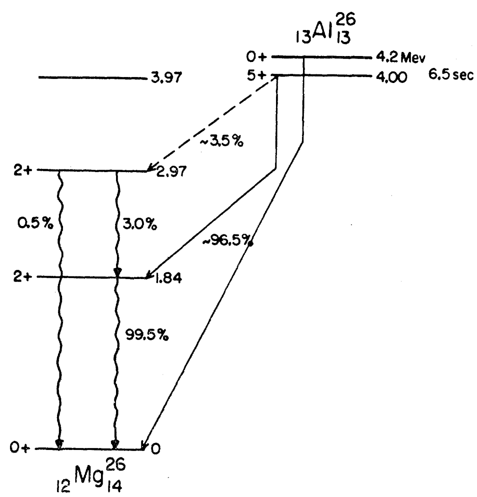

FIG. 5. Proposed Al^{26} decay scheme.

For the most part, the parities and angular momenta of the levels involved in the Al^{26} decay are not uniquely determined. We may, of course, assume the ground level of Mg^{26} is 0^+. The angular distribution work on the $Mg^{25}(d,p)Mg^{26}$ reaction by Holt and Marsham[7] indicates that the first and second excited levels of Mg^{26} each have even parity and angular momenta of 2 or 3. Two is more likely for the first excited level, since Mg^{26} is even-even. Assuming the first excited level is 2^+ and the second excited level is 2^+ or 3^+, the gamma transition from the second to the first excited level is magnetic dipole. The transition from the second excited level to the ground level is electric quadrupole if $I=2^+$ for the second level, magnetic octopole if $I=3^+$. Using Weisskopf's formula[8] for gamma-ray transition probabilities, the ratio of 1.1-Mev gamma rays to 2.97-Mev gamma rays would be 10^7 for an $I=3$ level, 25 for an $I=2$ level. The latter is in much better agreement with the observed ratio, so we assign $I=2$ to the second excited level.

The Al^{26} ground level has been predicted to be 5^+ by King and Peaslee,[9] on the basis of the systematics of odd-odd nuclei. This is in agreement with the observed forbidden positron decay to the first excited level, and the lack of any observable positron decay to the Mg^{26} ground level. The angular momentum and parity assignments are summarized in Fig. 5.

ACKNOWLEDGMENT

Thanks are due to Professor R. D. Evans of the Massachusetts Institute of Technology for suggesting this project and for many helpful discussions.

[7] J. R. Holt and T. N. Marsham, Proc. Phys. Soc. (London) **A66**, 249 (1953).

[8] V. F. Weisskopf, Phys. Rev. **83**, 1073 (1951).

[9] R. W. King and D. C. Peaslee, Phys. Rev. **90**, 1001 (1953).

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Beta-Gamma Correlation and Time-Reversal Invariance*

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The distribution function for the first-forbidden beta-gamma correlation for randomly oriented nuclei, including beta transverse polarization terms, is presented and discussed in connection with the question of time-reversal invariance. Coulomb field effects are included and it is found that even for relatively small Z the time-reversal testing asymmetry is reduced appreciably compared to that calculated for $Z=0$ by Curtis and Lewis. In the limit of high $(\alpha Z/2R)$, that is, for most first-forbidden decays, a definite relation exists between the ordinary directional correlation asymmetry and the beta polarization-dependent asymmetries. In this approximation it is found that terms which test time-reversal invariance appear in the same manner in all asymmetries but are dominated in general by contributions which do not test time-reversal invariance. For the particular case of Au^{198} it is shown that the experimental results are consistent with time-reversal invariance but are also consistent with an appreciable violation of time-reversal invariance. It is concluded that under favorable conditions it is barely possible that an investigation of the asymmetries for some other beta-gamma cascade could provide a test for time-reversal invariance. However, the *extent* to which this invariance is or is not violated could not be determined by such an investigation.

INTRODUCTION

CURTIS and Lewis[1] have suggested the possibility of testing the time-reversal invariance of the beta interaction Hamiltonian by an examination of the correlation between decay products in a beta-gamma cascade. The proposed test demands a measurement of the correlation between the transverse polarization of the beta particle and the momenta of the electron and the photon.

When the beta transition is allowed, the asymmetries which test time-reversal invariance in the theoretical distribution for the cascade process are negligible relative to the isotropic terms unless the

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[1] R. B. Curtis and R. R. Lewis, Phys. Rev. **107**, 543 (1957).

decaying nucleus is oriented[2] or unless the gamma circular polarization is observed.[3] To the extent that one may draw conclusions on the basis of the $Z=0$ approximation (the example of Curtis and Lewis is given in this approximation), the asymmetry which tests time-reversal invariance and which depends on the beta polarization would be expected to be appreciable relative to the isotropic terms when the beta transition is first forbidden—if the violation of time-reversal invariance is considerable.

Preliminary experimental results on the $2^- \rightarrow 2^+ \rightarrow 0^+$ decay of Au^{198} have been reported by Steffen[4] and a small effect depending on the electron transverse polarization has been found. Steffen quotes an upper bound of about 7% for the transverse polarization of the Au^{198} electrons in a direction perpendicular to both the electron and the photon momentum. Interpreted in accordance with the $Z=0$ calculation, the measured effect indicates a possible violation of time-reversal invariance.

