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Thermal Conductivity of an Electron Gas in a Gaseous Plasma*

T. SEKIGUCHI† AND R. C. HERNDON

Department of Electrical Engineering, University of Illinois, Urbana, Illinois

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The thermal conductivity of an electron gas in a gaseous plasma is determined by experimental techniques which have been improved over those reported in a previous article by Goldstein and Sekiguchi. A pulsed microwave is utilized to probe the plasma parameters as well as to selectively heat up the electron gas in a small volume of the elongated plasma. The photomultiplier tube detects the change in electron temperature by taking advantage of the phenomenon of "afterglow quenching." The experimental values for the thermal conductivity, which are determined by two different methods, are in good agreement. In the plasmas investigated, neon and helium, the degree of ionization is very low (10^{-5}–10^{-6}). However, the measured values of the thermal conductivity are still consistent with those obtained from the theoretical expression given by Landshoff, or Spitzer and Härm for a fully ionized gas.

INTRODUCTION

A PREVIOUS article by Goldstein and Sekiguchi,[1] referred to hereafter as GS, reported an experimental method for investigating the effect of electron-electron interactions on the thermal conductivity of an electron gas in a gaseous plasma. The thermal conductivity (K) was determined through the selective heating of the electron gas in a small volume of the decaying plasma by means of a pulsed microwave. Detection of the increase in electron temperature was accomplished by using the phenomenon of "afterglow quenching."[2]

It is necessary to know with considerable accuracy the value of the plasma parameters (electron concentration, effective collision frequency and initial electron temperature), both for the determination of the thermal conductivity, and for the quantitative comparison of the experimental results with the present theories. However, in the previous work reported by GS, the electron-molecule collision cross section was not precisely known as a function of electron energy for the gases used. This is directly related to the appearance of a rather strong "Ramsauer-Townsend effect" in the heavier noble gases such as xenon and krypton. Also, in the work of GS in which xenon was the principal gas employed, it apparently seemed that the thermal conductivity increased rapidly with increasing electron concentration, if the initial electron temperature was assumed constant at 300 degrees Kelvin. This was quite contrary to the theory of Spitzer and Härm[3] in which the value of K is supposed to increase rapidly with increasing initial electron temperature T_{e0}, but is a slowly varying function of electron density n_e. This apparent discrepancy was presumed due to an unknown variation of the initial electron temperature which had previously been assumed constant.

It was the above considerations which made it seem profitable to extend these investigations to the lighter noble gases, neon and helium, in which the "Ramsauer-Townsend effect" does not appear, and for which the electron-molecule cross section has been accurately determined as a function of electron temperature.[4,5] This extension was made feasible by the improved experimental techniques that were developed since the work previously done by GS.

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† On leave from the Faculty of Engineering, University of Tokyo, Tokyo, Japan.

[1] L. Goldstein and T. Sekiguchi, Phys. Rev. **109**, 625 (1958).

[2] Goldstein, Anderson, and Clark, Phys. Rev. **90**, 486 (1953).

[3] L. Spitzer, Jr., and R. Härm, Phys. Rev. **89**, 977 (1953); L. Spitzer, Jr., *Physics of Fully Ionized Gases* (Interscience Publishers, Inc., New York, 1956), p. 86.

[4] L. Gould and S. C. Brown, Phys. Rev. **95**, 897 (1954).

[5] A. L. Gilardini and S. C. Brown, Phys. Rev. **105**, 31 (1957).

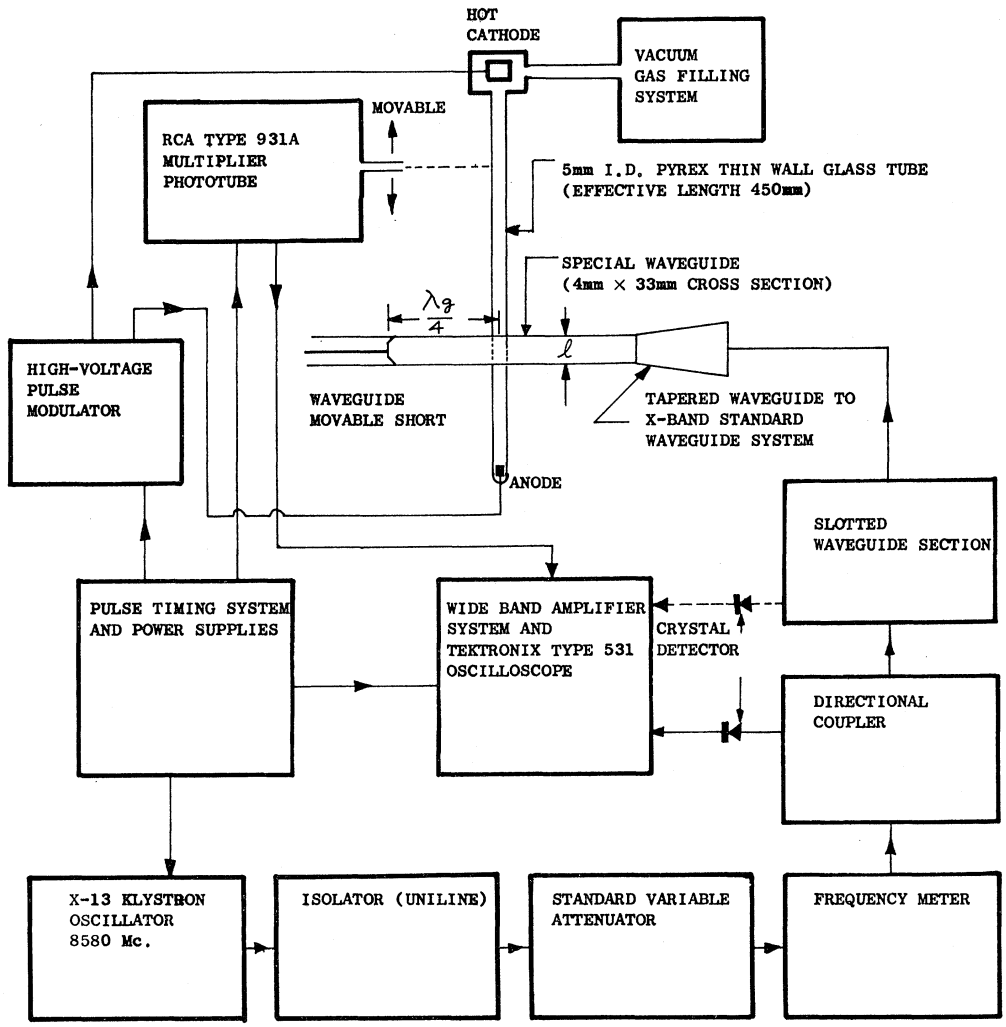

FIG. 1. Schematic diagram of the experimental apparatus.

