Collisions of Electrons with Hydrogen Atoms. III. Elastic Scattering

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The cross section for elastic scattering of electrons of less than 10-ev energy by free hydrogen atoms has been measured. The experimental approach was similar to that used by Bederson, Malamud, and Hammer in that a dc electron beam crossed an atomic hydrogen beam, which was chopped at a low frequency, and the signal was derived from the electrons scattered by the particles in the neutral beam. The electrons scattered by the beam were identified by their signal's appearing at the modulation frequency and at a specified phase. Direct measurements of the ratio of cross sections of the hydrogen atom and the molecule were made, and absolute values for the atomic cross section were obtained from knowledge of the absolute molecular cross section. The experimental results agree with theory.

I. INTRODUCTION

BEDERSON, Malamud, and Hammer[1] recently conducted an ingenious experiment to measure the absolute total cross section for elastic scattering of electrons by free hydrogen atoms below 10 ev, a process for which several theoretical predictions had previously been made.[2,3] The results of that measurement were in marked disagreement with all the predicted values, except in the region of 8 to 10 ev. At lower electron energies, the experimental values of the cross section greatly exceeded the theoretical values. This disagreement between theory and experiment led to improved calculations[4] of the cross section for elastic scattering, the values of which, however, did not resolve the discrepancy.

It therefore seemed desirable to remeasure the elastic scattering cross section on the atomic-beam apparatus at General Atomic. While experimental conditions were somewhat different from those of Bederson and his collaborators, the basic approach was the same. The agreement between theory and these experiments was acceptable. This paper summarizes the remeasurement of the elastic scattering cross section.

II. EXPERIMENTAL APPROACH

The basic General Atomic beam apparatus was described in the first paper of the present series.[5] The atomic beam is produced in a tungsten furnace in the first of three vacuum chambers, flows through a second chamber where it is modulated at 100 cps by a mechanical chopping wheel, and then enters the third chamber where the experiment is performed. The electron beam interacting with the atomic beam is run dc. In this manner, any signal arising from interactions of the electron beam with the atomic beam can be distinguished from the much larger effects arising from interactions of electrons with residual gas in the vacuum chamber, because the former signal is identifiable by its time characteristics, i.e., frequency and phase.

This experiment differs from those described in the earlier papers of this series in two respects. First, the signal in this case was the ac current at the modulation frequency carried by electrons which were scattered by particles in the modulated neutral beam. Second, since the detector was nondiscriminating it was necessary to add electrostatic deflection plates to prevent electrons and ions from the hot furnace from reaching the scattered-electron collector. A schematic diagram of the experimental arrangement is shown in Fig. 1. As is customary in our experiments, a mass spectrometer was used to monitor the neutral beam, so that for any given set of operating conditions, the relative proportions of atoms and molecules in the beam were known.

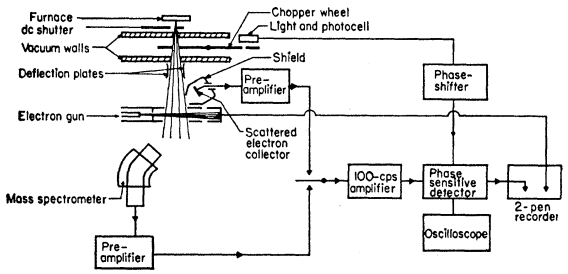

FIG. 1. Schematic diagram of elastic scattering experiment.

[1] Bederson, Malamud, and Hammer, Bull. Am. Phys. Soc. Ser. II, **2**, 122 (1957); see also Technical Report No. 2, Electron Scattering Project, College of Engineering, New York University (unpublished).

[2] J. McDougall, Proc. Roy. Soc. (London) **A136**, 549 (1932); P. M. Morse and W. P. Allis, Phys. Rev. **44**, 269 (1933); S. Chandrasekhar and F. H. Breen, Astrophys. J. **103**, 41 (1946); L. K. Hulthén, Kgl. Fysiograf. Sällskap. Lund, Förh. **14**, 21 (1944); W. Kohn, Phys. Rev. **74**, 1763 (1948); T. Kato, Progr. Theoret. Phys. (Japan) **6**, 394 (1951); S. Borowitz and H. Greenberg, Phys. Rev. **108**, 716 (1957).

[3] H. S. W. Massey and B. L. Moiseiwitsch, Proc. Roy. Soc. (London) **A205**, 483 (1951).

[4] Bransden, Dalgarno, John, and Seaton Proc. Phys. Soc. (London) **71**, 877 (1958).

[5] W. L. Fite and R. T. Brackmann, this issue [Phys. Rev. **112**, 1141 (1958)].

