$He^4$ and the closer orbits of the $K^-$-helium system; both of these differences favor $K^-$ capture from states of higher angular momentum than those which may be effective in $K^-$-proton capture. Since the direct observations necessary to establish whether or not a $\gamma$ ray accompanied an observed ${}_\Lambda H^4$ or ${}_\Lambda He^4$ decay event appear to be very difficult, the possibility of formation of ${}_\Lambda H^{4*}$, ${}_\Lambda He^{4*}$ may lead to considerable confusion concerning the interpretation of any observations of the ${}_\Lambda H^4$, ${}_\Lambda He^4$ hypernuclei following $K^-+He^4$ reactions.

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# Theory of the Internal Space*

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It is postulated that fermions with identical space-time properties have to obey the exclusion principle unless they differ dynamically in their interaction with other particles. This postulate is formulated within the framework of a theory of the internal degrees of freedom. When all possible Yukawa-type couplings are examined, this postulate together with some other specification singles out the $\boldsymbol{\phi}\cdot\boldsymbol{\tau}$ interaction between pion and nucleon. Finally, when applied to electric interactions it requires the displacement of the nucleon center of charge.

## 1. INTRODUCTION

An important prediction of quantum field theory is that the quanta of one field are identical and obey the Pauli principle. By Pauli principle we shall mean more generally the requirement of symmetric or antisymmetric wave functions for particles with integral or half-odd-integral spin, respectively. Different particles are conventionally represented by different fields and the formalism does not require the Pauli principle in this case. The difference between the particles may lie in their space-time properties like mass and spin or in internal properties like the electric charge. Nevertheless it is mathematically also perfectly consistent within the present theory to have several fields which differ in no respect. For instance, a theory characterized by

$$L=\bar{\psi}_{\mathrm{I}}(\partial-M)\psi_{\mathrm{I}}+\bar{\psi}_{\mathrm{II}}(\partial-M)\psi_{\mathrm{II}}+\tfrac{1}{2}(\phi_{,i}\phi^{,i}-m^2\phi^2)+g(\bar{\psi}_{\mathrm{I}}\psi_{\mathrm{I}}+\bar{\psi}_{\mathrm{II}}\psi_{\mathrm{II}})\phi \quad (1)$$

describes two kinds of fermions interaction with a scalar boson, the former having completely identical properties. Yet the theory asserts that the Pauli principle is not effective between them. Such a case does not seem to be realized in nature. As far as we know all particles with, for instance, unit electric charge, spin $\frac{1}{2}$, and mass of the electron actually obey the exclusion principle. This observation leads one to analyze the usual formulation of the exclusion principle more carefully. The statement "identical Fermi particles cannot occupy the same state" requires an explanation as to what identical particles are. One can adopt the two points of view:

(A) Nonidentical particles differ in their space-time properties or in their interaction with other particles.

(B) Identical Fermi particles obey the exclusion principle. The definition (B) leads to a circle and most people will agree that (A) makes more sense. However, conventional quantum field theory does not imply the exclusion principle in form (A) but only in form (B).

We shall now sketch a formulation of the Pauli principle which implies (A) and makes it more precise. For this purpose we consider a theory with $N_f$ different kinds of fermions and $N_b$ different kinds of bosons. Such a situation will be described by $N_f+N_b$ Hermitian fields. Now we imagine that in the Bureau of Standards they have $N_f+N_b$ boxes, each containing one of the different kinds of particles. (For bosons the box may contain a piece of a static field instead of a particle.) Now we require that with the aid of this set of boxes we can determine to which of the $N_f+N_b$ kinds any given particle belongs. We just do not permit to put for instance, a fermion into a box with fermions for this determination. For in this case we would use the exclusion principle as a criterion for identity. In other words, we shall not perform experiments in which the wave function of identical fermions (or bosons) overlap. If even with this restriction we can distinguish all par-

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ticles we shall say shortly that the "distinction principle" is satisfied. The distinction principle is thus a formulation of the Pauli principle which uses explanation (A). For instance, in the above-mentioned theory characterized by (1) it does not hold since in an external Bose field the two kinds of fermions behave in exactly the same way. To satisfy the distinction principle it is not enough that a state with two I fermions is different from a state with one I and one II fermion. In one case the exclusion principle is effective and in the other not, so that in any case there will be a difference in the scattering cross section in the two states. Furthermore, conservation laws may affect the two states differently so that the annihilation cross section may also be different in the two situations. But the distinction principle requires more. For theories of the Yukawa type it asserts that different isolated fermions interact differently with the Bose fields.

We have now to decide with respect to which group of interactions we want different particles to behave differently. By groups of interactions we mean the usual categories of strong, electromagnetic and weak interactions. Because of the fluid state of our knowledge about weak interactions we shall forget about them and about particles interacting only weakly. A little consideration shows that all other particles with identical space-time properties can be distinguished electromagnetically.[1] Since there are no charged particles of the same sign with identical masses ($\Sigma^+$ and $\bar{\Sigma}^-$ ought to have the same charge but different masses) we have to consider only neutral particles. Now $n$ and $\bar{n}$, $\Lambda^0$ and $\bar{\Lambda}^0$, $\Sigma^0$ and $\bar{\Sigma}^0$ differ in the sign of their magnetic moment. The neutral heavy mesons undergo the following virtual dissociations $k^0 \rightarrow p + \bar{\Sigma}^+$, $\bar{k}^0 \rightarrow \bar{p} + \Sigma^+$, so that their charge cloud will be different. We take the point of view that it is not a coincidence that the distinction principle holds if one considers only electric interactions. Of course, there is no logical necessity for this belief and we shall see that it is also possible to couple the electric field such that the distinction principle is violated. It will turn out that it is just the distinction principle which forces the displacement of the $(pn)$ center of charge.

Less obvious is the question whether also the strong interactions alone distinguish all particles. It is tempting to conjecture this to be true since one knows that the internal degrees of freedom dominate the dynamics of the pion-nucleon system, at least at low energies. Thus it seems plausible that the strong interactions are the mechanism which shows the differences in the internal properties of particles. A detailed study of this question is the content of the present paper and with our present knowledge the answer will be that also the strong interactions separately satisfy the distinction principle.