Because of the high-Z value of Au^{198} one might wonder whether it is legitimate to interpret the experiment in terms of the $Z=0$ approximation. It is well known from the study of ordinary beta-decay processes that the approximation which neglects the effect of the nuclear charge on the electron final-state wave function is adequate only when the first-forbidden transition is unique, i.e., $\Delta J=2$ (yes).[5] For example, except in a very few instances of low-Z decays, the energy spectrum associated with an ordinary first-forbidden transition has the allowed shape due to the dominance of energy-independent terms containing the nuclear charge. Even for small Z, these charge-dependent terms are at least comparable to the strongly energy-dependent terms which do not vanish with Z. Similarly the $Z=0$ approximation for the ordinary beta-gamma directional correlation anisotropy is inadequate even at relatively small Z.[6]

On the basis of these previous experiences it might be supposed that the nuclear charge radically alters the entire distribution function from its $Z=0$ form and thus completely modifies the interpretation of the experimental results on time-reversal invariance. Steffen points out that the Au^{198} case may be the only one with a sufficiently simple decay scheme to allow unambiguous interpretation. It is therefore of great interest to determine the effect of the Coulomb field on the complete beta-gamma distribution function for first-forbidden transitions, and in particular to discover to what extent the asymmetry which tests time-reversal invariance is altered from its $Z=0$ form.

It will be made evident in the following that, even for low Z, the modifications from the $Z=0$ limit are extensive and that the time-reversal testing asymmetry is reduced appreciably compared to that calculated for $Z=0$. It will be shown that in the limit of high $(\alpha Z/2R) \sim (Z/3)$, that is, for most first-forbidden transitions, a very definite relation exists between the ordinary directional correlation anisotropy and the beta polarization-dependent asymmetries so that an observation of one asymmetry implies the values of the others. In this approximation, terms which test time-reversal invariance appear in the same manner in all the asymmetries but are dominated in general by contributions which do not test time-reversal invariance and which can be separated from those which do only by their energy dependence.[7]

THE CASE CONSIDERED BY CURTIS AND LEWIS

The procedure followed in obtaining an angular correlation distribution function is well known and need not be elaborated.[8] The beta interaction Hamiltonian is taken to be[9]

$$H=\sum_i g_i H_i + g_i' H_i',$$

where the g_i are the "old" beta-interaction coupling constants, the g_i' are the corresponding "parity-conservation testing" coupling constants, and i ranges over the five interaction types. The partial Hamiltonians, H_i and H_i', have opposite behavior under the parity operation.

Curtis and Lewis[1] have given, in the $Z=0$ limit, the general form for the distribution function associated with a beta-gamma cascade from unoriented nuclei when the beta particle is transversely polarized and the gamma radiation is of pure multipole character and unpolarized. The inclusion of the final-state interaction between emitted beta particle and the nuclear Coulomb field introduces no *new* scalar or pseudoscalar products of observed vectors, but of course alters the coefficient of each such product from its $Z=0$ form.

After all common factors have been extracted, the distribution function associated with a beta transition to a final nucleus of atomic number Z followed by a pure 2^L-multipole gamma transition may be written as

$$W=S(Zpq\epsilon)+K(j_1j_2j_3L)[A(Zpq\epsilon)(\boldsymbol{\sigma}\cdot\hat{p}\times\hat{k})(\hat{p}\cdot\hat{k}) +B(Zpq\epsilon)P_2(\cos\theta_{pk})+C(Zpq\epsilon)(\boldsymbol{\sigma}\cdot\hat{k})(\hat{p}\cdot\hat{k})]. \quad (1)$$

Longitudinal beta polarization terms have been omitted. Here $\boldsymbol{\sigma}$ is a unit vector along the direction of the beta polarization and $\hat{p}$ and $\hat{k}$ are unit vectors along the

[2] Jackson, Treiman, and Wyld, Nuclear Phys. **4**, 206 (1957).
[3] M. Morita and R. S. Morita, Phys. Rev. **107**, 139, 1316 (1957).
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[5] E. J. Konopinski and L. M. Langer, in *Annual Review of Nuclear Science* (Annual Reviews, Inc., Stanford, 1953), Vol. 2; E. J. Konopinski, in *Beta- and Gamma-Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1955), Chap. X.
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[7] The presence of such "empty" Coulomb corrections is familiar in allowed beta decay. See Jackson, Treiman, and Wyld, reference 2.
[8] L. C. Biedenharn and M. E. Rose, Revs. Modern Phys. **25**, 759 (1953).
[9] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).

direction of the electron and photon momenta, respectively. Also p and q are the electron and neutrino momenta in units of m_ec and ϵ is the total electron energy in units of m_ec^2. The Legendre polynomial $P_2(\cos\theta_{pk})$ is a function of the angle θ_{pk} between the electron and photon. Further,

$$K(j_1j_2j_3L)=-6(2L+1)(2j_2+1)(-1)^{j_3-j_1}\times C(LL2;1,-1)W(j_2j_2LL;2j_3)W(j_2j_211;2j_1),$$

where j_1, j_2, and j_3, are the angular momenta of the initial, intermediate, and final nuclear states. The W's are Racah coefficients and $C(LL2;1,-1)$ is a vector addition coefficient.

The term in (1) proportional to $P_2(\cos\theta_{pk})$ represents the anisotropy in the ordinary beta-gamma directional correlation and is present whether or not the beta interaction Hamiltonian is invariant under any of the operations of space inversion, time reversal, or charge conjugation. The coefficient B has been calculated by Alder, Stech, and Winther[6] for first-forbidden transitions with the STP interactions and by Morita and Morita[10] with the combination STP as well as the combination AV.

Since the last term in (1) involves a pseudoscalar product, $(\boldsymbol{\sigma}\cdot\hat{k})(\hat{p}\cdot\hat{k})$, of observed vectors, its presence in an experimental distribution corroborates the known fact that the beta interaction does not conserve parity.

