P\frac{Z_1Z_2e^2}{r_0[A_1^{\frac{1}{3}}+A_2^{\frac{1}{3}}]1.6\times10^{-6}}\text{ Mev},$$

where r_0, the nuclear radius parameter, was taken as 1.3×10^{-13} cm and P, the penetrability factor for Be^7, was taken as 0.9 from an extrapolation of the data of Bethe and Konopinski,[11] and where A_1, Z_1, and A_2, Z_2 are mass and atomic numbers of residual nucleus and emitted particle, respectively.

By use of Eq. (5) the values of P_{Be^7i}/P_{ni} were found for each nucleus $(\bar{A}^*,\bar{Z}^*,\bar{E}^*)_i$. Calculations were not carried out below 50 Mev for Cu, 100 Mev for Ag, and 150 Mev for Au since it was shown that evaporation yields of Be^7 below these cutoff energies are extremely low.

The sum $(1/i_{\max})\sum_i(P_{Be^7i}/P_{ni})$ taken over all the nuclides $(\bar{A}^*,\bar{Z}^*,\bar{E}^*)_i$ in a given evaporation chain, i.e., for a given j, yields the average probability of Be^7 emission as compared with neutron emission over one

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[9] Metropolis, Bivins, Storm, Miller, Friedlander, and Turkevich, Phys. Rev. **110**, 204 (1958).

[10] A. G. W. Cameron, Atomic Energy of Canada Ltd. Report CRP–690, 1957 (unpublished).

[11] H. A. Bethe and E. J. Konopinski, Phys. Rev. **54**, 130 (1938).

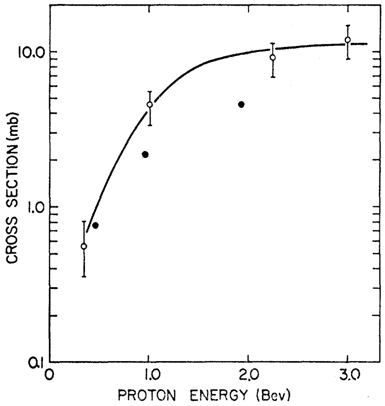

FIG. 1. Excitation function for the production of Be^7 from Cu. The open circles are the experimentally determined cross sections and the solid points are the calculated cross sections. The experimental point at 335 Mev is from reference 2.

complete evaporation cascade. If the number of neutrons emitted in each of the i energy intervals is equal,[12] then

$$\frac{N_{nj}}{i_{\max}}\sum_i \frac{P_{\mathrm{Be}^7 i,j}}{P_{ni,j}}=N_{\mathrm{Be}^7 j}, \tag{6}$$

where N_{nj}= the number of neutrons emitted in the jth evaporation cascade between the cutoff energy and the maximum energy, and $N_{\mathrm{Be}^7 j}$= the number of Be^7 nuclei emitted in the jth cascade between the cutoff energy and the maximum energy.

If f_j= the fraction of the total inelastic cross section resulting in the excited nuclides represented by $(\bar{A},\bar{Z},\bar{E})_j$, then the fraction F_{Be^7} of the total inelastic cross section resulting in Be^7 emission is

$$F_{\mathrm{Be}^7}=\sum_j N_{\mathrm{Be}^7 j}\times f_j. \tag{7}$$

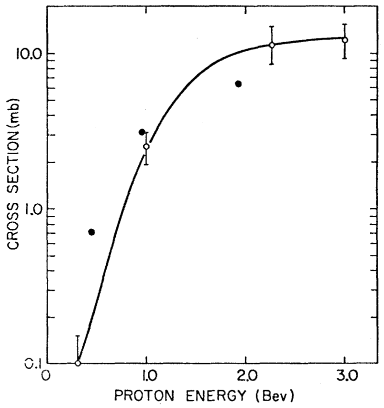

FIG. 2. Excitation function for the production of Be^7 from Ag. The open circles are the experimentally determined cross sections and the solid points are the calculated cross sections. The experimental point at 335 Mev is from reference 2.

At each incident proton energy the appropriate values of f_j obtained from the knock-on cascade data,[9] were inserted into Eq. (7).

The preceding calculation is based on the assumption that the Be^7 particles are evaporated in their ground state. The possibility of Be^7 formation in excited states has been taken into account in the following approximate manner. There is one known excited state of Be^7 which is stable towards particle emission. This state is 0.43 Mev above the ground state and has $J=\frac{1}{2}$,[13] whereas the spin of the ground state is most probably $\frac{3}{2}$.[14] Thus, the term $\gamma_{\mathrm{Be}^7}/\gamma_n$ in Eq. (5) has the value 14 for Be^7 in its ground state, and the value 7 for Be^7 in its excited state. Since Be^7 emission is probable only at high nuclear excitations, the energy difference between the ground state and excited state should not have a large effect on their relative emission probabilities. Therefore all values of the calculated Be^7 ground-state cross sections were multiplied by 1.5 to obtain the total Be^7 yield.