A striking feature of the strong interactions is their high symmetry with respect to the internal properties of particles. This is reflected, for instance, by the great number of different particles having (at least approximately) the same space-time properties. In fact, the guiding principle in the construction of interactions is usually to achieve maximum symmetry. This principle is, however, not too powerful since we shall see that the theories with maximum symmetry (which means that they allow for the largest invariance group) are not realized in nature. It is clear that by having too much symmetry the particles will lose their difference and we shall violate the distinction principle. In fact we shall see that the interactions we know to exist are such that they have the maximum symmetry which under the given circumstances is compatible with the distinction principle. It will turn out that for $N_f \leq 4$ and $N_b \leq 3$ the only theories of the Yukawa type which do not violate the distinction principle, conserve fermions and give no mass splitting are the electric and the $\boldsymbol{\phi}\cdot\boldsymbol{\tau}$ interaction. By not requiring the distinction principle one increases the number of possible theories by a factor 10. In particular the neutral or the charged meson theory do not satisfy the distinction principle since they do not distinguish between nucleon and anti-nucleon in the required fashion. As a consequence, for instance, the forces between $pn$ and $p\bar{p}$ are the same in those theories as long as $p\bar{p}$ do not touch each other so that they can annihilate. This is not the case in the $\boldsymbol{\phi}\cdot\boldsymbol{\tau}$ theory for pseudoscalar mesons but would also hold in this theory with vector mesons.

In the next section we shall develop the theory of internal degrees of freedom not connected with space-time. Although most of the material is well known it had to be included since there is no systematic account of it in the published literature.[2]

In the following section we shall investigate all possible Yukawa couplings for $N_f \leq 4$ and $N_b \leq 3$. The reasons for choosing the couplings linear in the Bose field and quadratic in the Fermi field are not very deep. It is just that at present it seems that the dominant features of strong interactions (like single boson emission) can be accounted for by them. We shall, however, not rely on arguments like renormalizability. One might hope that one can study the symmetry properties of the internal space independent of a possible modification of our space-time concepts at small distances. In fact, all our results would hold if we were to make the theory finite by, for instance, a nonrelativistic form factor. We shall use only the general features of relativistic field theories like the connection between spin and statistics. Thus we do not necessarily consider the form of the interaction as something fundamental but rather as a phenomenological description of observed

[1] We shall discuss later in what sense this statement does not contradict the observations of Wick, Wightman, and Wigner, Phys. Rev. 88, 101 (1952).

[2] This section is closely related in the corresponding part of J. Schwinger, Stanford Lectures summer 1956, mimeographed lecture notes. In the meantime this has been partly published in Ann. Phys. 2, 407 (1957).

processes. It is to be expected that our considerations could be carried over to a more detailed model where, for example, one replaces the bosons by bound states of an even number of fermions.

In the last section we shall discuss the introduction of an electromagnetic field into the systems considered before. It turns out that the distinction principle eliminates some possibilities but does not lead uniquely to the actual form of the interaction. For this one has to add the requirement that the charge of a fermion is $(-1,0,1)$ times the elementary charge.

The investigation of systems involving more particles than considered here becomes very complex and had to be deferred to a separate paper. There the principles proposed in this work will be applied to other systems, mainly to the $K$-meson baryon couplings. It will again turn out that the distinction principle helps somewhat but does not uniquely lead to the observed picture. There are several theories with many particles which comply with the distinction principle and it requires another point of view to explain why only a particular one is realized in nature.

## 2. GENERAL FORMALISM

In this section we shall develop the concepts relating to the internal symmetry properties of a system containing $N_b$ kinds of bosons and $N_f$ kind of fermions. We consider only the case where all Bose fields and all Fermi fields have the same transformation properties with respect to the full Lorentz group. The Fermi fields we take to have spin $\frac{1}{2}$ and the Bose fields to have spin 0 or 1. It is convenient to consider all Fermi fields $\psi_i$ as components of the Fermi field $\psi$ and similarly for the Bose fields $\phi$. Since supposedly each field component represents only one kind of particle of given space-time property we require that the field components be Hermitian:

$$\psi_i^\dagger=\psi_i, \quad \phi_i^\dagger=\phi_i. \tag{2.1}$$

For spin 0, one field component will be represented by one Hermitian scalar field which is known to describe one kind of particle. In the case of spin $\frac{1}{2}$, one component is represented by a four component Hermitian spinor field, that is to say by a Majorana field. This describes for given momentum two kinds of particles, differing by their spin direction. Here we need twice as many components for describing one particle since the Dirac equation is of first order. We shall usually suppress the labeling of spinor or vector components and be explicit only with respect to the internal space. A Hermitian vector field contains four kinds of particles. One of them has a negative energy and has to be eliminated by $\phi^k{}_{,k}=0$. This will always be understood in the following without explicit mention.

The free Lagrangian is a quadratic form of the Bose (and Fermi) field components. $L^0$ will for Bose fields be of the general form

$$\phi_i K\phi_j q_{ij} \quad \text{with} \quad fKg=f_{,i}g^{,i}-m^2fg.$$

Since the energy must be positive definite, the quadratic form $q$ must be positive. By a suitable rotation and scale transformation, it can be reduced to the unit form which we take as the convenient standard form. Thus we write $L^0$ as

$$L^0=\sum_{i=1}^{N_f}\psi_i(\alpha_\mu\partial^\mu-M\beta)\psi_i+\tfrac{1}{2}\sum_{i=1}^{N_b}(\phi_{i,k}\phi_i{}^{,k}-m^2\phi_i^2). \tag{2.2}$$

Here $\alpha$ and $\beta$ are the usual Dirac matrices in the Majorana representation:

$$\alpha_0=i,\quad \alpha_1=i\sigma_1,\quad \alpha_2=i\sigma_2\rho_2,\quad \alpha_3=i\sigma_3;\quad \beta=\sigma_2\rho_3. \tag{2.3}$$

The canonical commutation rules are

$$\begin{aligned}\{\psi_i(x),\psi_j(x')\}_{t=t'}&=\tfrac{1}{2}\delta_{ij}\delta(\mathbf{x}-\mathbf{x}'),\\ [\phi_k(x),\phi_j(x')]_{t=t'}&=\delta_{kj}\delta(\mathbf{x}-\mathbf{x}').\end{aligned} \tag{2.4}$$

