Hypernuclear Binding Energies and the Λ-Nucleon Interaction*

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A phenomenological analysis is made of the binding-energy data for light hypernuclei in terms of a two-body Λ-nucleon interaction, in which account is taken of the empirical information available on the structure of light nuclei. It is found that the observed binding energies can be interpreted in terms of a spin-dependent Λ-nucleon interaction; the agreement obtained is adequate for any force-range between 0.4×10^{-13} cm and 0.7×10^{-13} cm (for Yukawa shape) and any exchange character for this interaction. From observations on $_{\Lambda}H^4$ decay it is argued that the $_{\Lambda}H^4$, $_{\Lambda}He^4$ doublet has zero spin, which requires that the singlet Λ-nucleon interaction be more attractive than the triplet. The well-depth parameter for the singlet Λ-nucleon interaction has values from 0.90 to 0.85 for ranges between 0.4×10^{-13} cm and 0.7×10^{-13} cm, the triplet interaction being also attractive but only one-half to one-third as strong. Spin values are assigned to the light hypernuclei, and the angular correlations which could provide a check on these values are discussed. The absorption of K^- mesons in helium is discussed; it appears that the selection rules for the production of $A=4$ hypernuclei in this reaction may be confused by the existence of an excited state $_{\Lambda}H^{4*}$, $_{\Lambda}He^{4*}$. A number of uncertainties in the phenomenological analysis of the heavier hypernuclei are pointed out; the present data do not disagree with the conclusions based on the analysis of the lighter and better-known hypernuclei with $A\leqslant5$, but they add little weight to these conclusions.

1. INTRODUCTION[1]

THE established hypernuclear species have been listed in Table I. The values given there for the binding energy B_{Λ} of the Λ particle are based on the world survey of π^--mesic decay events recently reported by Levi Setti *et al.*[2] For several species, $_{\Lambda}H^4$ and $_{\Lambda}He^5$ in particular, the number of clearly identified events is now quite considerable, their B_{Λ} values being correspondingly well determined. It is the purpose of this paper to discuss a simple interpretation of the observed B_{Λ} values in terms of an elementary interaction between a Λ particle and a nucleon. This work extends the simple calculations which were reported earlier,[3] which were based on a representation of the Λ-nucleon interaction as a spin-dependent δ-function potential. In the present work more realistic assumptions are made about the range of the Λ-nucleon interaction, and the analysis is extended to include recent data on the heavier hypernuclei. The qualitative conclusions of the present work remain the same as those reported earlier,[3] however, and disagree with the conclusions reached by Brown and Peshkin[4] on the basis of the same assumptions as those of our earlier report.

The Λ particle is now known to be an isotopic-spin singlet ($T=0$) state. Charge symmetry for strong interactions then requires that the Λ-neutron and Λ-proton interactions be identical. That strong support for this situation can be derived from the approximate equality of the B_{Λ} values for $_{\Lambda}H^4$ and $_{\Lambda}He^4$ has been discussed previously[3,5] (see Sec. 2).

The major contributions to the Λ-nucleon interaction can be expected to arise from the exchange of pions and/or K mesons between the Λ particle and the nucleon, at least for sufficiently large separation. These contributions can be discussed in the following way:

(a) The exchange of pions alone. Since the emission of a single pion by a Λ particle is forbidden so long as charge symmetry holds ($\Lambda\not\rightarrow\Lambda+\pi^0$), this part of the Λ-nucleon interaction can only result from the transfer of two or more pions between a Λ particle and a nucleon; for example.

$$\Lambda+\mathfrak{N}\rightarrow\Lambda+\pi+\pi+\mathfrak{N}\rightarrow\Lambda+\mathfrak{N}. \qquad (1.1)$$

This transfer gives rise to an ordinary (nonexchange) interaction of range $1/2m_{\pi}\approx0.7$ fermi [1 fermi (f)$\equiv1\times10^{-13}$ cm] between a Λ particle and a nucleon. The form of the interaction generated by (1.1) and by more complicated pion exchanges has been discussed in some detail by Lichtenberg and Ross[6] and by Dallaporta

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[1] In this article, the units have been chosen such that $\hbar=c=1$.

[2] Levi Setti, Slater, and Telegdi, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Physics, 1957* (Interscience Publishers, Inc., New York, 1957), Vol. 7, Sec. 8, p. 6. The difference between the B_{Λ} values of Table I and those given by Levi Setti *et al.* arises mainly from the use of a larger Q value for the free Λ decay: $Q_{\Lambda}=37.22\pm0.22$ Mev. This value is the average (weighted by the number of events) of two determinations: $Q_{\Lambda}=36.75\pm0.2$ Mev [given by W. Slater, University of Chicago dissertation, 1958 (unpublished)] for 9 of the events reported by Friedlander, Keefe, Menon, and Merlin [Phil. Mag. 45, 533 (1954)] analyzed on the basis of the range-energy relation used in the world survey; and $Q_{\Lambda}=37.45\pm0.17$ Mev given for 18 events by Barkas, Giles, Heckman, Inman, Mason, and Smith [University of California Radiation Laboratory Report UCRL-3892, 1957 (unpublished)]. Additional shifts in B_{Λ} for $_{\Lambda}Li^7$ and $_{\Lambda}Li^8$ are due to the incorporation of new events. Further changes in Q_{Λ} (by ΔQ_{Λ}) will give rise to a change ΔQ_{Λ} in each of the B_{Λ} values.

[3] R. H. Dalitz, *Proceedings of the Sixth Annual Rochester Conference on High-Energy Physics, 1956* (Interscience Publishers, Inc., New York, 1956), Vol. 6, Sec. 5, p. 40; *Reports on Progress in Physics* (The Physical Society, London, 1957), Vol. 20, p. 163.

[4] L. M. Brown and M. Peshkin, Phys. Rev. **107**, 272 (1957).

[5] R. H. Dalitz, Phys. Rev. **99**, 1475 (1955).

[6] D. B. Lichtenberg and M. Ross, Phys. Rev. **103**, 1131 (1956).

TABLE I. Parameters for the identified hypernuclei.

Hyper-nucleus	B_Λ Mev	No. of events	U_n (Mev f^3) for $\kappa=m$	U_n (Mev f^3) for $\kappa=2m$	Remarks
$_\Lambda H^3$	0.6±0.4	9	$U_2=500(\pm10)$	795(±20)	See reference 16
$_\Lambda H^4$	1.8±0.3	21	$U_3=600(\pm10)$	820(±20)	$R_3=1.38$ f
$_\Lambda He^4$	2.0±0.3	9	$U_3=695(\pm10)$	915(±25)	$R_3=1.58$ f
$_\Lambda He^5$	2.9±0.3	15	$U_4=695(\pm25)$	910(±45)	...
$_\Lambda Li^7$	4.5±0.4	3	$U_6=1220\pm200\pm35$	1450±200±50	Errors quoted are due to
$_\Lambda Li^8$	5.4±0.8	1	$U_7=1230\pm200\pm65$	1480±200±85	(i) uncertainty in R, (ii) uncertainty in B_Λ.
$_\Lambda Be^8$	6.2±0.6	1	...	...	
$_\Lambda Be^9$	6.4±0.4	3	...	...	

and Farrari[7] for the particular case where the emission of pions by the Λ particle follows from successive elementary interactions of the Yukawa form $\Lambda\leftrightarrow\Sigma+\pi$, $\Sigma\leftrightarrow\Lambda+\pi$, the baryons being treated only in the static limit.

(b) The exchange of a single K meson, with or without the transfer of additional pions. The simplest exchange of this type is

$$\Lambda+\mathfrak{N}\to\mathfrak{N}+\bar{K}+\mathfrak{N}\to\mathfrak{N}+\Lambda, \quad (1.2)$$

which gives rise to an interaction of range $1/m_K\approx0.4$ fermi. The form of this interaction due to the exchange of a single K meson has been discussed by Wentzel.[8] The exchange of a K meson and a pion will, however, generate an additional interaction[9] with a range very little shorter $[1/(m_K+m_\pi)\approx0.3$ fermi$]$, which will generally have quite different form. These additional terms have recently been examined in detail by Lichtenberg and Ross.[10] The outstanding characteristic common to all these processes is that they involve transfer of "strangeness" from Λ particle to nucleon; consequently, they lead to an exchange interaction whose sign (for a given spin state of the particles) is proportional to the parity of the relative motion of the interacting Λ particle and nucleon.

(c) More complicated processes, involving the exchange of two or more K mesons with or without additional pions. The corresponding terms in the Λ-nucleon interaction will represent exchange or ordinary interactions depending upon whether the number of K mesons transferred is odd or even. Since these are contributions of very short range ($1/2m_K\approx0.2$ fermi, or less), however, their calculation can have very little reliability at the present stage of the art.

The calculations in this paper were carried through for two ranges of the Λ-nucleon interaction potential, that corresponding to the exchange of a K meson and that corresponding to the exchange of two pions. The analysis discussed in the body of the paper is based on the assumption of an ordinary interaction, the possibility of distinguishing the exchange character of the Λ-nucleon interaction being discussed in Appendix C.

There is the possibility that many-body forces between a Λ particle and nucleons are important. These may arise, for example, from the emission of two pions by the Λ particle, one of these pions being absorbed by each of two neighboring nucleons:

$$\mathfrak{N}+\Lambda+\mathfrak{N}\to\mathfrak{N}+(\pi+\Lambda+\pi)+\mathfrak{N}\to\mathfrak{N}+\Lambda+\mathfrak{N}. \quad (1.3)$$

Other many-body interaction mechanisms are also possible. Many-body forces among nucleons are, of course, also possible; these have been investigated by a number of authors.[11] From the fact that an adequate description of binding energies can be given both for the light nuclei and for bulk nuclear matter in terms of two-body interactions consistent with nucleon-nucleon scattering data, it has generally been concluded, however, that many-body forces among nucleons are relatively unimportant in nuclei. The introduction of many-body forces involving the Λ particle into the phenomenological analysis of hypernuclear binding energies would also involve more parameters than could be determined from the data at the present stage. For simplicity, our discussion of the light hypernuclei will therefore be based on a two-body interaction between a Λ particle and a nucleon; many-body forces involving the Λ particle are not discussed further in the present work.

It will be assumed that the Λ particle has spin $\frac{1}{2}$, this being the most likely value on the basis of the available data on angular correlations in Λ decay[12] and on the internal conversion coefficient for nonmesonic hypernuclear decay.[13] The Λ-nucleon interaction may depend on the relative orientation of Λ and nucleon spins. The triplet interaction potential will be denoted by V_p; the singlet, by V_a. With this notation the inter-

[7] N. Dallaporta and F. Ferrari, Nuovo cimento **5**, 111 (1957).
[8] G. Wentzel, Phys. Rev. **101**, 835 (1956).
[9] R. H. Dalitz, Mid-West Conference on Theoretical Physics, Iowa, 1957.
[10] D. B. Lichtenberg and M. Ross, Phys. Rev. **109**, 2163 (1958).
[11] E.g.: S. D. Drell and K. Huang, Phys. Rev. **91**, 1527 (1953); E. M. Gelbard, Phys. Rev. **100**, 1530 (1955).
[12] D. Glaser, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Physics*, 1957 (Interscience Publishers, Inc., New York, 1957), Vol. 7, Sec. 5, p. 24; Alvarez, Bradner, Falk-Variant, Gow, Rosenfeld, Solnitz, and Tripp, Nuovo cimento **5**, 1026 (1957); T. D. Lee and C. N. Yang, Phys. Rev. **109**, 1755 (1958).
[13] M. Ruderman and R. Karplus, Phys. Rev. **102**, 247 (1956); Schneps, Fry, and Swami, Phys. Rev. **106**, 1062 (1957).

action potential can be written

$$V=\frac{3+\boldsymbol{\sigma}_\Lambda\cdot\boldsymbol{\sigma}}{4}V_p+\frac{1-\boldsymbol{\sigma}_\Lambda\cdot\boldsymbol{\sigma}}{4}V_a, \tag{1.4}$$

where the coefficients of V_p and V_a are the spin projection operators for the states of total spin 1 and 0, respectively. The triplet interaction may include tensor force terms. Since the centrifugal barrier effective in a relative D state will prevent the very close Λ-nucleon approach necessary for the short-range tensor force to be effective, it is expected that any Λ-nucleon tensor force will be indistinguishable from an additional central potential; consequently, V_p and V_a will both be represented by equivalent central potentials whose low-energy scattering characteristics are the same as those of the correct interactions.

The binding energies B_Λ given in Table I are usually less than the corresponding neutron binding energy B_N for the stable nucleus of the same mass number; on this basis, the Λ-nucleon interaction appears to be less effective in binding than are the corresponding neutron-nucleon interactions. A quantitative measure of the relative strengths of these interactions is given in Sec. 3. The exceptions to this statement are ${}_\Lambda H^4$, ${}_\Lambda He^5$, and ${}_\Lambda Be^9$, the nuclei H^4 and He^5 being unstable against neutron emission, while B_N for Be^9 is only 1.67 Mev. These exceptions illustrate the operation of the Pauli principle for neutrons in nuclei; the Λ particle, being distinct from the nucleons, is not subject to the restrictions of the Pauli principle and can occupy the s state of lowest energy appropriate to the potential well provided by its interaction with the nucleons of the core nucleus. This last remark, in fact, is sufficient to lead to a qualitative account of the general trend of B_Λ values with increasing mass number A. Since the depth of the potential experienced by the Λ particle within nuclear matter depends primarily on the density of the nuclear matter (for given Λ-nucleon forces), the saturation property of nuclear matter implies that, in heavy hypernuclei, the Λ particle will see a potential well whose depth D is roughly independent of the mass number A but whose radius increases as $r_0A^{\frac{1}{3}}$. Estimates of this depth D, ranging from 30 to 40 Mev, are given in Sec. 4. Since the energy of the lowest s state for the Λ particle decreases with increasing radius of the well, it is natural to expect that B_Λ should ultimately increase monotonically with A, reaching quite large values[14] $[D-1/(2Mr_0^2A^{\frac{2}{3}})]$ for sufficiently large A. Although this asymptotic region lies far beyond the identified hypernuclei, this trend in binding energies already appears and is due primarily to this effect; this general trend, however, could well have been upset in detail for light hypernuclei by special features of the core nuclei (for example, the feature that ${}_\Lambda Be^9$ has a core nucleus with no stable ground state). The saturation property of nuclear matter, therefore, has the consequence that the binding energy values for the lightest hypernuclei can be expected to have a more sensitive dependence on the detailed properties of the Λ-nucleon interaction than do the B_Λ values for the heavier hypernuclei. For this reason, and also because the B_Λ values of the lighter hypernuclei are known with greater certainty, the analysis of the data for the hypernuclei with $A\leqslant 5$ is discussed first, in Sec. 2.

The data given for ${}_\Lambda He^4$ and ${}_\Lambda He^5$ in Table I suggested a qualitative argument[3] that the Λ binding is not due to a spin-independent, two-body Λ-nucleon interaction. The closeness of their B_Λ values, 2.0 and 2.9 Mev, respectively, and the expectation that the spatial extension of the He^3 nucleus should be comparable with (or larger than) that of He^4 together imply that the potential well seen by the Λ particle in ${}_\Lambda He^5$ has a depth comparable with (or less than) that seen by the Λ particle in ${}_\Lambda He^4$. A spin-independent, two-body Λ-nucleon interaction would require a well depth proportional to the number of nucleons (for the same spatial extension); that is, a well depth for ${}_\Lambda He^5$ about $\frac{4}{3}$ that for ${}_\Lambda He^4$. This would lead to a considerably larger B_Λ value for ${}_\Lambda He^5$ (about 6 Mev at least) than for ${}_\Lambda He^4$; this is obviously contrary to the observed situation. This argument is developed in a more quantitative way in Secs. 2 and 3.