In the $Z=0$ limit, the coefficient of $(\boldsymbol{\sigma}\cdot\hat{p}\times\hat{k})(\hat{p}\cdot\hat{k})$ vanishes unless the beta transition is a mixed Fermi-Gamow-Teller first-forbidden transition and unless the beta interaction Hamiltonian is not time-reversal invariant. That this last is necessary may be seen by recalling the ordinary behavior of linear and angular momentum vectors under the Wigner time-reversal transformation. If there are no final-state interactions (in our case, if $Z=0$), $\boldsymbol{\sigma}$, $\mathbf{p}$, and $\mathbf{k}$ all go into minus themselves under the time-reversal operation, so that $(\boldsymbol{\sigma}\cdot\hat{p}\times\hat{k})(\hat{p}\cdot\hat{k})$ is a pseudoscalar and can therefore only be present if the beta interaction is not time-reversal invariant. The presence of the nuclear Coulomb field alters the beta-particle final-state wave function in such a way that the simple argument based on the transformation properties of vectors is no longer valid.[2] The effect of the Coulomb interaction is to introduce to the asymmetry corrections which do not test time-reversal invariance and which have the $(\boldsymbol{\sigma}\cdot\hat{p}\times\hat{k})(\hat{p}\cdot\hat{k})$ dependence, and to introduce to the asymmetries corrections which do test time-reversal invariance and which have the $(\boldsymbol{\sigma}\cdot\hat{k})(\hat{p}\cdot\hat{k})$ and the $P_2(\cos\theta_{pk})$ dependences. For orientation purposes the effects of the Coulomb field will be illustrated by an approximate treatment of the example calculated by Curtis and Lewis.

Consider those terms in S, A, B, and C, that involve the first-forbidden vector and axial vector matrix elements characterized, respectively, by the irreducible tensors Y_1^M and $\boldsymbol{\sigma}\cdot\mathbf{T}_{11}{}^M$.[8] The function Y_1^M is a first-order spherical harmonic, $\mathbf{T}_{11}{}^M$ is a vector spherical harmonic,[11] and $\boldsymbol{\sigma}$ is the spin operator in the Dirac spinor space. In order to obtain simple expressions that are useful for qualitative estimates, the correct nuclear charge distribution will be approximated by a point charge and the replacements

$$(\alpha Z/2)(j_2\|Y_1\|j_1)\rightarrow(\alpha Z/2R)(j_2\|rY_1\|j_1),$$

and

$$(\alpha Z/2)(j_2\|\boldsymbol{\sigma}\cdot\mathbf{T}_{11}\|j_1)\rightarrow(\alpha Z/2R)(j_2\|r\boldsymbol{\sigma}\cdot\mathbf{T}_{11}\|j_1)$$

will be made. The double bars denote reduced matrix elements, α is the fine-structure constant, and R is a nuclear radius in units of the electron Compton wavelength ($R\sim10^{-2}$). Note that the nuclear radius is assumed to be the same in both replacements. Finally, it shall be assumed that $(\alpha Z/p)^2\ll1$ and $(\alpha Z/2R)>1$.

With these approximations and restrictions, the isotropic term in (1) is—disregarding all but the two matrix elements under consideration—

$$\begin{aligned}S=&\alpha_{VV}|M_V|^2[\xi^2+\tfrac{2}{3}\xi(p^2\epsilon^{-1}+q)+\tfrac{1}{3}(p^2+q^2+\tfrac{2}{3}qp^2\epsilon^{-1})]\\&+\alpha_{AA}|M_A|^2[\xi^2+\tfrac{2}{3}\xi(p^2\epsilon^{-1}-q)+\tfrac{1}{6}(p^2+q^2-\tfrac{4}{3}qp^2\epsilon^{-1})]\\&+\mathcal{R}\alpha_{VA}M_V{}^*M_A[\xi^2+\tfrac{2}{3}\xi p^2\epsilon^{-1}].\end{aligned}\quad(2)$$

For negatron decay, the coefficients of the asymmetric terms in (1) are proportional to

$$\begin{aligned}A=&\tfrac{3}{8}(p\epsilon^{-1})[2\alpha Z(\xi+\tfrac{1}{3}q)\alpha_{VV}|M_V|^2\\&-\alpha Z(\xi-\tfrac{1}{3}q)\alpha_{AA}|M_A|^2+\alpha Z(\xi-q)\mathcal{R}\alpha_{VA}M_V{}^*M_A\\&+\alpha Z(3\alpha Z\epsilon/p)[\xi+\tfrac{1}{3}\epsilon-(1/9)q]\mathcal{I}\alpha_{VA}M_V{}^*M_A\\&+\tfrac{4}{3}p\mathcal{I}\alpha_{VA}M_V{}^*M_A],\end{aligned}\quad(3)$$

$$\begin{aligned}B=&\tfrac{1}{3}(p^2\epsilon^{-1})[2(\xi+\tfrac{1}{2}\epsilon+\tfrac{1}{3}q)\alpha_{VV}|M_V|^2\\&-(\xi+\tfrac{1}{4}\epsilon-\tfrac{1}{3}q)\alpha_{AA}|M_A|^2+(\xi-q)\mathcal{R}\alpha_{VA}M_V{}^*M_A\\&+(9\alpha Z\epsilon/4p)[\xi+\tfrac{1}{3}p^2\epsilon^{-1}-(1/9)q]\mathcal{I}\alpha_{VA}M_V{}^*M_A],\end{aligned}\quad(4)$$

and

$$\begin{aligned}C=&\tfrac{1}{2}(p\epsilon^{-1})[-2(\xi+\tfrac{1}{3}q)\beta_{VV}|M_V|^2\\&+(\xi-\tfrac{1}{3}q)\beta_{AA}|M_A|^2-(\xi-q)\mathcal{R}\beta_{VA}M_V{}^*M_A\\&-(3\alpha Z\epsilon/p)[\xi-(1/9)q]\mathcal{I}\beta_{VA}M_V{}^*M_A],\end{aligned}\quad(5)$$

where

$$\alpha_{ik}=g_ig_k{}^*+g_i{}'g_k{}'^*=\mathcal{R}\alpha_{ik}+i\mathcal{I}\alpha_{ik},$$
$$\beta_{ik}=g_ig_k{}'^*+g_i{}'g_k{}^*=\mathcal{R}\beta_{ik}+i\mathcal{I}\beta_{ik},$$
$$M_V=(j_2\|rY_1\|j_1),\quad M_A=\sqrt{2}(j_2\|r\boldsymbol{\sigma}\cdot\mathbf{T}_{11}\|j_1),$$
$$\xi=\alpha Z/2R,$$

and it has been assumed that the interaction between *nuclear* particles is time-reversal invariant.