APPARATUS AND EXPERIMENTAL METHOD

The experimental apparatus and method employed to determine K and the plasma parameters are essentially as outlined in GS. A brief résumé with some supplementary explanations will be presented here.

Figure 1 shows the schematic diagram of the experimental arrangement. The region in which the microwave energy interacts with the plasma is as illustrated in Fig. 1 of GS. An RCA type 931A photomultiplier tube is mounted on the side of the movable wave guide No. 2 (see Fig.1 of GS). The attached photomultiplier tube can be moved along the discharge tube containing the plasma and detect the radiated light from a small volume of the plasma in the unheated region.[6,7] To detect the light output from the heated region, the photomultiplier is attached to wave guide No. 1. A wave-guide movable short is located in wave guide No. 1 and shown in Fig. 1. This movable short is positioned at a distance of one-quarter of a guide wavelength from the plasma post. The high-frequency admittance due to the plasma, referred to the plane of the plasma post, is determined by means of a standard X-band slotted wave-guide section. A vacuum system capable of evacuating the discharge tube to 10^{-6}–10^{-7} mm Hg was employed. Laboratory grade neon and helium gases were used (Linde, mass-spectrometer controlled).

The procedures for the determination of the electron concentration n_e, initial electron temperature T_{e0}, and the thermal conductivity are schematically presented in the form of a flow chart in Fig. 2.

[6] Wave guide No. 2 in Fig. 1 of GS was originally provided to detect the electron temperature deviation in the nonheated region by means of *microwave interaction techniques* [see J. M. Anderson and L. Goldstein, Phys. Rev. **100**, 1037 (1955); **102**, 933 (1956), and also reference 7]. However, owing to the small interaction region between the microwave energy and the plasma (less than 0.1 cm^3), the detected signal for this purpose was not sufficient to permit quantitative analysis of the photographically recorded data.

[7] A. A. Dougal and L. Goldstein, Phys. Rev. **109**, 615 (1958).

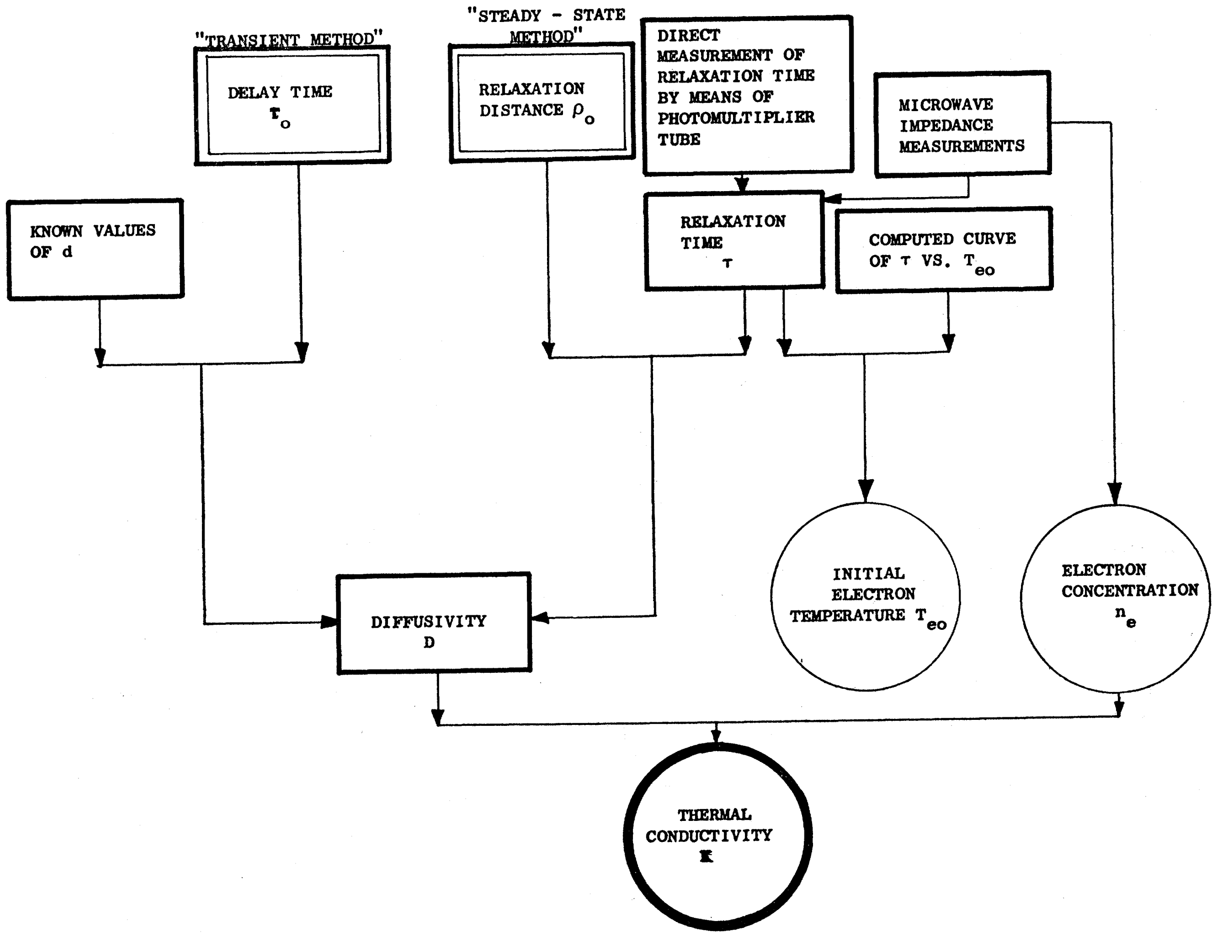

FIG. 2. Flow chart for the experimental procedures.

(1) Determination of the Thermal Conductivity (K)

Two separate methods were employed to determine K. These are referred to as the "transient" and "steady-state" methods. In the "transient" method, the time it takes for heat to propagate from a heated region of the plasma, where the electron temperature is suddenly raised by the heating microwave, to an unheated region of the plasma is determined with the help of the photomultiplier tube. The time element involved in this method is designated as the "delay time" (t_0). The diffusivity (D) can then be found from the expression

$$D = d^2/6t_0, \tag{1}$$

where d is the measured distance between the heated volume and a point in the nonheated volume where t_0 is measured.