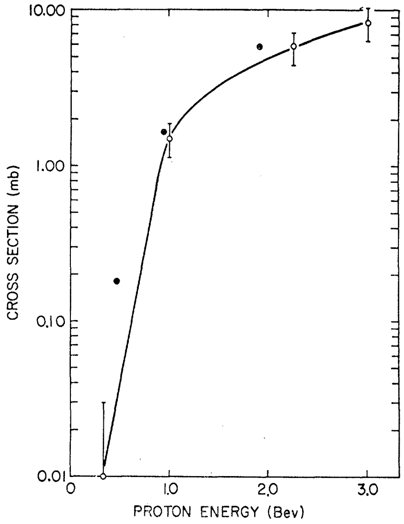

FIG. 3. Excitation function for the production of Be^7 from Au. The open circles are the experimentally determined cross sections and the solid points are the calculated cross sections. The experimental point at 335 Mev is from reference 2.

RESULTS AND DISCUSSION

In Figs. 1–3 are plotted the experimentally observed excitation functions for Be^7 production from copper, silver, and gold,[1] and in addition, the calculated cross sections. The agreement is satisfactory when one considers the averaging procedures which went into the calculations, and seems to be best with high-Z targets.

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[12] The evaporation cascade calculations of reference 8 indicate that this assumption is approximately correct.

[13] F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. **24**, 321 (1952).

[14] Segel, Kane, and Wilkinson, Phil. Mag. **3**, 204 (1958).

While this may be due to the fact that with high-Z targets a larger fraction of the observed Be^7 results from an evaporation mechanism, it should be noted that the calculations and assumptions employed are less reliable for copper than for silver or gold. It does seem evident, however, that at least a large fraction of the observed Be^7 formed when any of these elements interact with protons between 460 and 1840 Mev may be explained by an evaporation mechanism.

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Quasi-Elastic Scattering of Pions by Nuclei*

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This paper discusses a prescription, based upon the impulse approximation, for estimating the inelastic contamination generally present in attempted measurements of elastic pion-nucleus differential scattering cross sections because of the finite energy resolution of detectors. It is also pointed out that in such quasi-elastic measurements the interference portion of the inelastic scattering, for which the form factor is the Fourier transform of the important and unknown nucleon pair correlation density, is experimentally separated from the direct inelastic scattering over a large range of scattering angles.

1. INTRODUCTION

FOR lack of sufficient energy resolution, experimental studies of the elastic scattering[1] of high-energy pions from nuclei have generally included in what was designated as "elastic" some slightly inelastic scattering as well. It was only recently that Baker,[2] using counter techniques with greatly improved resolution (5%), managed for the first time to isolate truly elastically scattered 80-Mev pions well enough to observe clearly the diffraction minima at large scattering angles to be expected from the optical model of elastic scattering.[3]

It is the twofold purpose of the present theoretical analysis of quasielastic scattering (1) to provide a means of estimating the inelastic contamination in poorly resolved pion-nucleus elastic scattering data, and (2) to appraise the nuclear information obtainable from well-defined quasielastic scattering experiments in the hope that, with experimental improvements, pions might serve as radiation of sufficiently short wavelength to probe the details of nuclear structure. It will be suggested that experiments with Baker's degree of resolution might already be capable of providing information about nucleon pair correlation densities.

Following the necessary definitions, some simple cross section estimates will be discussed, and then in later sections a more general approach will be presented. Though the projectile and target will be referred to as pion and nucleus, many of the results are applicable to the scattering of any fast particle from a complex system.[4]

2. DEFINITIONS AND APPROXIMATIONS

The term "quasielastic" will include elastic scattering together with all inelastic scatterings in which the nucleus is excited by no more than some specified, small amount, ΔE. Then, the quasielastic differential cross section, $\sigma_q(\theta)$, is a weighted sum of partial cross sections, $\sigma_{fi}(\theta)$, leading from the initial nuclear state i, assumed for simplicity to be the ground state, to the

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‡ Now at the Oak Ridge National Laboratory, Oak Ridge, Tennessee.

[1] *Elastic* and *inelastic* are here defined with respect to the nucleus. A purely elastic collision is one in which the nucleus receives precisely zero excitation energy.

[2] W. F. Baker, Ph.D. thesis, Columbia University, March, 1957 (unpublished).

[3] Diffraction minima in the elastic scattering of other high-energy elementary particles by nuclei had already been observed: electrons, Hahn, Ravenhall, and Hofstadter, Phys. Rev. **101**, 1131 (1956); protons, K. Strauch and F. Titus, Phys. Rev. **103**, 200 (1956).

[4] For example, quasielastic scattering of high-energy protons from nuclei: K. Strauch and F. Titus, Phys. Rev. **104**, 191 (1956), and reference 3. H. Tyrén and Th. A. J. Maris, Nuclear Phys. **3**, 52 (1957); **4**, 637 (1957); **4**, 662 (1957); **6**, 82 (1958).