Two scattered-electron detectors were used. The first was the repeller plate of the mass spectrometer. It collected electrons scattered over slightly less than one-half the spherical zone extending between the polar angles of 45° and 135° from the direction of the electron beam. The second was a specially built scattered-electron collector which was placed, along with the one-tube preamplifier described in reference 5, inside a metal shield which was closed except for a circular aperture through which the scattered electrons entered. The observed electrons were scattered into a cone with a half-apex angle of 45° and with its axis perpendicular to the direction of the electron beam. (The schematic drawing in Fig. 1, because of its two-dimensionality, does not depict this detector accurately.) In both detection arrangements all electrodes in the vicinity of the region of interaction between the neutral beam and the electron beam were normally held at ground potential, so that electrostatic fields would influence neither the incident electron beam nor the scattered electrons.

It is important to note that with no fields in the collision and collection regions, all electrons scattered within the angular range determined by the detector were collected, irrespective of the scattered electron energy. Thus, the cross section under study was the elastic scattering cross section only for electron energies of less than the first excitation potential, i.e., 10.15 ev for the hydrogen atom.

Actually, the second detector was built after the failure of the first detecting arrangement. It was found that when the regular mass-spectrometer repeller plate was used to collect the scattered electrons, "stray" electrons from the cathode of the electron gun arrived at the scattered-electron collector. Although this would cause no difficulty, except for increased shot noise, if the electron current from the cathode were truly dc, the introduction of a modulated beam of hydrogen atoms into the third vacuum chamber was found to modulate the electron current from the oxide-coated cathode by a factor of $\sim 10^{-4}$. When $\sim 1\%$ of the total electron current was either straying to the electron collector or being scattered off the background gas in the vacuum chamber, the modulated current from the cathode produced a signal of the order of magnitude of the desired signal. Even after the removal of the second vacuum wall, so that the chopping of the neutral beam occurred in the experimental chamber, this cathode modulation was still quite serious.

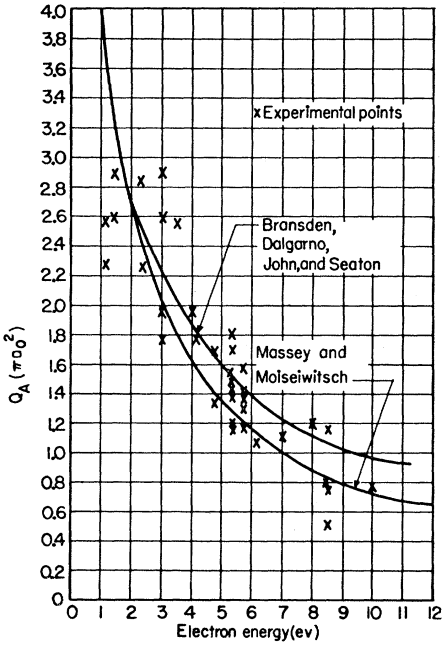

FIG. 2. Absolute experimental and theoretical cross sections for scattering of electrons into the experimental cone of observation.

This cathode-modulation effect manifested itself by a phase shift. The reference signal phase from an H_2 beam was adjusted to give a proper rectified output. Heating the furnace continued to give a proper output-signal phase up to the temperature at which hydrogen atoms began to appear in the beam, at which point the signal seen on the phase-monitoring oscilloscope shifted in the direction of a lag. When a highly pure atomic beam was run, a phase lag of $\sim 70°$ was seen.

It is presently presumed that the cathode modulation effect arose because hydrogen atoms, a small fraction of which did not recombine into molecules at the walls of the vacuum chamber, arrived at the cathode and slightly enhanced its emission. Supporting this contention is the observation that the dc electron current was increased by $\sim 5\%$ upon emission of the atomic beam. The fact that the phase lag of $\sim 70°$ was observed for 100-cps modulation suggests that the emission enhancement was a cumulative effect and had a time constant of a few milliseconds for the usual atomic-beam strengths used.

In the present experiments, replacing the oxide-coated cathode by a tungsten cathode appeared to eliminate the cathode modulation effect associated with atoms in the neutral beam. However, the broad energy spread accompanying the use of a tungsten-filament cathode (in our case 1.2 ev, as determined from stopping potential curves) made its use unsatisfactory in the present experiments where electron energies were less than 10 ev.

To allow the use of an oxide-coated cathode, with its improved electron energy resolution, the second collection arrangement was adopted. The shield was placed over the electron-collecting electrode and the preamplifier solely to reduce currents of stray electrons and by so doing to reduce the bothersome cathode modulation signal.