The interaction we take of the Yukawa type, that is to say of the form

$$L'=g\sum_{a,b=1}^{N_f}\sum_{i=1}^{N_b}\psi_a M_{ab}{}^i\Gamma\psi_b\phi_i, \tag{2.5}$$

where $g$ is the coupling constant and $M_{ab}{}^i$ is a set of numerical matrices which will be discussed below. The $\Gamma$ are a set of $\gamma$ matrices corresponding to which of the 5 types of space-time coupling we have. The scalar $\beta$ the pseudoscalar $\alpha_1\alpha_2\alpha_3\beta$ and the pseudovector $(\alpha_1\alpha_2\alpha_3, \alpha_2\alpha_3,\alpha_3\alpha_1,\alpha_1\alpha_2)$ are in the Majorana representation odd whereas the vector $\alpha_k$ and the tensor $\beta\frac{1}{2}[\alpha_k,\alpha_m]$ are even. We shall not consider mixtures of couplings which mix odd and even $\Gamma$'s and have therefore[3] $\Gamma^T=\pm\Gamma$. In view of the commutation relations we must have correspondingly

$$M_{ab}{}^i=\mp M_{ba}{}^i.$$

Furthermore, we shall take all the $\Gamma$'s to be purely imaginary. Then the hermiticity of $L'$ requires the $M$'s to be purely real. The total Lagrangian $L=L^0+L'$ and the commutation rules will be invariant under a group of linear transformations among the field components,

$$\psi_i\to\Lambda^F{}_{ij}\psi_j,\quad \phi_i\to\Lambda^B{}_{ij}\phi_j;\quad \Lambda^*=\Lambda. \tag{2.6}$$

For $g=0$ the invariance of $L^0$ and the commutation rules requires $\Lambda^T\Lambda=1$ and $\Lambda\Lambda^T=1$ respectively. Thus the $\Lambda^F$ and $\Lambda^B$ will be the matrices of the orthogonal group of $N_f$ and $N_b$ dimensions respectively. $L'$ will destroy this invariance to some extent so that the $\Lambda$'s will constitute only a subgroup of the orthogonal group.[4]

Let us first consider the case where this subgroup is a continuous group and the $\Lambda(\alpha_i)$ depend on, say, $n$ real

[3] This essentially excludes a scalar field coupled by a scalar and a vector coupling.

[4] That the study of the internal space is connected with the orthogonal group was first emphasized by A. Pais [Proc. Natl. Acad. Sci. U. S. 40, 484 (1954), and private conversation]. Salam, d'Espagnat, and Prentki [Nuclear Phys. (to be published)] have assumed that a theory can be invariant under a Lorentz transformation in the internal space. This does not seem to be possible since $L^0$ and the commutation laws are not invariant under such a transformation.

parameters $\alpha_i$. The elements in the infinitesimal neighborhood of the unit $[=\Lambda(0)]$ will have the form

$$\Lambda^B=1+\sum_{r=1}^{n}\alpha_r T_r^B,\quad \Lambda^F=1+\sum_{r=1}^{n}\alpha_r T_r^F, \tag{2.7}$$

where $T_r^T=-T_r$ and $T_r^*=T_r$ so that $\Lambda\Lambda^T=1$ holds to first order in $\alpha$ and $\Lambda^*=\Lambda$. For the full orthogonal group (e.g., for $g=0$) the $T$ constitute a set of $\frac{1}{2}N(N-1)$ linearly independent odd matrices. It is convenient to take them as the generators of an infinitesimal rotation in a coordinate plane and label them by two subscripts indicating in which plane the rotation occurs:

$$(i|T_{jk}|l)=\delta_{ij}\delta_{kl}-\delta_{ik}\delta_{jl}. \tag{2.8}$$

Correspondingly they obey commutation relations of the form,

$$[T_{ik},T_{lm}]=\delta_{kl}T_{im}+\delta_{im}T_{kl}-\delta_{il}T_{km}-\delta_{km}T_{il}. \tag{2.9}$$

For $N=2$ there is only one matrix

$$T_{12}=i\tau_2=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}, \tag{2.10}$$

and a finite rotation is represented by

$$\Lambda(\omega)=e^{i\tau_2\omega}. \tag{2.11}$$

For $N=3$ we introduce a vector notation, calling $T_{12}=\epsilon_3$, or generally

$$(i|\epsilon_k|j)=\epsilon_{ijk}. \tag{2.12}$$

A finite rotation depends on three parameters $\mathbf{n}$ and is given by

$$\Lambda(\mathbf{n})=\exp(\mathbf{n}\cdot\boldsymbol{\epsilon}). \tag{2.13}$$

The familiar commutation rules

$$[\epsilon_1,\epsilon_2]=\epsilon_3,\ \text{etc.}, \tag{2.14}$$

show that $\boldsymbol{\epsilon}$ transforms like a vector under $\Lambda(\mathbf{n})$.

It is conventional to represent the four-dimensional rotations by the product of two independent three-dimensional rotations. By forming the combinations

$$\begin{aligned}T_{12}&=s_3+r_3, & T_{13}&=s_2+r_2, & T_{23}&=s_1+r_1,\\ T_{34}&=s_3-r_3, & T_{24}&=s_2-r_2, & T_{14}&=s_1-r_1,\end{aligned} \tag{2.15}$$

we see that the $\mathbf{r}$ and $\mathbf{s}$ obey the same commutation laws as the $\boldsymbol{\epsilon}$

$$[r_i,s_j]=0,\quad [r_i,r_j]=\epsilon_{ijk}r_k,\quad [s_i,s_j]=\epsilon_{ijk}s_k. \tag{2.16}$$

The representation (2.8) for the $\mathbf{r}$ and $\mathbf{s}$ can also be expressed in terms of two sets of independent Pauli matrices $\boldsymbol{\sigma}$ and $\boldsymbol{\rho}$:

$$\mathbf{r}=\tfrac{1}{2}i\sigma_2(\rho_3,\sigma_2\rho_2,\rho_1),\quad \mathbf{s}=\tfrac{1}{2}i\rho_2(\sigma_3,\sigma_2\rho_2,\sigma_1). \tag{2.17}$$

A finite rotation depends on six parameters $\mathbf{n}$ and $\mathbf{n}'$

$$\Lambda(\mathbf{n},\mathbf{n}')=\exp(\mathbf{s}\cdot\mathbf{n}+\mathbf{r}\cdot\mathbf{n}'), \tag{2.18}$$

and $\mathbf{s}$ and $\mathbf{r}$ transform like vectors. In this way of writing, a rotation around $\mathbf{n}$ say with an angle of 360° corresponds to $\Lambda=-1$. In the following sections we will always use those representations of the orthogonal group.