It is appropriate to emphasize here that the limited extent of our understanding of the structure of nuclei in terms of the elementary nucleon-nucleon interactions necessarily implies a corresponding roughness in our estimates concerning the detailed structure of hypernuclei, particularly for the form of their wave functions. It may well be that the presence of a Λ particle with a strong, short-range attraction to nucleons will result in considerable distortion of the core nucleus. This may be the case especially for the heavier hypernuclei where the core nuclei have low-lying excited states and for ${}_\Lambda H^3$ where the core nucleus is lightly bound. The most favorable case in this respect appears to be ${}_\Lambda He^5$, which has a tightly-bound core and has no excited states below 20 Mev. It seems reasonable to anticipate, however, that our conclusions about the strength of the Λ-nucleon interaction should be relatively insensitive to these distortions of the core nucleus. The strength

[14] There is, at present, no empirical evidence available on this well depth D for large hypernuclei. Knowledge of D would provide an important independent parameter bearing on the Λ-nucleon interaction. F. C. Gilbert and R. S. White [Phys. Rev. **109**, 1770 (1958)] have pointed out that a lower limit on this parameter may be obtainable from the upper limit observed for π^- energies resulting from K^- capture by heavy nuclei, the most energetic pion resulting from the capture reaction $K^-+n\rightarrow\Lambda+\pi^-$ where the Λ particle remains bound in the ground state of the residual hypernucleus. Such events would be very rare, of course, on account of the high density of excited states of the residual system; since as much as 40 Mev additional energy may be gained by the pion in this way, however, a useful lower limit for D may ultimately be obtained in this way, many low-lying levels being available for the residual hypernucleus within 5 Mev of the ground state. An estimate of the well depth $D(\Sigma)$ seen by a $\Sigma^\pm$ particle in nuclear matter has already been obtained by the above authors in this way.

U_n of the total Λ-nucleon potential corresponding to a given energy B_Λ of binding of a Λ particle to a core of $n=(A-1)$ nucleons is related to the hypernuclear wave function through a minimum principle. The value obtained for U_n should, therefore, deviate from the true value only by terms of second order when the wave function has deviations of first order. This remark is illustrated in Sec. 2 by a discussion of a particular type of nuclear distortion (radial compression).

The nuclear parameters (for an undistorted core nucleus) to which U_n is most sensitive are the radius and shape of the nucleon density distribution. From the analysis of the electron scattering experiments of Hofstadter and collaborators[15] at Stanford, the proton distribution is now known empirically for many of the relevant core nuclei (although the parameters obtained are not well understood in terms of the nuclear forces); the neutron distribution is assumed to be the same, an assumption which seems reasonable for these light nuclei. The outstanding case where the nuclear structure is reasonably well known is the hypertriton $_\Lambda H^3$, which has a deuteron core. On account of the low total binding (∼2.8 Mev) of this system, its properties can be expected to depend mainly on the low-energy scattering characteristics for each pair of particles, being otherwise quite insensitive to the detailed forms of their interactions. This is also the hypernucleus for which distortion of the relative motion of the core nucleons by the Λ particle is expected to be most severe. The $_\Lambda H^3$ system has, for these reasons, been discussed in some detail separately,[16] the results being summarized here in Sec. 2.

A phenomenological analysis of the data relevant to the hypernuclei with $A\leqslant 5$ is given in Sec. 2. The results of this analysis are discussed in terms of spin dependence for the Λ-nucleon interaction in Sec. 3, where some of the consequences of the conclusions are followed through. Consideration is given to the structure of hypernuclei with $A\geqslant 6$ in Sec. 4, a discussion of the sensitivity of the analysis to the presence of an exchange component in the Λ-nucleon interaction being given in Appendix C.

2. DISCUSSION OF THE HYPERNUCLEI WITH $A\leqslant 5$

It is the information on the lightest hypernuclei which can be expected to reflect most sensitively any detailed properties of the Λ-nucleon interaction, such as its dependence on the Λ-nucleon spin state. The hypernuclei with $A\leqslant 5$ are the most firmly established of those listed in Table I; they are also the simplest, in that their core nuclei are almost pure S configurations. For this reason, and also because of the low binding energy B_Λ for the Λ particle in these nuclei, the Λ-nucleon interactions which take place are almost entirely s-wave interactions. This means that a pure exchange force will give almost as much attraction in these nuclei as would the ordinary force which gives the same s-wave interaction; it is shown in Appendix C that the difference between these two extreme cases is far less than the other uncertainties in the analysis.

The Λ-nucleon interaction being represented as a spin-dependent central potential, the wave function of the hypernucleus will consist of the product of a spin wave function and an orbital wave function. The orbital wave function obtained here corresponds to the motion of the Λ particle in an average potential provided by the interaction between the Λ particle and the nucleons of the core nucleus (which is assumed to suffer relatively little distortion). This average potential in which the Λ particle moves is given by the expectation value

$$U(r)=\left\langle S\left|\sum_{i=1}^{A-1}\int V_{\Lambda n_i}(|\mathbf{r}-\mathbf{r}_i|)\rho(r_i/R)d_3\mathbf{r}_i\right|S\right\rangle \quad (2.1)$$

in the spin state S considered for the system; $\rho(r/R)$ denotes the density distribution of a nucleon in the core nucleus, and the sum is over the $n=(A-1)$ nucleons of the core nucleus. If both V_p and V_a are assumed to have the same shape $v(r)$, this potential can be written

$$U(r)=U_n\int v(|\mathbf{r}-\mathbf{r}'|)\rho(r'/R)d_3\mathbf{r}'. \quad (2.2)$$

If $v(r)$ is normalized to unity for integration over all space, then U_n denotes the total volume integral for all the Λ-nucleon interactions in this state S. Two ranges are considered for the potential shape $v(r)$. These ranges are chosen to correspond to intrinsic ranges for this shape which are the same as those for a Yukawa potential $e^{-\kappa r}/\kappa r$ where: (i) $1/\kappa=1/2m_\pi\approx 0.7$ fermi, and (ii) $1/\kappa=1/m_K\approx 0.4$ fermi; these ranges (for a Yukawa potential) correspond to the two physical mechanisms which may contribute most to the Λ-nucleon interaction. For convenience in computation, the potential shape $v(r)$ has been taken to be of Gaussian form[17]

$$v(r)=\left(\frac{2.0604}{\pi b^2}\right)^{\frac{3}{2}}\exp(-2.0604r^2/b^2), \quad (2.3)$$

where the intrinsic range b is related to the range parameter by $b=2.1196/\kappa$.

[15] R. Hofstadter, Revs. Modern Phys. **28**, 214 (1956).

[16] R. H. Dalitz and B. W. Downs, Phys. Rev. **110**, 952 (1958).

[17] The physical conclusions reached in this paper are quite insensitive to the shape assumed for the Λ-nucleon potential $v(r)$ and to the shape assumed for the nucleon density distribution $\rho(r/R)$ (provided the rms radius R is adjusted to lead to a form factor for the nuclear charge distribution which provides an adequate fit to the empirical data[15]). For example, calculations made with a Yukawa shape for $v(r)$, the core nucleus being represented by the appropriate "modified" exponential form,[15] have led to results for the volume integral which deviate from those obtained assuming Gaussian shapes by less than the errors quoted herein. Similarly, calculations on the Λ-nucleon interaction required to account for the binding of the hypertriton have given (see reference 16) a volume integral for this interaction whose value was not sensitive to the shape assumed for $v(r)$.

Hypernucleus $_\Lambda He^5$.—This appears to be the most favorable case for a simple discussion. The core alpha particle has a spin-saturated structure with high total binding energy, and its distortion by the weakly bound Λ particle can be expected to be rather slight. Its shape and radius have been well determined by the Stanford experiments[15]; a Gaussian shape for its charge distribution with an rms radius of 1.61±0.05 fermi provides a good fit to the Stanford data. Since the proton charge distribution is known to have a finite size, the electron-proton scattering data being well fit by a Gaussian shape of rms radius $R_p=0.72\pm0.05$ fermi for the proton, the nucleon distribution $\rho(r/R)$ in the alpha particle corresponds to a Gaussian shape with rms radius $R=R_4=1.44\pm0.07$ fermi:

$$\rho\left(\frac{r}{R}\right)=\left(\frac{3}{2\pi R^2}\right)^{\frac{3}{2}}\exp\left(-\frac{3r^2}{2R^2}\right). \quad (2.4)$$

Since the alpha-particle core has zero spin, $_\Lambda He^5$ will have spin $\frac{1}{2}$, as does the Λ particle itself; the volume integral U_4 of the total Λ-nucleon potential in this system is obtained by summing the volume integral of the interaction (1.4) between a Λ particle and a single nucleon over all spin states for the core nucleons. This leads to

$$U_4=3\bar{V}_p+\bar{V}_a. \quad (2.5)$$

The potential seen by the Λ particle is then

$$U_4\int v(|\mathbf{r}-\mathbf{r}'|)\rho(r'/R_4)d_3\mathbf{r}'=U_4\rho(r/R_4'), \quad (2.6)$$

where $R_4'=(R_4^2+3b^2/4.121)^{\frac{1}{2}}$. From the known binding energy $B_\Lambda=2.9\pm0.3$ Mev for the Λ particle, the strength U_4 of the potential (2.6) can then be deduced from the solution of the Schrödinger equation for the motion of the Λ particle relative to the alpha particle. Numerical integration of this equation leads to the values $U_4=925\pm45$ and 715 ± 25 Mev f^3 for the parameters $\kappa=2m_\pi$ and m_K, respectively. The statistical error given arises mainly from the uncertainty in R_4, the random error in B_Λ being relatively unimportant.

A mode of distortion for the alpha-particle core of $_\Lambda He^5$ which can be considered quite readily is a uniform radial compression. The compressed core will have a nucleon distribution $\rho(\alpha r/R_4)$, which leads to a potential well for the Λ particle of form $\rho(r/R)$ with $R=\{(R_4/\alpha)^2+3b^2/4.121\}^{\frac{1}{2}}$. The energy of the distorted core will be higher than that of the undistorted core. If the compression of the core is relatively small, this energy increase $\Delta E(\alpha)$ is given by the quadratic approximation $\frac{1}{2}K(\alpha-1)^2$ Mev, where K is termed the stiffness of the core. For a given α, the volume integral $U_4(\alpha)$ obtained for this well shape must correspond to a Λ energy $-[B_\Lambda+\Delta E(\alpha)]$ Mev. Values of $\alpha<1$ always lead to a value of $U_4(\alpha)>U_4(1)$. As α increases from 1, $\Delta E(\alpha)$ increases slowly at first whereas R decreases rapidly, so that $U_4(\alpha)$ decreases at first. Later, for larger α, $\Delta E(\alpha)$ increases rapidly while R approaches a constant value, so that $U_4(\alpha)$ ultimately increases rapidly with α. The optimum value of α, at which $U_4(\alpha)$ has its minimum value, depends on the value of K; expressions for the position and value of this minimum are given in detail in Appendix A. Calculation of $U_4(\alpha)$ for $K=280$ Mev[18] shows that the optimum compression is 3% ($\alpha=1.03$) for $\kappa=2m_\pi$, the minimum value of U_4 being 910±45 Mev f^3, 1.5% lower than for the undistorted alpha particle. With the shorter range interaction, $\kappa=m_K$, it is natural to expect more severe distortion; the optimum compression is found to be 6%, U_4 being reduced by 3% to 695±25 Mev f^3 from the value given above. The improvement obtained for the value of U_4 in this way is at most comparable with the uncertainties in the problem; this is in accord with our general expectations discussed in the introduction.

Hypernuclei $_\Lambda H^4$, $_\Lambda He^4$.—The only empirical information bearing on the size and shape of the core nuclei H^3 and He^3 of these systems is the He^3 Coulomb energy $\mathcal{C}=0.76\pm0.01$ Mev, which is deduced from the difference in total binding energy between H^3 and He^3 under the assumption of charge symmetry for nuclear forces. In the absence of empirical information bearing directly on the shape of the charge distribution for these nuclei, it seems reasonable to assume a Gaussian shape for H^3 and He^3, as is known empirically for He^4. A convenient form of wave function which may be assumed for these nuclei and which leads to a Gaussian shape for their nucleon distributions is

$$\psi=N\exp[-\tfrac{1}{2}\lambda(r_{12}^2+r_{23}^2+r_{31}^2)]. \quad (2.7)$$

The parameter λ is then chosen to give the correct value of the Coulomb energy $\mathcal{C}$ for He^3; under the assumption of charge symmetry the same value of λ will be appropriate for H^3 also. The nucleon distribution corresponding to (2.7) then has the form (2.4) with $R=(3\lambda)^{-\frac{1}{2}}$. In the calculation of the Coulomb energy the finite size of the proton should be taken into account; with Gaussian shape and rms radius R_p for its charge distribution, the wave function (2.7) leads to the following expression for the Coulomb energy of He^3:

$$\mathcal{C}=\frac{e^2}{R}\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\left(1+\frac{2}{3}\frac{R_p^2}{R^2}\right)^{-\frac{1}{2}}. \quad (2.8)$$

[18] This estimate was obtained from a calculation of the He^4 energy by J. Irving [Phil. Mag. **42**, 338 (1951)]. It is given by the curvature of the total energy $E(\alpha)$ at $\alpha=1$, when parameters are chosen to give a minimum energy $E(1)$ close to the observed He^4 energy, the nuclear forces (Yukawa shape) being chosen to fit the low-energy scattering data. This value of K can be compared with other estimates for the stiffness of bulk nuclear matter. From semiempirical mass formulas, A. E. S. Green [Revs. Modern Phys. (to be published)] has obtained values between 175 and 218 Mev, while A. G. W. Cameron [Can. J. Phys. **35**, 1021 (1951)] found $K=302$ Mev. A recent theoretical calculation by K. A. Brueckner and J. L. Gammel [Phys. Rev. **109**, 1023 (1958)] using hard-core potentials leads to the value 172 Mev. Our results are, however, insensitive to variations in the value of K in this range.

With $R_p=0$, this expression leads to $R_3=1.49$ fermi; the finite proton size reduces this to $R_3=1.38$ fermi.

The radius value 1.38 fermi is smaller than the known rms radius $R_4=1.44$ fermi of the alpha particle. Variational calculations based on conventional nucleon-nucleon potentials of Yukawa or exponential forms have always led to wave functions which give a larger radius for H^3 and He^3 than for He^4. With these potentials, this result appears closely associated with the tight binding of He^4 relative to that of H^3, He^3. The radius calculated for He^4 from these wave functions is, however, much smaller[15] (by 25% or more) than the observed radius (even after inclusion of some of the D states, due to tensor forces), so that these theoretical calculations do not necessarily reflect the true situation. Similarly, the Coulomb energy computed for He^3 from these wave functions is generally of order 30% too large, indicating that they correspond to a nucleon distribution which is more compact than the physically correct one. It is now generally believed that the nucleon-nucleon interaction includes a strong short-range repulsion, a hard core with a radius of order 0.5 fermi, which has not been included in these calculations; this additional repulsion would certainly have the effect of increasing the mean radii for these systems. This view has been given support by recent calculations for H^3, He^3 by Kikuta *et al.*[19] and Ohmura *et al.*,[19] who find that it is possible to obtain the correct Coulomb energy and about the correct binding energy with the use of central potentials fitting the two-body data, provided there is a hard core of radius about 0.5 fermi. No corresponding calculations have yet been reported for the He^4 system. If the radii of these nuclei are more closely associated with the density permitted by the hard-core repulsions between nucleons than with the total binding energies, it is not impossible that the H^3, He^3 systems should have a radius smaller than does He^4. On the other hand, our estimate of R_3 from the He^3 Coulomb energy would be rather sensitively affected by any deviation from charge symmetry between the *n-n* and *p-p* nuclear forces. From Irving's calculation[18] of the binding energy for H^3, we can deduce that a 0.1% increase in the well depth for the $n-n$ interaction would increase the binding energy by 0.03 Mev; from this it follows that a 0.1% difference between the well depths for *n-n* and *p-p* nuclear forces would contribute 0.03 Mev to $\mathcal{C}$ (calculated with the assumption of exact charge symmetry) and would modify the estimate of R_3 from (2.8) by 4%. It is not well known empirically just how exactly charge symmetry holds for nuclear forces but, even with a charge symmetric pion-nucleon interaction, it is reasonable to expect deviations from charge symmetry of relative order $e^2/\hbar c\approx 0.7\%$ as a result of virtual electromagnetic effects in nuclear forces. In view of these uncertainties in our knowledge of the nuclear structure of the H^3, He^3 systems, calculations have been carried through for two values of the rms radius R_3, for $R_3=1.38$ fermi and for $R_3=1.58$ fermi. Comparison of these two cases then allows some estimate of the variations associated with the uncertainty in this parameter.