The interaction between hydrogen atoms and the cathode is believed to be the cause of a second troublesome effect found in this experiment. This effect was the drift in the apparent cathode work function as

determined from stopping potential measurements. It was found that running the atomic beam altered the apparent work function of an oxide-coated cathode by up to 0.4 ev and that of a tungsten cathode by almost 2 ev. Although data are far from complete, drift times appear to be of the order of one hour and with both types of cathodes an increase in apparent work function was noted upon running the hydrogen-atom beam. Since work-function drifts cause the actual energy of the electrons to drift also, and since long time constants were used in the final integration of the signal in order to obtain suitable signal-to-noise ratios, we cannot be certain that within each reading energy drifts did not occur. While such drifts are of relatively little importance for elastic scattering cross-section measure-

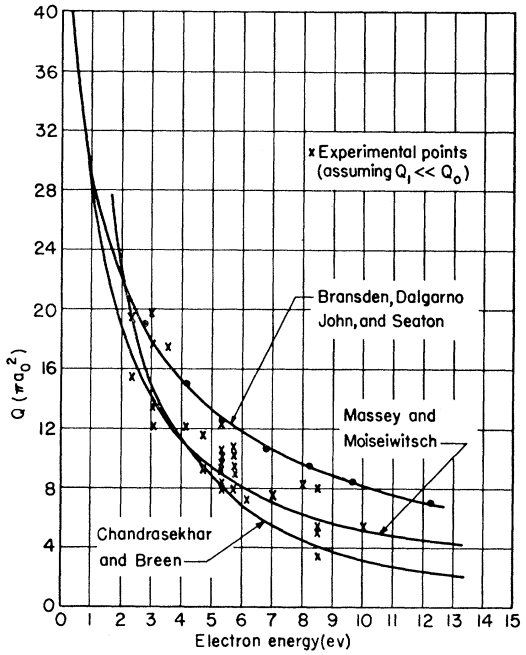

FIG. 3. Total scattering cross section assuming only S-wave scattering.

ments for hydrogen atoms above 3 or 4 ev, where the cross-section curve is rather flat, below 3 ev, where the cross-section curve is quite steep, even small energy uncertainties alter the apparent cross-section values by large amounts. Indeed, the large scatter of points on the atomic cross-section curves in Figs. 2, 3, and 4 is attributed primarily to energy uncertainties.

III. DETERMINATION OF ABSOLUTE CROSS SECTIONS

As in the case of the two experiments discussed in previous papers of the present series, it was possible to make direct measurements of the ratio of atomic and molecular cross sections, as well as to measure relative

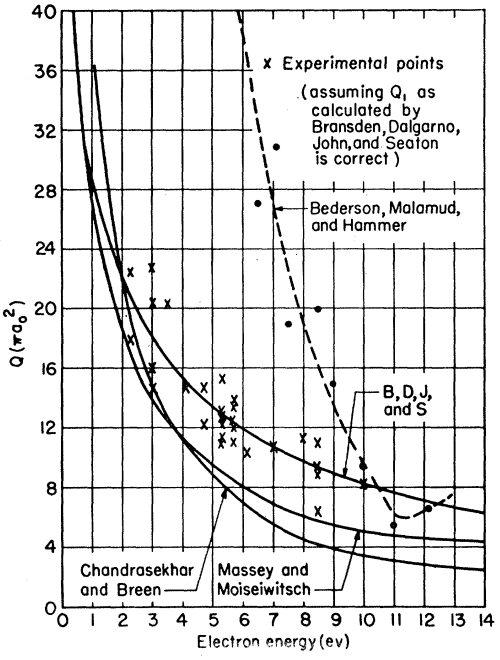

FIG. 4. Total scattering cross section assuming P-wave scattering as calculated by Bransden, Dalgarno, John, and Seaton.

atomic cross sections. The formulas used are analogous to those previously used since mass flow conditions in the beam were again constant.

First, it is convenient to define the signal S_0, representing the scattered electron signal which would have been seen with a furnace temperature T (in absolute degrees) had the hydrogen molecule not dissociated, by

$$S_0(T)=S_r(T_r/T)^{\frac{1}{2}}, \tag{1}$$

where T_r is any reference temperature at which the beam is actually purely molecular (usually taken as room temperature), and S_r is the signal observed at that reference temperature. Below dissociation temperatures, the observed signal was found to agree with this definition of S_0, which fact was used in the first paper of this series as a major piece of evidence that translational thermal equilibrium was being achieved in our furnace.