It is well known that to each infinitesimal transformation which leaves the action integral invariant there corresponds a constant of the equations of motion. In particular, transformations which are not connected with space-time yield invariants. By the latter we mean that they are given by an integral over a space-like surface and are in fact independent of this surface. Correspondingly we get $n$ invariants

$$\mathcal{T}_r=i\int d\sigma_k[\psi\alpha^k T_r^F\psi+\phi^{,k}T_r^B\phi]. \tag{2.19}$$

Being translationally invariant they commute with the Hamiltonian and can be simultaneously diagonalized with it. It is useful to redefine $\mathcal{T}_r$ by additional constants such that the ground state of the system $|0\rangle$ (vacuum) has eigenvalue zero:

$$\mathcal{T}_r|0\rangle=0. \tag{2.20}$$

For definiteness we shall now consider in more detail the case of highest symmetry, namely $g=0$. The modifications for lower symmetry when the interaction is present will be obvious. With the representation (2.8) for the $T$ we find that they obey the same commutation relations (2.9).

$$[\mathcal{T}_{ik},\mathcal{T}_{lm}]=\delta_{kl}\mathcal{T}_{im}+\delta_{im}\mathcal{T}_{kl}-\delta_{il}\mathcal{T}_{km}-\delta_{km}\mathcal{T}_{il}. \tag{2.21}$$

Thus there will be no eigenstate common to all of them with eigenvalue $\neq 0$. To construct the eigenstates of $\mathcal{T}$ explicitly we start from the commutation relation

$$[\mathcal{T}_{lm},\psi]=iT^F{}_{lm}\psi,\quad [\mathcal{T}_{lm},\phi]=iT^B{}_{lm}\phi. \tag{2.22}$$

$iT_{lm}$ has the eigenvalues $\pm 1$, 0 and is diagonalized by the combinations $(1/\sqrt{2})(\psi_l\pm i\psi_m)$, $\psi_k$ for $k\neq l$, $m$. Calling $\psi$ one of those combinations belonging to the eigenvalue $e$ of $T$ we find

$$\mathcal{T}_{lm}\psi|0\rangle=e\psi|0\rangle. \tag{2.23}$$

Thus the eigenvalues of $T$ are eigenvalues of $\mathcal{T}$ and similarly for $\phi$ and $T^B$. Furthermore the sum of two eigenvalues of $\mathcal{T}$ is again an eigenvalue since the $\mathcal{T}$ are additive quantities. To show this we remark that the structure of the commutation rules (2.22) implies that if $\mathcal{T}\psi|0\rangle=e\psi|0\rangle$ and $\mathcal{T}\psi'|0\rangle=e'\psi'|0\rangle$ then

$$\mathcal{T}\psi\psi'|0\rangle=(e+e')\psi\psi'|0\rangle. \tag{2.24}$$

This observation gives us a complete picture about the eigenstates and eigenvalues of the $\mathcal{T}_{lm}$.

Some or all of the $\Lambda$ may not be continuously connected with the unit. In the former case there is an ambiguity in the choice of the basic improper transformations since they can be combined with an arbitrary continuous operation. Since there is no improper transformation in the infinitesimal neighborhood of 1 we cannot use the usual procedure for constructing the generator in terms of field variables. Nevertheless the invariance of the formalism under $\Lambda_i$ assures the existence of a unitary operator $U_i$ such that

$$U_i\psi U_i^{-1}=\Lambda_i^F\psi,\quad U_i\phi U_i^{-1}=\Lambda_i^B\phi, \tag{2.25}$$

even if we know only in special cases the explicit form of $U$. For instance every theory of the type we are considering is invariant under[5]

$$\psi\rightarrow-\psi,\quad \phi\rightarrow\phi.$$

It is not hard to guess that $U$ is in this case $\exp i\pi$ [number of fermions]. However, since $U_i$ commutes with $L$ and therefore with the 10 generators of the Lorentz group it is an invariant like the $\mathcal{T}_{lm}$ even if the number of fermions is not. We shall consider only those $\Lambda_i$ which obey $\Lambda_i^4=1$ so that the $U_i$ have the eigenvalues $\pm1$, $\pm i$. Writing $U$ as $i^Q$ we see that $Q$ can change only by multiples of four. Thus the invariance under finite transformations yields those characteristic conservation laws where certain quantities can only change by some integer amounts. A familiar example is the $\pi^0$ decay which can go only into an even number of photons. A multiplicative factor left arbitrary by the definition of $U$ is conveniently normalized such that $U|0\rangle=|0\rangle$.

The structure of the relations (2.25) shows that the $U$'s are multiplicative quantities. Indeed, if $\psi$ and $\psi'$ are eigenvectors of $\Lambda_i$ with eigenvalues $e$ and $e'$ then one readily deduces

$$U_i\psi|0\rangle=e\psi|0\rangle,\quad U_i\psi'|0\rangle=e'\psi'|0\rangle \tag{2.26}$$

and

$$U\psi\psi'|0\rangle=ee'\psi\psi'|0\rangle.$$

It can happen that $L$ is only invariant under a finite subgroup of the orthogonal group. We shall find many examples of this kind in the next section, in which case there are only multiplicative invariants.[6]

To discuss the physical significance of our theory we assume as usual that the adiabatic hypothesis holds. Furthermore, we do not bother about renormalization because it has nothing to do with our problem and could be done easily. Accordingly we assume that a state $|v\rangle$ with one physical particle of a type $v$ is given by applying the following limit of a Heisenberg operator to the physical vacuum:

$$\begin{aligned}|v_F\rangle&=\lim_{t\to-\infty} v_F{}^i\int d^3xU(x)\psi_i(x)|0\rangle,\\ |v_B\rangle&=\lim_{t\to-\infty} v_B{}^i\int d^3xf(x)\phi_i(x)|0\rangle.\end{aligned} \tag{2.27}$$