[19] Kikuta, Morita, and Yamada, Progr. Theoret. Phys. (Japan) **15**, 222 (1956); Ohmura, Morita, and Yamada, Progr. Theoret. Phys. (Japan) **17**, 326 (1957).

There are two possible values for the spin J of the ${}_\Lambda H^4$, ${}_\Lambda He^4$ doublet, depending upon whether the spins of the Λ particle and the unpaired nucleon couple in the singlet or triplet configuration. In the former case one has $J=0$, and the volume integral $U_3(J=0)$ of the total Λ-nucleon interaction is

$$U_3(J=0)=\tfrac{1}{2}U_4+\bar{V}_a, \tag{2.9a}$$

where U_4 is given by (2.5). In the latter case one has $J=1$, and the volume integral $U_3(J=1)$ is

$$U_3(J=1)=\tfrac{1}{2}U_4+\bar{V}_p. \tag{2.9b}$$

If the H^3 (He^3) core is considered to be rigid, the potential seen by the Λ particle in the hypernucleus ${}_\Lambda H^4$ (${}_\Lambda He^4$) takes the form

$$U_3\int v(|\mathbf{r}-\mathbf{r}'|)\rho(r'/R_3)d_3\mathbf{r}'=U_3\rho(r/R_3'), \tag{2.10}$$

where $R_3'=(R_3^2+3b^2/4.121)^{\frac{1}{2}}$, and U_3 has the appropriate form (2.9). Since the Λ binding energies for ${}_\Lambda H^4$, ${}_\Lambda He^4$ are required by charge symmetry to be equal,[20]

[20] A difference between the B_Λ values for ${}_\Lambda H^4$ and ${}_\Lambda He^4$ may arise from violations of charge symmetry in a number of ways. The most important of these stems from the possibility that an interaction $\Lambda\to\Lambda+\pi^0$ may result from virtual electromagnetic effects. For example, with strict charge symmetry, the two terms resulting from the sequences

$$\Lambda\to\left\{\begin{matrix}p+K^-\to\pi^0+p+K^-\\ n+\bar{K}^0\to\pi^0+n+\bar{K}^0\end{matrix}\right\}\to\pi^0+\Lambda$$

will cancel precisely. There are clearly electromagnetic interactions possible between the particles in the upper sequence, however, which have no counterpart in the lower sequence, so that this cancellation will not be exact. The effective coupling strength for this interaction $\Lambda\to\pi^0+\Lambda$ will be at most of order $(e^2/\hbar c)G$, where G is the pion-nucleon coupling strength. Exchange of a π^0 meson between a Λ and a nucleon will contribute a Λ-nucleon interaction of opposite sign for proton and neutron. In the singlet Λ-proton state, this interaction will have the form of the proton-proton interaction with strength reduced by about 1/137; in the triplet state it is repulsive and weaker by a factor 3, so that the spin average of this interaction vanishes. This interaction may, therefore, contribute a difference of about 14 Mev f^3 between the $U_3(J=0)$ for H^3 and He^3; this contribution is relatively large as a result of the longer range associated with single pion exchange. This effect could therefore contribute as much as 0.2 Mev to the difference between the two B_Λ values.

Some decrease in B_Λ for ${}_\Lambda He^4$ may result from change in the Coulomb energy of the core nucleus. If the presence of the Λ particle causes compression of the core nucleus by a factor α, then $\mathcal{C}$ will increase to $\alpha\mathcal{C}$, and it seems reasonable to expect an increase of order 0.1 Mev in $\mathcal{C}$, from the numbers given in Table II. This would lead to a decrease in B_Λ by the same amount. Deviations from charge symmetry for the *n-n* and the *p-p* nuclear forces could also give rise to minor differences in structure between H^3 and He^3 which would also be reflected by small differences in B_Λ for ${}_\Lambda He^4$ and ${}_\Lambda H^4$, but this effect is probably smaller than those discussed above.

All of these deviations from charge symmetry can only lead to effects which are somewhat smaller than the present experimental uncertainties in the B_Λ values for ${}_\Lambda H^4$ and ${}_\Lambda He^4$.

TABLE II. Phenomenological analysis for $_\Lambda H^4$, $_\Lambda He^4$. The volume integral $\bar{U}_3$ Mev f³ is given for $B_\Lambda=0$ and 1.85 ± 0.3 Mev for $R_3=1.38$ and 1.58 fermi. Columns (i) and (ii) refer to $\kappa=m_K$ and $2m_\pi$ respectively; the bracketed numbers specify α at minimum where this is appropriate. The errors quoted are only those arising from the uncertainty in B_Λ.

$B_\Lambda=1.85\pm0.3$ Mev, and	$K=\infty$		$K=60$ Mev		$B_\Lambda=0$ $(K>K_m)$	
	(i)	(ii)	(i)	(ii)	(i)	(ii)
$R_3=1.38$ fermi	660±15	855±25	600±10(1.24)	820±20(1.11)	461	554
$R_3=1.58$ fermi	780±20	965±30	695±10(1.25)	915±25(1.12)	517	603

we have chosen to treat these two cases together, the mean value of B_Λ being 1.85 ± 0.3 Mev. The corresponding values of U_3 are given in Table II in the column $K=\infty$. The values of $U_3(B_\Lambda=0)$ for the potential (2.10) have also been given in this table since these provide an estimate of the strength above which the total Λ-nucleon interaction between a Λ particle and H^3 (or He^3) is sufficient for the formation of a bound state for this system.

The effect on U_3 of allowing radial compression for the nuclear core is calculated in the same way as it was for U_4. An estimate of $K=60$ Mev is obtained for the stiffness of the core nuclei H^3, He^3 by the method described above,[18] based on Irving's calculation of the energy of H^3 with Yukawa nucleon-nucleon forces. The values $U_3(\alpha)$ calculated for a nucleon distribution $\rho(\alpha r/R_3)$ have been plotted in Fig. 1 for a particular set R_3, B_Λ as function of α; curves are also shown for $K=40$ and 80 Mev in order to indicate how the final results depend upon the value chosen for K. The minima obtained for $U_3(\alpha)$ are tabulated in Table II for the two values of R_3 previously mentioned. The optimum distortions are quite large for the shorter range $1/m_K$ (a 20% reduction in radius with $K=60$ Mev), although the corresponding change in U_3 is somewhat smaller (a 10% reduction), for the reasons given in the introduction. It is natural to expect that a strong short-range Λ-nucleon interaction should result in quite strong correlations in position between the particle and the individual nucleons, especially for a nuclear core whose stiffness is as small as 60 Mev, and it is possible that, with a range parameter $\kappa=m_K$, correlations involving more complicated modes of distortion which take less energy from the Λ-nucleus relative motion may lead to further significant reductions in the value of U_3. For the longer interaction range $1/2m_\pi$, however, the optimum distortions are considerably less (about 10% reduction in radius for $K=60$ Mev) and involve much less reduction (about 5%) in U_3. For this case, it seems reasonable to expect that no really significant improvement in U_3 should result from consideration of more complicated distortions and correlations.

Hypernucleus $_\Lambda H^3$.—The hypertriton is believed to be an isotopic singlet $(T=0)$ state. The theoretical reasons for this belief have been given in some detail recently[16] and depend primarily on the fact that the *s*-wave nucleon-nucleon interaction is most strongly attractive in the $T=0$ configuration. In this configuration, the neutron-proton system forms a triplet spin state, and there are two possibilities for the spin of $_\Lambda H^3$. If the Λ-nucleon interaction is stronger in the triplet state, then the hypertriton will have spin $\frac{3}{2}$, the volume integral U_2 of the Λ-nucleon interactions being given by

$$U_2(J=\tfrac{3}{2})=2\bar{V}_p. \tag{2.11a}$$

If the Λ-nucleon interaction is stronger in the singlet spin state, the hypertriton will have spin $\frac{1}{2}$ and

$$U_2(J=\tfrac{1}{2})=\tfrac{3}{2}\bar{V}_a+\tfrac{1}{2}\bar{V}_p. \tag{2.11b}$$

Description of the hypertriton by a product wave function in which one factor represents the relative motion of the two nucleons, and a second factor represents the motion of the Λ particle relative to the center of mass of the two nucleons, leads to a considerable overestimate of U_2. Such a product wave function is inadequate here since it does not allow the strong Λ-nucleon correlations in position which can be expected to occur in this weakly-bound system as a result of the short-range Λ-nucleon attractions. This shortcoming is present already in the wave functions discussed above for the $A=4$ and 5 hypernuclei, of course, but it is much more serious for the hypertriton where the energy of relative motion between the nucleons is relatively little affected by the existence of Λ-nucleon correlations which distort this relative mo-

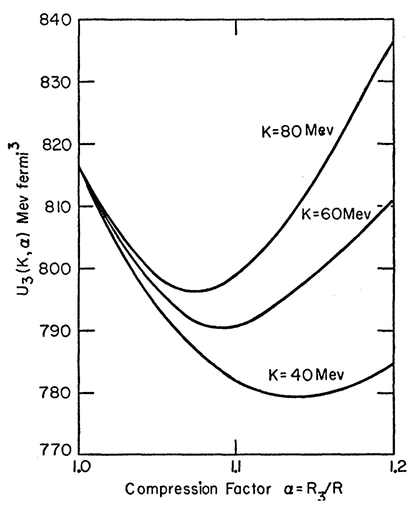

FIG. 1. The volume integral $U_3(K,\alpha)$ of the Λ-nucleon interactions corresponding to a binding energy $B_\Lambda=1.44$ Mev for a Λ particle to H^3, He^3 is plotted as function of the core compression factor α (with radius $R_3=1.38$ f for the undistorted core), for three values of the stiffness K for the core nucleus, the Λ-nucleon potential range being taken as $1/2m_\pi$.

tion. The hypertriton is, however, a sufficiently simple system that trial functions which include such correlations can be investigated without undue labor by making use of triangular coordinates (r_1,r_2,r_3), where r_3 denotes the neutron-proton separation and r_1, r_2 the Λ-neutron and Λ-proton separations. A trial function with these features, for which variational calculations can be carried through conveniently, has already been suggested[16]:

$$\psi=N(e^{-ar_1}+xe^{-br_1})(e^{-ar_2}+xe^{-br_2})(e^{-\alpha r_3}+ye^{-\beta r_3}). \quad (2.12)$$

Upper bounds for U_2, based on a simple version of (2.12) with $x=y=0$, have recently been reported.[16] With $B_\Lambda=0.6\pm0.4$ Mev, these calculations led to the value[21] $U_2=795$ Mev f^3 for $\kappa=2m_\pi$, the values for Yukawa and exponential shapes[22] lying within ±20 Mev f^3 of this value through the allowed range for B_Λ. With $\kappa=m_K$, a value $U_2=500$ Mev f^3 was obtained, with a corresponding uncertainty of ±10 Mev f^3.

The absence of empirical evidence for a bound $T=1$ hypernuclear state $A=3$, which would be exemplified by $_\Lambda$He^3 or $_\Lambda n^3$ decay events, appears consistent with the low B_Λ value observed for $_\Lambda$H^3. In this case, according to calculation with the simplified version of (2.12), the weakness of the nucleon-nucleon interaction in the $T=1$ state (about 68% of the $T=0$ interaction) is sufficient to insure that no such $T=1$ state should be bound.

3. SPIN-DEPENDENCE OF THE Λ-NUCLEON INTERACTION

The volume integrals U_n for the total interaction between the Λ particle and the $n=(A-1)$ nucleons of the core nucleus, listed in Table I, would be expected to be proportional to n if they were due to an elementary two-body Λ-nucleon interaction which is spin independent.[23] That this proportionality does not hold is clear from the fact that U_2 has been found to be much larger than $\frac{1}{2}U_4$. The use of a more complicated wave function for $_\Lambda$He^5 might even lead to further reduction in U_4, but already the present value $\frac{1}{2}U_4=455$ Mev f^3 for $\kappa=2m_\pi$ is so far below U_2 that there is no possibility that further refinements in the calculations for $_\Lambda$H^3 could bring agreement between U_2 and $\frac{1}{2}U_4$.[22,24] This same remark also holds true for the case $\kappa=m_K$. The most direct interpretation of this result is that the Λ-nucleon interaction is spin-dependent, since Eqs. (2.5) and (2.11) show that different spin combinations contribute to U_2 and to U_4.

The values given for U_3 in Table I similarly suggest that the Λ-nucleon interaction is spin-dependent. Even for the smaller radius $R_3=1.38$ f, the values of U_3 are considerably larger than the corresponding values for $\frac{3}{4}U_4$; and it seems unlikely that consideration of distortions of the core nucleus more elaborate than the radial compression discussed in Sec. 2 will lead to a reduction in U_3 large enough to bring it into agreement with $\frac{3}{4}U_4$. As we shall see below, the values for U_3 are not in disagreement with the values expected from the degree of spin dependence indicated by the comparison of U_2 and U_4. At the present time, however, there is no reason to exclude interpretations considerably more complicated (for example, interpretations which involve many-body forces) than that of a spin-dependent two-body interaction between the Λ particle and each nucleon of the hypernucleus.

The expressions for U_2 and U_3 in terms of $\bar{V}_p$ and $\bar{V}_a$ depend on whether V_p or V_a is the more attractive, whereas the expression for U_4 always has the form (2.5). As soon as it is specified whether $\bar{V}_p>\bar{V}_a$ or $\bar{V}_a>\bar{V}_p$, the values of $\bar{V}_p$ and $\bar{V}_a$ can be deduced from any two of these volume integrals. As discussed in Sec. 2, there is considerable uncertainty concerning the value of U_3, resulting from our lack of empirical knowledge of the H^3, He^3 radius. For this reason, the spin dependence of the Λ-nucleon interaction will be deduced from the values of U_2 and U_4, and the results of this analysis will then be compared with the values of U_3 obtained in Sec. 2. There are now two possible situations to be considered:

(a) $\bar{V}_p>\bar{V}_a$, the triplet Λ-nucleon spin state having the more strongly attractive interaction. In this case, $_\Lambda$H^3 will have spin $\frac{3}{2}$ and Eqs. (2.5) and (2.11a) lead to

$$\bar{V}_p=\tfrac{1}{2}U_2, \quad \bar{V}_a=U_4-\tfrac{3}{2}U_2. \quad (3.1a)$$

The values of $\bar{V}_p$ and $\bar{V}_a$ obtained from (3.1a) with the values of U_2 and U_4 given in Table I are listed in Table III.[25] The triplet potential $\bar{V}_p$ is relatively well determined whereas $\bar{V}_a$ (being the difference between two large numbers) is relatively poorly determined. For both ranges considered for the Λ-nucleon potential,

[21] Preliminary calculations carried through for the wave function (2.12) for the range parameter $\kappa=2m_\pi$ have led to a reduction of about 10% in this value, the improved value being about 700 Mev f^3 with the optimum choice for all six parameters.