The "steady-state" method is based upon the equilibrium temperature distribution in the nonheated region of the plasma, while continuous heating occurs in the heated region. In this method the characteristic distance involved is called the "relaxation distance" (ρ_0) and is likewise determined with the aid of the photomultiplier tube. Once ρ_0 is measured, the diffusivity can again be calculated from

$$D = \rho_0^2/\tau, \tag{2}$$

where τ is the "effective relaxation time."[7] τ represents the characteristic time associated with the equipartition of energy through the plasma by electron-ion and electron-molecule collisions. It may be expressed by

$$\frac{1}{\tau} = \frac{1}{\tau_{em}} + \frac{1}{\tau_{ei}}, \tag{3}$$

where τ_{em} and τ_{ei} are the electron-molecule and electron-ion relaxation times, respectively.

K can now be determined by use of the following equation, provided we know the electron number density (n_e):

$$K = \tfrac{3}{2} k n_e D, \tag{4}$$

in which k is Boltzmann's constant.

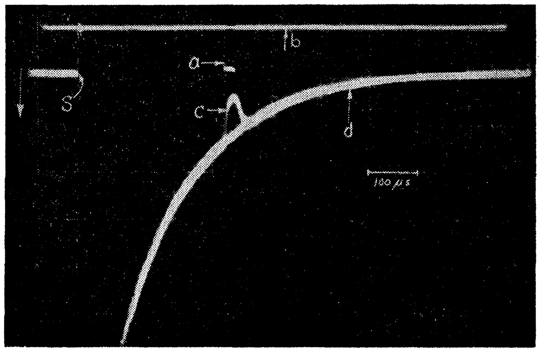

FIG. 3. Afterglow period in the presence of the heating pulse.

(2) Determination of the Plasma Parameters (n_e, τ, and T_{e0})

(a) The electron concentration and effective relaxation time (or effective collision frequency) are found through microwave admittance measurements. These measurements are made by means of a slotted wave-guide section and with a microwave of sufficiently low power so that no appreciable heating of the plasma is effected. For this purpose the plasma contained in the wave guide is treated as a cylindrical dielectric post whose high-frequency complex specific permittivity ϵ' is given by

$$\epsilon' = 1 - \frac{\omega_p^2}{\omega_s^2 + \nu^2}\left[1 + j\frac{\nu}{\omega_s}\right], \tag{5}$$

where $\omega_p = (n_e e^2/m\epsilon_0)^{\frac{1}{2}}$ is the plasma frequency (ϵ_0 being permittivity of a vacuum), $\nu = M/2m\tau$ is the effective collision frequency [M and m being the mass of a molecule (or an ion) and an electron, respectively], and ω_s is the microwave frequency. The microwave measurements of the normalized conductance and susceptance due to the plasma post enable us to calculate ϵ' through microwave circuit considerations.[8] The determination of ϵ', in turn, allows us to find n_e and τ (or ν) from Eq. (5).

The effective relaxation time τ can also be measured by a different method involving the use of the photomultiplier tube. For this purpose a microwave heating pulse of short duration (1–4 microsec) and low power level is used. The decay of the "quenched" light intensity of the plasma in the heated volume, immediately after the cessation of the heating pulse, is observed by the photomultiplier tube. For small electron temperature deviations the decay of the electron temperature can be expressed as a single exponential $e^{-t/\tau}$.[7]

(b) As will be seen later, the determination of the initial electron temperature is necessary in order to compare the experimental results with the present theory. If the dependence of τ on the electron temperature is known, we can then use this fact to help in the determination of the initial electron temperature.

The relaxation time for electron-molecule collisions (τ_{em}) has been experimentally determined as a function of electron energy for both helium[4] and neon.[5]

The relaxation time for electron-ion collisions (τ_{ei}), however, must be obtained from theoretical considerations. Various expressions have been derived by Landau,[9] Spitzer,[10] Chandrasekhar,[11] and Dougal and Goldstein.[7] Unfortunately, the different theories do not all give the same expression for the electron-ion

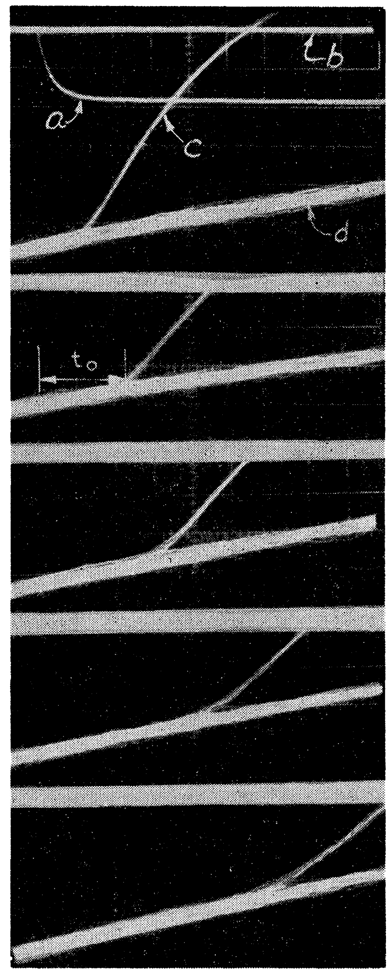

FIG. 4. Heat propagation phenomenon in neon plasma at 5.2 mm Hg pressure. The heating pulse (trace a) is of 20 μsec duration and is applied 150 μsec after the dc high-voltage breakdown pulse. The incident microwave power level $P_{\text{in}} = 275$ mw. Time scale = 1 μsec per division. (See Table I, experiment No. 4.)

[8] The equivalent circuit representation of a dielectric post in a rectangular wave guide for the dominant mode of wave propagation is shown in *Wave-guide Handbook*, Massachusetts Institute of Technology Radiation Laboratory Series (McGraw-Hill Book Company, Inc., New York, 1951), Vol. 10, p. 266.

[9] L. Landau, Physik. Z. Sowjetunion **10**, 154 (1936).

[10] L. Spitzer, Jr., Monthly Notices Roy. Astron. Soc. **100**, 396 (1940); *Physics of Fully Ionized Gases* (Interscience Publishers, Inc., New York, 1956), Chap. 5.