Here $v_F{}^i$ (or $v_B{}^i$) are a set of $N_f$ (or $N_b$) complex numbers which we normalize according to $v^iv^{i*}=1$. $U$ and $f$ are solutions of the Dirac and Klein-Gordon equation with the renormalized masses. Since there are only $N_f$ (or $N_b$) linearly independent $v$'s we have at most as many kinds of particles with the same mass as field components, not counting possible bound states. Particles corresponding to $v$'s which are connected by a transformation $\Lambda$ will have the same mass. To see this one has to apply the corresponding operation to $|v\rangle$ and remember that $|0\rangle$ is supposedly invariant. Hence, if the matrices $\Lambda$ are an irreducible set under unitary (not only orthogonal) transformation, all fermions (and bosons) will have equal masses. For reducible $\Lambda$, particles belonging to different invariant subspaces of the $\Lambda$ will, in general, have different masses. Since the problem of not being able to distinguish particles does not arise for particles with different mass we shall concentrate on theories where the $\Lambda$ are irreducible. There are exceptions to this rule owing to the fact that the self-energy does not depend on the sign of the coupling constant. For example, if $L'=g(\psi_1\psi_1\phi_1+\psi_2\psi_2\phi_2)$ then $\phi_1$ and $\phi_2$ particles will clearly have the same mass. Nevertheless, the invariance operations $\psi\rightarrow i\tau_2\psi$, $\phi\rightarrow i\tau_2\phi$ and $\psi\rightarrow\tau_3\psi$, $\phi\rightarrow\phi$ give a reducible $\Lambda^B$. However, the self-energy would also be the same for $L'=g(\psi_1\psi_1\phi_1-\psi_2\psi_2\phi_2)$ so that it is effectively invariant under $\psi\rightarrow\psi$, $\phi\rightarrow\tau_3\phi$ and $i\tau_2$ and $\tau_3$ are irreducible.

When we want to express the distinction principle in formal terms we have to remember that the different components of $\psi$ and $\phi$ in a one-particle state get mixed by virtual processes like $p\rightarrow n+\pi^+$. It is conventional to characterize the physical particle by some invariants $\mathcal{T}_s$. Of course, not all of them can be diagonalized simultaneously nor are they all needed for specifying a particle. For instance, for $N=3$ one of the $\epsilon$ and for $N=4$ one of the $s$ and one $r$ can be used. The sample boxes of our heuristic argument in the last section will be represented by external Bose fields acting on the physical particles.[7] In an external Bose field the number of invariants is reduced but in our cases there will be always enough left over to specify the particles. In meson theory, for instance, they will be the isotopic spin in the direction of the external field and the number of nucleons. With those explanations we shall say that a theory does not satisfy the distinction principle if even in the presence of arbitrary external fields there is an invariance operation which exchanges two or more physical particles. Our example (1) for instance, is invariant under $\psi_{\text{I}}\leftrightarrow\psi_{\text{II}}$ even in external Bose field. In a theory which satisfies the distinction principle like electrodynamics an external field enables us to distinguish the two kinds of particles. This does not mean that any combination of the $\psi$'s can be distinguished from any other. For instance, it is well

[5] This trivial case will not be mentioned again in the later analysis.

[6] Case, Karplus, and Yang have considered theories of this type [Phys. Rev. **101**, 874 (1956)]. Similarly B. d'Espagnat and J. Prentki [Phys. Rev. **102**, 1684 (1956)] tried to associate strangeness with a discontinuous operation to explain why there are no particles of higher strangeness. The present experimental data do not decide whether strangeness is an additive or the logarithm of a multiplicative quantity $U$ with $U^4=1$.

[7] That bosons and fermions are not treated on the same footing has its origin in the form of the coupling, as will be seen later.

known[1] that $\psi_1|0\rangle$ and $\psi_2|0\rangle$ cannot be distinguished. But they do not diagonalize the charge and are mixed by an external field. The $(\psi_1 \pm i\psi_2)|0\rangle$ diagonalize the charge and correspond to the physical particle. Those actually behave differently in an external field. In most cases which do not satisfy the distinction principle it happens that there is always an invariance operation which exchanges two groups of particles and lets one particular $\mathcal{T}_p$ go over into its negative. This implies also that this $\mathcal{T}_p$ is not an observable quantity.[8] The corresponding $T_p{}^B$ must clearly be zero since the external Bose field is fixed. The different position of the Bose fields in this formulation of the distinction principle has its origin in the fact that the theories we are considering do not allow for an invariance operation with $T^F=0$. Thus an arbitrary set of external sources will destroy any invariance connected with the Bose fields and will, therefore, enable us to identify all bosons.

### 3. CLASSIFICATION OF THEORIES

In this section we shall investigate the symmetry properties of all theories of the form (2.2) and (2.5) with $N_f \leq 4$ and $N_b \leq 3$. It is convenient to group them according to increasing symmetry. The group $A$ contains all theories where the $\Lambda$ are a reducible set of matrices, in which case not all the masses will remain degenerate. Theories of group $B$ have no mass splitting but satisfy the distinction principle. This group will be subdivided using as criterion whether or not the theory has an additive invariant containing only fermions. Finally, theories violating the distinction principle constitute group $C$. This classification is compiled in Table I, where $x$ means that the corresponding criterion is not relevant in this case.

Considering the $M_{ab}{}^i$ as a set of $N_b$ matrices $N_f \times N_f$ we see that we have to study all such sets of matrices for the $N_f$ and $N_b$ within our limits. Sets which can be transformed into each other by an orthogonal transformation lead to equivalent theories and need not be considered separately. Furthermore, those linear combinations of the $M^i$ which correspond to an orthogonal transformation of the Bose field give nothing new either. Also a factor common to all $M^i$ can be absorbed in $g$. Since the $M^i$ have to be even or odd it is convenient to represent them by a basic set of independent even or odd matrices which have a simple transformation property under the standard transformations (2.10) to (2.17). For the different dimensions we take

$$N=2\begin{cases}\text{odd } \; i\tau_2: & 1\\ \text{even } 1,\, \tau_1,\, \tau_3: & 3\end{cases}$$

$$N=3\begin{cases}\text{odd } \; \boldsymbol{\varepsilon}: & 3\\ \text{even } \tfrac{1}{2}\{\epsilon_i,\epsilon_k\}=\epsilon_{ik}: & 6\end{cases}$$

$$N=4\begin{cases}\text{odd } \; \mathbf{s},\, \mathbf{r}: & 6\\ \text{even } 1,\, s_i r_k: & 10\end{cases}$$

[8] This observation will serve as useful criterion for showing that a theory belongs to (**C**).