[22] In order to justify later comparison of these values of U_2 with the values U_n obtained above with Gaussian shape for the Λ-nucleon interaction, it is appropriate to remark here that the volume integral of a Gaussian potential is only 1.5% larger than that of an exponential potential of the same well-depth parameter and intrinsic range. This difference is less than the uncertainties in these U_n arising from the uncertainties in the initial data and in the form of the wave functions assumed for these hypernuclei.

[23] This is strictly true only if the interaction has no exchange component. For an exchange interaction of range as short as $1/m_K$, the largest value reasonable on physical grounds, the deviations from this proportionality should not exceed several percent for $n\leqslant4$, according to the calculations discussed in Appendix C.

[24] To emphasize this point, we may note that the potential $\frac{1}{4}U_4$ corresponds to a well-depth parameter $s=0.40$ for $\kappa=2m_\pi$ ($s=0.54$ for $\kappa=m_K$). In Appendix B, however, a lower limit (0.36) for the value of s in the Λ-nucleon spin state giving the stronger attraction is obtained, with only the assumption that three-body forces may be neglected; if the Λ-nucleon potential is further assumed to be central and of Gaussian shape, this lower limit can be improved to 0.43 ± 0.02 for $\kappa=2m_\pi$ (0.40 ± 0.01 for $\kappa=m_K$).

[25] The volume integrals listed in Table III can be compared with the volume integral of the triplet nucleon-nucleon interaction, which has the value 1403 Mev f^3 for a Yukawa potential with a range parameter $\kappa=0.848$ f^{-1}.

TABLE III. Summary of results.[a]

κ	$\bar{V}_p$ (Mev f^3)	$\bar{V}_a$ (Mev f^3)	$_\Lambda H^3$ spin	$_\Lambda H^4$ spin	U_3 value predicted	U_3^* value predicted
m_K	136±10	288± 8	$\frac{1}{2}$	0	636±12	484±22
$2m_\pi$	142±18	482±16	$\frac{1}{2}$	0	937±23	597±40
m_K	250± 5	−55±30	$\frac{3}{2}$	1	598±13	292±40
$2m_\pi$	398±10	−282±55	$\frac{3}{2}$	1	853±25	173±75

[a] The errors quoted in Table III are only those arising from the statistical deviation in the B_Λ and R_n determinations.

the potential $\bar{V}_a$ obtained corresponds to a repulsion in the singlet Λ-nucleon spin state.

(b) $\bar{V}_a > \bar{V}_p$, the singlet Λ-nucleon spin state having the more strongly attractive interaction. In this case, $_\Lambda H^3$ will have spin $\frac{1}{2}$, and Eqs. (2.5) and (2.11b) lead to

$$\bar{V}_p = \tfrac{3}{8}U_4 - \tfrac{1}{4}U_2, \quad \bar{V}_a = \tfrac{3}{4}U_2 - \tfrac{1}{8}U_4. \tag{3.1b}$$

In this situation both $\bar{V}_p$ and $\bar{V}_a$ are quite well determined, their values being listed in Table III. An attractive interaction is found for both singlet and triplet potentials, the ratio $\bar{V}_p/\bar{V}_a$ being 0.45 and 0.3 for interaction ranges $1/\kappa = 1/m_K$ and $1/2m_\pi$, respectively.

The values just obtained for $\bar{V}_p$ and $\bar{V}_a$ can now be used to predict values for U_3, following Eqs. (2.9). These predicted values can then be compared with the values of U_3 listed in Table I, which were obtained from the empirical data on the ($_\Lambda H^4$, $_\Lambda He^4$) doublet as discussed in Sec. 2. As before, there are two cases to be considered:

(*a*) $\bar{V}_p > \bar{V}_a$.—The ground state of $_\Lambda H^4$, $_\Lambda He^4$ will have spin 1 and U_3 is given by expression (2.9a). For $\kappa = m_K$, this predicted value of U_3 is 598 Mev f^3; it is to be compared with the empirical values of 600 for $R_3 = 1.38$ fermi and 695 for $R_3 = 1.58$ fermi. Similarly, with $\kappa = 2m_\pi$, the predicted value for U_3 is 853 Mev f^3 compared with empirical values 820 and 915 for these two values of R_3. There is no essential disagreement to be found in this comparison, but the agreement can be regarded as satisfactory only for a radius R_3 somewhat larger than 1.38 fermi. More elaborate calculations than those discussed in Sec. 2 are expected to result in a greater reduction for U_3 than for U_2 and U_4, which means that the decrease in the predicted value of U_3 will then be less than the decrease in the value of U_3 calculated from the binding energy data.

The volume integral $U_3^* = \frac{1}{2}U_4 + \bar{V}_a$ appropriate to an excited state of the ($_\Lambda H^4$, $_\Lambda He^4$) doublet with spin 0 is only about 30 to 60% of $U_3(B_\Lambda = 0)$, the critical value required for a bound state of this system to exist. In the present case, there will be no bound $J=0$ state for these hypernuclei; this is primarily a consequence of the repulsion present in the singlet Λ-nucleon interaction.

(*b*) $\bar{V}_a > \bar{V}_p$.—The ground state of the ($_\Lambda H^4$, $_\Lambda He^4$) doublet will have zero spin, and the values of U_3 predicted by (2.9b) are 636 Mev f^3 for $\kappa = m_K$ and 937 Mev f^3 for $\kappa = 2m_\pi$. Considering the uncertainties of the calculation, these values do not disagree with the values of U_3 listed in Table I, but the agreement is again satisfactory only for R_3 somewhat larger than 1.38 fermi.

In the present case it appears that an excited state of the ($_\Lambda H^4$, $_\Lambda He^4$) doublet might exist with spin $J=1$ since the triplet potential V_p is attractive and $U_3^* = \frac{1}{2}U_4 + \bar{V}_p$. For the case $\kappa = m_K$, U_3^* has the value 484 Mev f^3, which is to be compared with the critical values $U_3(B_\Lambda = 0) = 461$ Mev f^3 for $R_3 = 1.38$ fermi and 517 Mev f^3 for $R_3 = 1.58$ fermi. For the longer range case $\kappa = 2m_\pi$, however, U_3^* has the value 597 Mev f^3; this is to be compared with $U_3(B_\Lambda = 0) = 554$ Mev f^3 for $R_3 = 1.38$ fermi and 603 Mev f^3 for $R_3 = 1.58$ fermi. It follows from these numbers that, if the $J=1$ excited state is bound, its binding energy B_Λ^* does not exceed 0.1 Mev. The existence of this excited state would be of considerable importance for certain types of experiments (see below). Such a state would not be observed to undergo a characteristic hypernuclear decay, but would be expected to emit a photon of energy about 1.8 Mev in an $M1$ transition to the $J=0$ ground state of this doublet. A rough estimate of the lifetime for this decay is given as function of B_Λ^* by

$$(\mu_\Lambda - \mu_c)^{-2}(B_\Lambda^* \text{ Mev})^{-\frac{1}{2}} \times 10^{-13} \text{ sec}, \tag{3.2}$$

where μ_Λ, μ_c denote the magnetic moments (in nuclear magnetons) for the Λ particle and the core nucleus. This lifetime is much shorter than the hypernuclear decay lifetime. Even if this excited $J=1$ state is not bound, the values given above for U_3^* imply that, if $\bar{V}_a > \bar{V}_p$, a low-energy resonance should exist in the triplet s-wave scattering of Λ particles by He^3 or H^3. Such a resonance would have a strong influence on the energy spectra for a reaction such as

$$K^- + He^4 \to He^3 + \Lambda + \pi^-, \tag{3.3}$$

for which this low-energy resonance would give rise to a strong peaking of the π^- energy distribution toward the upper end of its spectrum.

Direct information on the interaction in a Λ-nucleon system is not yet available experimentally. Several events representing secondary Λ-proton collisions have been observed[26] in bubble-chamber studies of Λ-particle production processes, but it will be some time yet before scattering cross sections for such collisions are known as function of energy and angle. No unambiguous evidence has yet been reported for the existence of a bound Λ-nucleon system. The world survey of Levi Setti *et al.*[2] does list nine events which are compatible with the decay of a $Z=1$ hypernucleus, but which give a B_Λ value significantly lower than that for $_\Lambda H^3$; some of these events could be due to the decay of a $\Lambda - p$ system, but other interpretations are also possible for these events. On the basis of the present analysis, it appears unlikely that a Λ-nucleon bound state should exist. This is most clear for the case $\bar{V}_p > \bar{V}_a$, where the

[26] G. Puppi, (private communication, 1957).

triplet interaction is the more attractive. This is because the Λ-nucleon interactions effective in $_\Lambda H^3$ then refer only to V_p; in fact $U_2=2\bar{V}_p$ according to (2.11a). This situation has been discussed in detail recently[16]; in all physically reasonable cases, the strength deduced for V_p from the $_\Lambda H^3$ data does not exceed 77% of that needed for binding of a Λ-nucleon system. For the case $\bar{V}_a>\bar{V}_p$, where the lowest bound state would have $J=0$, the analysis of $_\Lambda H^3$ leads to the quantity $\frac{1}{2}U_2=\frac{3}{4}\bar{V}_a+\frac{1}{4}\bar{V}_p$, so that a conclusion on Λ-nucleon binding must appeal to the data on other hypernuclei for the relative strengths of $\bar{V}_p$ and $\bar{V}_a$. The largest well-depth parameter obtained from $\frac{1}{2}U_2$ is 0.77, corresponding to the shorter range $1/m_K$, so that the well-depth parameter for V_a can exceed unity only if V_p is repulsive. If this were the case, the Λ-binding observed for $_\Lambda He^5$ would be difficult to understand since $U_4=3\bar{V}_p+\bar{V}_a$ depends quite strongly on the triplet Λ-nucleon potential. With the spin dependence which has been deduced from the values for U_2 and U_4, the largest well-depth parameter found for V_a is 0.88, corresponding to $\kappa=m_K$. The absence of evidence for $A=2$ Λ-hypernuclei therefore appears natural in terms of the interpretation proposed in this paper for the binding energies of light hypernuclei.[27]

It should be added here that the existence of spin dependence in the Λ-nucleon interaction strengths the argument against the existence of a bound $_\Lambda He^3$ or $_\Lambda n^3$ state. Not only is the nucleon-nucleon interaction weaker in the $T=1$ state than in the $T=0$ state possible for $_\Lambda H^3$, but also the Λ-nucleon interaction U_2' effective in this state is $\frac{1}{2}U_4$, which is much weaker than the total Λ-nucleon interaction in the $T=0$ ground state. This spin-dependence also excludes[16] the binding of $T=0$ excited states of $_\Lambda H^3$.

The comparison of U_2, U_3, and U_4 made in this section clearly allows no decision to be made on the exchange character or range parameter (in the physically reasonable region) appropriate to the Λ-nucleon interaction. This comparison also does not distinguish between the two spin-dependence possibilities discussed above. The existence of excited states $_\Lambda H^{4*}$, $_\Lambda He^{4*}$ would require the singlet state to be favored, but it appears relatively difficult to establish the existence of these excited states. These spin dependences could be distinguished by a knowledge of the spin value for the ground state of $_\Lambda H^4$, $_\Lambda He^4$ or $_\Lambda H^3$, which may be obtained from the observation of angular correlations between their production and decay processes or from the relative branching ratios for their various decay modes. If the spin of the ($_\Lambda H^4$, $_\Lambda He^4$) doublet is zero, these systems cannot carry any information from their production processes; consequently, the observation of any spatial anisotropy for any decay product of $_\Lambda H^4$, $_\Lambda He^4$ would require $J=1$ for these systems. Since about half of $_\Lambda H^4$ decay events follow

$$_\Lambda H^4\rightarrow\pi^-+He^4, \tag{3.4}$$

it is not inconvenient to confine attention to these events, especially as specific statements can be made for this decay process. Since both products have spin zero, the relative orbital angular momentum between them is $l=0$ or 1 according as $J=0$ or 1. This means that the final state in (3.4) has a definite partity, so that none of the features characteristic of parity nonconservation in Λ decay can appear in these decay events; there can be no up-down asymmetry of the final pions relative to the $_\Lambda H^4$ production plane, and the pion angular distribution relative to the $_\Lambda H^4$ production direction cannot be more complicated than $(1+A\cos^2\theta)$. The absence of such angular correlations ($A=0$), however, does not necessarily imply $J=0$. It was pointed out earlier[3] that the relatively simple production process

$$K^-+He^4\rightarrow {}_\Lambda H^4+\pi^0 \tag{3.5}$$

is of special interest in this respect because the π^- angular distribution from the subsequent $_\Lambda H^4\rightarrow\pi^-+He^4$ decay (in the $_\Lambda H^4$ rest system) is predictable for K^- capture from an atomic orbit of angular momentum l. These angular distributions have been given previously,[3,28] but they will be summarized again here. Spin zero will be assumed for the K particle, this being consistent with all the available data on angular correlations in K decay and on the internal correlations in the τ mode of decay.

If $_\Lambda H^4$ has $J=1$, the values of the orbital angular momentum L in the final state of (3.5) are limited to $l-1$, l, $l+1$. For a *pseudoscalar* K meson (the parity being specified relative to positive parity for the Λ particle), parity conservation for strong interactions allows only $L=l$. The value $l=0$ is forbidden by angular momentum conservation, since $L=0$ requires total angular momentum 1 in the final state, and $_\Lambda H^4$ production then cannot result from reaction (3.5) for s-wave capture of the K particle. Capture from higher atomic orbits can be expected to compete favorably with radiative transitions of the K particle to lower orbits. The π^- angular distribution in the subsequent two-body decay (3.4) is given by

$$\sum_l P(l)\{\sum_m |c(l1l,0m)|^2(Y_m^1(\theta))^2\}, \tag{3.6a}$$

where $P(l)$ is a weighting factor representing the relative probability for this reaction to take place from the orbit l. Since $c(l1l,00)\equiv 0$, only terms with $m=\pm1$

[27] If attractive many-body forces involving the Λ particle were to play a significant role in the binding of these light hypernuclei, this would allow the possibility of fitting the binding energy data by means of a two-body interaction with a stronger spin-dependence than has been found here, since these many-body forces would have a greater effect on U_4 than on U_2. If a bound hyperdeuteron is ever established, this would appear to require that strongly attractive many-body forces should be effective in the hypernuclei $A\geqslant 3$.

[28] M. Gell-Mann, Phys. Rev. **106**, 1296 (1957); R. H. Dalitz, Brookhaven National Laboratory Report BNL-3405, 1957 (unpublished).

contribute and the expected distribution is $\sin^2\theta$ regardless of the orbit from which capture takes place. For a *scalar* K meson only $L=l+1$ and $l-1$ are allowed. If a_{l+} and a_{l-} denote the amplitudes for this reaction to proceed through these two channels after capture from the orbit l, the corresponding π^- distribution has the form

$$\begin{aligned}\sum_l P(l) \sum_m |a_{l+}c(l+1\ 1\ l, 0m) \\ +a_{l-}c(l-1\ 1\ l, 0m)|^2 |Y_m^1(\theta)|^2 \\ =\sum_l P_l\{l(l+1)(A_{l+}+A_{l-})^2 \sin^2\theta \\ +4[(l+1)A_{l+}-lA_{l-}]^2 \cos^2\theta\}, \quad (3.6b)\end{aligned}$$

where $A_{l\pm}=a_{l\pm}[(2l+1\pm1)(2l+1\pm2)]^{-\frac{1}{2}}$. A nonisotropic distribution is to be expected ($\cos^2\theta$ for capture from an s orbit) in general but it is possible for the amplitudes a_{l+}, a_{l-} to be such as to give cancellations and a correspondingly weak angular correlation.