[11] S. Chandrasekhar, *Principle of Stellar Dynamics* (University of Chicago Press, Chicago, 1942), p. 48.

collision frequency or relaxation time. This difference has its origin in the nature of the assumptions made by each author. However, they all do have a similar functional form for singly charged ions:

$$\tau_{ei}=f(n_e,T_e)=\frac{A}{F(n_e,T_e)}(M_iT_e^{\frac{3}{2}}/n_e), \tag{6}$$

where M_i, A, $F(n_e,T_{e0})$ are the mass of an ion, a constant, and a slowly varying function of n_e and T_{e0}, respectively. The expressions given by each of the above mentioned authors differ only in the constant A and in the form of the function F. (The largest value of τ_{ei} calculated from the various expressions is only about twice as large as the smallest.) Dougal and Goldstein[7] have recently performed experiments in which they verified the theoretical dependence of τ_{ei} on M_i and n_e in Eq. (6). However, they were unable to verify as closely the dependence on T_{e0}, because of several secondary processes which existed in their experiments. As Dougal and Goldstein's experimental results for the electron-ion relaxation time were found to be most consistent with their own derived expression for τ_{ei} (around room temperature), their expression has been used in this article for the estimation of the initial electron temperature.

EXPERIMENTAL OBSERVATIONS AND RESULTS

The general nature of the afterglow period is illustrated in Fig. 3. The detected envelope of the heated microwave pulse is labeled *a*. The horizontal trace *b* is the baseline for the microwave signal. Point *s* denotes the time at which the high-voltage dc pulse is applied to break down the gas. In the case illustrated in Fig. 3, the 20-μsec heating pulse was applied 300 μsec after the 2-μsec breakdown pulse. Trace *d* shows the photo-output current (detected by the photomultiplier tube) in the decaying plasma in the absence of the heating pulse. The small signal *c* standing on trace *d* is due to the heating pulse *a*. It is the quenched light output arising from the electron temperature increase which was effected by either direct heating, or by heat conduction to an unheated region of the plasma. In Fig. 3

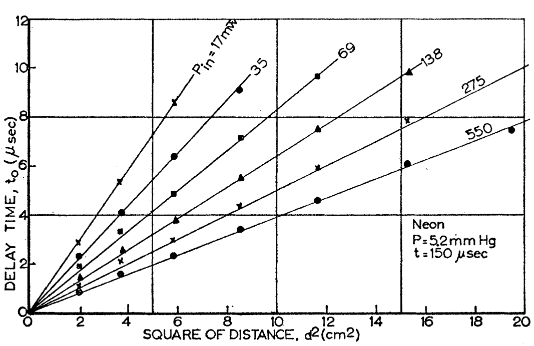

FIG. 5. Curves of the delay time *versus* the square of the distance from the heated volume for various incident power level (P_{in}).

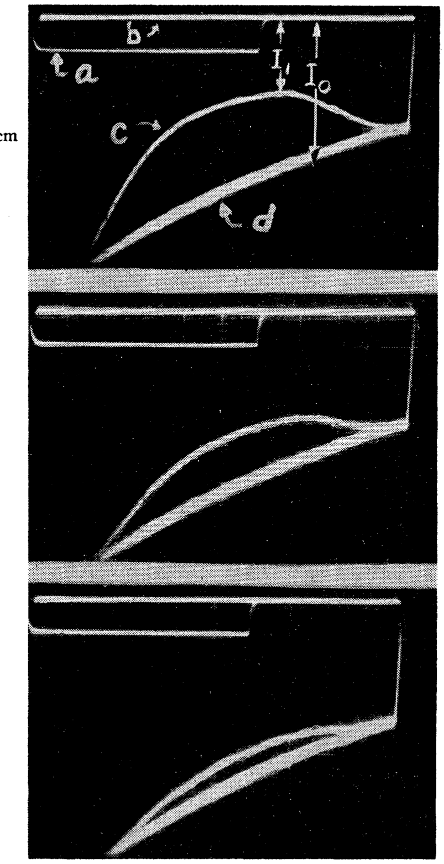

FIG. 6. The decrease in the light intensity, as a function of the distance from the heated volume, due to the heat conduction in the plasma, for the "steady-state method." (Heating power level P_{in}=110 mw; see Table I, experiment No. 4.)

both the detected amplitude of the heating microwave and the magnitude of the light output are increasing with downward deflection of the oscilloscope trace. Since, in general, n_e and T_{e0} vary in time in the afterglow period, K may be obtained as a function of these two parameters by applying the heating pulse at various times during the afterglow. The time scale in Fig. 3 is 100 microseconds per division which unfortunately is too large to see the details of the phenomena in the vicinity of the quenched light signal.

Figure 4 presents a typical series of photographs which magnify the region near the leading edge of the heating pulse. These photographs clearly illustrate how the delay time (t_0) varies as a function of the distance (d) for a fixed heating power level.[12] [d is as defined for Eq. (1).] The symbols in Fig. 4 are the same as used in Fig. 3. The sequence of photographs, such as shown in Fig. 4, has been taken at various power levels of the heating microwave, as well as for different values of d. In Fig. 5 the delay time (t_0) is plotted

[12] Figure 4, which is for neon, is of the same nature as Fig. 2 of GS which is for xenon. The reduction of the noise which existed in the previous experiment permits the determination of t_0 more precisely.

TABLE I. Experimental results compared with theoretical calculations.

No.	Gas	Pressure (mm Hg)	Time position in afterglow t (μsec)	$n_e\times10^{-12}$ (cm^{-3})	τ (μsec)	ρ_0 (mm)	10^6K (meas.) (watt/cm deg)	T_{e0} (estimated) (deg Kelvin)	10^6K (theo.)
1	Ne	1.0	40	1.07	3.5	5.6	1.99	365-400	1.06-1.30
2	Ne	1.0	80	0.44	5.7	7.0	0.784	~300	0.63
3	Ne	5.2	100	1.50	3.4	5.1	2.38	490-550	2.10-2.65
4	Ne	5.2	150	0.74	4.4	5.4	1.03	340-390	0.92-1.20
5	Ne	5.2	200	0.49	5.8	7.0	0.858	325-365	0.70-0.96
6	Ne	5.2	250	0.36	6.8	7.7	0.650	~300	0.58
7	Ne	9.5	100	1.15	3.8	5.1	1.63	450-520	1.7-2.4
8	Ne	9.5	150	0.595	4.3	5.8	0.963	300-320	0.65-0.71
9	Ne	9.5	200	0.41	5.6	6.9	0.722	~300	0.61
10	Ne	9.5	250	0.31	9.4	9.2	0.578	~300	0.57
11	Ne	9.5	300	0.25	11.0	9.8	0.452	~300	0.56
12	Ne	14.5	130	1.40	2.5	4.0	1.855	325-385	0.86-1.23
13	Ne	14.5	180	0.74	3.8	4.9	0.968	310-360	0.69-0.99
14	Ne	14.5	230	0.57	4.3	5.2	0.741	~300	0.63
15	Ne	14.5	280	0.41	7.9	7.0	0.527	~300	0.60
16	Ne	21.4	120	0.93	3.2	4.5	1.22	330-400	0.84-1.30
17	Ne	21.4	170	0.43	4.7	5.9	0.659	~300	0.61
18	Ne	21.4	220	0.27	6.8	8.3	0.567	~300	0.57
19	He	5.3	70	0.95	0.49	1.3	0.680	~300	0.69
20	He	5.3	90	0.60	0.72	1.7	0.499	~300	0.63