At temperatures above that at which dissociation occurs, the contribution to the signal from the molecules remaining in the beam is given by

$$S_2=(1-D)S_0, \tag{2}$$

where D is the dissociation fraction, and the contribution of atoms in the beam to the total signal is given by

$$S_1=\sqrt{2}D(Q_A/Q_M)S_0, \tag{3}$$

where Q_A and Q_M are the measured scattering cross sections of the hydrogen atom and molecule, respectively. The total signal, $S(T)$, is the sum of Eqs. (2)

and (3), which give, on substitution from Eq. (1),

$$S(T)=S_r\left(\frac{T_r}{T}\right)^{\frac{1}{2}}\left[\sqrt{2}D\frac{Q_A}{Q_M}+1-D\right], \quad (4)$$

from which the ratio of cross sections is given by

$$\frac{Q_A}{Q_M}=\frac{1}{\sqrt{2}D}\left[\frac{S(T)}{S_r}\left(\frac{T}{T_r}\right)^{\frac{1}{2}}+D-1\right]. \quad (5)$$

The dissociation fraction D is again measurable by the mass spectrometer, as was outlined in the second paper of this series, so that all quantities on the right side of Eq. (5) are directly measurable. In this experiment, D was usually kept from 0.90 to 0.96.

It will be recalled that in our experimental arrangement only electrons scattered into a cone of 45° half-apex angle, with its axis at 90° with respect to the electron beam, were detected. Thus, the cross sections, Q_A and Q_M, introduced in ratio as a proportionality constant in Eq. (3), are not total cross sections but are those for scattering into the cone of observation. In order to make absolute determinations of the atomic cross section for scattering of electrons into the observation cone, it is necessary to know the absolute cross section for scattering into this cone by the molecule.

This molecular cross-section curve was determined by numerical integration from the differential cross-section data of Ramsauer and Kollath.[6]

Agreement to within 5% between this calculated cross section and a measured relative cross-section curve for the molecule, normalized to give a "best fit" to the calculated curve, was found. This gave confidence in our relative-cross-section measurements of atomic scattering cross sections.

The absolute cross sections for scattering by molecules into the observation cone were used to obtain the absolute atomic cross section for scattering into the cone. Experimental points for the absolute cross sections for scattering into the observation cone are shown in Fig. 2, which also shows the curves of Massey and Moiseiwitsch[3] and Bransden, Dalgarno, John, and Seaton,[4] as calculated from their total cross-section curves. These two curves are chosen for comparison because similar variational methods were used in both cases, although the former authors consider only S-wave scattering while the calculations of the latter authors include a fairly large P-wave component as well.

Clearly, the present experimental data cannot distinguish between the two calculations on the basis of the cross section for scattering into the cone of observation, but give qualitative agreement with either calculation.

It is of interest to consider the relation between the experimental values for the cross section of the hydrogen atom for scattering into the cone of observation and the total scattering cross-section values. The total cross section may be expressed as the sum of partial scattering cross sections, Q_l, where l is the angular momentum quantum number of the incident electrons, i.e.,

$$Q_{\text{total}}=Q_0+Q_1+\cdots. \quad (6)$$

When electrons are collected only in the angular range of this experiment, the cross section measured is related to the partial cross sections by

$$Q_A=(2\pi/4\pi)[1-(1/\sqrt{2})](Q_0+0.40Q_1+\cdots). \quad (7)$$

The derivation of this equation is discussed in the appendix. Neglecting all but Q_0 and Q_1, and combining Eqs. (6) and (7) so as to eliminate Q_0,

$$Q_{\text{total}}=6.85Q_A+0.60Q_1 \quad (8)$$

relates the total, measured, and P-wave partial cross sections.

If it is assumed that the scattering is isotropic so that Q_1 may be neglected, the experimental points predict a total cross section as given in Fig. 3. If it is assumed that Q_1 takes values calculated by Bransden, Dalgarno, John, and Seaton (BDJS), the total cross section would be as shown in Fig. 4. The fact that the experimental points, especially those at 5.3 and 5.7 ev, map over to fit the BDJS total cross-section curve, using their Q_1, better than the Massey and Moiseiwitsch curve, neglecting Q_1, appears to indicate that the experimental data are more consistent with the BDJS calculations.

IV. CONCLUSION

While this measurement is not sufficiently refined to allow a choice between the various theories of low-energy scattering of electrons by hydrogen atoms, it seems eminently clear that above 3 ev the theories are not in substantial disagreement with the experimental facts, as ascertained in these measurements.

V. ACKNOWLEDGMENTS

We are indebted to Professor Benjamin Bederson of New York University for a number of very helpful discussions on this experiment and for making available the experimental details of his work, and to Dr. M. J. Seaton of University College London both for general discussions of this problem and for communication of the results of the Bransden, Dalgarno, John, and Seaton calculations prior to their publication.