TABLE I. Classification of theories.

| | $A$ | $B_1$ | $B_2$ | $C$ |
|---|---|---|---|---|
| I. $\Lambda$'s irreducible | no | yes | yes | yes |
| II. For a $\Lambda$ $T_b=0$ $T_f\neq 0$ | $x$ | no | yes | yes |
| III. Violates distinction principle | $x$ | $x$ | no | yes |

In Table II we shall summarize the properties of the theories under consideration. If a coupling can be written as the sum of two parts such that each part has the same or higher symmetry properties than the sum, the sum will not be mentioned separately. Since we are not interested in class $A$, some cases belonging trivially to $A$ will not be listed. The material we arrange so that we give first the matrices $M^i$ which characterize the theory. Then we quote all independent invariance operations and abbreviate a continuous operation by rot. and a discontinuous operation by ref. Finally, on the right in boldface we tell the class to which the theory belongs, like "(**C**)" and say why unless it is obvious. If a coupling is one of the usually considered ones we shall mention it.

Naturally most couplings belong to (**A**). ($\mathbf{B}_2$) contains only three couplings; the two odd ones of this class are $N_b=1$, $N_f=2$, $M=i\tau_2$ and $N_b=2$, $N_f=4$, $M_{1,2}=i\rho_2, i\sigma_2$. The first one is the coupling $e$-$\gamma$ or $\mu$-$\gamma$ and the second one is created from it by a doubling process. We shall see in the following paper that this process creates from any theory of class ($\mathbf{B}_2$) another theory of the same class with twice as many particles. The even coupling of class ($\mathbf{B}_2$) is the symmetric $\pi$-$N$ interaction.[9]

Many theories have no continuous invariance group but enough symmetry to give no mass splitting. Since they do not have any additive invariant they all belong to ($\mathbf{B}_1$). It is good to keep in mind that it is the existence of additive invariants and not the mass degeneracy which allows one to infer the invariance under a rotation group.

Of the ten theories belonging to (**C**) none is realized in nature. The theories with the largest invariance groups belong to (**C**). For instance, for $N_f=4, N_b=3$ the coupling with $\mathbf{M}=\mathbf{S}$ has an invariance group isomorphic to the four-dimensional rotation group. It belongs to (**C**) since, in conventional language, it does not distinguish between nucleon and antinucleon.

### 4. ELECTRIC INTERACTIONS

We shall finally investigate the possibilities of introducing an electromagnetic interaction in the conventional way into the systems considered previously. They require the existence of a current $j^i$ which satisfies $j^i{}_{,i}=0$. Such a current is only available in theories with a continuous invariance group. Systems with a finite invariance group only cannot have an electro-

[9] Note that our result implies that the space-time form of the pion-nucleon coupling must be $1, \gamma_5, \gamma_\mu\gamma_5$. It cannot be, for instance, a vector coupling.

TABLE II. Properties of the theories.

| Type of theory | | Invariance operations | Class |
|---|---|---|---|
| $N_b=1$ | | | |
| $N_f=2$ | odd: | $M=\begin{pmatrix}0&1\\-1&0\end{pmatrix}$ rot. $\psi\to\psi e^{i\tau_2\omega}\psi$, $\phi\to\phi$ <br> ref. $\psi\to\tau_1\psi$, $\phi\to-\phi$ | ($B_2$), is electron-photon coupling |
| | even: | $M=\begin{pmatrix}a&0\\0&b\end{pmatrix}$ (in diagonal form) | Belongs to (A) except for $\lvert a\rvert=\lvert b\rvert$ |
| | | $M=\begin{pmatrix}1&0\\0&1\end{pmatrix}$ rot. $\psi\to e^{i\tau_2\omega}\psi$, $\phi\to\phi$ <br> ref. $\psi\to\tau_1\psi$, $\phi\to\phi$ | (C) since for ref. $\phi\to\phi$ but $\mathcal{T}_\omega\to-\mathcal{T}_\omega$ |
| | | $M=\begin{pmatrix}1&0\\0&-1\end{pmatrix}$ rot. none <br> ref. $\psi\to\tau_3\psi$, $\phi\to\phi$ <br> $\psi\to\tau_1\psi$, $\phi\to-\phi$ | ($B_1$) |
| $N_f=3$ | odd: | $M=\mathbf{v}\cdot\boldsymbol{\varepsilon}$ is equiv. to $M=\epsilon_1$ <br> rot. $\psi\to e^{\epsilon_1\omega}\psi$, $\phi\to\phi$ | (A), is similar to pion-photon system |
| | even: | $M=\begin{pmatrix}a&0&0\\0&b&0\\0&0&c\end{pmatrix}$ (in diagonal form) | Belongs to (A) except for $\lvert a\rvert=\lvert b\rvert=\lvert c\rvert$. In this case it is (C) |
| $N_f=4$ | odd: | $M=\mathbf{v}_1\cdot\mathbf{r}+\mathbf{v}_2\cdot\mathbf{s}$ is equiv. to $as_2+br_2$ <br> $a\neq0$ rot. $\psi\to e^{i(\rho_2\omega+\sigma_2\omega')}\psi$, $\phi\to\phi$ <br> $b\neq0$ ref. $\psi\to\rho_1\sigma_1\psi$ $\phi\to-\phi$ reduc. | (A) |
| | | $a=0$ or $b=0$ is equivalent to $M=\rho_2$: <br> rot. $\left.\begin{array}{l}\psi\to e^{i\rho_2\omega}\psi\\ \psi\to\exp(\mathbf{s}\cdot\mathbf{n})\psi\end{array}\right\}\phi\to\phi$ <br> ref. $\psi\to 2r_3\psi$ $\phi\to-\phi$ | (C) since, for instance, $\psi\to 2s_1\psi$, $\phi\to\phi$, $\mathcal{T}_{n2}\to-\mathcal{T}_{n2}$ |
| | even: | $M=\begin{bmatrix}a&0&0&0\\0&b&0&0\\0&0&c&0\\0&0&0&d\end{bmatrix}$ | Belongs to (A) except for $\lvert a\rvert=\lvert b\rvert=\lvert c\rvert=\lvert d\rvert$; in those cases it belongs to (C) (neutral $\pi-N$ theory) |
| $N_b=2$ | | | |
| $N_f=1$ | even: | $M_1=a$, $M_2=b$ is equiv. to $M_1=1$, $M_2=0$ | (A) |
| $N_f=2$ | odd: | $M_1=i\tau_2=M_2\times$const. | All theories with $M_1=M_2\times$const. belong to (A) |
| | even: | $M_1=1$, $M_2=\tau_3$ is equiv. to $L'\cong\psi_1\psi_1\phi_1+\psi_2\psi_2\phi_2$ no rot. <br> ref. $\psi_1\leftrightarrow\pm\psi_2$, $\phi_1\leftrightarrow\phi_2$ <br> This theory is exceptional as it gives obviously no mass-splitting although $\Lambda_B$ is reducible | ($B_1$) |
| | | $M_1=\tau_1$ $M_2=\tau_3$: <br> rot. $\psi\to e^{i\tau_2\omega/2}\psi$, $\phi\to e^{i\tau_2\omega}\phi$, <br> ref. $\psi\to\tau_3\psi$, $\phi\to-\tau_3\phi$, | ($B_1$) |
| $N_f=3$ | odd: | $M_1=\mathbf{v}_1\cdot\boldsymbol{\varepsilon}$, $M_2=\mathbf{v}_2\cdot\boldsymbol{\varepsilon}$ | (A) |
| | even: | $\left.\begin{array}{l}M_1=1,\ M_2=b\epsilon_1^2\\ M_1=\epsilon_1^2,\ M_2=b\epsilon_2^2\\ M_1=\epsilon_{12},\ M_2=b\epsilon_{23}\end{array}\right\}$ | (A) Even for $b=1$ the invariance operations are reducible |
| $N_f=4$ | odd: | $M_1=\mathbf{v}_1\cdot\mathbf{s}$, $M_2=\mathbf{v}_2\cdot\mathbf{s}$ or $\mathbf{v}_2\cdot\mathbf{r}$ | Belong to (A) except for: |
| | | $M_1=s_1$, $M_2=s_2$: rot. $\left\{\begin{array}{ll}\psi\to e^{s_3\omega/2}\psi, & \phi\to e^{i\tau_2\omega}\phi\\ \psi\to\exp(\mathbf{r}\cdot\mathbf{n})\psi, & \phi\to\phi\end{array}\right.$ <br> ref. $\psi\to 2s_1\psi$, $\phi\to\tau_3\phi$ | (C) Since there are operations $\mathcal{T}_n\to-\mathcal{T}_n$, $\phi\to\phi$ |
| | | $M_1=s_2$, $M_2=r_2$ <br> rot. $\left\{\begin{array}{l}\psi\to e^{i\sigma_2\omega}\psi\\ \psi\to e^{i\rho_2\omega}\psi\end{array}\right\}$, $\phi\to\phi$ <br> ref. $\left\{\begin{array}{ll}\psi\to\dfrac{1+\boldsymbol{\sigma}\cdot\boldsymbol{\rho}}{2}\psi, & \phi\to\tau_1\phi\\ \psi\to 2r_1\psi, & \phi\to\tau_3\phi\end{array}\right.$ | ($B_2$) |