against the square of d; the resulting straight lines confirm the use of Eq. (1). The slope of each straight line determines the diffusivity (D) for a particular heating power level (P_{in}).

The photographs in Fig. 6 show the decay of the "quasi-steady-state" electron temperature deviation in the nonheated region of the plasma as a function of the distance d from the heated portion. The heating pulse has a duration long enough for the electron temperature to reach an equilibrium condition (for Fig. 6 the heating pulse is 30 microsec long). The symbols are the same as for Fig. 3 and 4. The ratio of I_0/I_1 in Fig. 6 determines the relative electron temperature deviation $\Delta T_e/T_{e0}$ by means of Eq. (11) of GS. Figure 7 depicts

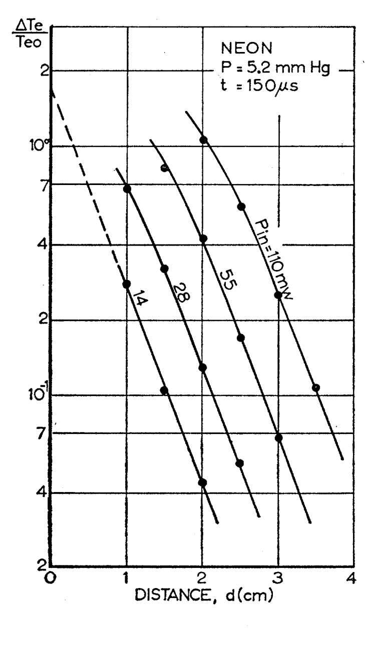

FIG. 7. Curves of the relative electron temperature deviation *versus* the distance from the heated volume.

the temperature distribution in the unheated region along the plasma, for the particular case given by the photographs in Fig. 6.[13] The slope of the straight lines determines the relaxation distance ρ_0.

The experimental results for K, as well as for the measured values of n_e, τ, and ρ_0 obtained for neon and helium, are tabulated in Table I, together with the experimental conditions under which K was determined. The values given for τ are those obtained from the direct method using the photomultiplier tube.[14] Figure 8 presents some of the measured values of K determined by both the "steady-state" and the "transient" methods as a function of the heating microwave power level.[15] The numbers labeling the different curves correspond to those shown in the column designated by "No." in Table I. The microwave heating power level (P_{in}), which is the abscissa in Fig. 8, is a measure either of the electron temperature deviation in the heated volume or the electron temperature gradient in the unheated volume of the plasma. The strong dependence of K on P_{in}, determined by the steady-state method, implies that the value of K increases with increasing electron temperature and its gradient. The steady-state values for K are shown as short, solid horizontal lines, since this method is justified only when the tempera-

[13] The reasons why some of the curves deviate from a straight line have been described in Sec. 4 of GS.

[14] Because of the limitations imposed by the microwave measuring apparatus, and the fact that the interaction volume of the microwave with the plasma was quite small (see reference 6), the measured value of the high-frequency conductance, which primarily determines τ, was comparable to the error involved. However, the use of the photomultiplier tube led to a more precise method for determining τ. Nevertheless, n_e was determined quite precisely by microwave measurements, since it is primarily dependent on the measured high-frequency susceptance, which is at least one order of magnitude larger than the conductance.

[15] Figure 8 corresponds to Fig. 3 of GS, in which neon and xenon were used, respectively.

ture gradient is small (see Sec. 4 of GS). An inspection of Fig. 8 shows that the values of K determined by the two different methods are in good agreement for smaller temperature gradients, that is, for low values of P_{in}. The values of the thermal conductivity listed in the column designted as "K(meas.)" in Table I were found by the steady-state method.

COMPARISON WITH THEORY

The thermal conductivity for electrons in a *fully ionized gas* has been calculated by Cowling,[16] Landshoff,[17,18] Chapman and Cowling,[19] Spitzer and Härm,[3] and Chapman.[20] The most extensive calculations are those of Landshoff, and Spitzer and Härm who have taken into account the electron-electron interaction. For this reason the values of K obtained here will be compared with the theoretical values given by these theories.[21] This comparison is justified even though the degree of ionization is quite small (10^{-5}–10^{-6}) in the experiments performed, the reason for this having been presented in Sec. 5 of GS.

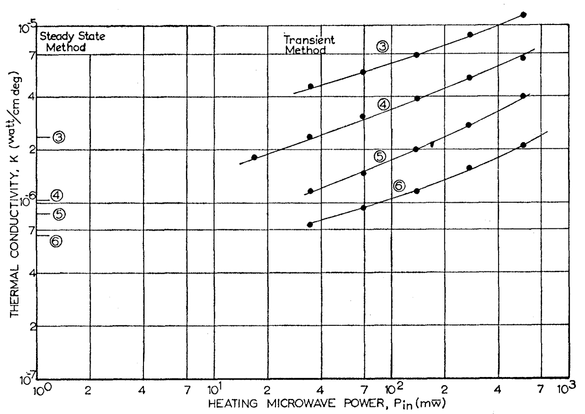

FIG. 8. Experimental results for the thermal conductivity as a function of the heating power level. (See Table I, experiment No. 3, 4, 5, and 6 for the experimental conditions.)