By setting $Z=0$, one obtains the vector-axial vector analog of the distribution function considered by Curtis and Lewis.[1] The correspondence with Curtis and Lewis'

[10] M. Morita and R. S. Morita, Phys. Rev. **109**, 2048 (1958).

[11] M. E. Rose, *Multipole Fields* (John Wiley and Sons, Inc., New York, 1955), Chap. II.

results is accomplished by the replacements

$$M_V \to M_S \to +(3/4\pi)^{\frac{1}{2}}(f\|\beta\mathbf{r}\|i),$$
$$M_A \to M_T \to -i(3/4\pi)^{\frac{1}{2}}(f\|\beta\boldsymbol{\sigma}\times\mathbf{r}\|i),$$
$$g_{V,A} \to g_{S,T}, \quad g_{V,A}' \to -g_{S,T}', \quad q \to -q,$$

and by the identification $K(j_1j_2j_3L) = -\frac{3}{2}f(j_1j_2j_3L)$.

Since, in the $Z=0$ limit, only the imaginary part of a product, α_{ik}, of coupling constants appears in the asymmetry proportional to $(\boldsymbol{\sigma}\cdot\hat{p}\times\hat{k})(\hat{p}\cdot\hat{k})$, Curtis and Lewis suggested that time-reversal invariance could possibly be tested by determining the presence or absence of this asymmetry in an experimental distribution function.

By examining the corresponding term in the $Z=0$ theoretical distribution function, one detects an immediate objection to the proposed test. For most conceivable cascades involving a first-forbidden transition, the coefficient A multiplying $(\boldsymbol{\sigma}\cdot\hat{p}\times\hat{k})(\hat{p}\cdot\hat{k})$ contains terms which do not test time-reversal invariance and which are at least comparable to the terms which do test time-reversal invariance. Hence, merely the presence, even significant presence, of the asymmetry having the $(\boldsymbol{\sigma}\cdot\hat{p}\times\hat{k})(\hat{p}\cdot\hat{k})$ dependence is not necessarily an indication of a violation of time-reversal invariance.

A more adverse consequence of the interaction between the emitted beta particle and the nuclear charge is that, due to the dominance of the quantity $(\alpha Z/2R)^2$ in that part of the distribution function which determines the total probability of the beta-gamma coincidence, the importance of the asymmetric terms relative to the isotropic terms is greatly reduced from its importance in the $Z=0$ distribution function. Even if the terms which vanish with Z in the expressions multiplying $(\boldsymbol{\sigma}\cdot\hat{p}\times\hat{k})(\hat{p}\cdot\hat{k})$ cancel with one another in some fortuitous fashion for some particular values of Z and q, the maximum relative magnitude of the term which tests time-reversal invariance is considerably reduced. The prime reason for considering first-forbidden transitions as opposed to allowed transitions, where the anisotropic terms are always reduced from the isotropic terms by a factor on the order of R, is thus weakened.

To the extent that the first-forbidden spectrum exhibits the allowed shape, all but terms of highest order in $(\alpha Z/2R)$ may be discarded in S and in A, B, and C. In this limit, it is apparent from (3), (4), and (5) that almost precisely the same information about the nature of the coupling can be derived from an experimental investigation of the energy dependence of any one of the three asymmetries. For instance, the detection of a relatively large energy-independent component in the coefficient of $(\boldsymbol{\sigma}\cdot\hat{p}\times\hat{k})(\hat{p}\cdot\hat{k})$ and/or in the coefficient of $(\boldsymbol{\sigma}\cdot k)(p\cdot k)$ would be an indication of the failure of time-reversal invariance. The same conclusion could be drawn from the detection of a component linear in p in the coefficient of $P_2(\cos\theta_{pk})$.

It may be observed further that, if one makes the reasonable assumption that $\beta_{ik}=\pm\alpha_{ik}$, then the coefficients A, B, and C in the experimental distribution function should satisfy approximately the ratio

$$|A|:|B|:|C| = \tfrac{1}{2}\alpha Z : (4/9)p : \tfrac{2}{3}. \tag{6}$$

These conclusions reached on the basis of the large $(\alpha Z/2R)$ approximation are not at all affected by the inclusion of all other first-forbidden matrix elements and are not affected appreciably when the finite size of the nuclear charge distribution and the terms in $(\alpha Z/p)^2$, which were neglected in (3), (4), and (5), are taken into account.

GENERAL RESULT IN THE HIGH-Z LIMIT

Let a nuclear matrix element which depends on light-particle radial functions only through

$$\omega_1(r) = (\alpha Zr/2R)[1-\tfrac{1}{5}(r/R)^2], \quad r\le R,$$

be called large and let it be denoted by $M(i)$, where i specifies the particular irreducible tensor characterizing the matrix element as well as the interaction type to which it belongs. For example,

$$M(V,Y_1) = (j_2\|\omega_1(r)Y_1\|j_1),$$

and

$$M(T,\beta\boldsymbol{\sigma}\cdot\mathbf{T}_{01}) = (j_2\|\omega_1(r)\beta\boldsymbol{\sigma}\cdot\mathbf{T}_{01}\|j_1).$$