If $_\Lambda H^4$ has $J=0$, the final orbital angular momentum L must equal l, and parity conservation can then be satisfied only if the K meson has the same parity as the pion and is therefore *pseudoscalar*. If it is found empirically that $_\Lambda H^4$ production never occurs as a result of K^- capture by He^4, this would imply that the K meson is *scalar* and that $J=0$ for $_\Lambda H^4$. Before this conclusion can be drawn, however, it is necessary to have some estimate of the relative frequency of $_\Lambda H^4$ production when this is allowed by all the selection rules; this question is considered in Appendix D. The converse conclusion, that observation of ground state $_\Lambda H^4$ decay following the K^-+He^4 reaction implies that the K meson is *pseudoscalar*, can only be drawn (even when it is known that $_\Lambda H^4$ has spin 0) if it is established that no excited state $_\Lambda H^{4*}$ exists, for ground state $_\Lambda H^4$ can still appear with capture of a *scalar* K meson through the sequence

$$K^-+He^4\rightarrow\pi^0+{}_\Lambda H^{4*}, \quad {}_\Lambda H^{4*}\rightarrow{}_\Lambda H^4+\gamma. \quad (3.7)$$

It can be stated unambiguously, however, that with $J=0$ for the ground state of $_\Lambda H^4$, no angular correlations can ever appear for the (π^-+He^4) decay following the K^-+He^4 reaction.

Spatial anisotropy for the π^- meson in $_\Lambda H^3$ decay is possible with either spin value. With spin $\frac{1}{2}$, this possibility depends on parity nonconservation in Λ decay; an up-down asymmetry in the decay may be observed if the $_\Lambda H^3$ system is produced in a state of polarization. With spin $\frac{3}{2}$, an angular distribution $(1+A\cos^2\theta)$ for the π^- meson relative to the production direction of the $_\Lambda H^3$ particle is possible in addition to the up-down asymmetry. For the two-body decay alone,

$$_\Lambda H^3\rightarrow He^3+\pi^-, \quad (3.8)$$

up-down asymmetry is possible only for spin $\frac{1}{2}$, since spin $\frac{3}{2}$ allows only p-wave pions in the final state; an angular distribution relative to production direction is possible only for spin $\frac{3}{2}$.

A qualitative argument which indicates $J=0$ for $_\Lambda H^4$ (and therefore $\bar{V}_a>\bar{V}_p$) can be derived from the large branching ratio observed for the two-body mode (3.4) for $_\Lambda H^4$ decay relative to other π^- modes. This depends on the fact that, since the Λ particle in $_\Lambda H^4$ moves in an s wave relative to the H^3 core, the s-wave pion of the two-body $_\Lambda H^4$ decay for $J=0$ can only result from the s-wave channel of Λ decay, whereas the p-wave pion of the two-body decay for $J=1$ comes only from the p-wave channel. Both channels of Λ decay can contribute to the three-body modes of $_\Lambda H^4$ decay. The partial lifetime for all π^- modes of $_\Lambda H^4$ decay will clearly be very close to that for free π^-+p decay on account of the low binding of the Λ particle, the corrections due to the Pauli principle then being quite small. With $J=1$, not all of the p-wave emissions can possibly lead to the He^4 state for the nucleons. Quantizing along the outgoing pion direction, it is clear that only the $m=0$ initial state can lead to He^4; that is, at most $\frac{1}{3}$ of the p-wave emissions give the correct spin configuration to allow He^4 formation. This is further reduced by a factor of about 2 when the sticking probability for the recoil proton (momentum $\sim$100 Mev/c) to form He^4 is included (see Fig. 2 and Appendix D). A comparison of the ratio of the number of mesonic to nonmesonic hypernuclear decays[29] with the internal conversion ratios Q_l calculated by Karplus and Ruderman[30] as a function of the orbital angular momentum l of the pion emitted in Λ decay next provides an estimate of the relative strengths of the p- and s-wave channels of Λ decay. With parity nonconservation in Λ decay, both s- and p-wave channels are generally effective in both mesonic and nonmesonic hypernuclear decay modes: these lead to final states of opposite parity, which do

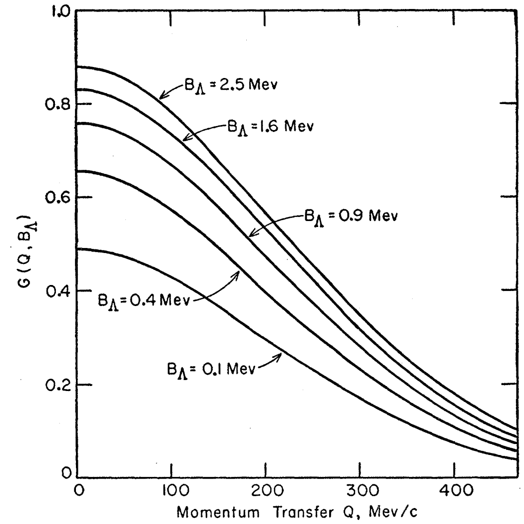

FIG. 2. The overlap integral $G(Q,B_\Lambda)$ of Eq. (D3) is plotted as function of recoil momentum Q Mev/c for five values of B_Λ. Values of $G(Q,B_\Lambda)$ for $B_\Lambda<0.1$ Mev can be obtained by multiplying the values for $B_\Lambda=0.1$ Mev by $(10B_\Lambda)^{\frac{1}{2}}$.

[29] Fry, Schneps, and Swami, Phys. Rev. **106**, 1062 (1957).
[30] M. Ruderman and R. Karplus, Phys. Rev. **102**, 247 (1956).

not interfere in the total probability for each of these modes of decay. Denoting by x the proportion of free Λ decays which occur through the p-wave channel, the internal conversion coefficient $Q(\pi^-)$ takes the form

$$Q(\pi^-)=(1-x)Q_0(\pi^-)+xQ_1(\pi^-). \qquad (3.9)$$

For $_\Lambda$He hypernuclei, Fry *et al.*[29] obtained an experimental value of about 1.5 for $Q(\pi^-)$, compared with the theoretical estimates $Q_0=1.1$, $Q_1=20$. There are appreciable uncertainties both in the experimental estimate and in these theoretical values, especially for the p-wave term[31] when x is small; but, allowing a factor 5 to cover all these uncertainties, the comparison indicates $x\lesssim 0.4$. Similarly, for hypernuclei $Z>2$, Fry *et al.* obtain $Q\sim 43(\pm 20)$ compared with theoretical estimates $Q_0=50$ and $Q_1=800$; a similar safety factor requires $x\sim 0.2$. Recent observations[32] on the up-down asymmetry in Λ decay have established that x must lie within the range 0.18 and 0.82 so that an estimate $x\sim\frac{1}{3}$ seems reasonably consistent with all the data. Collecting these factors together, a reasonable estimate of the proportion of π^- decays of $_\Lambda H^4$ which lead to the two-body decay (3.4) is about 5%, if ground state $_\Lambda H^4$ has $J=1$. This is to be compared with the observed proportion of 12 two-body decays in a total of between 21 and 27 $_\Lambda H^4$ decays in the world survey of Levi Setti *et al.*[2] On the other hand, with $J=0$, the two-body decays can occur through the s-wave channel of Λ decay, and the sticking probability is the main factor reducing the proportion of (π^-+He^4) decays; a reasonable estimate for this proportion is 0.5 $(1-x)=0.33$, which is consistent with the observed proportion. This provides a powerful argument favoring $J=0$ for the spin of $_\Lambda H^4$, with the stronger Λ-nucleon attraction in the singlet state and the other consequences shown in Table III.

4. CONSIDERATION OF THE HYPERNUCLEI WITH $A\geqslant 6$

Although hypernuclei of charge $Z\geqslant 3$ predominate among the observed hyperfragments, relatively few have been identified unambiguously and have allowed a B_Λ determination. This is due largely to the high value of the internal conversion coefficient Q for these heavier systems, energetic and relatively complicated nonmesonic modes predominating in their decay. Even the mesonic modes of the heavier hypernuclei are more difficult to establish because there are generally several heavy final particles whose identity and momenta must be established, and there is frequently the possibility that several neutrons may be emitted.

No examples of $A=6$ hypernuclei have yet been established, although it is reasonable to expect a bound state for the $T=\frac{1}{2}$ doublet ($_\Lambda He^6$, $_\Lambda Li^6$). The $p_{\frac{3}{2}}$ nucleon is known to have a strong attraction to an alpha particle, a resonance at 0.95 Mev being known for the neutron and at 2.1 Mev for the proton. The binding energy of a Λ particle to an alpha particle is 2.9 Mev and, since the interaction between Λ particle and nucleon is known to be attractive (note that it is still the s-wave interaction which predominates in the interaction between the Λ particle and the p-shell nucleon; see Appendix C), it appears that there would be sufficient attraction in these systems for a bound state to be formed. With $\bar{V}_p>\bar{V}_a$, this doublet would have spin 2 and the effective interaction would be $U_4+\bar{V}_p$; with $\bar{V}_a>\bar{V}_p$, its spin would be 1, the effective interaction being $U_4+(\frac{2}{3}\bar{V}_a+\frac{1}{3}\bar{V}_p)$. No detailed calculations have yet been made to predict B_Λ values for these hypernuclear states. If it should happen that B_Λ for $_\Lambda Li^6$ ($_\Lambda He^6$) is less than it is for $_\Lambda He^5$, then $_\Lambda Li^6$ ($_\Lambda He^6$) would dissociate to $_\Lambda He^5+p(n)$ with a lifetime many orders of magnitude too short to allow the possibility of hypernuclear decay for $_\Lambda Li^6$ ($_\Lambda He^6$). Owing to the additional Coulomb repulsions, $_\Lambda Li^6$ will have a B_Λ value less (by $\sim$1 Mev) than that of $_\Lambda He^6$; even if there is no bound state for $_\Lambda Li^6$, a bound state leading to hypernuclear decay events is still possible for $_\Lambda He^6$. Identification of $_\Lambda He^6$ decay events might often be difficult, however, because of the neutron emitted. If $_\Lambda Li^6$ is stable, the π^--mesonic decay mode

$$_\Lambda Li^6\rightarrow\pi^-+p+p+He^4,$$

should appear with relatively high frequency and could be easily identified.

Three examples of $_\Lambda Li^7$ have been established. This hypernucleus is almost certainly an isotopic spin singlet for the same reasons that were given for $_\Lambda H^3$; the lowest state of the core nucleus Li^6 has $T=0$ and allows parallel spins for the two nucleons so that the Λ-nucleon spin dependence can be effective. The radius and shape of Li^6 have recently been measured[33]; the conclusion is that a Gaussian shape is adequate, the rms radius of the nucleon distribution then being 2.1 ± 0.2 fermi. From these data and the observed $B_\Lambda=4.5\pm 0.4$ Mev, the volume integrals $U_6=1220$ Mev f^3 and 1450 Mev f^3 of Table I have been obtained for the range parameters $\kappa=m_K$ and $2m_\pi$, respectively; the relatively large uncertainty in U_6 given in Table I is due primarily to the uncertainty in the radius of Li^6. The values to be expected for U_6 depend to some extent on the structure of the core nucleus and the degree to which its coupling scheme may be modified by the presence of the Λ particle. For the light p-shell nuclei, it appears that neither $L-S$ nor $j-j$ coupling give general agreement with the energy level data.[34] For Li^6, however, the magnetic moment agrees quite well

[31] S. Treiman (private communication, 1958).

[32] Crawford, Cresti, Good, Gottstein, Lyman, Solmitz, Stevenson, and Ticho, Phys. Rev. **108**, 1102 (1957); Plano, Prodell, Samios, Schwartz, Steinberger, Bassi, Borelli, Puppi, Tanaka, Woloschek, Zoboli, Conversi, Franzini, Mannelli, Santangelo, Silvestrini, Glaser, Graves, and Perl, Phys. Rev. **108**, 1353 (1957).

[33] R. Hofstadter and G. R. Burleson, Bull. Am. Phys. Soc. Ser. II, **2**, 390 (1957).

[34] D. Kurath, Phys. Rev. **101**, 216 (1956); D. R. Inglis, Revs. Modern Phys. **25**, 390 (1953).

with expectation for $L-S$ coupling with $L=0$. For this configuration the expected value of U_6 is U_4+U_2, the p-shell nucleons being in a deuteron configuration, with spin $\frac{3}{2}$ or $\frac{1}{2}$ for ${}_{\Lambda}\mathrm{Li}^7$ according as $\bar{V}_p>\bar{V}_a$ or $\bar{V}_a<\bar{V}_p$. The values of U_4+U_2, 1195 and 1705 Mev f^3 for $\kappa=m_K$ and $2m_\pi$, respectively, do not lie outside the empirical errors for U_6; according to Appendix C, an exchange component in the Λ-nucleon interaction would reduce the volume integral expected for the p-shell nucleons (here U_2) by not more than about 10%, which would not modify appreciably the present degree of agreement. It is of interest to mention the other extreme, $j-j$ coupling, for Li^6 since the true situation is intermediate between these limiting cases. With the configuration $(\frac{3}{2})^2$, the expected value of U_6 is $[U_4+(5/3)\bar{V}_p+\frac{1}{3}\bar{V}_a]$ with spin $\frac{3}{2}$ for ${}_{\Lambda}\mathrm{Li}^7$ or $[U_4+(7/6)\bar{V}_p+\frac{5}{6}\bar{V}_a]$ with spin $\frac{1}{2}$. In either case the value of U_6 for the ground state is $U_6=U_4+\frac{1}{3}(U_4+U_2)$ (which is less than the value U_4+U_2 for $L-S$ coupling), the values being 1093 Mev f^3 for $\kappa=m_K$, and 1478 for $\kappa=2m_\pi$. If the 2.19-Mev (spin 3) excited state of Li^6 also has configuration $(\frac{3}{2})^2$, the potential seen by a Λ particle bound to this excited state would be $U_6^*=U_4+2\bar{V}_p$ for the case $\bar{V}_p>\bar{V}_a$, the spin of the system being $\frac{7}{2}$. This value of U_6^* is larger than the value of U_6 just obtained for binding to the ground state of this $(\frac{3}{2})^2$ configuration (for the present case U_6^* is the same as the value of U_6 discussed above for $L-S$ coupling). For the longer range $1/2m_\pi$, U_6^* is about 230 Mev f^3 larger than U_6 for this configuration. On account of this stronger attraction, the Λ particle would have about 2 Mev greater binding to this excited state of Li^6 than it would to the ground state, and this would be almost enough to bring the $J=\frac{7}{2}$ state below the $J=\frac{3}{2}$ state just discussed in the $j-j$ coupling limit. This example illustrates the point that it is not necessarily true that the ground state of a hypernucleus should allow description simply as the attachment of a Λ particle to the ground state of the core nucleus. As more data on the nuclear parameters associated with these p-shell hypernuclei become available, possibilities of this kind will have to be investigated in more detail.