Figure 9 shows the values for the thermal conductivity (for singly charged ions) calculated from Spitzer and Härm's theory. It should be mentioned that the results of this theory are for the case of small electron temperature gradients, and it is therefore appropriate to compare these theoretical values with the experimental ones shown in Table I. In order to make this comparison, it is necessary first to know the absolute value of the initial electron temperature T_{e0} and the electron concentration n_e. T_{e0} is estimated by means

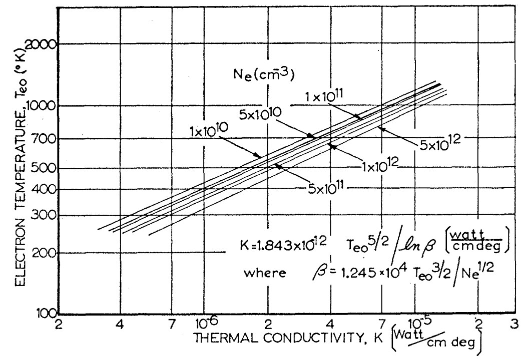

FIG. 9. Theoretical curves for the thermal conductivity as a function of electron temperature for various values of electron concentration, evaluated from the expression given by Spitzer and Härm.

of the measured value of τ listed in Table I, and the method described previously. The theoretical values of K, obtained from Fig. 9, and the estimated values of T_{e0} are also tabulated in Table I. To facilitate comparison, Fig. 10 is a graph of the experimental *versus* the theoretical values of K. If both values were the same, all of the points would lie on the straight line of 45° slope. This figure shows that the theory and experiment give consistent results.

DISCUSSION

The experimental results shown in Table I and Fig. 10 confirm the statements made in the previous article by GS, Sec. 5. The value of K has no appreciable dependence either on the gas pressure in the pressure range considered, or on the kind of gas employed. The present set of experiments also gives further support to the statement made in GS that "the rate at which thermal energy is transferred from a small volume of

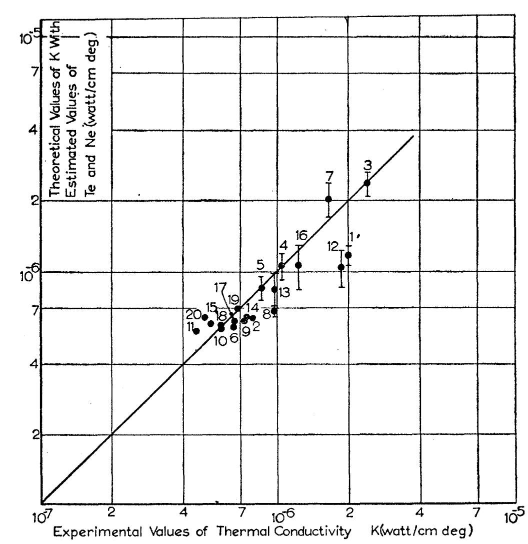

FIG. 10. The experimental results for the thermal conductivity compared with the values from Spitzer and Härm's theory.

[16] T. G. Cowling, Proc. Roy. Soc. A, **183**, 453 (1945).
[17] R. Landshoff, Phys. Rev. **76**, 904 (1949).
[18] R. Landshoff, Phys. Rev. **82**, 442 (1951).
[19] S. Chapman and T. G. Cowling, *Mathematical Theory of Nonuniform Gases* (Cambridge University Press, Cambridge, 1953), second edition, Chaps. 9, 10, 13, and 18.
[20] S. Chapman, Astrophys. J. **120**, 151 (1954).
[21] Landshoff has employed an approach different from that of Spitzer and Härm. However, the result of the former, in the case of zero external magnetic field, coincides with that of the latter.

warm electron gas to equal volumes of cool electrons within the same plasma is considerably faster (at least one order of magnitude faster) than the rate of heat transfer to any of the two other heavy plasma constituents (ions and neutral atoms)." From Eqs. (1) and (2), the delay time t_0, which is a measure of how rapidly heat flows through the plasma, can be written as

$$t_0=(d^2/6\rho_0^2)\tau. \tag{7}$$

As an example, consider two small volumes of the plasma which are about 1 mm apart. Putting $d=1$ mm in Eq. (7), and using the values for ρ_0 from Table I, we have the ratio of $d^2/6\rho_0^2$ ranging from 0.0015 to 0.01 for neon and from 0.1 to 0.05 for helium.

It is worth while to consider one aspect of the role of the heat conduction phenomena which has not always been realized thus far. If the effect of heat conduction can be assumed negligible or the gradient of the electron temperature does not exist, then the electron temperature deviation in the heated region, in the case of microwave heating, is given by Eq. (5) of GS under the equilibrium condition. However, if this is not the case, the temperature increase should be different from that given by the above-mentioned expression. Since a general discussion of such a situation is complicated, only a particular case which has appeared in our experiments will be considered here.

As shown in Fig. 1, the electron gas in the limited volume of length l (4 mm) is heated by the microwave energy. This limited volume of the electron gas is a portion of the elongated plasma and is contained in the specially designed wave guide. The high-frequency electrical conductivity $[\sigma=\epsilon_0\omega_p^2\nu/(\omega_s^2+\nu^2)]$, which is a function of n_e and T_{e0} in general, may be considered as nearly constant over the heated volume of the plasma. The reasons for this are as follows:

(1) The major disintegrating mechanism of the decaying plasma in our experiments has been found to be the electron-ion recombination process rather than diffusion. Since this is the case, the electron concentration (n_e) is almost uniformly distributed along and across the discharge tube, except in the very near vicinity of the glass wall where n_e is considered to have a very small value.

(2) As long as the heating level is low, the effect of thermal diffusion to the glass wall may be neglected to a first approximation, in view of the thermal barrier existing in the vicinity of the wall.[7,22]

Furthermore, since the diameter of the discharge tube (5 mm) is much smaller than the width of the wave guide (33 mm), and since the dominant mode (H_{01}) of the wave propagation is used, the effective high-frequency electric intensity E_h (Eq. 5 of GS) may be considered as constant over the entire heated volume.

By means of the above considerations the problem can be greatly simplified. The microwave power absorbed per unit volume is σE_h^2 and this energy is uniformly dissipated over the entire heated volume of the plasma, thus raising the electron temperature. A portion of this energy is transferred to the molecules and ions through elastic collisions. The remainder is removed from the heated region to the cooler regions by *one-dimensional heat flow*, which arises from the thermal conductivity of the electrons. What we wish to investigate is how significant this heat conduction by the electron gas is.