We are again indebted to Miss Eugenia L. Rossell for her assistance in carrying out these measurements.

APPENDIX

To derive Eq. (7), one has merely to integrate the differential cross section over the angular range observed in this experiment. As is shown in any textbook on quantum mechanics, the differential cross section is

[6] C. Ramsauer and R. Kollath, Ann. Physik **12**, 529 (1932).

given in terms of phase shifts, η_l, by

$$\sigma(\theta)=\frac{1}{4k^2}\left|\sum_{l=0}^{\infty}(2l+1)(e^{2i\eta l}-1)P_l(\cos\theta)\right|^2. \quad \text{(A-1)}$$

In integrating over a right circular cone whose axis lies in the equatorial plane of the scattering sphere and which has a half-apex-angle $\pi/4$, the cross section Q_A will be given by

$$Q_A=\int_{\pi/4}^{3\pi/4}\sigma(\theta)\sin\theta\phi(\theta)d\theta, \quad \text{(A-2)}$$

where

$$\phi(\theta)=\cos^{-1}(\cot^2\theta). \quad \text{(A-3)}$$

On expansion of Eq. (A-1) and integration, the interference terms vanish because of the symmetry of the integration limits around the equator of the scattering sphere, leaving

$$Q_A=(1/k^2)(I_0\sin^2\eta_0+9I_1\sin^2\eta_1+\cdots), \quad \text{(A-4)}$$

where

$$I_n=\int_{\pi/4}^{3\pi/4}(\cos\theta)^{2n}\sin\theta\cos^{-1}(\cot^2\theta)d\theta. \quad \text{(A-5)}$$

After integration by parts to eliminate the arccosine from the integrand, changing variables by

$$\cos\theta=(1/\sqrt{2})\sin(\psi/2) \quad \text{(A-6)}$$

and by $z=e^{i\psi}$ transforms the integrals to

$$I_n=\frac{1}{2n+1}\frac{(-1)^{n+1}}{2^{3n+1/2}}\times\frac{1}{i}\oint\frac{(z-1)^{2n+2}}{z^{n+1}(z^2+6z+1)}dz, \quad \text{(A-7)}$$

where the integration is taken about the unit circle in the complex plane. Evaluation of the residues at the two poles inside the unit circle yields

$$\begin{aligned}I_0&=2\pi[1-(1/\sqrt{2})],\\ I_1&=\pi/3[2-(5/2\sqrt{2})].\end{aligned} \quad \text{(A-8)}$$

Substituting (A-8) into (A-4) and recalling that the total partial cross sections are given by

$$\begin{aligned}Q_0&=(4\pi/k^2)\sin^2\eta_0,\\ Q_1&=(4\pi/k^2)3\sin^2\eta_1,\end{aligned}$$

yields Eq. (7) immediately.

The analytical evaluation of the cross section for scattering into a right circular cone of *any* apex angle whose axis lies in the equatorial plane of the scattering sphere may be carried out similarly.

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Charge Exchange in Proton-Hydrogen-Atom Collisions*

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The charge-exchange cross sections for the reactions $p+H\rightarrow H+p$, and $H_2^++H\rightarrow H_2+p$ have been examined over the energy range 200 to 14 000 ev. In the experiment a dc fast-ion beam crossed a slow atomic hydrogen beam which was chopped at 100 cps. The desired signal thus was separable from the much larger signal arising from interaction of the ions with the residual gas in the vacuum chamber, because the signal arising from the interaction of the two beams occurred at the chopping frequency and in a specified phase. The signals used were the saturated slow-ion currents, recorded at a detector which did not discriminate the ion mass, and the slow-ion currents after mass analysis. The measured values at high energies agree very satisfactorily with the Born approximation calculations by Bates and Dalgarno and, at low energies, with calculations by Dalgarno and Yadav using the method of perturbed stationary states. Experimental comparison of cross sections for proton and deuteron collisions is presented.

I. INTRODUCTION

As a target system in a collision experiment, the free hydrogen atom possesses the virtue that its wave functions are completely and exactly known. Thus, in the theoretical treatment of a collision between the hydrogen atom and any elementary particle, the only error which can arise occurs through the intrinsic failure of the particular scattering approximation used in the theory. As a result, comparison of a theoretical prediction with experimental results of scattering cross-section measurements for those collisions in which the hydrogen atom is the target serves to evaluate the validity and degree of failure of the scattering approximation only.

In addition to this basic interest, collision cross sections of the hydrogen atom (or deuterium atom, since the extranuclear properties are virtually identical) are of importance in understanding the operation of experimental controlled thermonuclear devices, es-

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