The lightest hypernuclear triplet for which stable systems may be expected is ${}_{\Lambda}\mathrm{He}^7$, ${}_{\Lambda}\mathrm{Li}^{7*}$, ${}_{\Lambda}\mathrm{Be}^7$ with $J=\frac{1}{2}$. The nucleus He^6 is bound ($B_N=0.93$ Mev), and the Λ-nucleon interaction available has volume integral $U_6'=\frac{3}{2}U_4$, which is sufficient for binding of the particle. However the identification of this hypernucleus ${}_{\Lambda}\mathrm{He}^7$ would obviously be difficult since its decay would generally give rise to several neutrons (although a decay $\pi^-+p+\mathrm{He}^6$ could possibly be established from the beta-decay of the residual nucleus). The $T=1$ hypernuclear state ${}_{\Lambda}\mathrm{Li}^{7*}$ would be unstable with respect to γ radiation to the ground state of ${}_{\Lambda}\mathrm{Li}^7$ (and also for dissociation to ${}_{\Lambda}\mathrm{He}^5+\mathrm{H}^2$ if $B_\Lambda^*<5.1$ Mev), with a lifetime short compared to the Λ decay lifetime. Although Be^6 is not a known nucleus, a bound state is possible for ${}_{\Lambda}\mathrm{Be}^7$ although no examples of this system have been unambiguously identified (${}_{\Lambda}\mathrm{Be}^7$ will be stable against dissociation to ${}_{\Lambda}\mathrm{He}^5+p+p$ only if $B_\Lambda>2.9$ Mev). Even if there were any real possibility of establishing B_Λ for this $T=1$ state of ${}_{\Lambda}\mathrm{Li}^{7*}$, comparison of the three B_Λ values for this isotopic triplet would depend only weakly on charge-independence for the Λ particle interactions; charge symmetry already requires the two-body Λ-neutron and Λ-proton forces to be the same, and charge independence gives rise to further conditions only when three-body forces involving the Λ particle contribute to Λ-binding in hypernuclei.

The hypernuclear doublet ${}_{\Lambda}\mathrm{Li}^8$, ${}_{\Lambda}\mathrm{Be}^8$ has been identified. The B_Λ values of these hypernuclei listed in Table I agree within the large errors quoted, as is required by charge symmetry (after allowing for differences which may exist in the structure of Li^7 and Be^7 as a result of Coulomb forces). Electron scattering experiments have established that the shape of Li^7 is approximately Gaussian, its radius[15] being (2.5 ± 0.8)% less than that of Li^6. From these data, the volume integral U_7 for ${}_{\Lambda}\mathrm{Li}^8$ is calculated to be 1230 Mev f^3 and 1480 Mev f^3 for $\kappa=m_K$ and $2m_\pi$, respectively. The large errors given in Table I are due to the large radius uncertainty for Li^7, but the errors in U_6 and U_7 from this cause are correlated because the ratio of Li^6 and Li^7 radii is relatively well known. The effective potential U_7 expected for the Λ particle will depend on the coupling scheme appropriate for the core nucleus. The magnetic moment[35] of Li^7 lies between the Schmidt value and the value for the $T=\frac{1}{2}$ configuration $(\frac{3}{2})^3$, $J=\frac{3}{2}$ implied by the $j-j$ coupling model for Li^7. The wave functions for the three p-wave nucleons can be written in the form

$$(1-a^2)^{\frac{1}{2}}\psi((\nu\tfrac{3}{2})^2j=0,\ (\pi\tfrac{3}{2});\ J=\tfrac{3}{2}) +a\psi((\nu\tfrac{3}{2})^2j=2,\ (\pi\tfrac{3}{2});\ J=\tfrac{3}{2}),\quad (4.1)$$

where $a=0$ gives the Schmidt configuration and $a=1/\sqrt{6}$, the $T=\frac{1}{2}$ configuration. The calculation of the mean potential U_7 for the Λ particle can be carried out easily for the Schmidt configuration. The two neutrons contribute $2(\frac{3}{4}\bar{V}_p+\frac{1}{4}\bar{V}_a)$, their spins being randomly oriented, and the $p_{\frac{3}{2}}$ proton contributes $\bar{V}_p$ if $\bar{V}_p>\bar{V}_a$ ($J=2$ for ${}_{\Lambda}\mathrm{Li}^8$) or $(\frac{2}{3}\bar{V}_a+\frac{1}{3}\bar{V}_p)$ if $\bar{V}_a<\bar{V}_p$ ($J=1$ for ${}_{\Lambda}\mathrm{Li}^8$). The total interactions are then

$$U_7=\tfrac{3}{2}U_4+\begin{cases}\bar{V}_p & \text{for } J=2 \quad (4.2\text{a})\\ \tfrac{2}{3}\bar{V}_a+\tfrac{1}{3}\bar{V}_p & \text{for } J=1. \quad (4.2\text{b})\end{cases}$$

For the general configuration (4.1), the separate contributions from the two neutrons and from the proton generally differ from those for $a=0$, but they lead to the same expressions (4.2) regardless of the value of a. With $J=2$, the values expected for U_7 are 1292 and 1763 Mev f^3 for $\kappa=m_K$ and $2m_\pi$, respectively. With

[35] M. G. Mayer and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structure* (J. Wiley and Sons, Inc., New York, 1955), pp. 157 and 247.

$J=1$, the corresponding U_7 values are 1280 and 1734 Mev f^3. The present empirical uncertainties preclude any possibility of distinguishing between these cases. With $\kappa=2m_\pi$, the comparison for both U_6 and U_7 would be improved if the radii of both Li^6 and Li^7 were larger (but still within present uncertainties); on the other hand, the existence of some exchange component would reduce the p-shell contribution and also improve the agreement for this case.

The presence of the low-lying $(\frac{1}{2}-)$ state Li^{7*} at 0.43 Mev should also be mentioned. States with $J=0$ and 1 can be formed by combination of the Λ particle with this configuration. With $\bar{V}_a>\bar{V}_p$, the ground state of $_\Lambda Li^8$ will generally consist of a linear superposition of this $J=1$ state with the $J=1$ state discussed above; here again, it is no longer true that the ground state hypernucleus should be correctly described as a particle bound to the ground state of the core nucleus. It also appears likely from these remarks that $_\Lambda Li^8$ may have low-lying excited states; however, if these states are such as to give an energy release for hypernuclear decay appreciably different from that for ground state $_\Lambda Li^8$, they will necessarily have γ-decay lifetimes short compared to the Λ decay lifetime. Finally, it can be remarked that observation of the decay (π^-+Be^8) for $_\Lambda Li^8$ would establish its spin as $J=0$ or 1 and exclude one of the possibilities mentioned above.

Three events have established the existence of $_\Lambda Be^9$ clearly, on account of its characteristic decay to π^-+p+Be^8, followed by $Be^8\rightarrow He^4+He^4$; its B_Λ value is quite reliably known. Discussion of this system is complicated, however, by the fact that its core nucleus Be^8 does not form a bound state. On account of the fact that $_\Lambda Be^9$ is strongly bound by an s-wave Λ particle, the structure of its nuclear core can be expected to be more compact than that for Be^9, in which the core is (weakly) bound by a p-wave neutron. It seems unreasonable to assume, as Brown and Peshkin[4] have done, that the nucleon distribution in $_\Lambda Be^9$ can be represented by a nucleon core with the same radius as that determined for the charge distribution of Be^9 by electron scattering experiments. No reasonable estimate of U_8 can be made on the basis of the kind of analysis presented in this paper since nothing is known of the structure of the nuclear core of $_\Lambda Be^9$; consequently this system is not included in the present discussion.†

In this section it has been emphasized that the analysis of the data on p-shell hypernuclei is obscured by uncertainties in the nature of these hypernuclear states, in the structure of their core nuclei and in our empirical knowledge of the nuclear parameters needed even for the simple discussion given here. These data do not lead to any conclusions inconsistent with our quantitative conclusions from the analysis of the hypernuclei with $A\leqslant 5$, nor do they shed any additional light at the present stage on the detailed nature of the elementary Λ-nucleon interaction.‡

Finally, it is of interest to estimate the well-depth D of the Λ-nucleus potential for a Λ particle in a heavy nucleus. The proton distributions of many heavy nuclei have been determined by electron scattering experiments. They are adequately represented by the density function $\rho_P(0)[1+\exp(4.40(r-c)/t)]^{-1}$, where the parameters c and t have the values determined empirically by Hahn *et al.*[36] and $\rho_P(0)$ normalizes the charge density to total charge Z. The assumption that the neutron density distribution has the same form leads to a mean central nucleon density

$$\rho(0)=\rho_P(0)+\rho_N(0)=3A/[4\pi c(c^2+0.51t^2)]. \quad (4.3)$$

The value of $\rho(0)$ is almost independent of the nucleus considered, the values obtained for nuclei from Ca to Bi lying within 5% of 0.17 f^{-3}, the value for Bi. Since nuclear matter is spin-saturated, the average potential acting on the Λ particle has volume integral $\frac{1}{4}U_4$ per nucleon per unit volume; the well depth D is therefore given by

$$D=\tfrac{1}{4}\rho(0)U_4. \quad (4.4)$$

With the above value for $\rho(0)$, this estimate for D still depends on the range parameter κ assumed for the Λ-nucleon interaction. With $\kappa=2m_\pi$, the volume in-

† *Note added in proof.*—K. S. Suh (Phys. Rev., to be published) has now completed a discussion of $_\Lambda Be^9$, assuming this to consist of a Λ particle and two alpha particles. The potential between the two alpha particles is taken to fit the low-energy alpha-alpha scattering, and Suh then finds that the observed B_Λ for $_\Lambda Be^9$ corresponds to a Λ-alpha potential whose strength agrees well with that obtained here from analysis of $_\Lambda He^5$, for both of the cases $1/m_K$ and $1/2m_\pi$ for the range of the Λ-nucleon potential. See also the calculation of H. Wilhelmsson and P. Zielinski [Nuclear Phys. 6, 219 (1958)], who have neglected the nuclear potential between the alpha particles.

‡ *Note added in proof.*—Recent measurements (private communication from R. Hofstadter) on the absolute cross section for electron scattering by Li^6 have shown that the Gaussian shape can no longer be regarded as an adequate fit for the charge distribution of Li^6. An adequate shape for this, which is convenient for our numerical calculations, is provided by a composite shell model form

$$\rho(r)\sim\{(3/a_1^3)\exp(-r^2/a_1^2)+(r^2/a_2^5)\exp(-r^2/a_2^2)\},$$

with $a_1=2.65$ f and $a_2=1.07$ f. These parameters correspond to an rms radius 2.73 ± 0.16 f for the nucleon distribution in Li^6, substantially larger than was used in the above work. The use of these new parameters leads to a substantial increase in the values U_6 and U_7, as follows:

$$\kappa=1/m_K:\quad U_6=1650\pm140\pm\ 45 \text{ Mev f}^3,\quad U_7=1665\pm140\pm\ 80 \text{ Mev f}^3,$$

$$\kappa=1/2m_\pi:\quad U_6=1915\pm145\pm\ 60 \text{ Mev f}^3,\quad U_7=1930\pm150\pm100 \text{ Mev f}^3,$$

where the uncertainties given are, in succession, those due to the uncertainty in nuclear radius and in B_Λ. Allowance for the distortions of the core nuclei by the Λ particle may lead to some reduction in these values of U_6 and U_7, perhaps by as much as 10%, but even with this allowance, these values are much larger than those predicted for U_6 and U_7 above from the analysis of the s-shell hypernuclei, assuming a range $1/m_K$ for the Λ-nucleon potential. However these values for U_6 and U_7 agree, within the errors quoted, with the values predicted assuming a range $1/2m_\pi$ for the Λ-nucleon potential. This comparison therefore implies that the Λ-nucleon potential must have a range of order $1/2m_\pi$ rather than of order $1/m_K$, and must arise predominantly from the exchange of pions.

[36] Hahn, Ravenhall, and Hofstadter, Phys. Rev. **101**, 1131 (1956).

tegral U_4 of Table I leads to the well depth $D=38$ Mev; with $\kappa=m_K$, the corresponding well depth is $D=29$ Mev. These estimates indicate that binding energies in the range 25 to 35 Mev appear reasonable for a Λ particle attached to a medium-weight or heavy nucleus. Modifications of these estimates will be necessary if the Λ-nucleon potential includes an exchange component, or if many-body forces involving the Λ particle contribute significantly to Λ-nucleus binding. For these reasons, an empirical estimate of D would provide independent information bearing on the nature of the nuclear interaction of the Λ particle.

5. CONCLUSION

By means of phenomenological analysis of the B_Λ binding energies observed for the $A=3$, 4, and 5 hypernuclei in terms of a two-body Λ-nucleon interaction, we have been led to conclude that this interaction must be taken to depend on the spin-state of the Λ-nucleon system. By itself, this analysis still allows two possibilities for this spin dependence, with two corresponding sets of conclusions concerning the spin values for these hypernuclei. An argument from the branching ratios observed for the decay modes of ${}_\Lambda H^4$, however, establishes spin zero for this hypernucleus; this means that the singlet Λ-nucleon interaction is stronger than the triplet. The strength of this singlet interaction corresponds to a volume integral 480 Mev f^3 for a Yukawa shape of range parameter $\kappa=2m_\pi$ or 290 Mev f^3 for $\kappa=m_K$. The neutron-proton ${}^3S$ potential with range parameter $\kappa=0.848$ f^{-1} corresponds to a volume integral 1403 Mev f^3 for Yukawa shape; taking into account the shorter range of the Λ-nucleon interaction, it is clear, therefore, that the Λ-nucleon interaction must be the consequence of elementary interactions with coupling strength comparable to that for the pion-nucleon interaction. The weakness of the Λ binding to light nuclei relative to the neutron binding (see Introduction) must be attributed to the shorter range of the Λ-nucleon interaction rather than to any weakness of the elementary interactions generating the Λ-nucleon force. The triplet Λ-nucleon interaction is weaker than the singlet by a factor ranging from 0.3 to 0.45 as the range parameter κ decreases from m_K to $2m_\pi$. Some check on these conclusions may be provided in due course by observations on angular correlations in the decay of light hypernuclei (especially following K^- absorption in He^4) or on the energy distributions of the final particles following K^- capture by deuterium or helium.

This analysis gives no information on the range of the Λ-nucleon interaction within the range which appears physically appropriate. Nor does it allow any conclusion to be reached concerning the exchange character of the interaction. For the p-shell hypernuclei, the analysis can proceed at present only on a rather uncertain basis and has added relatively little so far to our knowledge of the nature of the Λ-nucleon interaction.