A relatively simple calculation results in the following expressions for the equilibrium electron temperature deviation ΔT_e at the center and boundary of the heated volume (Appendix I):

$$\Delta T_e=\begin{cases}\Delta T_{e0}(1-e^{-q}) & \text{at center} \quad (8)\\ \Delta T_{e0}\tfrac{1}{2}(1-e^{-2q}) & \text{at boundary,} \quad (9)\end{cases}$$

where

$$q=l/2\rho_0=(3k/8)^{\frac{1}{2}}(n_e/K\tau)^{\frac{1}{2}}l. \tag{10}$$

ΔT_{e0} is given by Eq. (5) of GS. The factors in the brackets in the above expressions represent the effect of the heat conduction of the electrons. The ratio (R) of the total power swept away from the heated volume due to heat conduction, to the total power supplied by the microwave energy source to the entire heated volume of the plasma is given by

$$R=(\sinh q/q)e^{-q}. \tag{11}$$

In our experiments the electron concentration and effective collision frequency are known quantities ($\nu=M/2m\tau$). The high-frequency electrical conductivity $[\sigma=\epsilon_0\omega_p^2\nu/(\omega_s^2+\nu^2)]$ can then be calculated, which enables us to estimate the effective high-frequency electric intensity E_h through the relation $P_0=V\sigma E_h^2$. (V is the net volume of the heated region of the plasma.) In addition, of course, the microwave power (P_0) absorbed by the electron gas in the heated region must be known. The microwave power absorbed, P_0, is measured by microwave measurements so that E_h and ΔT_{e0} can be computed by Eq. (5) of GS. The value of q can also be computed from the experimental results shown in Table I and Eq. (10). Finally, the effective temperature deviation in the heated volume and the value of R can be found from Eqs. (8)–(10).

The equilibrium temperature in the heated volume can also be estimated roughly from the experimental results alone. Figure 7 shows, for example, the relative temperature distribution $\Delta T_e/T_{e0}$ in the unheated region along the axis of the plasma at 150 μsec after the breakdown pulse. Since we already know the value of T_{e0} from Table I, the extrapolation of these curves to the plane $d=0$ permits us to obtain an idea as to the order of magnitude of the temperature deviation in the heated volume.

The computed values of ΔT_{e0}, ΔT_e, R, and also ΔT_e estimated by means of the latter method, are given in Table II for the 150-μsec case. It was previously ex-

[22] H. Schirmer, Appl. Sci. Research B5, 196 (1955).

TABLE II. Equilibrium electron temperature deviation in the heated volume of plasmas.

Time position in afterglow t (μsec)	P_{in} (mw)	P_0 (mw)	T_{e0} (°K)	T_e (comp.) Center	T_e (comp.) Boundary	T_e (meas.) boundary	R
			Neon, pressure 5.2 mm Hg				
100	23	3.5	4960	1610	1350	1720($\pm$100)	0.69
150	14	0.73	2670	820	700	640($\pm$50)	0.71
200	29	0.48	3530	880	770	660($\pm$40)	0.73

plained that it is possible to describe the decay of the electron temperature as a single exponential only for the case of small temperature deviation, that is, for low heating power P_{in}. Therefore in Fig. 7, which corresponds to 150 μsec, as is true for the 100- and 200-μsec cases, the curves which are linear over the largest range of d correspond to the lowest value for P_{in}. For this reason the lowest power level curve in Fig. 7 was used for the extrapolation purpose mentioned above. It can be seen that the values of ΔT_{e0} which have been obtained by the two different methods are in fair agreement. An interesting result is that R turns out to be around 0.7, which implies that 70% of the total power absorbed by the electrons from the energy source is swept away from the heated volume of the plasma to the unheated region, due to the thermal conductivity of the electron gas.

There are two additional points to examine. In our experiments the microwave frequency which has been used to probe and heat up the plasma was fixed at 8580 Mc/sec. This frequency corresponds to a plasma frequency with $n_e = 9.78\times10^{11}$ cm^{-3}. However, as seen from Table I, some of the measured values of n_e will give values for the plasma frequency which are larger than the microwave frequency. The question appears as to the validity of these measured values of n_e, since electromagnetic waves are not supposed to propagate in a plasma in which the plasma frequency is greater than the frequency of the electromagnetic wave. Fortunately, a semiquantitative analysis shows that the skin depth of the plasma, under the conditions described in Table I, is much larger than the radius of the plasma cross section used, so that the high-frequency electric field penetrates the plasma sufficiently (Appendix II).

Also it was found that for the manner in which these experiments were performed, there was no need to consider the effect of the direct transfer of the microwave energy from wave guide No. 1, through the plasma, to wave guide No. 2. The reason is that such transfer occurs only when the value of n_e in the afterglow is much larger than that for which the measurements of K were taken.

CONCLUSION

Improved experimental techniques have enabled us to verify the results of the previous paper by GS, and also to determine the thermal conductivity of an electron gas in a gaseous plasma more precisely. The results are in good agreement with the theoretical values given by the theories of Landshoff, and Spitzer and Härm. The experimental values for the thermal conductivity were determined by two separate methods. The results of these different methods are consistent and in fair agreement. It has also been shown that thermal conduction in a plasma is determined largely by the thermal conductivity of the electron gas of the plasma.

ACKNOWLEDGMENTS

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APPENDIX I. EQUILIBRIUM TEMPERATURE DISTRIBUTION IN THE PLASMA IN THE PRESENCE OF HEATING

In the equilibrium condition and for the situation in our experiment, Eq. (3) of GS becomes

$$D\frac{\partial^2 T_e}{\partial x^2}-\frac{1}{\tau}(T_e-T_0)=-\frac{\Delta T_{e0}}{\tau}.$$

Under the assumption that the electron temperature deviation is not large, so that the quantities D and τ can be regarded as constants, the above equation reduces to a linear differential equation with constant coefficients.

Taking the coordinate x along the plasma and its origin at the center plane of the heated region with length l, the boundary conditions are as follows:

(1) As $x\rightarrow\infty$, $T_e-T_0\rightarrow 0$.
(2) At $x=0$, $\partial(T_e-T_0)/\partial x=0$ (owing to the symmetry).
(3) At $x=l/2$ (at the boundary plane), T_e-T_0 is continuous.

The solution of the equation for the temperature deviation $\Delta T_e = T_e - T_0$ is given by

$$\Delta T_e=\Delta T_{e0}[1-e^{-q}\cosh(x/\rho_0)] \quad (|x|\leqq l/2, \text{ in the heated region}),$$
$$=\Delta T_{e0}(\sinh q)e^{-x/\rho_0} \quad (|x|\geqq l/2, \text{ in the unheated region}).$$

Putting $x=0$ into the first equation gives Eq. (8). Likewise, Eq. (9) is found by putting $x=l/2$ in either of the equations above. Next, from the definition of the ratio R, we obtain

$$R=-2K\left[\frac{\partial(\Delta T_e)}{\partial x}\right]_{x=l/2}\Big/ l\sigma E_h^2.$$

By means of the expression for R, and with the solution shown above, we get Eq. (11).