Let the corresponding matrix element with r replacing $\omega_1(r)$ be called small and let it be denoted by $m(i)$; e.g.,

$$m(V,Y_1) = (j_2\|rY_1\|j_1),$$

and

$$m(T,\beta\boldsymbol{\sigma}\cdot\mathbf{T}_{01}) = (j_2\|r\beta\boldsymbol{\sigma}\cdot\mathbf{T}_{01}\|j_1).$$

Both $\omega_1(r)$ and r arise from the angular momentum eigenfunctions of an electron in the potential corresponding to the nuclear charge distributed uniformly over a spherical volume of radius R. If the radial functions are evaluated at the nuclear radius and then extracted from the nuclear matrix elements, it is clear that, in order of magnitude, corresponding large and small matrix elements are in the ratio

$$M(i)/m(i) \sim (4/5)(\alpha Z/2R).$$

In the limit of large Z, such that $(2\alpha Z/5R)\gg 1$, the coefficients of the isotropic and the anisotropic terms in the distribution function associated with a nonunique first-forbidden beta transition followed by a pure-multipole gamma ray are proportional to

$$\begin{aligned}
S &= \sum_{ik} f_{ik}M^*(i)M(k)\mathcal{R}\alpha_{ik},\\
A &= \sum_{ik} g_{ik}M^*(i)m(k)(i\mathfrak{M}^-\alpha_{ik}+\text{c.c.})F^{-1},\\
B &= -\tfrac{2}{3}\sum_{ik} g_{ik}M^*(i)m(k)(\mathfrak{M}^+\alpha_{ik}+\text{c.c.})F^{-1},\\
C &= \pm\sum_{ik} g_{ik}M^*(i)m(k)(\mathfrak{M}^-\beta_{ik}+\text{c.c.})F^{-1},
\end{aligned} \tag{7}$$

where the sign before the summand in C is $+$ when

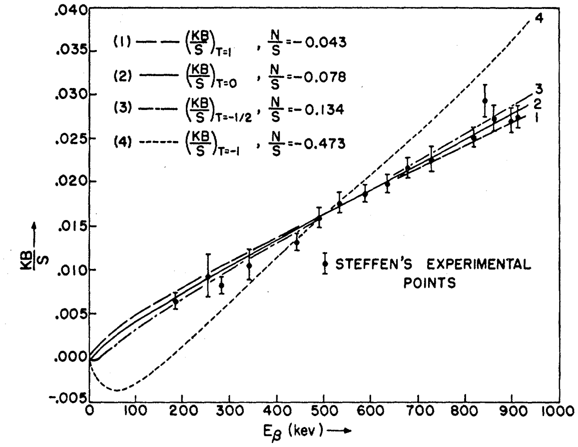

FIG. 1. Coefficient of $P_2(\cos\theta_{pk})$ relative to the isotropic terms in the distribution function for the Au^{198} decay.

i, k are V and/or A and $-$ when i, k are S and/or T. Longitudinal polarization terms have been omitted and it has again been assumed that nucleon-nucleon interactions are time-reversal invariant. The symbols f_{ik} and g_{ik} are numerical factors on the order of unity such that $f_{ik}M^*(i)M(k)$ and $g_{ik}M^*(i)m(k)$ are real quantities. The function $F(Z,\epsilon)$ is the standard Fermi correction factor.[12] Exact expressions for $\mathfrak{N}^-$, $\mathfrak{N}^+$, and $\mathfrak{M}^-$ are given in the Appendix. By neglecting $(\alpha Z)^2/4$ in comparison with one, one finds that

$$\begin{aligned}(i\mathfrak{N}^-\alpha_{ik}+\text{c.c.})&=2F_{12}\epsilon^{-1}\ \tfrac{3}{4}\alpha Z(\mathcal{R}\alpha_{ik}-\lambda\mathcal{J}\alpha_{ik}),\\ (\mathfrak{N}^+\alpha_{ik}+\text{c.c.})&=-2F_{12}p\epsilon^{-1}(1+\tfrac{1}{4}\lambda\alpha Z\epsilon p^{-1})\\ &\qquad\times(\mathcal{R}\alpha_{ik}-\lambda'\mathcal{J}\alpha_{ik}),\\ (\mathfrak{M}^-\beta_{ik}+\text{c.c.})&=-2F_{12}\epsilon^{-1}(\mathcal{R}\beta_{ik}-\lambda\mathcal{J}\beta_{ik}),\end{aligned}\tag{8}$$

where F_{12}, λ, and λ' are given in the appendix.

To the extent that one may neglect $(\alpha Z/p)^2$ in comparison with unity, one has

$$\begin{aligned}F_{12}&\to\tfrac{2}{3}pF(Z,\epsilon),\\ \lambda&\to\alpha Z\epsilon/p,\end{aligned}\tag{9}$$

and

$$\lambda'\to 3\alpha Z\epsilon/4p.$$

Recent investigations on the β^+ decay of A^{35},[13] on the β^- decay of He^6,[14] and on the orbital electron capture in Eu^{152m} [15] indicate that the dominant interactions in beta decay are vector and axial vector. In deference to this evidence, the general expressions (7) shall be written explicitly only in terms of the six major vector and axial vector first-forbidden matrix elements. With the approximations (8) one has, for negatron decay,

$$S=\alpha_{VV}|x|^2+\alpha_{AA}(2|y|^2+3|z|^2)+2\sqrt{2}\mathcal{R}\alpha_{VA}x^*y,\tag{10}$$

$$A=(F_{12}/\epsilon F)\tfrac{3}{4}\alpha ZN(1+\lambda T),\tag{11}$$

$$B=(F_{12}/\epsilon F)\tfrac{2}{3}p(1+\tfrac{1}{4}\lambda\alpha Z\epsilon/p)N(1+\lambda' T),\tag{12}$$