It is of interest to compare the conclusions reached in this paper with the limited theoretical calculations which have been made to date. There is general agreement with the estimate of the Λ-nucleon forces made by Lichtenberg and Ross[6] for the pion-exchange mechanism on the basis of a static model of the universal pion-baryon interaction proposed by Gell-Mann.[28] With this model, the K-exchange process would contribute relatively little to the forces on account of its relatively short range unless the coupling strength for K mesons were unreasonably large.[10] Lichtenberg and Ross conclude that the singlet interaction should be the stronger, the triplet interaction having strength about 0.65 that of the singlet. The coupling strength required by the strength deduced here for the singlet Λ-nucleon interaction corresponds to a pion-baryon coupling strength comparable with (perhaps a little larger than) the pion-nucleon coupling strength. This general agreement, therefore, provides evidence in qualitative support of Gell-Mann's proposal for a universal pion-baryon interaction. In this case, three-body forces arising from the pion exchange process (1.3) may be expected to contribute to the binding force between a Λ particle and a nucleus and a quantitative estimate of their influence on the binding energy analysis of this paper will now be desirable.§

Lichtenberg and Ross[10] have also considered the forces arising from K exchange, including the additional contributions implied by the pion-nucleon coupling but neglecting the pion-hyperon coupling. They found that neither scalar K nor pseudoscalar K

§ *Note added in proof.*—Calculations of the three-body forces corresponding to the process (1.3) have now been carried out by H. Weitzner [Phys. Rev. **110**, 593 (1958)], by R. Spitzer [Phys. Rev. **110**, 1190 (1958)], and by G. Bach (private communication from E. Lomon). The forces obtained have complicated noncentral forms; however, their central part does not depend on the Λ-spin vector and is proportional to $\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\ \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$, which takes the value -3 for two nucleons in relative s-wave motion. Spitzer and Bach find the central three-body potential to be attractive, but Weitzner points out that the slow variation of U_n with n for $n\leq 4$ could be due to the increasing effectiveness of a repulsive three-body potential, rather than to a spin dependence for the two-body potential, as considered here. With the above properties for the central three-body potential, the volume integrals U_n are given by

$$U_4=3\bar{V}_p+\bar{V}_a+6W,$$
$$U_3=\tfrac{3}{2}\bar{V}_p+\tfrac{3}{2}\bar{V}_a+3(1-\eta)W,$$
$$U_2=\tfrac{1}{2}\bar{V}_p+\tfrac{3}{2}\bar{V}_a+(1-\epsilon)W,$$

where W denotes the volume integral of the three-body potential for two nucleons with spatial correlation appropriate to He^4, and η, ϵ are corrections due to the difference of the correlation functions for two nucleons in He^3 and H^2 from that for He^4. The values of η and ϵ are estimated as $0\lesssim\eta\lesssim 0.1$, $\epsilon\approx 0$. The solution of these equations depends sensitively on the values of U_n and is therefore rather indeterminate in view of the large uncertainty in U_3. However $\bar{V}_a\cong(U_3-\tfrac{1}{2}U_4)$ still holds and $\bar{V}_a>\bar{V}_p$ still follows from spin 0 for ${}_\Lambda H^4$, so that the singlet potential cannot be appreciably stronger than has been derived here. In contrast, the triplet potential is almost undetermined, and the choice $\bar{V}_p\approx\bar{V}_a$ with W repulsive is acceptable within present uncertainties, as pointed out by Weitzner, although the above work shows that $W\approx 0$ with $\bar{V}_p$ considerably less than $\bar{V}_a$ is also acceptable. At present, it appears that Λ-proton scattering data will be needed for the determination of the ${}^3S$ Λ-nucleon potential, before the contributions of the two-body and three-body forces in hypernuclei can be convincingly distinguished.

leads to a Λ-nucleon interaction which corresponds even qualitatively to either of the spin-dependences suggested by the analysis given above; in each case the triplet Λ-nucleon interaction obtained is either repulsive or so weak that if the coupling strength is adjusted to correspond to the B_Λ observed for ${}_\Lambda H^3$, the attraction in the ${}_\Lambda He^5$ system is insufficient to account for its observed binding.

Finally, we may remark briefly on the possibility of binding two Λ particles to a nucleus. This would be of particular interest insofar as it depends on the strength of the interaction between two Λ particles, an interaction of range $1/2m_\pi$ or less. One simple possibility is the double hypernucleus ${}_{\Lambda\Lambda}He^6$, which could result from a reaction of the type

$$\Xi^- + Li^7 \rightarrow {}_{\Lambda\Lambda}He^6 + n. \tag{5.1}$$

Since the $\Lambda - He^4$ attraction is quite strong, binding for this ${}_{\Lambda\Lambda}He^6$ system might occur even if there is a strong short-range $\Lambda-\Lambda$ repulsion.‖ The Pauli principle allows two s-wave Λ particles coupled to zero total spin, so that ${}_{\Lambda\Lambda}He^6$ would have $J=0$, $T=0$. The decay of such a system would consist of two steps; first, one Λ particle would decay giving, possibly, the products $\pi^- + p + {}_\Lambda He^5$; then, the recoil hypernucleus ${}_\Lambda He^5$ would undergo its normal mode of decay after coming to rest. This sequence of events, ${}_{\Lambda\Lambda}He^6$ production (5.1) followed by its decay, would give rise to a triple-centered star. There are many other possibilities for such "double hypernuclei" and, with the artificial production of Ξ^- particles now observed at the Bevatron, it seems appropriate to mention the possibility and interest of such objects.

APPENDIX A. COMPRESSION OF THE HYPERNUCLEAR CORE

The simplest kind of distortion which can be considered for the core of a hypernucleus is a uniform radial compression corresponding to a nucleon distribution $\rho(\alpha r/R)$, where $\alpha=1$ refers to the undistorted core configuration. If both the core nucleon distribution and the Λ-nucleon interaction potential are of Gaussian shape (as for the cases considered in this paper), then the potential seen by the Λ particle is

$$V = -\frac{U}{\pi^{\frac{3}{2}}}\left(\frac{1.4354}{b'}\right)^3 \exp\left\{-\left(1.4354\frac{r}{b'}\right)^2\right\}; \tag{A1}$$

$$b' = \left\{b^2 + 1.3736\left(\frac{R}{\alpha}\right)^2\right\}^{\frac{1}{2}}, \tag{A2}$$

where b' is the intrinsic range of this potential. With $\alpha=1+\delta$, the energy of the distorted core is higher than that of the undistorted core by $\Delta E(\delta)=\frac{1}{2}K\delta^2$ Mev.

‖ With the "global symmetry" hypothesis of a universal pion-baryon coupling (see reference 28), the $\Lambda-\Lambda$ force in the ${}^1S$ state is predicted to equal the ${}^1S$ $\Lambda - p$ force, and therefore to be quite strongly attractive.

For a given δ, the potential (A1) must provide the effective binding energy $B=B_\Lambda+\Delta E(\delta)$ Mev. The well-depth parameter s required for V to produce this binding is

$$s = 1 + s_1\eta + s_2\eta^2 + \cdots, \tag{A3}$$

in terms of $\eta = b'(2\mu_\Lambda B)^{\frac{1}{2}}$, where μ_Λ is the Λ reduced mass and the coefficients s_i have been given by Blatt and Jackson.[37] In terms of this well-depth parameter the volume integral of the total Λ-nucleon interaction is

$$U(\alpha) = 215.86\frac{M}{\mu_\Lambda}sb' \text{ Mev f}^3, \tag{A4}$$

where M is the nucleon mass and b' is expressed in fermis. In order to show how the volume integral $U(\alpha)$ depends on the core compression, (A4) is now expanded in powers of δ. The expansion of b' can be obtained directly from (A2); that for s is obtained from the MacLaurin series for $s(\delta)$, with

$$\eta = \eta_0\left\{1 - x\delta + \left[\frac{K}{4B_\Lambda} + \frac{3}{2}x - \frac{1}{2}x^2\right]\delta^2 + \cdots\right\}, \tag{A5}$$

where $x = 1.3736R^2/(b^2+1.3736R^2)$. The subscript 0 is used here to designate the value of the function at $\delta=0$. The expansion obtained for U in this manner is

$$U(\delta) = U_0\left\{1 - y\delta + \left(z + w\frac{K}{B_\Lambda}\right)\delta^2 + \cdots\right\}, \tag{A6}$$

with

$$y = x + x\frac{\eta_0}{s_0}\frac{ds_0}{d\eta_0},$$

$$z = \frac{3}{2}y + \frac{1}{2}x^2\left(\frac{\eta_0}{s_0}\frac{ds_0}{d\eta_0} + \frac{\eta_0^2}{s_0}\frac{d^2s_0}{d\eta_0^2} - 1\right),$$

$$w = \frac{\eta_0}{4s_0}\frac{ds_0}{d\eta_0}.$$

To this approximation, $U(\delta)$ has its minimum value at $\delta = y/[2(z+wK/B_\Lambda)]$, giving

$$\frac{\Delta U}{U_0} = \frac{U_{\min} - U_0}{U_0} = -\frac{y^2}{4(z+wK/B_\Lambda)}. \tag{A7}$$

The derivation given above is not correct for $B_\Lambda=0$. In this case (A5) must be replaced by

$$\eta = (\mu_\Lambda K)^{\frac{1}{2}}|\delta|b', \tag{A8}$$

and the expression for $U(\delta)$ may be obtained by making use of the expansion (A3),

$$U(\delta) = U_0\{1 - x\delta + (\mu_\Lambda K)^{\frac{1}{2}}b_0's_1|\delta| - 2(\mu_\Lambda K)^{\frac{1}{2}}xb_0's_1\delta|\delta| + (\tfrac{3}{2}x - \tfrac{1}{2}x^2 + \mu_\Lambda K s_2 b_0'^2)\delta^2 + \cdots\}. \tag{A9}$$

[37] J. M. Blatt and J. D. Jackson, Phys. Rev. 76, 18 (1949).

Just as in the general case $B_\Lambda>0$, core expansion ($\delta<0$) leads to $U(\delta)>U_0$ for $B_\Lambda=0$. In the case of core compression ($\delta>0$), however, a minimum for $U(\delta)$ exists only if $K<K_m=x^2/[\mu_\Lambda(b_0's_1)^2]$; if $K>K_m$, the least value of $U(\delta)$ is U_0. For the case of the (${}_\Lambda H^4$, ${}_\Lambda He^4$) doublet discussed in this paper, K_m has the values 15 Mev for $\kappa=m_K$ and 5 Mev for $\kappa=2m_\pi$, when $R_3=1.38$ f.

APPENDIX B. A LOWER BOUND FOR THE STRENGTH OF THE Λ-NUCLEON INTERACTION

In this appendix a useful lower bound for the strength of the Λ-nucleon interaction will be obtained from the existence of the hypertriton ${}_\Lambda H^3$, by extending an argument given by Nishijima.[38] In the center-of-mass system, the Hamiltonian can be written

$$H(np\Lambda)=\left\{-\frac{\hbar^2}{M}\nabla_{np}^2+V(np)\right\}+\left\{-\frac{\hbar^2(2M+M_\Lambda)}{8MM_\Lambda}\nabla_\Lambda^2+V(\Lambda p)\right\}+\left\{-\frac{\hbar^2(2M+M_\Lambda)}{8MM_\Lambda}\nabla_\Lambda^2+V(\Lambda n)\right\}. \quad \text{(B1)}$$

Designating the least eigenvalue of the operator A by $\min(A)$, the inequality $\min(A+B)\geqslant\min(A)+\min(B)$ leads to

$$\min\{H(np\Lambda)\}=-B_d-B_\Lambda\geqslant\min\left\{-\frac{\hbar^2}{M}\nabla_{np}^2+V(np)\right\}+2\min\left\{-\frac{\hbar^2(2M+M_\Lambda)}{8MM_\Lambda}\nabla_\Lambda^2+V(\Lambda\mathfrak{N})\right\}, \quad \text{(B2)}$$

because the total energy $-(B_d+B_\Lambda)$ of the hypertriton represents the minimum value of $H(np\Lambda)$ with respect to all functions of three variables $\mathbf{r}_p$, $\mathbf{r}_n$, $\mathbf{r}_\Lambda$ related by the center of mass condition

$$M(\mathbf{r}_p+\mathbf{r}_n)+M_\Lambda\mathbf{r}_\Lambda=0. \quad \text{(B3)}$$

The last two terms in (B1) have been set equal, following the requirement of charge symmetry. The two operators whose minima occur in (B2) are each functions of only two of the three variables $\mathbf{r}_p$, $\mathbf{r}_n$, $\mathbf{r}_\Lambda$, such that their minima are not affected by the restriction (B3). Since the first term on the right hand side of (B2) is just $-B_d$, it follows that

$$\min\left\{-\frac{\hbar^2(M+M_\Lambda)}{2MM_\Lambda}\nabla_\Lambda^2+\frac{4(M+M_\Lambda)}{2M+M_\Lambda}V(\Lambda\mathfrak{N})\right\}\leqslant-\frac{2(M+M_\Lambda)}{2M+M_\Lambda}B_\Lambda. \quad \text{(B4)}$$

[38] K. Nishijima, Progr. Theoret. Phys. (Japan) 14, 526 (1956).

Apart from the factor multiplying $V(\Lambda\mathfrak{N})$, the Hamiltonian in (B4) is just that for the Λ-nucleon system; consequently, (B4) provides an inequality for the well-depth parameter s of the Λ-nucleon system. Even if B_Λ is taken to be zero, this inequality is

$$\frac{4(M+M_\Lambda)}{2M+M_\Lambda}s\geqslant1; \quad \text{(B5)}$$

that is, $s\geqslant0.364$. This result holds regardless of the detailed form of $V(\Lambda\mathfrak{N})$ and assumes only that three-body forces are unimportant in the hypertriton. If the interaction terms in (B1) are confined to central potentials, the inequality (B5) will hold even for the Λ-nucleon potential effective on the average in the spin configuration appropriate to ${}_\Lambda H^3$; insertion of the empirical value of B_Λ on the right of (B4) allows some improvement in the bound (B5) when the Λ-nucleon potential is assumed to have a given shape and range. For example, with $B_\Lambda=0.6$ Mev and a Gaussian potential corresponding to $\kappa=2m_\pi$ for the Λ-nucleon interaction, the inequality (B5) can be replaced by $s\geqslant0.429$. If spin $\frac{3}{2}$ holds for ${}_\Lambda H^3$, this inequality refers to the triplet potential; if ${}_\Lambda H^3$ has spin $\frac{1}{2}$, the inequality holds for the mean potential $(3V_a+V_p)/4$.

APPENDIX C. CONTRIBUTION OF EXCHANGE FORCES TO THE Λ-BINDING POTENTIAL

Since it is the s-wave Λ-nucleon interaction which makes the predominant contribution to Λ binding in the hypernuclei considered here, a comparison between the potentials acting on the Λ particle due to an ordinary or an exchange interaction which give the same s-wave interaction between Λ and nucleon can be made by comparing their expectation values in the relevant hypernuclear configuration. For this purpose the Λ wave function will be approximated by a Gaussian form $\exp(-\beta r^2)$, and the expectation value of the interaction of the Λ particle with an s-wave and with a p-wave nucleon will be considered. For the nucleon wave functions the forms $\exp(-\alpha r^2)$ for an s-wave, and $\mathbf{r}\exp(-\alpha r^2)$ for a p-wave nucleon will be considered. For convenience the Λ-nucleon potential will be taken to be of the form

$$V(r)=U(\lambda/\pi)^{\frac{3}{2}}\exp(-\lambda r^2). \quad \text{(C1)}$$

The interaction of the Λ particle with an s-wave nucleon is considered first. The two integrals which are to be compared are

$$I_{\text{ord.}}=\int\psi_\Lambda(r)\phi_n(s)V(|\mathbf{r}-\mathbf{s}|)\psi_\Lambda(r)\phi_n(s)d_3\mathbf{r}d_3\mathbf{s}, \quad \text{(C2)}$$

$$I_{\text{exch.}}=\int\psi_\Lambda(r)\phi_n(s)V(|\mathbf{r}-\mathbf{s}|)\psi_\Lambda(s)\phi_n(r)d_3\mathbf{r}d_3\mathbf{s}, \quad \text{(C3)}$$

where $\psi_\Lambda(r)=\exp(-\beta r^2)$, $\phi_n(s)=\exp(-\alpha s^2)$, and V is given by (C1). These integrals can be evaluated by use of the result

$$\int \exp(-Ar^2-2B\mathbf{r}\cdot\mathbf{s}-Cs^2)d_3\mathbf{r}d_3\mathbf{s} = \pi^3/(AC-B^2)^{\frac{3}{2}}. \quad \text{(C4)}$$

From this result the ratio

$$\frac{I_{\text{exch.}}^{s}}{I_{\text{ord.}}^{s}} = \left\{\frac{(2\alpha+\lambda)(2\beta+\lambda)-\lambda^2}{(\alpha+\beta+\lambda)^2-\lambda^2}\right\}^{\frac{3}{2}} \quad \text{(C5)}$$

is obtained directly. This ratio reduces to unity in two cases of interest: (i) $\alpha=\beta$ and (ii)$\lambda\to\infty$. In the first case exchange of the two particles leaves them each in the same wave function, which is possible because the Pauli principle is not effective between Λ particle and nucleon. In the second case, exchange of the two particles leaves the system unchanged because they must coincide in position (in the limit $\lambda\to\infty$) in order to interact.