APPENDIX II. SKIN DEPTH OF A PLASMA WHEN SIGNAL FREQUENCY IS LOWER THAN PLASMA FREQUENCY

Under the assumptions, which are here evident, that (1) the signal frequency is much greater than the cyclotron frequency of ions, (2) the pressure gradient and the gravitational force are neglected, and (3) the magnetic field is zero, so that the macroscopic current i may be taken parallel to the high-frequency wave front, in which case no charge accumulation occurs, Maxwell's equation [Eq. (4-1) in reference 3] for a plane wave becomes (in mks rationalized units)

$$\frac{\partial^2 E}{\partial x^2}=\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}+\mu_0\frac{\partial i}{\partial t}.$$

With the same assumptions, the "generalized Ohm's law" [Eq. (2-12) in reference 3] reduces to

$$\frac{1}{\epsilon_0\omega_p^2}\frac{\partial i}{\partial t}=E-\frac{i}{\kappa},$$

where κ is the dc conductivity of a plasma and is equal to $n_e e^2/m\nu=\epsilon_0\omega_p^2/\nu$.

The complex propagation constant γ for the continuous sinusoidal wave, which can be derived from the above two equations, enables us to calculate the skin depth of a plasma. The skin depth d is defined by the reciprocal of the real part of γ. The result is

$$d=\left(\frac{\sqrt{2}c}{\omega_p}\right)\left[\frac{1}{1+\eta^2}-\zeta^2+\left(\frac{\eta^2\zeta^4+(1-\zeta^2)^2}{1+\eta^2}\right)^{\frac{1}{2}}\right]^{-\frac{1}{2}},$$

where $\eta\equiv\nu/\omega_s$ and $\zeta\equiv\omega_s/\omega_p$. For the special case of $\nu=0$ (no collisions), the above expression for the skin depth reduces to Eq. (4-9) of reference 3.

For $n_e=1.5\times10^{12}$ cm^{-3}, $\tau=3.4$ μsec (or $\nu=5.45\times10^9$ sec^{-1}), and $f_s=8580$ Mc/sec, the value of d is calculated to be 6.89 mm. If we assume $\nu=0$, with the same value of n_e, the value of d becomes 6.94 mm. It should be noted that this value of n_e chosen from Table I is the largest value given. The values of d for most of the cases shown in Table I were found to be much larger than this value. The use of the above expression of the skin depth is justified only for the propagation of a plane wave. However, this expression seems to determine semiquantitatively whether or not the high-frequency electric field penetrates sufficiently into the plasma in our experiments.

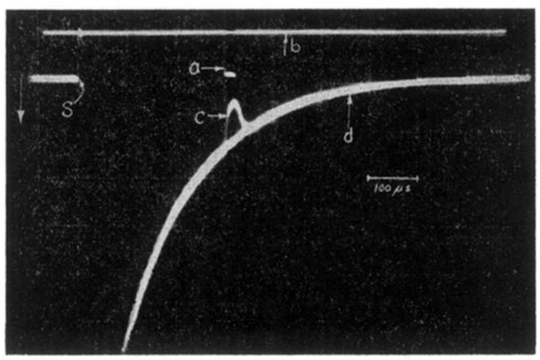

FIG. 3. Afterglow period in the presence of the heating pulse.

$d = 1.4$ cm

1.9

2.4

2.9

3.4

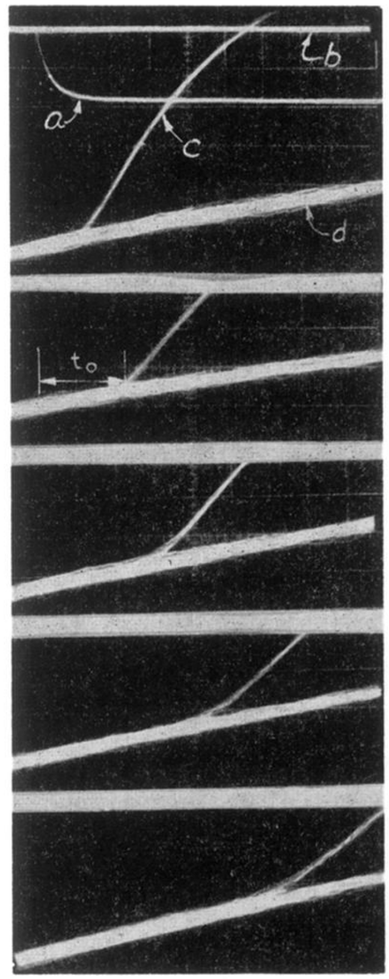

FIG. 4. Heat propagation phenomenon in neon plasma at 5.2 mm Hg pressure. The heating pulse (trace a) is of 20 μsec duration and is applied 150 μsec after the dc high-voltage breakdown pulse. The incident microwave power level $\bar{P}_{\rm in} = 275$ mw. Time scale $= 1$ μsec per division. (See Table I, experiment No. 4.)

d = 2.9 cm

3.4

3.9

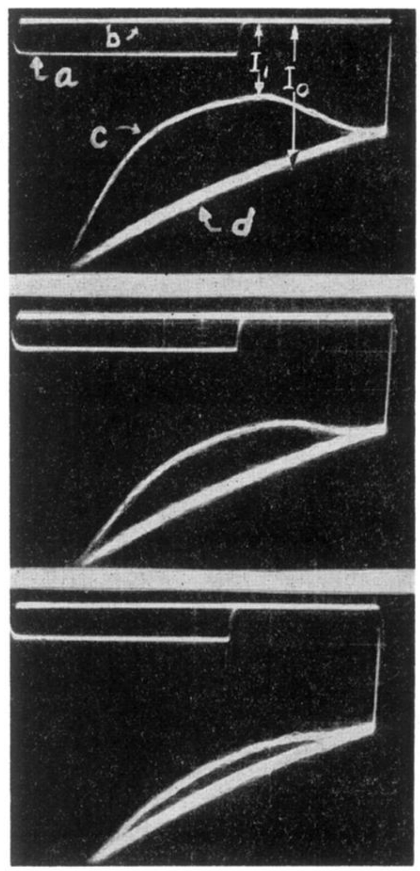

FIG. 6. The decrease in the light intensity, as a function of the distance from the heated volume, due to the heat conduction in the plasma, for the "steady-state method." (Heating power level $P_{\text{in}} = 110$ mw; see Table I, experiment No. 4.)