$$C=-(F_{12}/\epsilon F)N'(1+\lambda T'),\tag{13}$$

where

$$\begin{aligned}N=\tfrac{3}{2}[\alpha_{VV}x^*a-\alpha_{AA}y^*(b+\bar{W}_{12}c)+\alpha_{AA}\bar{W}_{02}z^*c\\ -\mathcal{R}(\alpha_{VA}/\sqrt{2})\{x^*(b+\bar{W}_{12}c)-2y^*a\}],\end{aligned}\tag{14}$$

$$T=\tfrac{3}{2}N^{-1}\mathcal{J}(\alpha_{VA}/\sqrt{2})[x^*(b+\bar{W}_{12}c)+2y^*a],\tag{15}$$

and N' and T' may be obtained from N and T by replacing all α_{ik} with β_{ik}. Further,

$$\begin{aligned}x&=(j_2\|\omega_1Y_1\|j_1)+i\sqrt{3}(j_2\|\boldsymbol{\sigma}\cdot\mathbf{T}_{10}\|j_1),\\ y&=(j_2\|\omega_1\boldsymbol{\sigma}\cdot\mathbf{T}_{11}\|j_1),\\ z&=(j_2\|\omega_1\boldsymbol{\sigma}\cdot\mathbf{T}_{01}\|j_1)-i(j_2\|\gamma_5Y_0\|j_1),\\ a&=(j_2\|rY_1\|j_1),\\ b&=(j_2\|r\boldsymbol{\sigma}\cdot\mathbf{T}_{11}\|j_1),\\ c&=\sqrt{3}(j_2\|r\boldsymbol{\sigma}\cdot\mathbf{T}_{21}\|j_1),\\ \bar{W}_{ik}&=W(j_2j_2ik;2j_1)/W(j_2j_211;2j_1).\end{aligned}\tag{16}$$

For positron emission, make the replacement $g_V\to+g_V^*$, $g_V'\to-g_V'^*$, $g_A\to-g_A^*$, $g_A'\to+g_A'^*$, and $Z\to-Z$ in all preceding formulas.

APPLICATION TO AU^{198}

A comparison of Steffen's measurements of the Au^{198} beta-gamma directional anisotropy with the calculated anisotropy coefficient B, expression (12), gives no information concerning time-reversal invariance.

In Fig. 1, (KB/S) is plotted as a function of electron kinetic energy, E_β, for four values of T. In each case (N/S) is chosen to give the best fit with Steffen's experimental points. The exact form for $\mathfrak{N}^+$ is used in the calculations.

In view of the abundance of matrix elements in N as compared to those in NT, values of T greater than one are not to be expected in the general case. Since the corresponding curves cannot be fitted to Steffen's points, values of T less than $-\frac{3}{4}$ are excluded for the case of Au^{198}. Further, the values of (N/S) necessary to secure even an approximation to Steffen's points for $T<-\frac{1}{2}$ are unreasonably large.

It is evident from Fig. 1 that Steffen's points do not distinguish between curves with T in the range $-\frac{1}{2}<T<1$. Since nothing can be said theoretically $\mathcal{J}$bout the matrix elements in T, no limits can be set on aα_{VA}, and hence nothing concerning time-reversal invariance can be concluded.

Measurements of the beta polarization-dependent anisotropies for Au^{198} are under way.[16] By using the

[12] See, for example, J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), p. 682.

[13] Herrmansfeldt, Maxson, Stähelin, and Allen, Phys. Rev. **107**, 641 (1957).

[14] Herrmannsfeldt, Burman, Stähelin, Allen, and Braid, Phys. Rev. Letters **1**, 61 (1958).

[15] Goldhaber, Grodzins, and Sunyar, Phys. Rev. **109**, 1015 (1958).

[16] P. C. Simms (private communication).

values of (N/S) necessary to fit Steffen's directional correlation measurements, it is possible to predict the expected behavior of the time-reversal asymmetry having the $(\boldsymbol{\sigma}\cdot\hat{p}\times\hat{k})(\hat{p}\cdot\hat{k})$ dependence. The coefficient (KA/S) is plotted in Fig. 2 for the same values of T and $(N/S)_T$ for which the directional anisotropy coefficient, (KB/S), is drawn in Fig. 1. Again there is very little distinction either in order of magnitude (of order $\frac{1}{2}\%$) or in energy dependence between the coefficients predicted with different values of T in the range $-\frac{1}{2}<T<1$. The curve for $T=-1$ shows a rather different energy behavior, but may be excluded on the basis of the directional correlation shown in Fig. 1.

The coefficient, (KC/S), of the parity asymmetry having the $(\boldsymbol{\sigma}\cdot\hat{k})(\hat{p}\cdot\hat{k})$ dependence can be gotten from (KA/S) if one assumes that $\beta_{ik}=\alpha_{ik}$ when i and k are A and/or V. This assumption is supported by experiments on the angular distribution of electrons from polarized Co^{60} by Wu *et al.*,[17] and on the longitudinal polarization of electrons from Co^{60} and P^{32} by Frauenfelder *et al.*[18] and of positrons from Na^{22} by Page and Heinberg.[19] In the case of Au^{198}, the choice of $\beta_{ik}=\alpha_{ik}$ leads to the relation $C=-2.02A$. At $E_\beta\sim 500$ kev the curves of Fig. 2 for $-\frac{1}{2}<T<1$ indicate that $(KC/S)\sim-0.01$. This value is consistent with the $\lesssim 2\%$ effect reported by Steffen as a result of preliminary measurements[4] and with the $1\%\pm1\%$ effect reported most recently by Steffen and Simms.[16]

CONCLUSION

In summary it may be stated that, in conjunction with reasonable order-of-magnitude estimates, the experimental determination of the coefficient of $P_2(\cos\theta_{pk})$ in the Au^{198} beta-gamma distribution function restricts T [Eq. (15)] to values in the range $-\frac{1}{2}<T<1$. However, since the coefficients of $(\boldsymbol{\sigma}\cdot\hat{p})\times\hat{k})(\hat{p}\cdot\hat{k}$ and $(\boldsymbol{\sigma}\cdot\hat{k})(\hat{p}\cdot\hat{k})$ as well as the coefficient of $P_2(\cos\theta_{pk})$ are essentially insensitive to variations in T within this range, it must be concluded that, within the framework of present beta-decay theory, no information concerning time-reversal invariance can be drawn from a determination of either the magnitude or the energy dependence of these coefficients in the Au^{198} distribution function.