The most suitable Gaussian wave function to represent the motion of a Λ particle in a Gaussian potential of the form $U(\sigma/\pi)^{\frac{3}{2}}\exp(-\sigma r^2)$ can be determined from a variational calculation of U for given B_Λ. From this calculation the appropriate choice for β is

$$\beta=\{\sigma+[\sigma(\sigma+16z)]^{\frac{1}{2}}\}/8, \quad \text{(C6)}$$

where $z=2\mu_\Lambda B_\Lambda$ and μ_Λ is the reduced mass for the Λ particle. This leads to the estimate

$$U=(B_\Lambda/z)(\pi^3/2\beta)^{\frac{1}{2}}(1+2\beta/\sigma)^{\frac{5}{2}}. \quad \text{(C7)}$$

It was explained in the introduction that only a short-range exchange force is physically reasonable; consequently, the comparison between an exchange and an ordinary force is made here for the range $1/m_K$. For $_\Lambda$He^5, (C6) and (C7) lead to $\beta=0.226$ f^{-2} and $U=792$ Mev f^3 (for an undistorted core), which is about 11% larger than the computed value U_4. For $_\Lambda$Li^7, the corresponding values are $\beta=0.173$ f^{-2} and $U=1288$ Mev f^3, which is about 6% too large.

The parameter α indicated by the observed nucleon distribution in He^4 is $\alpha=0.362$ f^{-2}. For $\kappa=m_K$, the interaction parameter $\lambda=2.91$ f^{-2} is relatively large. In this case the reduction factor (C5) is essentially $\{1-3(\alpha-\beta)^2/[4(\alpha+\beta)\lambda]\cdots\}$, which has the value 0.99 for $_\Lambda$He^5. In $_\Lambda$He^5 even the difference between a pure exchange and a pure ordinary force is quite unimportant in comparison with other uncertainties; this is also the case for the s-wave nucleons in the $_\Lambda$Li hypernuclei.

For interaction of the Λ particle with a p-shell nucleon, $\phi_n(s)=\mathbf{s}\exp(-\alpha s^2)$. The corresponding integrals (C2) and (C3) can then be obtained from suitable derivatives of the integral (C4). The result for the ratio,

$$\frac{I_{\text{exch.}}^{p}}{I_{\text{ord.}}^{p}} = \frac{\lambda}{\lambda+2\beta}\left\{\frac{(2\alpha+\lambda)(2\beta+\lambda)-\lambda^2}{(\alpha+\beta+\lambda)^2-\lambda^2}\right\}^{\frac{5}{2}}, \quad \text{(C8)}$$

then follows. This ratio will be evaluated for the Li p-shell nucleons, for which case[39] $\alpha=11/12R^2$, R being the rms radius of the nucleon distribution in Li^6 or Li^7. In this case the expectation value of an exchange potential is appreciably less than that of an ordinary potential, the ratio (C8) having the value 0.90. Even if the Λ-nucleon interaction were entirely of exchange character, the strength of the potential provided by the interaction of the Λ particle with the p-shell nucleons in $_\Lambda$Li would be reduced by at most 10% from that arising from an ordinary interaction of the same spin-dependence.

APPENDIX D. HYPERFRAGMENT YIELD FROM K^- ABSORPTION IN HELIUM

A brief discussion of K^- absorption in He^4 is given here in order to estimate the expected yield of $_\Lambda$H^4, $_\Lambda$He^4 and their excited state (if any) from this reaction. Such an estimate is necessary before experiment can decide whether the production of this doublet is really forbidden if no hyperfragments are observed after some number of K^- absorption events. Only the simplest assumptions will be considered in the present discussion.

Experimental evidence on the absorption cross section for slow K^- particles in hydrogen shows that this cross section rises steeply with decreasing K^- energy, the experimental determinations being consistent with a $1/v$ law. This evidence establishes that there is a strong s-wave absorption of K^- particles by protons (but does not exclude strong absorption also in other partial waves). In this preliminary discussion, we will consider only this s-wave interaction. This means that the reaction amplitude for

$$K^-+\mathfrak{N}\to\Lambda+\pi \quad \text{(D1)}$$

has the form $A\boldsymbol{\sigma}\cdot\mathbf{q}_\pi$ for a scalar K meson (parity specified relative to + parity for Λ particle) or B for a pseudoscalar K meson. The impulse approximation will be made in this calculation, secondary scattering being neglected for the outgoing particles. In this approximation three factors must be considered: (a) the relative frequency of Λ production in the elementary interaction, (b) the proportion of final states in which the spins are suitably oriented for formation of the hypernuclear state, and (c) the sticking probability of the Λ particle for the formation of the recoil hyperfragment.

With the assumption that K^- capture from atomic orbits in hydrogen occurs mainly from s states (not necessarily correct since they could be absorbed from p states before reaching an s orbit[40]), the first factor (a) can be estimated from the Berkeley data[41] on the basis

[39] This is based on a shell model wave function corresponding to an oscillator potential. This wave function leads to a proton distribution of the form $(1+\frac{2}{3}\alpha r^2)\exp(-2\alpha r^2)$, which corresponds to an rms radius $R^2=11/12\alpha$.

[40] R. Gatto, Nuovo cimento 3, 1142 (1956).

[41] Alvarez, Bradner, Falk-Variant, Gow, Rosenfeld, Solmitz, and Tripp, Nuovo cimento 5, 1026 (1957).

of charge independence. For absorption of K^- particles incident equally frequently on p and n, formation of the system $(\Lambda+\pi^0)$ is then expected for 3.7% of these interactions; $(\Lambda+\pi^-)$, for 7.5% of these interactions. As for the second factor (b), no spin flip occurs for capture of a pseudoscalar K^- meson so that the spin state reached is always appropriate for formation of $_\Lambda He^4$, $_\Lambda H^4$ with $J=0$. Capture of a scalar K^- meson, however, can only lead to a $J=1$ state of $_\Lambda H^4$, $_\Lambda He^4$; the assumption that this capture takes place from s orbits of the K^--helium atom means that the spins of the heavy particles after the reaction can sum to unity only for $\frac{1}{3}$ of the transitions. The sticking probability (c) is given by $F^2(Q)$, where Q is the momentum of the recoil hypernucleus and F denotes the overlap integral

$$F(Q)=\int \psi_{\mathrm{He}^4}(\Lambda,2;3,4) \times\exp[i\mathbf{Q}\cdot(3\mathbf{r}_\Lambda-\mathbf{r}_2-\mathbf{r}_3-\mathbf{r}_4)/4] \times\psi_{_\Lambda\mathrm{H}^4}(\Lambda,2;3,4)d\tau_\Lambda d\tau_2 d\tau_3 d\tau_4. \quad \text{(D2)}$$

For the evaluation of F we have used the product wave function for ($_\Lambda H^4$, $_\Lambda He^4$) computed as described in the body of this paper, and the wave function $\exp(\frac{1}{2}\alpha_4\sum r_{ij}^2)$ with $\alpha_4=9/(32R_4^2)$ for He^4. Taking the H^3 core of $_\Lambda H^4$ to have the wave function (2.9), $F(Q)$ can be written in the form

$$F(Q)=\left(\frac{48\alpha_3\alpha_4}{(3\alpha_3+4\alpha_4)^2}\right)^{\frac{3}{2}} G(Q,B_\Lambda), \quad \text{(D3)}$$

where the first factor arises from the integration over the coordinates of the H^3 core, and $G(Q,B_\Lambda)$ denotes the overlap integral

$$G(Q,B_\Lambda)=\left(\frac{3\alpha_4}{\pi}\right)^{\frac{3}{4}} \times\int \exp(-\tfrac{3}{2}\alpha_4 R^2)\frac{\sin(\frac{3}{4}QR)}{\frac{3}{4}QR}u_\Lambda(R)d_3R, \quad \text{(D4)}$$

$u_\Lambda(R)$ being the normalized wave function describing the Λ motion in $_\Lambda H^4$, $_\Lambda He^4$. The overlap integral G has been plotted as function of Q in Fig. 2 for various values of B_Λ. For sufficiently small values of B_Λ, the form of $u_\Lambda(R)$ in the region contributing to the integral (D4) is independent of B_Λ, while its normalization is determined almost entirely by its asymptotic form

$$u_\Lambda(R)\sim C\exp[-R(2\mu_\Lambda B_\Lambda)^{\frac{1}{2}}]/R.$$

In this situation, with B_Λ less than about 0.1 Mev, $G(Q,B_\Lambda)$ has the form

$$G(Q,B_\Lambda)\simeq(B_\Lambda)^{\frac{1}{4}}\phi(Q), \quad \text{(D5)}$$

so that the sticking probability for given Q falls off only with $B_\Lambda^{\frac{1}{2}}$ as the binding energy B_Λ approaches zero.

The expected frequency of production of $_\Lambda H^4$ and $_\Lambda He^4$ per K^- helium absorption event can be estimated on the basis of the three factors discussed above. For pseudoscalar K^- mesons, formation of $_\Lambda H^4$ and $_\Lambda He^4$ with spin 0 is allowed by the selection rules; since it appears most likely that the ground state of this doublet has spin 0, only this case will be considered here. For capture from an s orbit, the production of $_\Lambda H^4$ and $_\Lambda He^4$ with spin 0 will have relative frequencies[42] of the order of magnitude of 6×10^{-3} and 13×10^{-3}, respectively, per K^- capture event. In this estimate, indirect hypernucleus production arising from secondary interactions of the recoil Σ particles with the other nucleons has been neglected. About 90% of the primary K^- interactions can be expected to lead to Σ particles (of momentum about 150 Mev/c) which can interact with other nucleons transforming to produce additional Λ particles. This latter reaction, however, releases $\sim$80 Mev kinetic energy to two final particles (the Λ particle and the interacting nucleon), which will then each have momentum of order 300 Mev/c; the possibility that both of these particles fail to escape but stick to form an $A=4$ hypernucleus will therefore have negligible probability.

It is of interest to discuss also the K^--helium absorption for a scalar K^- meson. The formation of $_\Lambda H^4$ and $_\Lambda He^4$ with spin 0 is forbidden for this reaction (3.5); but if the excited states $_\Lambda H^{4*}$ and $_\Lambda He^{4*}$ with spin 1 exist (as appears relatively probable), there is still the possibility of observing $_\Lambda H^4$, $_\Lambda He^4$ decay events which occur following γ decay of $_\Lambda H^{4*}$, $_\Lambda He^{4*}$ produced in this reaction. With binding energy B_Λ^* ($\lesssim$0.1 Mev) for this excited state, $F^2(Q)$ has the value 0.15 (B_Λ^* Mev)$^{\frac{1}{2}}$ for the relevant recoil momentum, and the frequency of formation of either $_\Lambda H^{4*}$ or $_\Lambda He^{4*}$ is of order $5(B_\Lambda^*)^{\frac{1}{2}}\times10^{-3}$ per K^--helium interaction. For a reasonable value of B_Λ^* (say 0.01–0.1 Mev), this rate is a little less than one order of magnitude below that estimated above for an allowed ground-state reaction. It must be emphasized that there is considerable uncertainty in the absolute value of these estimates because the capture processes in He^4 may be quite unrelated to the capture processes observed for K^- in hydrogen on account of the appreciable nuclear size of

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[42] $_\Lambda H^4$ production is also possible in the production reaction

$$\pi^-+He^4\rightarrow{}_\Lambda H^4+K^0,$$

which has selection rules closely related to those for the reaction (3.4); this was pointed out by J. J. Sakurai [Phys. Rev. **107**, 1119 (1957)]. The cross section for Λ production in $\pi^- - p$ collisions at 0.95 Bev is only about 0.6 mb, however, compared with the total cross section of 47 mb. Taking into account the $\pi^- - n$ interactions also, the proportion of $\pi^- - He^4$ interactions leading to a Λ particle will be of the order of 1%. At this π^- energy, the Λ particles are produced typically with momenta $\sim$400 Mev/c in the laboratory system; the corresponding sticking probability is about 2%. These remarks indicate that, when it is allowed by the selection rules, $_\Lambda H^4$ production can be expected to occur in $\pi^- - He^4$ collisions of $\sim$1 Bev with a frequency of the order $\sim10^{-4}$ per π^- interaction.

He^4 and the closer orbits of the K^--helium system; both of these differences favor K^- capture from states of higher angular momentum than those which may be effective in K^--proton capture. Since the direct observations necessary to establish whether or not a γ ray accompanied an observed ${}_\Lambda H^4$ or ${}_\Lambda He^4$ decay event appear to be very difficult, the possibility of formation of ${}_\Lambda H^{4*}$, ${}_\Lambda He^{4*}$ may lead to considerable confusion concerning the interpretation of any observations of the ${}_\Lambda H^4$, ${}_\Lambda He^4$ hypernuclei following K^-+He^4 reactions.

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Theory of the Internal Space*

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It is postulated that fermions with identical space-time properties have to obey the exclusion principle unless they differ dynamically in their interaction with other particles. This postulate is formulated within the framework of a theory of the internal degrees of freedom. When all possible Yukawa-type couplings are examined, this postulate together with some other specification singles out the $\boldsymbol{\phi}\cdot\boldsymbol{\tau}$ interaction between pion and nucleon. Finally, when applied to electric interactions it requires the displacement of the nucleon center of charge.

1. INTRODUCTION

An important prediction of quantum field theory is that the quanta of one field are identical and obey the Pauli principle. By Pauli principle we shall mean more generally the requirement of symmetric or antisymmetric wave functions for particles with integral or half-odd-integral spin, respectively. Different particles are conventionally represented by different fields and the formalism does not require the Pauli principle in this case. The difference between the particles may lie in their space-time properties like mass and spin or in internal properties like the electric charge. Nevertheless it is mathematically also perfectly consistent within the present theory to have several fields which differ in no respect. For instance, a theory characterized by

$$L=\bar{\psi}_{\mathrm{I}}(\partial-M)\psi_{\mathrm{I}}+\bar{\psi}_{\mathrm{II}}(\partial-M)\psi_{\mathrm{II}} +\tfrac{1}{2}(\phi_{,i}\phi^{,i}-m^2\phi^2)+g(\bar{\psi}_{\mathrm{I}}\psi_{\mathrm{I}}+\bar{\psi}_{\mathrm{II}}\psi_{\mathrm{II}})\phi \quad (1)$$

describes two kinds of fermions interaction with a scalar boson, the former having completely identical properties. Yet the theory asserts that the Pauli principle is not effective between them. Such a case does not seem to be realized in nature. As far as we know all particles with, for instance, unit electric charge, spin $\frac{1}{2}$, and mass of the electron actually obey the exclusion principle. This observation leads one to analyze the usual formulation of the exclusion principle more carefully. The statement "identical Fermi particles cannot occupy the same state" requires an explanation as to what identical particles are. One can adopt the two points of view:

(A) Nonidentical particles differ in their space-time properties or in their interaction with other particles.

(B) Identical Fermi particles obey the exclusion principle. The definition (B) leads to a circle and most people will agree that (A) makes more sense. However, conventional quantum field theory does not imply the exclusion principle in form (A) but only in form (B).

We shall now sketch a formulation of the Pauli principle which implies (A) and makes it more precise. For this purpose we consider a theory with N_f different kinds of fermions and N_b different kinds of bosons. Such a situation will be described by N_f+N_b Hermitian fields. Now we imagine that in the Bureau of Standards they have N_f+N_b boxes, each containing one of the different kinds of particles. (For bosons the box may contain a piece of a static field instead of a particle.) Now we require that with the aid of this set of boxes we can determine to which of the N_f+N_b kinds any given particle belongs. We just do not permit to put for instance, a fermion into a box with fermions for this determination. For in this case we would use the exclusion principle as a criterion for identity. In other words, we shall not perform experiments in which the wave function of identical fermions (or bosons) overlap. If even with this restriction we can distinguish all par-

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