TABLE II.—(*Continued*).

| Type of theory | Invariance operations | Class |
|---|---|---|
| even: | Not belonging to (**A**) are | |
| | $M_1=1$, $M_2=r_1s_1$ | (**C**) is a doubling of $N_b=2$, $N_f=2$,<br>$M_1=1$, $M_2=\tau_3$ |
| | $M_1=r_2s_3$, $M_2=r_3s_2$ no rot.<br>ref. $\left.\begin{matrix}\psi\rightarrow\sigma_3\psi\\ \psi\rightarrow\rho_3\psi\end{matrix}\right\}\phi\rightarrow\phi,\ \begin{matrix}\psi\rightarrow\rho_1\psi, & \phi\rightarrow\tau_3\phi\\ \psi\rightarrow\frac{1+\boldsymbol{\sigma}\cdot\boldsymbol{\varrho}}{2}\psi, & \phi\rightarrow\tau_1\phi\end{matrix}$ | (**B**$_1$) |
| | $M_1=r_2s_3$, $M_2=r_2s_1$<br>rot. $\begin{cases}\psi\rightarrow e^{i\sigma_2\omega/2}\psi, \ \phi\rightarrow e^{i\tau_2\omega}\phi\\ \psi\rightarrow e^{i\rho_2\omega'}\psi, \ \phi\rightarrow\phi\end{cases}$<br>ref. $\begin{cases}\psi\rightarrow\sigma_1\psi, \ \phi\rightarrow\tau_3\phi\\ \psi\rightarrow\rho_1\psi, \ \phi\rightarrow\phi\end{cases}$ $(\mathcal{T}_{\omega'}\rightarrow-\mathcal{T}_{\omega'})$ | (**C**) (Charged $\pi$-$N$ theory) |
| | $M_1=r_2s_3$, $M_2=r_1s_1$, no rotations<br>ref. $\left.\begin{matrix}\psi\rightarrow\sigma_1\psi\\ \psi\rightarrow i\sigma_2\psi\end{matrix}\right\}\phi\rightarrow-\phi,\ \begin{matrix}\psi\rightarrow\rho_1\psi, \ \phi\rightarrow\tau_3\phi\\ \psi\rightarrow\rho_3\psi, \ \phi\rightarrow\phi\end{matrix}$<br>This is again exceptional since $\Lambda_B$ is reducible but there is no mass-splitting since $L'\cong\psi(\sigma_3\phi_1+\sigma_3\rho_3\phi_2)\psi$ | (**B**$_1$) |
| $N_b=3$ | | |
| $N_f=1$ and $N_f=2$ | odd, belong trivially to (**A**) | |
| $N_f=2$ even: | $M_1=1$, $M_2=\tau_1$, $M_3=\tau_3$: rot. $\psi\rightarrow e^{i\tau_2\omega/2}$, $\phi=e^{\epsilon_2\omega}\phi$<br>ref. $\psi\rightarrow\tau_1\psi$, $\phi_3\rightarrow-\phi_3$ | (**A**) since $\Lambda_B$ = reducible |
| $N_f=3$ odd: | Not to (*A*): $\mathbf{M}=\boldsymbol{\varepsilon}$: rot. $\psi\rightarrow\exp(\boldsymbol{\varepsilon}\cdot\mathbf{n})\psi$, $\phi\rightarrow\exp(\boldsymbol{\varepsilon}\cdot\mathbf{n})\phi$<br>ref. none independent of $\psi\rightarrow-\psi$ | (**B**$_1$) |
| even: | not to (**A**):<br>no rotations<br>$\left.\begin{matrix}M_1=\epsilon_1^2, & M_2=\epsilon_2^2, & M_3=\epsilon_3^2\\ M_1=\epsilon_2^2+\epsilon_3^2, & M_2=\epsilon_3^2+\epsilon_1^2, & M_3=\epsilon_1^2+\epsilon_2^2\\ M_1=\epsilon_{12}, & M_2=\epsilon_{23}, & M_3=\epsilon_{13}\end{matrix}\right\}$ ref. $\begin{matrix}\psi_1\leftrightarrow\psi_2, \ \phi_1\leftrightarrow\phi_2;\\ \psi_2\leftrightarrow\psi_3, \ \phi_2\leftrightarrow\phi_3;\\ \psi_3\leftrightarrow\psi_1, \ \phi_3\leftrightarrow\phi_1;\end{matrix}$ | (**B**$_1$) |
| $N_f=4$ odd: | Not to (**A**). $\mathbf{M}=\mathbf{S}$, rot. $\psi\rightarrow\exp(\mathbf{s}\cdot\mathbf{n})\psi$, $\phi\rightarrow\exp(\boldsymbol{\varepsilon}\cdot\mathbf{n})\phi$<br>$\psi\rightarrow\exp(\mathbf{r}\cdot\mathbf{n}')\psi$, $\phi\rightarrow\phi$ | (**C**) since for all three $n'$ there is an operation $\mathcal{T}n'_i\rightarrow-\mathcal{T}n'_i$, $\phi\rightarrow\phi$ |
| | $M_1=S_1$, $M_2=S_2$, $M_3=r_3$ belongs to (**A**) | |
| even: | not to (**A**)<br>$\mathbf{M}=r_2\mathbf{s}$ rot. $\begin{cases}\psi\rightarrow\exp(\mathbf{s}\cdot\mathbf{n})\psi, & \phi\rightarrow\exp(\boldsymbol{\varepsilon}\cdot\mathbf{n})\phi\\ \psi\rightarrow e^{r_2\omega}\psi, & \phi\rightarrow\phi,\end{cases}$<br>ref. $\psi\rightarrow 2r_3\psi$, $\phi\rightarrow-\phi$ | (**B**$_2$) (sym. $\pi$-$N$ theory) |
| | $\mathbf{M}=\mathbf{r}\times\mathbf{s}$: rot. $\psi\rightarrow\exp[(\mathbf{r}+\mathbf{s})\cdot\mathbf{n}]\psi$, $\phi\rightarrow\exp(\boldsymbol{\varepsilon}\cdot\mathbf{n})\phi$ | (**B**$_1$) |
| | $M_1=r_1s_1$, $M_2=r_2s_2$, $M_3=r_3s_3$. No rotations<br>ref. $\psi\rightarrow\frac{1}{2}(1+2s_3)(1+2r_3)\psi$, $\phi_1\leftrightarrow\phi_2$<br>$\psi\rightarrow\frac{1}{2}(1+2r_2)(1+2s_2)\psi$, $\phi_1\leftrightarrow\phi_3$ | (**B**$_1$) |