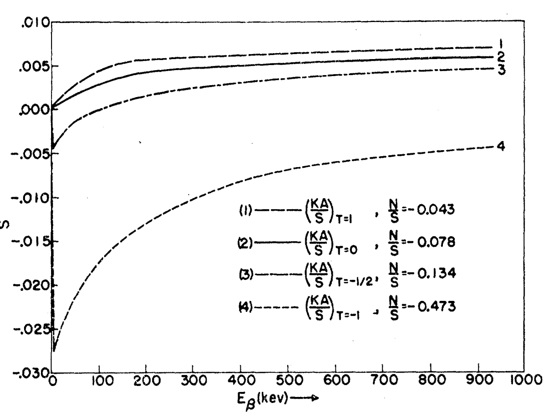

FIG. 2. Coefficient of $(\boldsymbol{\sigma}\cdot\hat{p}\times\hat{k})(\hat{p}\cdot\hat{k})$ relative to the isotropic terms in the distribution function for the Au^{198} decay.

This does not imply that time-reversal invariance cannot be tested by examining the distribution function for other beta-gamma cascades. Although the $T=-1$ coefficients plotted in Figs. 1 and 2 are excluded for Au^{198}, they correspond to a situation that might prevail for some other case if time-reversal invariance were violated in the extreme. For Au^{198} the functions $(KA/S)_{T=-1}$ and $(KC/S)_{T=-1}$ exhibit relatively sharp extrema at low energies and are of opposite sign to the corresponding $T=0$ functions.

If time-reversal invariance were violated to a considerable extent, it is quite probable that, for at least one first-forbidden beta-gamma cascade, the coefficients of both the parity asymmetry having the $(\boldsymbol{\sigma}\cdot\hat{k})(\hat{p}\cdot\hat{k})$ dependence and of the time-reversal asymmetry having the $(\boldsymbol{\sigma}\cdot\hat{p}\times\hat{k})(\hat{p}\cdot\hat{k})$ dependence might exhibit detectable extrema at low energies. If, further, the coefficient of the parity asymmetry and that of the directional correlation asymmetry for this hypothetical case were of the same sign or if, equivalently, the coefficient of the time-reversal asymmetry and the coefficient of $P_2(\cos\theta_{pk})$ were of opposite sign, it could be concluded that time-reversal invariance is violated. Even then, however, the improbability of obtaining a direct experimental or a reliable theoretical determination of the matrix elements in the terms in the asymmetry coefficients which do and those which do not test time reversal invariance, prevents any statement as to the *extent* to which time-reversal invariance is violated. On the other hand, as the experience with Au^{198} points out, even if the observed asymmetries may be all accounted for by assuming that the beta interaction is time-reversal invariant, one may not in general conclude that time-reversal invariance holds, but only that the experimental data are consistent with time-reversal invariance.

APPENDIX

When the condition $(\alpha Z/2R\epsilon_0)\gg 1$ holds, it is necessary to use accurate expressions for the dominant Z-dependent terms in the asymmetry coefficients.

The functions $\mathfrak{N}^-$, $\mathfrak{N}^+$, and $\mathfrak{M}^-$ appearing in the high-Z approximation to the asymmetry coefficients, expressions (11), (12), and (13), may be given explicitly in terms of known functions. With the definitions

$$F_{12}=\frac{e^{\pi\beta\epsilon}|\Gamma_1\Gamma_2|}{(2\gamma_1)!(2\gamma_2)!}16(1+\gamma_1)p(2pR)^{-(\delta_1+\delta_2)}\frac{\gamma_1\cos u}{(1+\beta^2)^{\frac{1}{2}}},$$

$$u=(\pi/2)(\delta_1-\delta_2)-\arg(\Gamma_1/\Gamma_2)-\tan^{-1}(\beta\epsilon/\gamma_1),$$

[17] Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. **105**, 1413 (1957); Ambler, Hayward, Hoppes, Hudson, and Wu, Phys. Rev. **106**, 1361 (1957).

[18] Frauenfelder, Bobone, von Goeler, Levine, Lewis, Peacock, Rossi, and DePasquali, Phys. Rev. **106**, 386 (1957); Frauenfelder, Hanson, Levine, Rossi, and DePasquali, Phys. Rev. **107**, 643 (1957).

[19] L. A. Page and M. Heinberg, Phys. Rev. **106**, 1220 (1957).

$$\bar{\gamma}_1=\gamma_1-\beta\epsilon\tan u,\quad \lambda=(\beta\epsilon+\gamma_1\tan u)/\bar{\gamma}_1,$$
$$\lambda'=[\lambda\{1+\tfrac{1}{4}(\delta_1-\delta_2)\}-\tfrac{1}{4}\beta\epsilon(\delta_1-\delta_2)/\delta_2]$$
$$\div[1+\tfrac{1}{4}(\delta_1-\delta_2)+\tfrac{1}{4}\beta\epsilon\lambda(\delta_1-\delta_2)/\delta_2],$$
$$\beta=\alpha Z/p,\quad \gamma_\kappa=[\kappa^2-(\alpha Z)^2]^{\frac{1}{2}},\quad \delta_\kappa=\kappa-\gamma_\kappa,$$
$$\Gamma_\kappa=(\gamma_\kappa-1+i\beta\epsilon)!,$$

one has

$$\mathfrak{N}^-=+F_{12}\epsilon^{-1}\ \tfrac{1}{4}\alpha Z[(\delta_1+\delta_2)/\delta_2](\lambda-i),$$
$$\mathfrak{N}^+=-F_{12}p\epsilon^{-1}[1+\tfrac{1}{4}(\delta_1-\delta_2)+\tfrac{1}{4}\beta\epsilon\lambda(\delta_1-\delta_2)/\delta_2](1+i\lambda'),$$
$$\mathfrak{M}^-=-F_{12}\epsilon^{-1}[1-\tfrac{1}{4}(\delta_1+\delta_2)](1+i\lambda).$$

For rapid computation, one may use the approximation

$$u=(\delta_1-\delta_2)[\tfrac{1}{2}\pi+\tan^{-1}(\tfrac{1}{2}\beta\epsilon)],$$

which is correct to the extent that $(\alpha Z)^2/24\ll 1$.

When the condition $\alpha Z/2R\epsilon_0\gg 1$ does not hold, additional terms must be included in the first-forbidden beta-gamma distribution function. The complete expressions for the isotropic terms in the $(\alpha Z/p)^2\ll 1$ approximation may be obtained from the literature.[20] Conformity to the normalization used in the text is achieved by matching leading $(2\alpha Z/5R)^2$ terms in the text with the leading $(\alpha Z/2R)^2$ terms found in the references, and by everywhere replacing $(\alpha Z/2R)$ by $(2\alpha Z/5R)$.

To obtain complete expressions for the asymmetry coefficients in the small-$(\alpha Z/p)^2$ approximation, add

$$A'=\tfrac{1}{4}\alpha Zqp\epsilon^{-1}[\alpha_{VV}|a|^2+\alpha_{AA}\{|b|^2-d^*c\bar{W}_{02}+b^*c\bar{W}_{12}\}$$
$$-\mathfrak{R}(\alpha_{VA}/\sqrt{2})a^*(3b+c\bar{W}_{12})]$$
$$+\tfrac{1}{3}p^2\epsilon^{-1}\mathfrak{I}(\alpha_{VA}/\sqrt{2})a^*(3b+c\bar{W}_{12})$$

to A, expression (11); add

$$B'=\tfrac{2}{3}p^2\epsilon^{-1}[\alpha_{VV}|a|^2(\tfrac{1}{2}\epsilon+\tfrac{1}{3}q)-\alpha_{AA}|b|^2(\tfrac{1}{4}\epsilon-\tfrac{1}{3}q)$$
$$-\alpha_{AA}d^*c\bar{W}_{02}(\tfrac{1}{3}\epsilon+\tfrac{1}{3}q)-\alpha_{AA}b^*c\bar{W}_{12}(\tfrac{1}{2}\epsilon-\tfrac{1}{3}q)$$
$$-\mathfrak{R}(\alpha_{VA}/\sqrt{2})a^*(3b+c\bar{W}_{12})\tfrac{1}{3}q]$$
$$+\tfrac{1}{6}\alpha Zp\mathfrak{I}(\alpha_{VA}/\sqrt{2})a^*[3b(p^2\epsilon^{-1}-\tfrac{1}{3}q)$$
$$+c\bar{W}_{12}(p^2\epsilon^{-1}+q)]+\tfrac{1}{6}(7/3)^{\frac{1}{2}}\alpha_{AA}|c|^2\bar{W}_{22}p^2$$

to B, expression (12); and add

$$C'=-\tfrac{1}{3}qp\epsilon^{-1}[\beta_{VV}|a|^2+\beta_{AA}\{|b|^2-d^*c\bar{W}_{02}+b^*c\bar{W}_{12}\}$$
$$-\mathfrak{R}(\beta_{VA}/\sqrt{2})a^*(3b+c\bar{W}_{12})]-\tfrac{1}{3}\alpha Z\mathfrak{I}(\beta_{VA}/\sqrt{2})a^*$$
$$\times[3b(\tfrac{1}{4}p^2\epsilon^{-1}-\tfrac{1}{3}q)+c\bar{W}_{12}(\tfrac{1}{4}p^2\epsilon^{-1}+q)]$$

to C, expression (13). Here $d=(j_2\|r\boldsymbol{\sigma}\cdot\mathbf{T}_{01}\|j_1)$.

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Note added in proof.—The most recent experimental results on the Au^{198} β-γ correlation[21] are consistent with the analysis presented in this paper. It is suggested[21] that the observed sign of the coefficient C relative to that of B gives evidence for the V-A coupling in preference to the S-T coupling. This is true only in conjunction with the results of the Goldhaber, Grodzins, and Sunyar[15] experiment on the helicity of neutrinos.

If one assumes [see Eq. (7) *et seq.* in the body of this paper] that

$$\beta_{VV}/\alpha_{VV}=\beta_{AA}/\alpha_{AA}=\beta_{VA}/\alpha_{VA},$$

or that

$$\beta_{SS}/\alpha_{SS}=\beta_{TT}/\alpha_{TT}=\beta_{ST}/\alpha_{ST},$$

then

$$A:B:C\cong\tfrac{3}{4}\alpha Z:\tfrac{2}{3}p:(-1)^r\beta_{ik}/\alpha_{ik},$$

where $r=1$ when i and k are V and/or A and $r=0$ when i and k are S and/or T. The Goldhaber *et al.*[15] experiment then says that $\beta_{ik}/\alpha_{ik}\cong 1$.

It is apparent that one may obtain the same information from an observation of the relative sign of B and C as one obtains from the Wu *et al.*[17] experiment and from the many longitudinal polarization experiments.[18,19]

[20] L. C. Biedenharn and M. E. Rose, reference 8; M. Morita and R. S. Morita, reference 10.

[21] P. C. Simms and R. M. Steffen, Phys. Rev. Letters 1, 289 (1958).