magnetic interaction. In theories with a continuous invariance group of several parameters there are several independent currents satisfying the continuity equation. Correspondingly, the introduction of the electric interaction is not unique in this case. Any linear combination of the currents can be used without leading to formal inconsistencies. To get some restriction we postulate that the distinction principle has to hold for the electric interactions separately. To show how this works we consider the case $N_F=4, N_B=3$, even: the pion-nucleon interaction. There are four additive invariants, isotopic spin and nucleon number. Correspondingly the current can generally be

$$j_i=\psi\alpha_i(\mathbf{a}\mathbf{s}+br_2)\psi+\phi_{,i}\mathbf{a}\cdot\boldsymbol{\varepsilon}\phi,$$

where $\mathbf{a}$ and $b$ are 4 arbitrary numbers. We shall now show that $\mathbf{a}=0$ or $b=0$ is forbidden by the distinction principle when required for the electric interaction separately.[10] For $b=0$ we characterize the particles by the nucleon number and the isotopic spin in direction of $\mathbf{a}$. $L'=eAj$ will then be invariant under the operation $\psi\rightarrow 2r_3\psi, \phi\rightarrow\phi$ which changes the sign of nucleon

[10] This means, as mentioned before, that we can distinguish all particles using an external electric field only.

number. Similarly we see that for $\mathbf{a}=0$ the distinction principle is violated. It is clear that for $p, n$ positive and $\bar{p}, \bar{n}$ negative and $\pi$ neutral, the electric field cannot distinguish all particles. Thus the distinction principle forces the center of charge of the nucleons to be displaced. However, any other current with $\mathbf{a}\neq 0$ and $b\neq 0$ would satisfy the distinction principle. To get the observed interaction we have to postulate that $q=\mathbf{a}\cdot\mathbf{S}+b$ has the eigenvalue $\pm 1, 0$ or $q^3=q$. This requires $|a|=|b|=1$, which gives the neutron and antineutron charge zero.[11] There seems to be at present no theoretical argument why $q^3$ must be $q$. It would, for instance, be mathematically possible to have an additional charge which is proportional to the nucleon number. Why this is absent seems not to be understood, although several people thought about this point.[12]

Let us finally try to couple to the electric field this theory of $(B_2)$ which is not realized in nature. This was $N_f=4, N_b=2$ with $L'\cong\psi(r_2\phi_1+s_2\phi_2)\psi$. Correspondingly $j$ in $L''=eAj$ will be $\psi\alpha q\psi$ with $q=ar_2+bs_2$. $q^3=q$ is satisfied for $a=0$, $|b|=2$, $b=0$, $|a|=2$, $|a|=1$, $|b|=1$. This gives two nonequivalent cases $q=2r_2$ and $q=r_2+s_2$. A short consideration shows that both do not satisfy the distinction principle.

To summarize we can say the following. The theory of multicomponent fields, being a phenomenological description of different particles, is too wide a framework to lead uniquely to the observed particles and interactions. Some conditions have to be added to exclude many possibilities which are not realized in nature, though they are mathematically possible. One might hope that those conditions can be formulated in a simple fashion like the quantum conditions in Bohr's theory. We have seen that for the systems we considered this is actually possible. The requirement of a conservation law for fermions, $q^3=q$ and the distinction principle lead to the observed systems. A more fundamental theory which leads to our principles and renders any other theory inconsistent requires, of course, an entirely new approach.

[11] Thus the distinction principle for the electric field is satisfied only if the pion-nucleon interaction exists and gives the neutron a magnetic moment.

[12] See T. D. Lee and C. N. Yang, *Proceedings of the Fifth Annual Rochester Conference on High-Energy Physics, 1955* (Interscience Publishers, Inc., New York, 1955), p. 66.

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